Novel results on decentralized H_∞ controller design for structural vibration control of large buildings

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ABSTRACT: In this paper, we present a novel control design methodology for structural vibration control of large buildings. The main idea consists in decomposing the overall building system into decoupled single-story subsystems and modeling the subsystem interactions as external disturbances. Then, a complete set of local decentralized controllers can be efficiently computed using the existing LMI solvers. In the proposed approach, two different levels of decentralization can be distinguished: decentralized design and decentralized implementation, which are both of critical importance in large-scale control problems. From the design point of view, the local controllers are independently synthesized using only the low-dimensional subsystem models. The implementation phase is through the overall decentralized controller defined by the set of local controllers, which can drive the actuation devices using only local state-feedback information. To illustrate the proposed methodology, decentralized H_∞ controllers are designed for the seismic protection of a five-story building and a twenty-story building. A proper set of numerical simulations is carried out to demonstrate the effectiveness of the proposed decentralized controllers and the computation times are considered to assess the computational effectiveness of the decentralized design methodology.

1 INTRODUCTION

For vibration control of large structures, the idea of a distributed control system formed by a large number of small control devices that work jointly to mitigate the overall vibration response is certainly an appealing concept. From a technical point of view, designing control devices that combine actuation mechanisms, sensors, communication units and computational capabilities seems to be a clearly solvable problem (Housner et al. 1997, Spencer and Nagarajaiah 2003). In contrast, designing suitable controllers to drive a large number of such devices is certainly a challenging and complex problem (Zečević and Šiljak 2010), which is characterized by three basic elements: large dimensionality, high computational cost, and severe information constraints (Palacios-Quiñonero et al. 2010, Karimi et al. 2013). For this kind of problems, design strategies based on linear matrix inequality (LMI) formulations make it possible to compute advanced controllers (Boyd et al. 1994, Wang et al. 2009, Palacios-Quiñonero et al. 2014). However, these strategies are only computationally effective in problems of moderate dimension. Moreover, the centralized design of decentralized controllers by setting a particular zero-nonzero structure on the LMI variable matrices frequently leads to infeasibility issues.

In this paper, we present a novel controller design methodology for vibration control of large buildings equipped with a distributed system of interstory control devices. The main objective is to provide a decentralized design methodology to compute decentralized controllers for the considered large-scale control problem. The underlying idea consists in decomposing the building model into a set of single-story decoupled subsystems and modeling the subsystem interactions as external disturbances. For each subsystem, a local controller is computed, which drives the corresponding interstory actuation device using the information provided by the local sensor. To illustrate the proposed methodology, decentralized H_∞ controllers are designed for the seismic protection of a five-story building and a twenty-story building. Also, centralized H_∞ controllers are designed and taken as a natural reference in the performance assessment

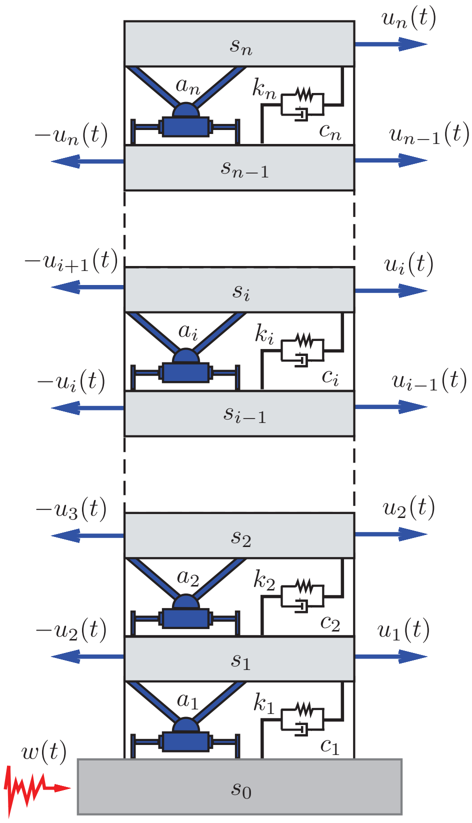

Figure 1: n-story building structure equipped with a complete system of interstory actuation devices.

of the proposed decentralized controllers and in the study of the computational effectiveness of the decentralized design procedure. The rest of the paper is organized as follows: In Section 2, a mathematical model for an n-story building is provided. In Section 3, the derivation of the decoupled subsystems is presented. In Section 4, decentralized and centralized H_∞ controllers for a five-story building and a twenty-story building are designed and compared. Finally, some brief conclusions are presented in Section 5.

