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ESTIMATION OF SOIL WATER RETENTION CURVE BASED ON PORE FREEZING KINETICS

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Abstract

The effect of pore radius can be observed during different processes that happen in geological media, taking a fundamental role during inter-pore ice formation and soil freezing. This can be expressed by the freezing characteristic function that relates chemical potential equilibrium and soil pore size distribution through the use of the water retention curve and the equilibrium temperature. By the analysis of temperature time evolution obtained for an undrained laboratory freezing test, and a Finite Element thermo – hydraulic model calibration, the estimation of the soil water retention curve was performed. A comparison between laboratory and the best fitted results were made and an acceptable agreement was observed between the estimated SWRC and the measured one, supporting the hypothesis of SWRC prediction through a relatively simple and rapid test. Furthermore, this relation allows to propose a novel test to estimate the soil water retention curve based on energy balance equations and freezing kinetics.

1. Freezing process in porous media and SWRC effect

In general, free water experiences liquid-to-solid phase change when temperature reaches the freezing point at a specific pressure and the chemical potential of ice and liquid water become equilibrated. In case of pore water, due to the presence of grain/water interfaces and the rise in the chemical potential to equilibrate, water freezing point depression is observed as smaller pores tend to freeze with minor temperatures than the larger ones. The microfabric relationship with the freezing process can be observed in Figure 1 where soil thermal behavior can be accurately simulated through the use of the respective pore size distribution. This result is exploited in this paper to estimate the pore size distribution and thus the retention curve of the material from temperature measurements taken during a simple constant volume freezing test.

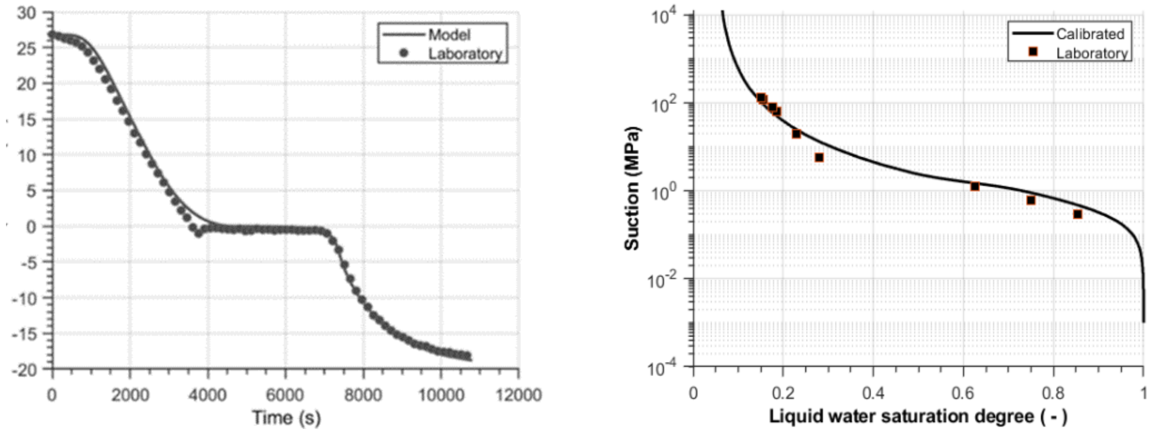

Figure 1. SWRC effect on freezing process. a) Temperature time evolution saturated constant volume test. b) Calibrated SWRC and laboratory test comparison.

From a theoretical point of view, pore radius can be formally related to the difference in chemical potential prevailing between the solid (ice) and liquid water phases through the Young-Laplace equation. Under the assumption of the reach of equilibrium between the two phases, the difference in chemical potential is related to the temperature through the Clausius – Clapeyron equation. The combination of the two equations leads to Equation 1, which expresses the temperature T required to freeze water in a pore of radius R for a given liquid pressure P_L (Nishimura et al., 2009; Macias, 2023).

$$R = \frac{2\sigma_{T_{\alpha L}}}{1 - \left(1 - \frac{\rho_i}{\rho_l}\right) P_l - \rho_i L ln\left(\frac{T}{273.15}\right)} \qquad (1)$$

ρ_i and ρ_l are the ice and liquid water densities, L is the latent heat of fusion and $\sigma_{T_{\alpha L}}$ is the surface tension between water solid and liquid phases. On the other hand, the balance of energy within the sample allows to back-analyze the amount of energy used in the latent heat of phase changes and thus the amount of liquid water that freezes at the current time. This provides a relationship between pore radius and pore volume at any time of the experiment.

