Traumatic Brain Injury in Pedestrian–Vehicle Collision: Convexity and Suitability of some functionals used as Injury Metrics

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Abstract

Background and Objective: Abrupt accelerations or decelerations can cause large strain in brain tissues and, consequently, different forms of Traumatic Brain Injury (TBI). In order to predict the effect of the accelerations upon the soft tissues of the brain, many different *injury metrics* have been proposed (typically, an injury metric is a real valued functional of the accelerations). The objective of this article is to make a formal and empirical comparison, in order to identify general criteria for reasonable injury metrics, and propose a general guideline to avoid ill-proposed injury metrics.

Methods: A medium-size sample of vehicle-pedestrian collisions, from Post Mortem Human Subject (PMHS) tests, is analyzed. A statistical study has been conducted in order to determine the discriminant power of the usual metrics. We use Principal Component Analysis to reduce dimensionality and to check consistency among the different metrics. In addition, this article compares the mathematical properties of some of these functionals, trying to identify the desirable properties that any of those functionals needs to fulfill in order to be useful for optimization.

Results: We have found a pair-wise consistency of all the currently used metrics (any two injury metrics are always positively related).

In addition, we observed that two independent principal factors explain about 72.5% of the observed variance among all collision tests. This is remarkable because it indicates that despite high number of different injury metrics, a reduced number of variables can explain the results of all these metrics. With regard to the formal properties, we found that essentially all injury mechanisms can be accounted by means of scalable, differentiable and convex functionals (we propose to call *minimization suitable injury metric* to any metric having these three formal properties). In addition three useful functionals, usable as injury metrics, are identified on the basis of the empirical comparisons.

Conclusions: The commonly used metrics are highly consistent, but also highly redundant. Formal minimal conditions of a reasonable injury metric has been identified. Future proposals of injury metrics can benefit from the results of this study.

KEYWORDS: Traffic collision, Traumatic Brain Injury, Injury metrics, HIC, HIP, BRIC.

1 Introduction

Traumatic brain injury (TBI) is a major global health problem. Country-based estimate of incidences range from 108 to 332 new cases admitted to the hospital per 100 000 population per year [1]. On average, 39% of patients with severe traumatic brain injury die from their injury [2]. On the other hand, the design of restraint systems has had an impact on the number and type of injuries in traffic collisions. Currently, the design of restraint systems is assessed using some injury metric. Indeed, a large number of different injury metrics have been proposed for different purposes [3].

This study presents a theoretical overview of *Injury Metrics* and considers what kind of mathematical properties are desirable for such a metric to be suitable for damage minimization and the optimization of restrain systems. The existent metrics are systematically considered from a formal point of view and its mathematical properties are explored. Finally, a comparison of the prediction of different metrics is made using a medium-size sample of vehicle-pedestrian collision with Post Mortem Human Subjects (PMHS).

Sections 2 and 3 provide a mathematical overview and proper definitions of the commonly used Injury Metrics for TBI. In section 4, the empirical predictions are presented and three new Injury Metrics are introduced. The new metrics are suggested by physical arguments and by the results obtained. Some discussion of the results is provided in section 5. Most of the mathematical details are provided in the final Appendix.

2 Injury metrics

2.1 General description

An injury metric is a real valued functional of the "acceleration curve" $(\mathbf{a}(t), \boldsymbol{\alpha}(t))$, where $\mathbf{a}(t)$ represents the linear acceleration of the center of mass of the head and $\boldsymbol{\alpha}(t)$ the rotational acceleration of the skull. In order to properly define an injury metric we need to specify the domain of definition for this injury metric. Being the arguments $\mathbf{a}(t)$ and $\boldsymbol{\alpha}(t)$, we consider first the vector space of all possible linear and rotational accelerations satisfying some regularity conditions. Mathematically, it is convenient for each component of the acceleration to be integrable over time. For these reasons, we consider the *Hilbert vector space* of [equivalence classes of] square-integrable functions $L^2(\mathbb{R})$ for each component. A function $f(t) \in L^2(\mathbb{R})$ satisfies:

$$\int_{\mathbb{R}} |f(t)|^2 \, dt < \infty \tag{1}$$

Thus for the linear accelerations we consider the Hilbert space [given by the Cartesian product $\mathbf{L}^2(\mathbb{R}) = L^2(\mathbb{R}) \times L^2(\mathbb{R}) \times L^2(\mathbb{R})$] and similarly for the rotational accelerations. The squared value in the equation (1) is needed in order to ensure that we can define an abstract inner product in the space of accelerations (in practice, this technical mathematical condition is not a restriction, because accelerations are different from zero only during a finite time interval).