2 BUILDING MODEL

Let us consider the n-story building schematically depicted in Figure 1, where k_i and c_i are the stiffness and damping coefficient of the ith story, respectively, $w(t)$ denotes the seismic ground acceleration and $u_i(t)$ is the control action exerted by the interstory actuation device a_i implemented between the stories s_{i-1} and s_i, which produces a pair of opposite structural forces as indicated in the figure. By considering the vector of displacements

$$\mathbf{q}(t) = \left[q_1(t), \dots, q_n(t)\right]^T, \tag{1}$$

where $q_i(t)$ is the lateral displacement of the story s_i with respect to the ground level s_0, the lateral motion of the structure can be described by the differential equation

$$\mathbf{M}\,\ddot{\mathbf{q}}(t) + \mathbf{C}_d\,\dot{\mathbf{q}}(t) + \mathbf{K}\,\mathbf{q}(t) = \mathbf{T}_u\mathbf{u}(t) + \mathbf{T}_w w(t), \tag{2}$$

where

$$\mathbf{u}(t) = \left[u_1(t), \dots, u_n(t)\right]^T \tag{3}$$

is the vector of control actions, $\mathbf{M}$, $\mathbf{C}_d$ and $\mathbf{K}$ are the mass, damping and stiffness matrices, respectively, $\mathbf{T}_u$ is the control location matrix and $\mathbf{T}_w$ is the excitation input matrix. The mass matrix is a diagonal matrix

$$\mathbf{M} = \begin{bmatrix} m_1 & & & \\ & \dots & & \\ & & \dots & \\ & & & m_n \end{bmatrix}, \tag{4}$$

where m_i, $i = 1, \dots, n$, denotes the ith story mass and the stiffness matrix has the following tridiagonal structure:

$$\mathbf{K} = \begin{bmatrix} k_1+k_2 & -k_2 & & & & \\ -k_2 & k_2+k_3 & -k_3 & & & \\ & \dots & \dots & \dots & & \\ & & \dots & \dots & \dots & \\ & & & -k_{n-1} & k_{n-1}+k_n & -k_n \\ & & & & -k_n & k_n \end{bmatrix}. \tag{5}$$

When the damping coefficients are known, a tridiagonal damping matrix can be obtained from (5) by replacing the stiffness coefficients k_i by the damping coefficients c_i. However, in most practical situations the damping coefficients cannot be properly determined and an approximate damping matrix $\mathbf{C}_d$ is computed from $\mathbf{M}$ and $\mathbf{K}$ by setting a suitable damping ratio on the building modes (Chopra 2007). The control location matrix is a square matrix of size n with the following upper-diagonal band structure:

$$\mathbf{T}_u = \begin{bmatrix} 1 & -1 & & & \\ & 1 & -1 & & \\ & & \dots & \dots & \\ & & & \dots & \dots \\ & & & 1 & -1 \\ & & & & 1 \end{bmatrix}, \tag{6}$$

and the excitation input matrix has the form

$$\mathbf{T}_w = -\mathbf{M}\,[\mathbf{1}]_{n\times 1}, \tag{7}$$

where $[\mathbf{1}]_{n\times 1}$ denotes a vector of dimension n with all its entries equal to 1. Next, by introducing the interstory drifts

$$\begin{cases} r_1(t) = q_1(t) \\ r_i(t) = q_i(t) - q_{i-1}(t) \quad \text{for } \ i = 2, \dots, n, \end{cases} \tag{8}$$

