On the statement and numerical solution of the thermal problem within inversion methods for the study of lithospheric structure. EGU23 1845. *Supplementary material.*

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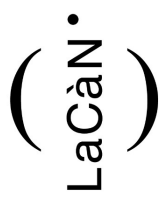

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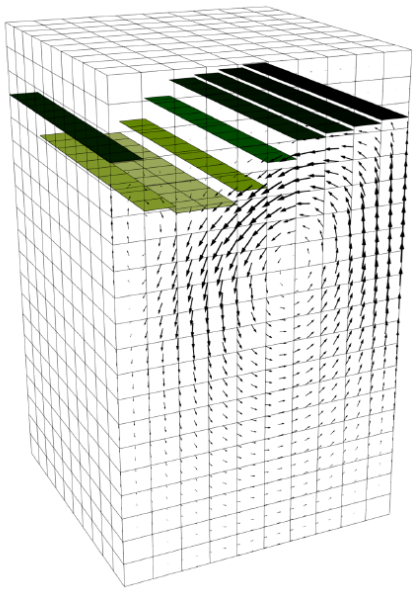
Extend previous work

Ortega-Gelabert et al. [OGZAD20] finds LAB location by solving a multi-observable inverse problem. Each forward problem is a Stokes problem given by,

$$\begin{cases} -\nabla \cdot (2\mu \nabla^{s} u) + \nabla p = \rho g, & \text{in } \Omega, \\ \nabla \cdot u = 0, & \text{in } \Omega, \\ u \cdot \mathbf{n} = 0, & \text{on } \partial\Omega, \\ t \times \mathbf{n} = \mathbf{0}, & \text{on } \partial\Omega. \end{cases} \tag{1}$$



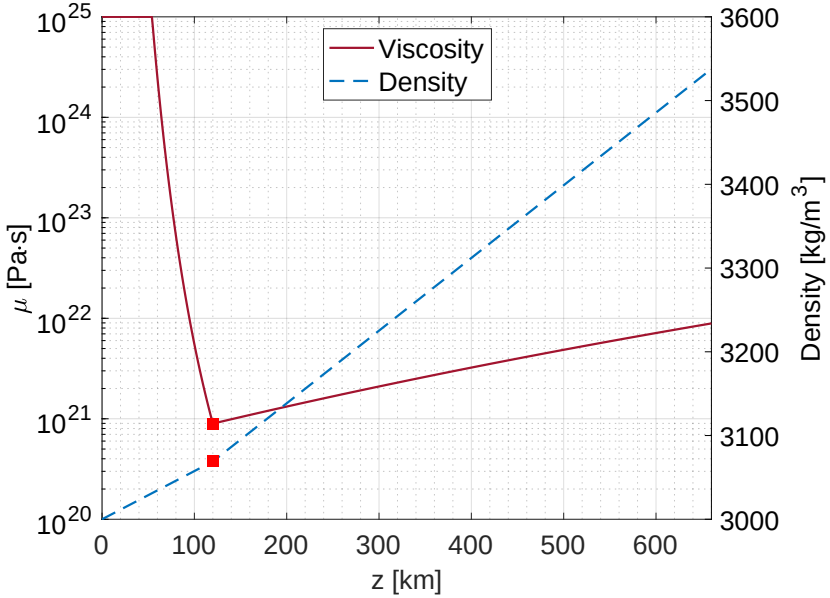
Additionally,

- Implements reduced basis;
- solves the inverse problem using Metropolis-Hastings (MCMC).

What can be done differently?

Density and viscosity parameters are calculated according to,

$$\begin{cases} \rho(z,T) = \rho_0(1-\alpha(T-T_0)+\beta(p(z)-p_0)), \\ \mu(z,T) = \min\left(\begin{array}{l} \mu_{max}; \\ A_D^{-1/n}\dot{\varepsilon}_{II}^{(1/n)-1}\exp\left(\frac{E+p(z)V}{nRT}\right) \end{array} \right) \end{cases}$$



- where strain rate $\dot{\varepsilon}_{II}$ is considered constant,
- pressure variates linearly with depth $p(z) = \rho\ g\ z$,
- and temperature is approximated, column by column, with a bi-linear distribution (where the discontinuity is at the LAB position).

We choose to improve the thermal description.