A typical injury metric can be represented by a functional, defined on a [convex] set of the Hilbert space $\mathbf{L}^2(\mathbb{R}) \times \mathbf{L}^2(\mathbb{R})$. Typically this type of functional involves computing integrals, taking maxima or particular values of the acceleration curves $(\mathbf{a}(t), \boldsymbol{\alpha}(t)) \in \mathbf{L}^2(\mathbb{R}) \times \mathbf{L}^2(\mathbb{R})$. We can ask for the reasonable mathematical properties of an injury metric to be useful (continuity, existence of optimal curves, differentiability, convexity, existence of minima, etc.). In particular we are interested in comparing different processes of the impact of a human head against the structure of a vehicle or an abrupt deceleration of the head. In order to compare severity, we are particularly interested in curves that imply a complete deceleration after a distance d in the direction of the initial velocity $\mathbf{v}_0$. This distance is given by:

$$\begin{aligned} d - \|\mathbf{v}_0\| T &= \int_0^T \int_0^\tau \hat{\mathbf{u}} \cdot \mathbf{a}(\bar{\tau}) \; d\bar{\tau} \\ &= \int_0^T (T-\tau) \hat{\mathbf{u}} \cdot \mathbf{a}(\tau) \; d\tau = \langle (T-\tau), \hat{\mathbf{u}} \cdot \mathbf{a}(\tau) \rangle \end{aligned} \tag{2}$$

where the versor $\hat{\mathbf{u}} = \mathbf{v}_0 / \|\mathbf{v}_0\|$ is aligned with the initial velocity $\mathbf{v}_0$, and $\mathbf{a}$ represents the linear acceleration (which is different from zero only in the time interval $[0, T]$). Notice that the second member can be expressed in terms of the inner product $\langle \cdot, \cdot \rangle$ of $L^2(\mathbb{R})$. For this reason we consider the convex set of $\mathbf{L}^2(\mathbb{R}) \times \mathbf{L}^2(\mathbb{R})$ given by:

$$V_{d,\mathbf{v}_0} = \{ (\mathbf{a}, \boldsymbol{\alpha}) \in \mathbf{L}^2(\mathbb{R}) \times \mathbf{L}^2(\mathbb{R}) | \langle T - t, \mathbf{a}(t) \cdot \hat{\mathbf{u}} \rangle \le d - \|\mathbf{v}_0\| T \} \tag{3}$$

$V_{d,\mathbf{v}_0}$ is a half-space of $\mathbf{L}^2(\mathbb{R}) \times \mathbf{L}^2(\mathbb{R})$ and, therefore, it is convex (indeed, a half-space is always convex). The requirement for the dominion of comparison to be convex is a crucial technical condition for some comparison of metrics.

2.2 Desirable properties for injury metrics

An injury metric functional $\mathrm{Inj} : \mathbf{L}^2(\mathbb{R}) \times \mathbf{L}^2(\mathbb{R}) \to \mathbb{R}$ is *scalable* if for any $\lambda > 1$, and $\mathbf{a} \in \mathbf{L}^2(\mathbb{R}) := L^2(\mathbb{R}) \times L^2(\mathbb{R}) \times L^2(\mathbb{R})$, we have

$$\mathrm{Inj}(\mathbf{a}) \le \mathrm{Inj}(\lambda \mathbf{a}) \tag{4}$$

This condition ensures that "all else being equal, injury does not decrease if the acceleration increases for each time t". Another convenient condition is *continuity* [or *differentiability*], this additional condition implies that small changes in the acceleration imply small changes in the effect of the brain tissues. Finally we introduce the notion of convexity related to the existence of minima and/or optimal curves. An injury metric $\mathrm{Inj}(\cdot)$ is *convex* if it is defined on [a convex subset of] $\mathbf{L}^2(\mathbb{R}) \times \mathbf{L}^2(\mathbb{R})$ and if for any $0 \le \mu \le 1$, we have

$$\mathrm{Inj}(\mu \mathbf{a}_1 + (1-\mu)\mathbf{a}_2) \le \mu \mathrm{Inj}(\mathbf{a}_1) + (1-\mu)\mathrm{Inj}(\mathbf{a}_2) \tag{5}$$

This last property is important because it entails the existence of a minimum (if the functional Inj(·) is strictly convex this minimum is unique), (see theorem 4 of section 6 for details).