the interstory velocities

$$v_i(t) = \dot{r}_i(t), \quad i = 1, 2, \dots, n \tag{9}$$

and the state vector

$$\mathbf{x}(t) = [r_1(t), v_1(t), \dots, r_n(t), v_n(t)]^T \tag{10}$$

$$
\mathbf{A} = 10^3 \times \begin{bmatrix}
0 & 0.0010 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.6831 & -0.0023 & 0.5251 & 0.0011 & 0 & 0.0002 & 0 & 0.0001 & 0 & 0.0000 \\
0 & 0 & 0 & 0.0010 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.6831 & 0.0014 & -1.0652 & -0.0029 & 0.4732 & 0.0009 & 0 & 0.0002 & 0 & 0.0001 \\
0 & 0 & 0 & 0 & 0 & 0.0010 & 0 & 0 & 0 & 0 \\
0.0000 & 0.0003 & 0.5402 & 0.0011 & -0.9515 & -0.0028 & 0.4300 & 0.0009 & 0 & 0.0001 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0010 & 0 & 0 \\
0.0000 & 0.0001 & 0.0000 & 0.0002 & 0.4783 & 0.0009 & -0.8645 & -0.0027 & 0.4102 & 0.0008 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0010 \\
0 & 0.0001 & 0 & 0.0001 & 0 & 0.0002 & 0.4346 & 0.0009 & -0.7258 & -0.0026
\end{bmatrix},
$$

$$
\mathbf{B} = 10^{-5} \times \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0.4647 & -0.4647 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
-0.4647 & 0.9427 & -0.4780 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & -0.4780 & 0.9611 & -0.4831 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & -0.4831 & 0.9714 & -0.4883 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.4883 & 0.8641
\end{bmatrix}, \quad
\mathbf{E} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.
$$

Figure 2: System matrices for the five-story building model.

Table 1: Parameter values for the five-story building model.

story	1	2	3	4	5
mass ($\times 10^5$ Kg)	2.152	2.092	2.070	2.048	2.661
stiff. ($\times 10^8$ N/m)	1.470	1.130	0.990	0.890	0.840
relative damping	5%				

that groups together the interstory drifts and velocities of a same level, one obtains the following first-order state-space model:

$$
\dot{\mathbf{x}}(t) = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\,\mathbf{u}(t) + \mathbf{E}\,w(t), \tag{11}
$$

where

$$
\mathbf{A} = \mathbf{P}\widehat{\mathbf{A}}\mathbf{P}^{-1}, \quad \mathbf{B} = \mathbf{P}\widehat{\mathbf{B}}, \quad \mathbf{E} = \mathbf{P}\widehat{\mathbf{E}}, \tag{12}
$$

$$
\widehat{\mathbf{A}} = \begin{bmatrix} [\mathbf{0}]_{n\times n} & \mathbf{I}_n \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C}_d \end{bmatrix}, \tag{13}
$$

$$
\widehat{\mathbf{B}} = \begin{bmatrix} [\mathbf{0}]_{n\times n} \\ \mathbf{M}^{-1}\mathbf{T}_u \end{bmatrix}, \quad \widehat{\mathbf{E}} = \begin{bmatrix} [\mathbf{0}]_{n\times 1} \\ -[\mathbf{1}]_{n\times 1} \end{bmatrix}, \tag{14}
$$

$\mathbf{I}_n$ is the identity matrix of order n, $[\mathbf{0}]_{n\times m}$ represents a zero-matrix of the indicated dimensions and $\mathbf{P}$ is the change-of-basis matrix corresponding to the state transformation

$$
\mathbf{x}(t) = \mathbf{P} \begin{bmatrix} \mathbf{q}(t) \\ \dot{\mathbf{q}}(t) \end{bmatrix}. \tag{15}
$$

For the five-story building parameter values in Table 1 (Kurata et al. 1999), we obtain the system matrices presented in Figure 2. It should be observed that, in these matrices, the value 0.0000 indicates a relatively small but nonzero element.

3 DECOUPLED MODEL

The dynamics of the ith story can be described by the first-order model

$$
\dot{\mathbf{x}}^{(i)}(t) = \sum_{j=1}^{n} \mathbf{A}_{ij}\mathbf{x}^{(j)}(t) + \sum_{j=1}^{n} \mathbf{B}_{ij}u_j(t) + \mathbf{E}_i w(t), \tag{16}
$$

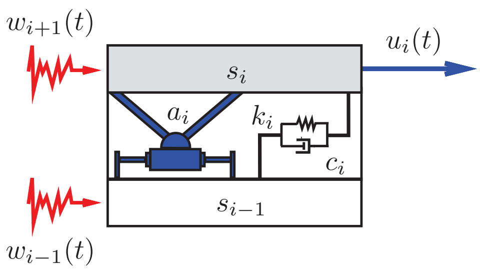

Figure 3: Fully decoupled one-story subsystem. The interactions with neighboring stories are included in the external disturbances w_{i-1} and w_{i+1}.