What is it done in the bibliography regarding the thermal field?

- Divide the domain:
  - lithosphere, mainly diffusion heat;
  - upper mantle, convection heat (approximately adiabatic gradient);
  - transition region, *smear out differences*.
- this strategy is not consistent and do not solves any PDE;
- **can we get the T results as the solution of an energy balance equation without smearing out the results?**

How do we obtain the thermal field results?

Hypothesis:

- steady-state,
- material properties discontinuity (coefficients K_L, K_A, etc.),
- if convection is considered: constant gravity, linear variation of pressure with depth, and density and viscosity according to the eq. in slide 3.

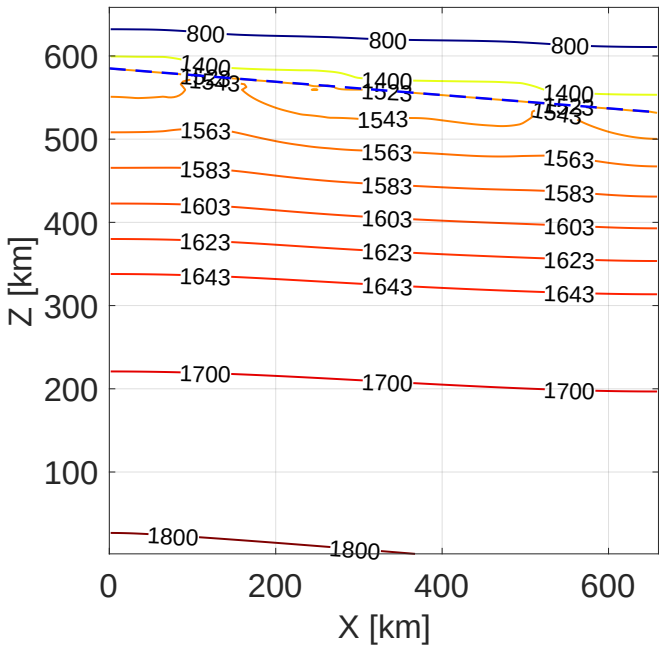
How do we expect our results to be?

- we want the LAB isotherm to match the LAB position,
- we expect the upper mantle gradient to be in the range $\nabla T \approx 0.3 - 0.5$ K/km [AFR+08],
- heat flux direction: from the mantle to the surface.

Ill-posed problem: Over-constrained problem

This is an old case, that we abandon quickly. This is just an example to show how these results look like.

$$\begin{cases} -\nabla \cdot (k\ \nabla T) = s, & \text{in } \Omega, \\ T = T_{\text{surface}}, & \text{on } \Gamma_{\text{surface}}, \\ -k\ \nabla T \cdot \mathbf{n} = g, & \text{on } \Gamma_{\text{bottom}}, \\ -k\ \nabla T \cdot \mathbf{n} = 0, & \text{on } \Gamma_{\text{walls}}, \\ T_{\text{LAB}} = 1523\ \text{K}, & \text{on } \Gamma_{\text{LAB}}. \end{cases}$$



Note in the following slides we started using $T_{\text{LAB}} = 1500\ \text{K}$. Regardless of the value this is just a code input and can therefore be changed easily.

Pure diffusion

Problem in Ω_L:

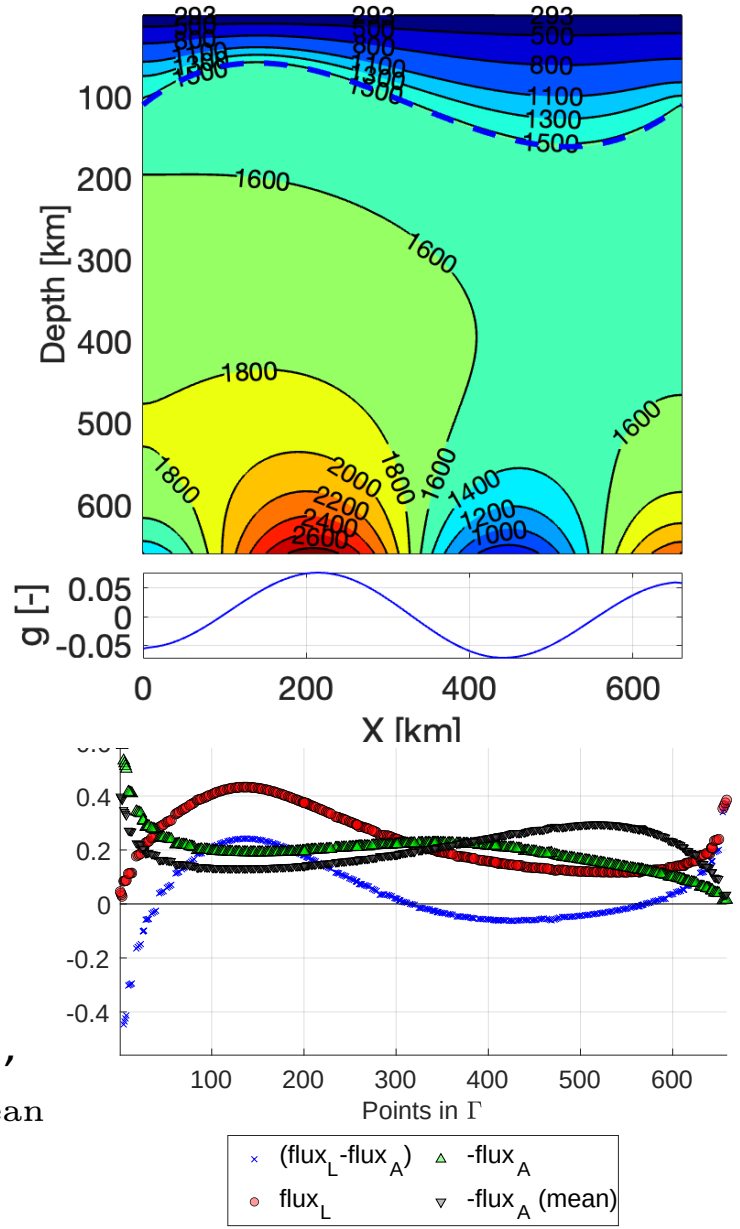
$$\begin{cases} -\nabla \cdot (\kappa_L \nabla T_L) = s_L, & \text{in } \Omega_L, \\ T_L = T_{\text{surface}}, & \text{on } \Gamma_{\text{surface}}, \\ -k_L \nabla T_L \cdot \mathbf{n} = 0, & \text{on } \Gamma_{\text{walls}}, \\ T_L = T_{LAB}, & \text{on } \Gamma_{\text{LAB}}. \end{cases}$$

Problem in Ω_A

$$\begin{cases} -\nabla \cdot (\kappa_A \nabla T_A) = s_A, & \text{in } \Omega_A, \\ -k_A \nabla T_A \cdot \mathbf{n} = g^*, & \text{on } \Gamma_{\text{bottom}}, \\ -k_A \nabla T_A \cdot \mathbf{n} = 0, & \text{on } \Gamma_{\text{walls}}, \\ T_A = T_{LAB}, & \text{on } \Gamma_{\text{LAB}}. \end{cases}$$

Flux minimization:

$g^* = \text{arg. min.}_g ||\kappa_L \nabla T_L \cdot \mathbf{n}_L + \kappa_L \nabla T_A\ (g) \cdot \mathbf{n}_A||^2$,
$g = g_{\text{mean}} + p$, *flux*$_A$(*mean*) *associated with* g_{mean}
$g_{\text{mean}} = \kappa_A$ 0.5 K/km.



Convection-diffusion without restrictions

Problem in Ω_L:

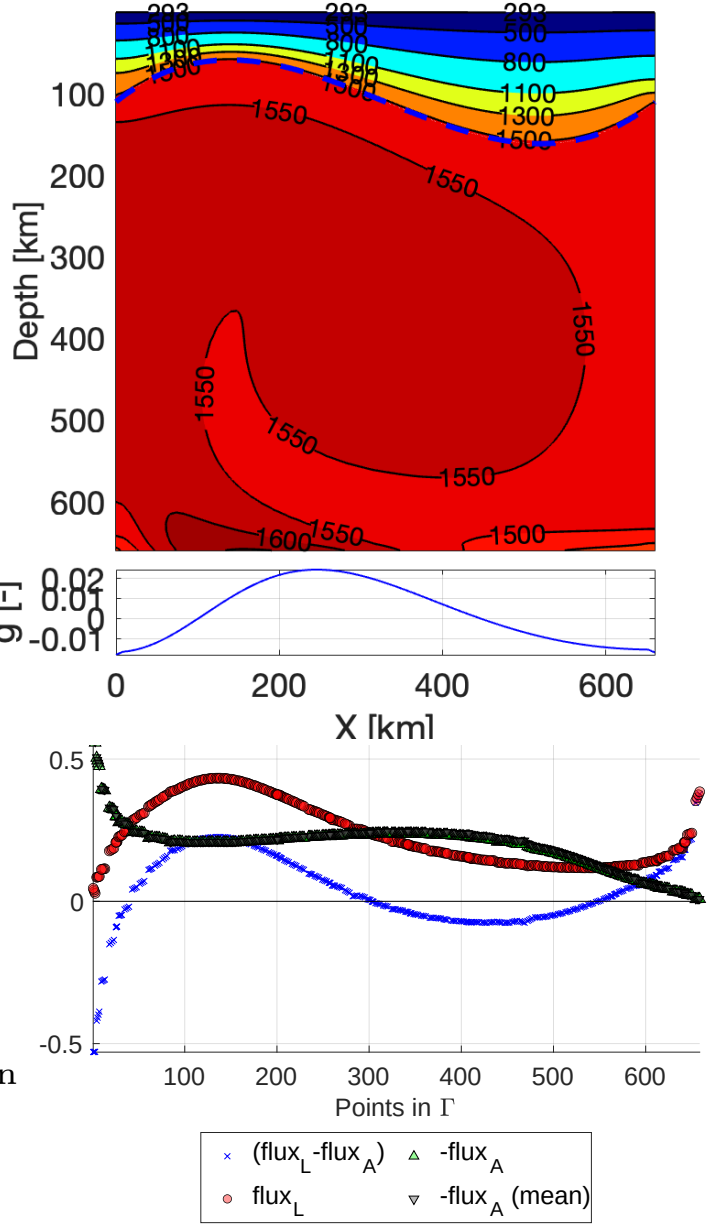
$$\begin{cases} -\nabla\cdot(\kappa_L\nabla T_L)+\rho c\ \mathbf{u}_L\cdot\nabla T_L = s_L, & \text{in } \Omega_L, \\ T_L = T_{\text{surface}}, & \text{on } \Gamma_{\text{surface}}, \\ -k_L\ \nabla T_L\cdot\mathbf{n} = 0, & \text{on } \Gamma_{\text{walls}}, \\ T_L = T_{LAB}, & \text{on } \Gamma_{\text{LAB}}. \end{cases}$$

Problem in Ω_A

$$\begin{cases} -\nabla\cdot(\kappa_A\nabla T_A)+\rho c\ \mathbf{u}_A\cdot\nabla T_A = s_A, & \text{in } \Omega_A, \\ -k_A\nabla T_A\cdot\mathbf{n} = g^*, & \text{on } \Gamma_{\text{bottor}} \\ -k_A\ \nabla T_A\cdot\mathbf{n} = 0, & \text{on } \Gamma_{\text{walls}}, \\ T_A = T_{LAB}, & \text{on } \Gamma_{\text{LAB}}. \end{cases}$$

Flux minimization:

$$g^* = \text{arg. min.}_g ||\kappa_L\nabla T_L\cdot\mathbf{n}_L + \kappa_L\nabla T_A\ (g)\cdot\mathbf{n}_A||^2,$$
$g = g_{\text{mean}} + p$, *flux*$_A$(*mean*) *associated with* g_{mean}
$g_{\text{mean}} = \kappa_A\ 0.5\ \text{K/km}.$



Convection-diffusion with minimization restrictions

Problem in Ω_L:

$$\begin{cases} -\nabla\cdot(\kappa_L\nabla T_L)+\rho c\ \mathbf{u}_L\cdot\nabla T_L = s_L, & \text{in } \Omega_L,\\ T_L = T_{\text{surface}}, & \text{on } \Gamma_{\text{surface}},\\ -k_L\ \nabla T_L\cdot\mathbf{n}=0, & \text{on } \Gamma_{\text{walls}},\\ T_L = T_{LAB}, & \text{on } \Gamma. \end{cases}$$