An injury metric is *suitable for minimization (or simply* suitable*)* if it is scalable, continuous and convex. In fact, we will see in the next section that many of the commonly used injury metrics are suitable. This suggests that it is mathematically convenient for other new possible proposals of injury metrics to be suitable, [and probably also for additional physical reasons].

3 Commonly used injury metrics

3.1 Head Injury Criterion (HIC) and derived metrics (RIC, KLC)

The Head Injury Criterion HIC_Δ is a very commonly used injury metric, it is given by [3]:

$$\mathrm{HIC}_\Delta(\mathbf{a}) = \max_{t_1,t_2,t_2-t_1\leq\Delta}\left\{\left\|\frac{1}{t_2-t_1}\int_{t_1}^{t_2}\mathbf{a}(\tau)\ d\tau\right\|^{2.5}(t_2-t_1)\right\} \tag{6}$$

The above formula is a functional over the possible acceleration/decelerations curves $\mathbf{a}(t)\in\mathbf{L}^2(\mathbb{R})$ and Δ is the time span for computation (in literature $\Delta = 36$ ms and $\Delta = 15$ ms are used). This metric is a scalable, differentiable and convex functional (see section 6). It is important to note that this definition of HIC was preceded by other alternative forms, for example *Severity Index* (SI) [4], and *Versace's Head Injury Criterion* HIC_T [5]. It can be shown that $\mathrm{HIC}_T \leq \mathrm{HIC}_\Delta$ for the unidimensional case. It is well established that rotational acceleration is relevant for the prediction of TBI, and that the HIC-type measures fail to capture this fact [6, 3]. Because of this, it is necessary to consider functionals on $(\mathbf{a}(t),\boldsymbol{\alpha}(t))\in\mathbf{L}^2(\mathbb{R})\times\mathbf{L}^2(\mathbb{R})$.

Some authors suggested that fast rotational accelerations could produce large stresses in the brain. For this reason, some authors introduced injury metrics which tried to take into account the rotations. For example in [8], a modified formula of HIC was introduced by using rotational acceleration instead of linear acceleration, known as the Rotation Injury Criterion (RIC):

$$\mathrm{RIC}_\Delta(\boldsymbol{\alpha}) = \max_{t_1,t_2,t_2-t_1\leq\Delta}\left\{\left\|\frac{1}{t_2-t_1}\int_{t_1}^{t_2}\boldsymbol{\alpha}(\tau)\ d\tau\right\|^{2.5}(t_2-t_1)\right\} \tag{7}$$

Another metric derived from HIC is the Kleiven Linear Combination (KLC) [9]:

$$\text{KLC}_\Delta(\mathbf{a},\boldsymbol{\omega}) = 0.004718 \cdot \omega_{\max} + 0.000224 \cdot \text{HIC}_\Delta \tag{8}$$

These two latter functionals are continuous, differentiable and convex as is shown in the Appendix. In order, to improve the HIC-like metrics we will propose a new metric, named GHIC (see section 4.3), which generalizes HIC in a physically justified way and has other convenient properties.

3.2 Head Injury Power (HIP)

This functional takes into account rotational accelerations [10], the functionals HIP_t and HIP are defined by:

$$\begin{cases} \text{HIP}_t(\mathbf{a},\boldsymbol{\alpha}) = \sum_{i=1}^3 C_i a_i(t) \int_0^t a_i(\tau) d\tau + \sum_{j=1}^3 \hat{C}_j \alpha_j(t) \cdot \int_0^t \alpha_j(\tau) d\tau \\ \text{HIP}(\mathbf{a},\boldsymbol{\alpha}) = \max_t \text{HIP}_t(\mathbf{a},\boldsymbol{\alpha}) \end{cases} \tag{9}$$

where $C_L = C_1 = C_2 = C_3 = 4.5$ kg and $C_4 = 0.016$ N $\cdot$ m; $C_5 = 0.024$ N $\cdot$ m; $C_6 = 0.022$ N $\cdot$ m [11]. These last two functionals are scalable but they are not suitable, because neither HIP_t nor HIP are convex, although the first one is differentiable. The problem is that the second [functional] derivative of HIP_t could fail to be strictly positive if the components of $(a_x, a_y, a_z, \alpha_x, \alpha_y, \alpha_z)$ change in sign (however if these components are a monotonic function the problem disappears). We will proceed to slightly redefine the HIP functional in order to avoid this problem. Recall the ramp function $\langle x \rangle^+ = (x + |x|)/2 = \max(x, 0)$ that is continuous and convex (and $\langle x \rangle^- = \max(-x, 0)$ is also continuous and convex). Then for any component of acceleration $f \in \{a_x, a_y, a_z, \alpha_x, \alpha_y, \alpha_z\}$, we define the positive and negative component HIP:

$$\begin{cases} \text{HIP}^+_{c,t}(f) = \langle f(t) \rangle^+ \int_0^t \langle f(\tau) \rangle^+ \, d\tau \geq 0 \\ \text{HIP}^-_{c,t}(f) = \langle f(t) \rangle^- \int_0^t \langle f(\tau) \rangle^- \, d\tau \geq 0 \end{cases} \tag{10}$$

Then we define $\text{HIP}_{c,t}(f) = \max(\text{HIP}^+_{c,t}(f), \text{HIP}^-_{c,t}(f))$ and finally the redefinition of HIP, replacing the original equation (9) is:

$$\begin{cases} \overline{\text{HIP}}_t(\mathbf{a},\boldsymbol{\alpha}) = \sum_{i=1}^3 C_i \text{HIP}_{c,t}(a_i(t)) + \sum_{j=1}^3 \hat{C}_j \text{HIP}_{c,t}(\alpha_j(t)) \\ \overline{\text{HIP}}(\mathbf{a},\boldsymbol{\alpha}) = \max_t \overline{\text{HIP}}_t(\mathbf{a},\boldsymbol{\alpha}) \end{cases} \tag{11}$$

For monotonic accelerations, this new definition coincides with the old one. This slight redefinition implies that $\overline{\text{HIP}}$ is now scalable, continuous and convex, and thus it is a suitable metric. Computationally, there is not much difference between usual HIP and modified $\overline{\text{HIP}}$, but with the second one it is guaranteed that an HIP-minimal curve exists.

A more sophisticated attempt to combine the functional form of HIC and HIP, is *Power Rotation Head Injury Criterion* [8]:

$$\text{PRHIC}_{\Delta}(\boldsymbol{\alpha}) = \max_{t_1,t_2,t_2-t_1\leq\Delta}\left\{\left|\frac{1}{t_2-t_1}\int_{t_1}^{t_2}\text{HIP}_t(\mathbf{0},\boldsymbol{\alpha}(\tau))\,d\tau\right|^{2.5}(t_2-t_1)\right\} \quad (12)$$

This metric is scalable and differentiable. In addition, if in this definition the term HIP_t is changed for $\overline{\text{HIP}}_t$ the functional is also convex, and the resulting metric is $\overline{\text{PRHIC}}_{\Delta}$ (this last metric is also suitable).

3.3 Cumulative Strain Damage Metric (CSDM) and Brain Injury Criterion (BrIC)

CSDM is a metric which requires a FEM computation. CSDM was intended as a predictor for *diffuse axonal injury* (DAI) [12, 13], *i. e.* it is an indicator of the probability of damages due to excessive tensile stress in the axons of the neurons. For CSDM, we use the explicit for given in [3]:

$$\text{CSDM}_t^{\varepsilon_0}(\mathbf{a},\boldsymbol{\alpha}) = \frac{\sum_{k=1}^{N} V_k\phi_k^{\varepsilon}(\varepsilon_0,t)}{\sum_{k=1}^{N} V_k} \leq 1 \quad (13)$$

where N is the total number of finite elements, and the function $\phi_k^{\varepsilon}(\cdot,\cdot)$ is given by

$$\phi_k^{\varepsilon}(\varepsilon_0,t) = \max_{\tau\in[0,t]} H(\varepsilon_k(\tau)-\varepsilon_0) \leq 1 \quad (14)$$

where H is the Heaviside step function and ε_0 is the prescribed threshold of strain (usually $\varepsilon_0 = 0.05,\ 0.10,\ 0.015$ or 0.25). The cumulative character of the function implies that $0 \leq \text{CSDM}_r^{\varepsilon_0} \leq \text{CSDM}_s^{\varepsilon_0} \leq 1$, for all $r < s$. Note in addition that $\phi_k^{\varepsilon}(\varepsilon_0,t) = 1$ only if $\varepsilon_k(t) > \varepsilon_0$ for some t, being 0 otherwise. Note that equation 13 gives the proportion of volume that has experienced strains greater than ε_0 in any instant. For an elastic or viscoelastic linear material this function is scalable, but it is neither continuous nor convex (with minor changes a continuous metric that approximates CSDM can be constructed, and by reducing the domain it could also be convex.). The risk curve for CSDM was constructed using survival analysis and Weibull distribution:

$$\text{Injury Risk} = 1 - \exp\left[\left(-\frac{\text{CSDM}}{\lambda}\right)^k\right] \tag{15}$$

where $\lambda = 0.6162$ and $k = 2.7667$ [13]. The main difficulty with the computation of CSDM is the requirement of numerical FEM computation. For avoiding this computation, two empirical alternative metrics were proposed: both of them named Brain Injury Criterion (BRIC for the first version, BrIC for the second). The firsts proposal of BRIC [13] was defined as the best linear estimator of CSDM using just the peak variables $\alpha_{\max}$ and $\omega_{\max}$:

$$\text{BRIC}_a = \frac{\omega_{max}}{\omega_{cr}} + \frac{\alpha_{max}}{\alpha_{cr}} \tag{16}$$

where $\omega_{max} = \max_t \|\boldsymbol{\omega}(t)\|$ and $\alpha_{max} = \max_t \|\boldsymbol{\alpha}(t)\|$ and ω_{cr}, α_{cr} are two coefficients obtained by linear regression [13]. As it is shown in the Appendix 6, this metric is scalable, continuous and convex. In section 4.3, we generalize this last metric for pedestrian-vehicle collisions. In a later study [14], the first author of [13], reconsidered the from of this criterion (renamed as BrIC) which distinguishes rotation about different axes and excludes angular accelerations, namely:

$$\text{BrIC}_b = \left[\left(\frac{\omega_x}{\omega_{x,cr}}\right)^2 + \left(\frac{\omega_y}{\omega_{y,cr}}\right)^2 + \left(\frac{\omega_z}{\omega_{z,cr}}\right)^2\right]^{1/2} \tag{17}$$

In our study, we did not find any evidence of the effect of angular accelerations either. For this reason, our generalization does not contain angular accelerations terms, see equation (24).

3.4 Relative Motion Damage Metric (RMDM)

This metric is an indicator of the probability of damage in the bridge veins between the skull and the brain. A failure in these veins frequently implies a subdural hematoma. It is defined by:

$$\mathrm{RMDM}_t(\mathbf{a},\boldsymbol{\alpha}) = \max_{k\in N_v} \frac{\langle \varepsilon_k(t)\rangle^+}{\varepsilon_u(\dot{\varepsilon}_k(t))} \tag{18}$$

where $\langle x\rangle^+ = \max(x,0)$ and ε_u is the failure strain which is a function of strain rate ($\dot{\varepsilon}$), $\varepsilon_k(t)$ is the strain in the k−th blood vessel, and N_v is the number of blood vessels in the model. Experimentally for the failure strain we have:

$$\varepsilon_u(\dot{\varepsilon}) = 0.0608\dot{\varepsilon}^2 - 0.4414\dot{\varepsilon} + 0.9872 \tag{19}$$

The condition RMDM > 1 (where RMDM $= \max_{t\in\mathbb{R}} \mathrm{RMDM}_t$) is an intended predictor for subdural hematoma. This metric is scalable and continuous (under mild assumptions, it is also convex, see the Appendix).

Collectively, all these measures or metrics have been shown to incorporate tissue-level evaluations of injury that are dependent on the duration, magnitude, and direction of applied linear and angular accelerations.

4 Results of the empirical study

4.1 Data and Methods

The empirical data used for this study were a series of pedestrian collisions with Post Mortem Human Subjects (PMHS) performed at the Center for Applied Biomechanics of the University of Virginia (CAB-UVA). The experimental setting of the tests was described in detail in [15]. A set of accelerometers rigidly attached to the skulls of the pedestrians provided the local accelerations. These were filtered to eliminate noise. From a set of different accelerometers strategically located in the head it is possible to compute linear accelerations and angular velocities. The acceleration curves were used for computing the empirical and analytical injury metrics. Empirical metrics using only kinematic data were computed by a macro. This macro uses data generated by LS-Dyna, for each element in the FE model it is verified if at some instant t, the condition of strain $\varepsilon_{\max}(t) > \varepsilon_0$ holds, then the volume of the elements satisfying this condition of strain is computed, this provides the numerator of equation (13), and enables to compute directly CSDM. For computing the analytical metrics, the computed curves were used as an input for the SIMon model (a finite FE model developed with the support of NHTSA [16]).