where $\mathbf{x}^{(i)}(t) = [r_i(t), v_i(t)]^T$ is the local state vector, and $\mathbf{A}_{ij}$, $\mathbf{B}_{ij}$ and $\mathbf{E}_i$ are submatrices of $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{E}$, respectively, with the following form:

$$
\mathbf{A}_{ij} = \begin{bmatrix} a_{2i-1,\,2j-1} & a_{2i-1,\,2j} \\ a_{2i,\,2j-1} & a_{2i,\,2j} \end{bmatrix}, \tag{17}
$$

$$
\mathbf{B}_{ij} = \begin{bmatrix} b_{2i-1,\,j} \\ b_{2i,\,j} \end{bmatrix}, \quad \mathbf{E}_i = \begin{bmatrix} e_{2i-1} \\ e_{2i} \end{bmatrix}. \tag{18}
$$

The matrices $\mathbf{A}_{ij}$ with $j \neq i$ represent mechanical interactions with other stories and the matrices $\mathbf{B}_{ij}$ with $j \neq i$ correspond to the effect of control actions exerted by actuation devices located in other building levels. As it can be clearly appreciated in the system matrices presented in Figure 2, a natural approximation of the model in (16) can be obtained by removing the interactions with non-neighboring stories. Accordingly, we consider the following approximate local models:

$$
\dot{\mathbf{x}}^{(1)}(t) = \sum_{j=1}^{2} \mathbf{A}_{ij}\mathbf{x}^{(j)}(t) + \sum_{j=1}^{2} \mathbf{B}_{ij}u_j(t) + \mathbf{E}_1 w(t), \tag{19}
$$

$$
\dot{\mathbf{x}}^{(i)}(t) = \sum_{j=i-1}^{i+1} \mathbf{A}_{ij}\mathbf{x}^{(j)}(t) + \sum_{j=i-1}^{i+1} \mathbf{B}_{ij}u_j(t), \tag{20}
$$

$$\mathbf{G}_c = 10^7 \times \begin{bmatrix} -1.7423 & -0.8344 & -0.3992 & -0.2537 & 0.7455 & -0.1031 & 0.1025 & -0.0594 & -0.5784 & -0.0876 \\ 0.7055 & -0.3504 & -0.5307 & -0.7770 & -0.7766 & -0.3094 & 0.4851 & -0.1328 & -0.0248 & -0.0870 \\ 2.8608 & -0.2258 & -0.5375 & -0.3924 & -2.0913 & -0.8686 & 1.0442 & -0.1412 & -0.0417 & -0.1141 \\ 1.6864 & -0.1962 & 2.0728 & -0.2071 & -3.5912 & -0.4925 & 2.1535 & -0.4925 & -1.3030 & -0.1957 \\ 1.2026 & -0.1179 & 1.0279 & -0.1453 & -0.5679 & -0.2365 & -1.3511 & -0.2112 & 0.9141 & -0.4272 \end{bmatrix}$$

Figure 4: Centralized H_∞ control gain matrix for the five-story building model.

for $i = 2, \ldots, n-1$, and

$$\dot{\mathbf{x}}^{(n)}(t) = \sum_{j=n-1}^{n} \mathbf{A}_{ij}\mathbf{x}^{(j)}(t) + \sum_{j=n-1}^{n} \mathbf{B}_{ij}u_j(t). \tag{21}$$

Next, by interpreting the interactions with neighboring stories as external disturbances (see Figure 3), a set of n fully decoupled one-story models can be obtained:

$$\dot{\mathbf{x}}^{(i)}(t) = \mathbf{A}_i\,\mathbf{x}^{(i)}(t) + \mathbf{B}_i\,u_i(t) + \bar{\mathbf{E}}_i\bar{\mathbf{w}}^{(i)}(t), \tag{22}$$

where $\mathbf{x}^{(i)}(t)$ is the local state vector, $u_i(t)$ is the local control action, the local system matrices are

$$\mathbf{A}_i = \mathbf{A}_{ii},\ \mathbf{B}_i = \mathbf{B}_{ii},\ i = 1, 2, \ldots, n, \tag{23}$$

the disturbance vectors have the form

$$\bar{\mathbf{w}}^{(1)}(t) = \begin{bmatrix} r_2(t) \\ v_2(t) \\ u_2(t) \\ w(t) \end{bmatrix},\ \bar{\mathbf{w}}^{(n)}(t) = \begin{bmatrix} r_{n-1}(t) \\ v_{n-1}(t) \\ u_{n-1}(t) \end{bmatrix}, \tag{24}$$