Problem in Ω_A

$$\begin{cases} -\nabla\cdot(\kappa_A\nabla T_A)+\rho c\ \mathbf{u}_A\cdot\nabla T_A = s_A, & \text{in } \Omega_A,\\ -k_A\nabla T_A\cdot\mathbf{n}=g^*, & \text{on } \Gamma_{\text{bottor}}\\ -k_A\ \nabla T_A\cdot\mathbf{n}=0, & \text{on } \Gamma_{\text{walls}},\\ T_A = T_{LAB}, & \text{on } \Gamma_{\text{LAB}}. \end{cases}$$

Flux minimization:

$$g^* = \text{arg. min.}_g ||\kappa_L\nabla T_L\cdot\mathbf{n}_L+\kappa_L\nabla T_A\ (g)\cdot\mathbf{n}_A||^2,$$
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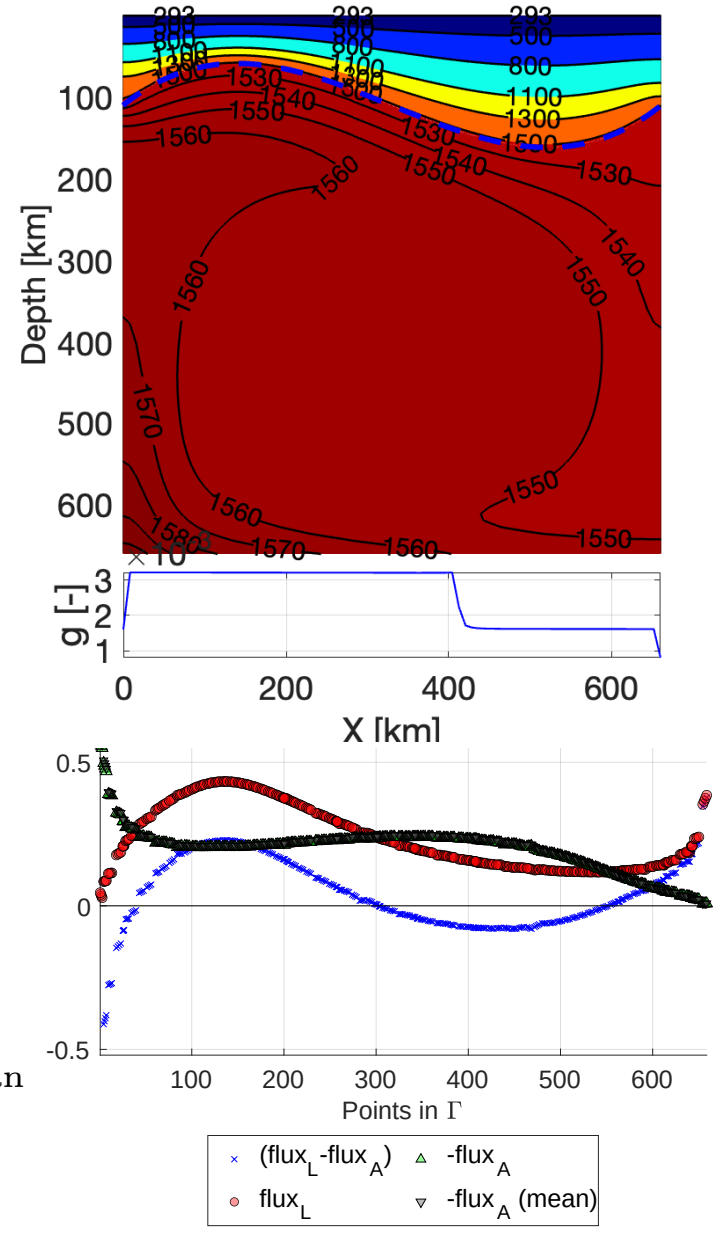

References I

Juan Carlos Afonso, Manel Fernàndez, Giorgio Ranalli, William Griffin, and James Connolly, *Integrated geophysical-petrological modeling of the lithosphere and sublithospheric upper mantle: Methodology and applications*, Geochemistry, geophysics, geosystems **9** (2008), no. 5.

O. Ortega-Gelabert, S. Zlotnik, J. C. Afonso, and P. Díez, *Fast stokes flow simulations for geophysical-geodynamic inverse problems and sensitivity analyses based on reduced order modeling*, Journal of Geophysical Research: Solid Earth **125** (2020), no. 3.