The procedure of estimation of the retention curve is based on the back-analysis of the time evolution of temperature at the center of a saturated soil sample under a prescribed temperature evolution at the lateral boundaries. Firstly, the back-analysis has been carried out using a thermo-hydraulic Finite Element model considering a priori expressions for the retention curve. Figure 1b shows a comparison of the retention curve back-analyzed by the model (solid line) and the one obtained experimentally (black dots), using an expression for the RC based on the proposal by Casini et al. (2012) for double-structure materials:

$$S_l = 1 - S_i = (1 - w)\left[\frac{1}{1 + \left(\frac{S_{cr}}{P_M}\right)^{\frac{1}{1-M}}}\right]^M + w\left[\frac{1}{1 + \left(\frac{S_{cr}}{P_m}\right)^{\frac{1}{1-m}}}\right]^m \qquad (2)$$

Where S_l and S_i are liquid water and ice saturation degrees, S_{cr} is the cryogenic suction, P_M and M are the parameters proposed by van Genuchten (1980) for single-structure material and applied here to soil macrostructure. P_m and m are van Genuchten's parameters applied to the retention characteristics of the microstructure. w is a weighting factor to combine the micro- and macro-retention curves into the retention curve of the whole material.

The method is further extended to provide a step-by-step determination of the retention curve without defining a priori expression of the retention curve. It is based on the development of an analytical solution for the equation of energy balance considering the geometry and boundary conditions of the test. Agreement between experimental and computed retention curves for different soils is acceptable and validates the considered approach that links in a simple way microstructure patterns like the Pore Size Distribution to macroscopic variables like the temperature.

2. Analytical solution development

Considering a multi-phased saturated porous medium composed of soil grains (*s*), liquid water (*l*) and ice (*i*), the energy balance described by Fourier's law can be expressed through the apparent heat capacity (C_a) and the heat flux divergence term (Koorevaar et al., 1983; An et al., 2016; Tubini et al., 2021) as indicated in Equation 3.

$$C_a \frac{dT}{dt} = \nabla \cdot (\lambda \nabla \mathrm{T}) \tag{3}$$

Expanding the apparent heat capacity term by its phase components, including the effect of the latent heat of fusion, Equation 3 can be written as:

$$\left[(\rho_s c_s \theta_s + \rho_l c_l \theta_l + \rho_i c_i \theta_i) + \rho_l L_f \frac{\partial \theta_l}{\partial T}\right] \frac{dT}{dt} = \nabla \cdot (\lambda \nabla \mathrm{T}) \tag{4}$$

Where θ_α, ρ_α and c_α correspond to the phase α volumetric fraction, density and heat capacity, respectively. T is the medium equilibrium temperature at the current state and λ is the medium thermal conductivity which is considered constant during the process. Taking an incremental framework, Equation 4 is expressed as:

$$(\rho_s c_s \theta_s + \rho_l c_l \theta_l + \rho_i c_i \theta_i)\Delta T + \rho_l L_f \Delta \theta_l = [\nabla \cdot (\lambda \nabla \mathrm{T})]\Delta t \tag{5}$$

Rearranging the terms and solving for the liquid water content increment Equation 6 is obtained.

$$-\frac{(\rho_s c_s \theta_s + \rho_l c_l \theta_l + \rho_i c_i \theta_i)\Delta T - [\nabla \cdot (\lambda \nabla \mathrm{T})]\Delta t}{\rho_l L_f} = \Delta \theta_l \tag{6}$$

If one-dimensional heat flux and axisymmetric conditions are considered, the flux divergence term reads:

$$\nabla \cdot (\lambda \nabla \mathrm{T}) = \lambda(\nabla \cdot \nabla \mathrm{T}) = \lambda\left(\nabla \cdot \frac{\partial \mathrm{T}}{\partial r}\right) = \lambda\left(\frac{\partial^2 \mathrm{T}}{\partial r^2} + \frac{1}{r}\,\frac{\partial \mathrm{T}}{\partial r}\right) \tag{7}$$