$$\bar{\mathbf{w}}^{(i)}(t) = \begin{bmatrix} r_{i-1}(t) \\ v_{i-1}(t) \\ r_{i+1}(t) \\ v_{i+1}(t) \\ u_{i-1}(t) \\ u_{i+1}(t) \end{bmatrix},\ i = 2, \ldots, n-1, \tag{25}$$

and the disturbance input matrices have the following block structure:

$$\bar{\mathbf{E}}_1 = [\mathbf{A}_{12}\,\mathbf{B}_{12}\,\mathbf{E}_1],\ \bar{\mathbf{E}}_n = [\mathbf{A}_{n,n-1}\,\mathbf{B}_{n,n-1}], \tag{26}$$

$$\bar{\mathbf{E}}_i = [\mathbf{A}_{i,i-1}\mathbf{A}_{i,i+1}\mathbf{B}_{i,i-1}\mathbf{B}_{i,i+1}], i = 2, \ldots, n-1. \tag{27}$$

In order to compensate the different order of magnitude in the disturbance vector components, the following formulation of the decoupled models can be considered:

$$\dot{\mathbf{x}}^{(i)}(t) = \mathbf{A}_i\,\mathbf{x}^{(i)}(t) + \mathbf{B}_i\,u_i(t) + \widetilde{\mathbf{E}}_i\widetilde{\mathbf{w}}^{(i)}(t), \tag{28}$$

where the disturbance vectors

$$\widetilde{\mathbf{w}}^{(i)}(t) = \mathbf{S}_i\,\bar{\mathbf{w}}^{(i)}(t),\ i = 1, \ldots, n, \tag{29}$$

are properly scaled by means of the diagonal matrices

$$\mathbf{S}_1 = \mathrm{diag}(\beta_r, \beta_v, \beta_u, \beta_w),\ \mathbf{S}_n = \mathrm{diag}(\beta_r, \beta_v, \beta_u), \tag{30}$$

$$\mathbf{S}_i = \mathrm{diag}(\beta_r, \beta_v, \beta_r, \beta_v, \beta_u, \beta_u),\ i = 2, \ldots, n-1 \tag{31}$$

and the scaled input disturbance matrices have the form

$$\widetilde{\mathbf{E}}_i = \bar{\mathbf{E}}_i\,\mathbf{S}_i^{-1},\ i = 1, 2, \ldots, n. \tag{32}$$

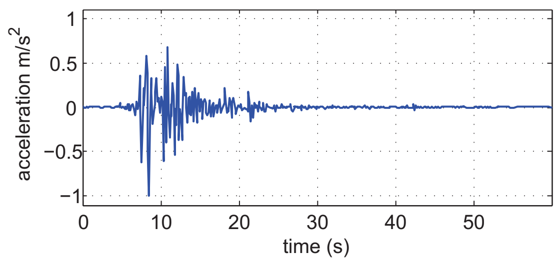

Figure 5: North-South Kobe 1995 seismic record scaled to an acceleration peak of $1\mathrm{m/s^2}$.

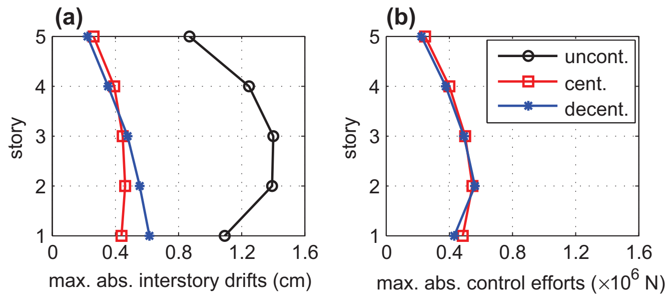

Figure 6: Response of the five-story building model corresponding to the scaled North-South Kobe 1995 seismic record for the uncontrolled configuration (black line with circles), the centralized controller $\mathbf{u}(t) = \mathbf{G}_c\mathbf{x}(t)$ (red line with squares) and the decentralized controller $\mathbf{u}(t) = \mathbf{G}_d\,\mathbf{x}(t)$ (blue line with asterisks). (a) Maximum absolute interstory drifts. (b) Maximum absolute control efforts.

4 NUMERICAL RESULTS

4.1 *Five-story building controller design*

To compute a centralized state-feedback H_∞ controller

$$\mathbf{u}(t) = \mathbf{G}_c\,\mathbf{x}(t) \tag{33}$$

for the first-order system (11) under the bounded-energy external disturbances, we introduce the controlled output

$$\mathbf{z}(t) = \mathbf{C}\,\mathbf{x}(t) + \mathbf{D}\,\mathbf{u}(t), \tag{34}$$

defined by the matrices

$$\mathbf{C} = \begin{bmatrix} \mathbf{I}_{2n} \\ [\mathbf{0}]_{n\times 2n} \end{bmatrix},\quad \mathbf{D} = \alpha \begin{bmatrix} [\mathbf{0}]_{2n\times n} \\ \mathbf{I}_n \end{bmatrix}, \tag{35}$$

where α is a scaling factor that compensates the different magnitude of the state components and the control forces. The control gain matrix $\mathbf{G}_c$ can be computed by solving the following LMI optimization problem (Boyd et al. 1994):

$$\mathcal{P}_\infty : \begin{cases} \text{maximize } \eta \\ \text{subject to } \mathbf{X} > 0,\ \eta > 0 \text{ and the LMI in (36)} \end{cases}$$

Table 2: Parameter values for the twenty-story building model.

story	1-5	6-11	12-14	15–17	18-19	20
mass ($\times 10^6$ Kg)	1.10	1.10	1.10	1.10	1.10	1.10
stiff. ($\times 10^8$N/m)	8.62	5.54	5.54	2.91	2.56	1.72
relative damping	5%					

$$\begin{bmatrix} \mathbf{AX}+\mathbf{XA}^T+\mathbf{BY}+\mathbf{Y}^T\mathbf{B}^T+\eta\mathbf{EE}^T & * \\ \mathbf{CX}+\mathbf{DY} & -\mathbf{I} \end{bmatrix} < 0, \tag{36}$$

where $\mathbf{X}$ and $\mathbf{Y}$ are variable matrices, $\mathbf{I}$ is an identity matrix of suitable dimension and * represents the transpose of the element in the symmetric position. If an optimal value is attained for the LMI matrices $\left(\widetilde{\mathbf{X}},\widetilde{\mathbf{Y}}\right)$, then an optimal control matrix can be written in the form

$$\mathbf{G}=\widetilde{\mathbf{Y}}\widetilde{\mathbf{X}}^{-1}. \tag{37}$$

By solving the LMI optimization problem $\mathcal{P}_\infty$ with the system matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{E}$ presented in Figure 2, and the matrices $\mathbf{C}$ and $\mathbf{D}$ in (35) with $\alpha=10^{-6.7}$, we obtain the control gain matrix $\mathbf{G}_c$ displayed in Figure 4. Next, to compute a set of decentralized local H_∞ controllers

$$u_i(t)=\mathbf{G}^{(i)}\mathbf{x}^{(i)}, i=1,2,\dots,n, \tag{38}$$

for the decoupled local models in (28), we consider the controlled outputs

$$\mathbf{z}^{(i)}(t)=\mathbf{C}_i\mathbf{x}^{(i)}(t)+\mathbf{D}_iu_i(t),\ i=1,2,\dots,n, \tag{39}$$

defined by the matrices

$$\mathbf{C}_i=\begin{bmatrix}\mathbf{I}_2\\ [\mathbf{0}]_{1\times 2}\end{bmatrix},\quad \mathbf{D}_i=\alpha_i\begin{bmatrix}[\mathbf{0}]_{2\times 1}\\ 1\end{bmatrix}. \tag{40}$$

Solving the n LMI optimization problems defined by the matrices $\mathbf{A}_i$, $\mathbf{B}_i$ in (23); the matrix $\widetilde{\mathbf{E}}_i$ in (32) corresponding to the scaling coefficients $\beta_r=10^2$, $\beta_v=10$, $\beta_u=10^{-6}$ and $\beta_w=1$; and the matrices $\mathbf{C}_i$ and $\mathbf{D}_i$ in (40) with $\alpha_i=10^{-6.3}$, $i=1,\dots,5$, we obtain the following set of local control matrices:

$$\begin{aligned}
\mathbf{G}^{(1)} &= 10^7\times[-0.0334\ \ -0.9009],\\
\mathbf{G}^{(2)} &= 10^7\times[-0.6997\ \ -1.2932],\\
\mathbf{G}^{(3)} &= 10^7\times[-0.7954\ \ -1.3296],\\
\mathbf{G}^{(4)} &= 10^7\times[-0.6732\ \ -1.3836],\\
\mathbf{G}^{(5)} &= 10^7\times[-0.2296\ \ -1.2987],
\end{aligned} \tag{41}$$

which define a decentralized controller for the overall structure with the block diagonal control matrix

$$\mathbf{G}_d=\mathrm{blockdiag}\left[\mathbf{G}^{(1)}\ \mathbf{G}^{(2)}\ \mathbf{G}^{(3)}\ \mathbf{G}^{(4)}\ \mathbf{G}^{(5)}\right]. \tag{42}$$

To demonstrate the behavior of the proposed controllers, we have carried out a suitable set of numerical simulations using the scaled North-South Kobe 1995 seismic record as ground acceleration input (see Figure 5). The maximum absolute interstory drifts obtained in these simulations are displayed in Figure 6 (a), where the black line with circles represents the uncontrolled response, the red line with squares corresponds to the centralized H_∞ controller defined by the control matrix $\mathbf{G}_c$ and the blue line with asterisks describes the decentralized controller defined by the block diagonal matrix $\mathbf{G}_d$ in (42). The corresponding maximum absolute control efforts are displayed in Figure 6 (b), using the same colors and symbols. Looking at the plots in Figure 6, it can be appreciated that both controllers attain a good level of reduction in the interstory drift peak-values with practically the same force peak-values. A moderate performance loss can be observed for the decentralized controller at the first-story level, which is consistent with the slightly smaller force peak-value in this level.

Table 3: Local control matrices for the twenty-story building.

i	$\widetilde{\mathbf{G}}^{(i)}$ ($\times 10^7$)	i	$\widetilde{\mathbf{G}}^{(i)}$ ($\times 10^7$)
1	$[-0.1083\ \ -1.5687]$	11	$[-0.6089\ \ -2.6124]$
2	$[-0.3358\ \ -2.1744]$	12	$[-1.2012\ \ -2.8793]$
3	$[-0.3316\ \ -2.1254]$	13	$[-1.5448\ \ -2.8967]$
4	$[-0.3302\ \ -2.1089]$	14	$[-1.2972\ \ -2.8347]$
5	$[-0.2882\ \ -2.0726]$	15	$[-1.4760\ \ -3.3913]$
6	$[-1.3724\ \ -2.6726]$	16	$[-0.8543\ \ -3.6934]$
7	$[-0.6221\ \ -2.6328]$	17	$[-0.7325\ \ -3.7018]$
8	$[-0.6209\ \ -2.6558]$	18	$[-0.0966\ \ -3.6969]$
9	$[-0.6203\ \ -2.6225]$	19	$[-1.2106\ \ -3.6909]$
10	$[-0.6197\ \ -2.6188]$	20	$[-0.6788\ \ -4.0830]$

Table 4: Computation time (in seconds) corresponding to centralized and decentralized controller designs.

controller	centralized	decentralized
five-story building model	0.6655	0.05756
twenty-story building model	163.0967	0.19885

4.2 *Twenty-story building controller design*

To provide a better insight into the characteristics of the proposed controller design methodology, we consider in this section a twenty-story building model corresponding to the parameter values presented in Table 2 (Wang et al. 2009). Proceeding as in the previous section, we first compute a centralized H_∞ controller

$$\mathbf{u}(t)=\widetilde{\mathbf{G}}_c\,\mathbf{x}(t) \tag{43}$$

by solving the LMI optimization problem $\mathcal{P}_\infty$ with the matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{E}$ corresponding to the new set of building parameters and the controlled-output matrices $\mathbf{C}$ and $\mathbf{D}$ in (35) with $n=20$ and $\alpha=10^{-7.3}$. Next, we design a set of decentralized local controllers

$$u_i(t)=\widetilde{\mathbf{G}}^{(i)}\mathbf{x}^{(i)}, i=1,2,\dots,20, \tag{44}$$

by solving $\mathcal{P}_\infty$ with the corresponding system matrices $\mathbf{A}_i$ and $\mathbf{B}_i$ in (23); the matrices $\widetilde{\mathbf{E}}_i$ in (32) with the