Following the incremental approach, the derivatives of the divergence term can be computed using direct temperature measurements along the sample. At this stage, a numerical model has been used for this purpose. An axisymmetric finite element model was developed with a geometry and boundary conditions in which the heat flux can be considered one-dimensional since a constant prescribed temperature is applied over the perimeter of the sample. This can be achieved by introducing a sample into a freezing bath while isolating the top and the bottom of the sample container to avoid vertical heat flux and having only the radial component.

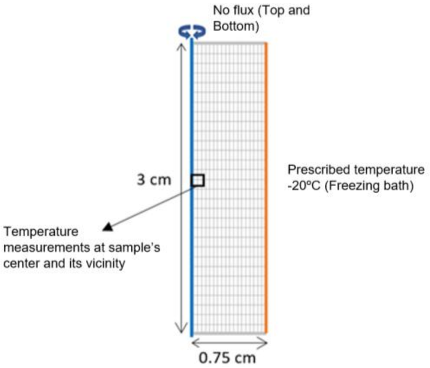

Figure 2. Scheme of the FE model used as a source for the temperature measurements required for the SWRC estimation method.

Taking the conditions exposed in Figure 2, a set of synthetic cases was calculated considering different soil types (clay, sand and silt) reflected on its retention curves. For each model, the temperature was measured at the center of the sample as well as at some points in its proximity. Equation 6 was then used to calculate the water content during the freezing process and thus the liquid saturation degree. Combining the liquid saturation results and the cryogenic suction values according to the Clausius – Clapeyron equation, the estimated retention curves were obtained exhibiting excellent agreement with the retention curves imposed in the FEM, particularly in the case of fine-grained soils. Some shift can be observed for the sandy and silty soils at very high and low water contents, this can be attributed to the loss of precision in the computation of the derivatives in the zones of low variations in temperature (flat parts of the curve). Nonetheless, the results encourage the pursuit of a relatively simple process to obtain a key soil parameter as is the retention curve as well as highlighting that through the use of a macroscopic variable (Temperature) and pore freezing kinetics, the microscopic state of the soil can be obtained through the close relationship that the pore size distribution and retention curve exhibits.

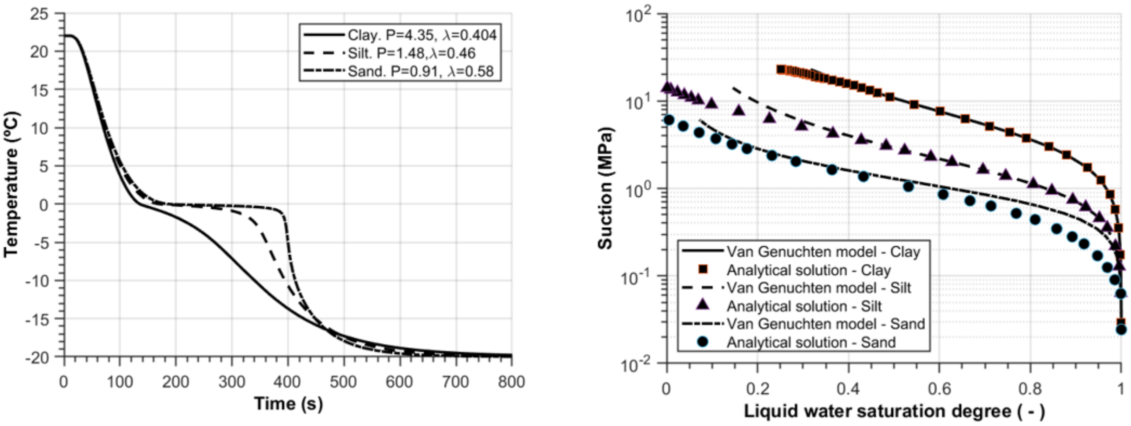

Figure 3. (a) Temperature time evolution at the center of the FEM for different soil types. (b) Comparisons between imposed SWRC and estimated ones.

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