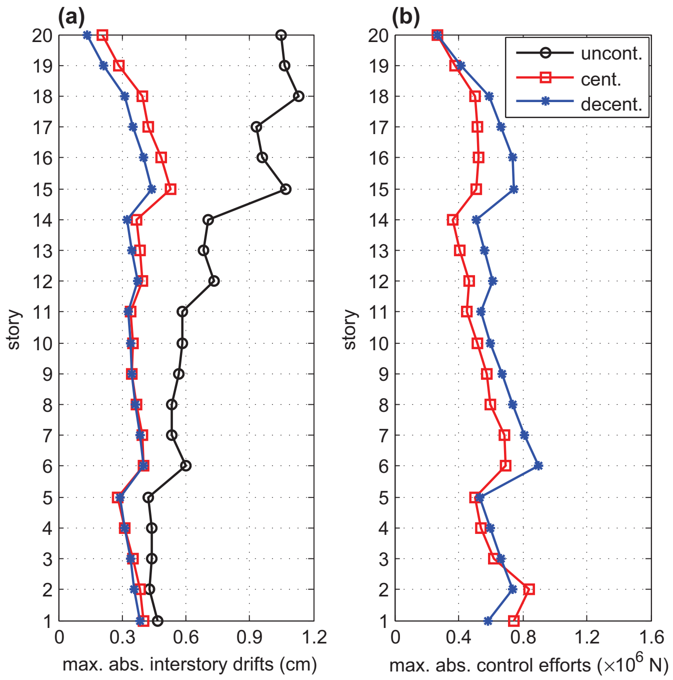

Figure 7: Response of the twenty-story building model corresponding to the scaled North-South Kobe 1995 seismic record for the uncontrolled configuration (black line with circles), the centralized controller $\mathbf{u}(t) = \widetilde{\mathbf{G}}_c\mathbf{x}(t)$ (red line with squares) and the decentralized controller $\mathbf{u}(t) = \widetilde{\mathbf{G}}_d\mathbf{x}(t)$ (blue line with asterisks). (a) Maximum absolute interstory drifts. (b) Maximum absolute control efforts.

same scaling β_i coefficients used in the previous section; and the local controlled-output matrices $\mathbf{C}_i$ and $\mathbf{D}_i$ in (40) with $\alpha_i = 10^{-6.8}$, $i = 1, 2, \ldots, 20$. Table 3 lists the obtained local control matrices, which define a decentralized controller with the following control matrix:

$$\widetilde{\mathbf{G}}_d = \text{blockdiag}\big[\widetilde{\mathbf{G}}^{(1)}\ \widetilde{\mathbf{G}}^{(2)} \ldots\ \widetilde{\mathbf{G}}^{(20)}\big]. \tag{45}$$

The twenty-story building response to the scaled Kobe seismic excitation is presented in Figure 7, with the same colors and symbols used in the five-story building plots. Also in this case, we can observe that the overall behavior of the proposed decentralized controller and the fully centralized controller are quite similar. Slightly larger force peak-values are produced by the decentralized controller, what is consistent with the slightly smaller interstory drift peak-values attained by this controller. The total computation time, in seconds, required to compute the control matrices $\mathbf{G}_c$, $\mathbf{G}_d$, $\widetilde{\mathbf{G}}_c$ and $\widetilde{\mathbf{G}}_d$ are collected in Table 4, where it can be clearly appreciated the computational effectiveness of the proposed decentralized design methodology. All the controllers in this paper have been computed with the LMI solver included in the MATLAB Robust Control Toolbox (Balas et al. 2012) and using a personal computer with a two-core Intel i5 processor.

5 CONCLUSIONS

In this paper, a decentralized control design strategy for structural vibration control of large buildings has been presented. Following the proposed methodology, the overall system can be decomposed into low-dimensional decoupled subsystems. Then, a complete set of decentralized local controllers can be efficiently computed using the existing LMI solvers. This approach has been applied to design decentralized H_∞ controllers for the seismic protection of a five-story building and a twenty-story building with positive results. In summary, two main facts can be highlighted: (i) despite the reduced feedback information, the proposed decentralized controllers present a good level of performance when compared with the full-state centralized controllers and (ii) the computation times required by the decentralized design procedure are remarkably small and present only a linear increment with the dimension increase.

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