Twenty seven PHMS were used for the testing. For each test, a set of curves $(\mathbf{a}(t), \boldsymbol{\omega}(t)) \in \mathbb{R}^3 \times \mathbb{R}^3$ was obtained. In addition, four computed curves were added to the sample representing a head and body falling from a height. Thus, most of the curves were measured for collisions of a pedestrian with the hood/front of the vehicle. For each of the thirty-one curves, thirteen Injury Metrics were calculated: HIC_{36}, RIC_{36}, $\overline{\mathrm{HIP}}$, $\overline{\mathrm{PRHIC}}_{36}$, BrIC, KLC, $\mathrm{CSDM}_{0.05}$, $\mathrm{CSDM}_{0.10}$, $\mathrm{CSDM}_{0.15}$, $\mathrm{CSDM}_{0.25}$, RMDM, and the new proposed metric GHIC_{36} (generalized HIC, see section 4.3). A matrix of 372 ($= 31 \times 12$) values of Injury Metrics were obtained and statistically analyzed for verifying independence, and underlying dimensionality of the data. The objective was to determine which metrics are more distinctive and more useful for predicting TBI. The Fig. 1 shows the correlations between the metrics. All the computed values are shown in tables 1 and 2.

A conventional Principal Component Analysis (PCA) was performed with the 31 different acceleration curves: 20 cases were experimental curves from sedan vehicles, and 7 cases were from sport utility vehicles). Four additional curves more were computed integrating the equations of motion (representing TBI caused by falls from a height, $h = 2,50$ m), for introducing variability in the sample. It can be seen in 3 that all the injury metrics are positively correlated with the first principal factor PC_1, this is an important condition of consistency among the metrics.

4.2 Results for Pedestrian-Vehicle collision

The PCA allowed to differentiate clearly all the three categories ([se]: sedan vehicles, [su]: sport utility vehicles, and [fh]: falls from a height) as shown in Fig. 2. The first and second Principal Components PC_1 and PC_2 can explain roughly 70% of the observed variance among all the sample. Thus, theoretically, we can construct two independent Injury Metrics explaining 70% of the observed variance (namely, the first principal component PC_1 and the second PC_2). We have observed in the sample, that a collision involving SUVs is generally more serious than a typical collision involving a sedan vehicle (see Fig. 2). Note that this is not a general rule, in practice. There are reported cases in the literature of pedestrian impacts, where the head-to-vehicle impact is more severe for sedans than for SUVs [17].

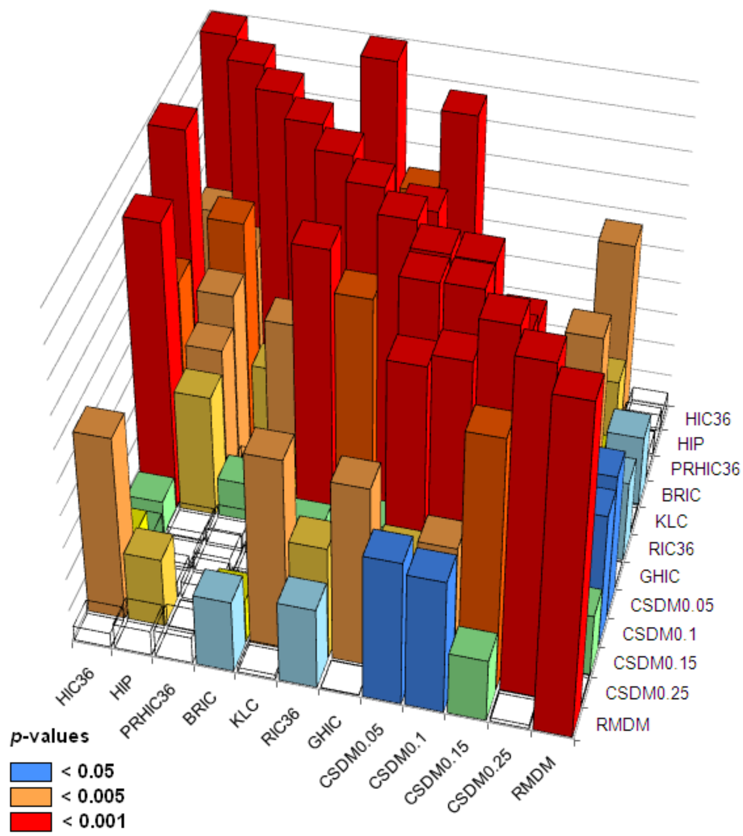

Figure 1: A plot showing the correlations among the variables, height indicates the value, orange-red color indicate high p-value, blue moderate p-value.

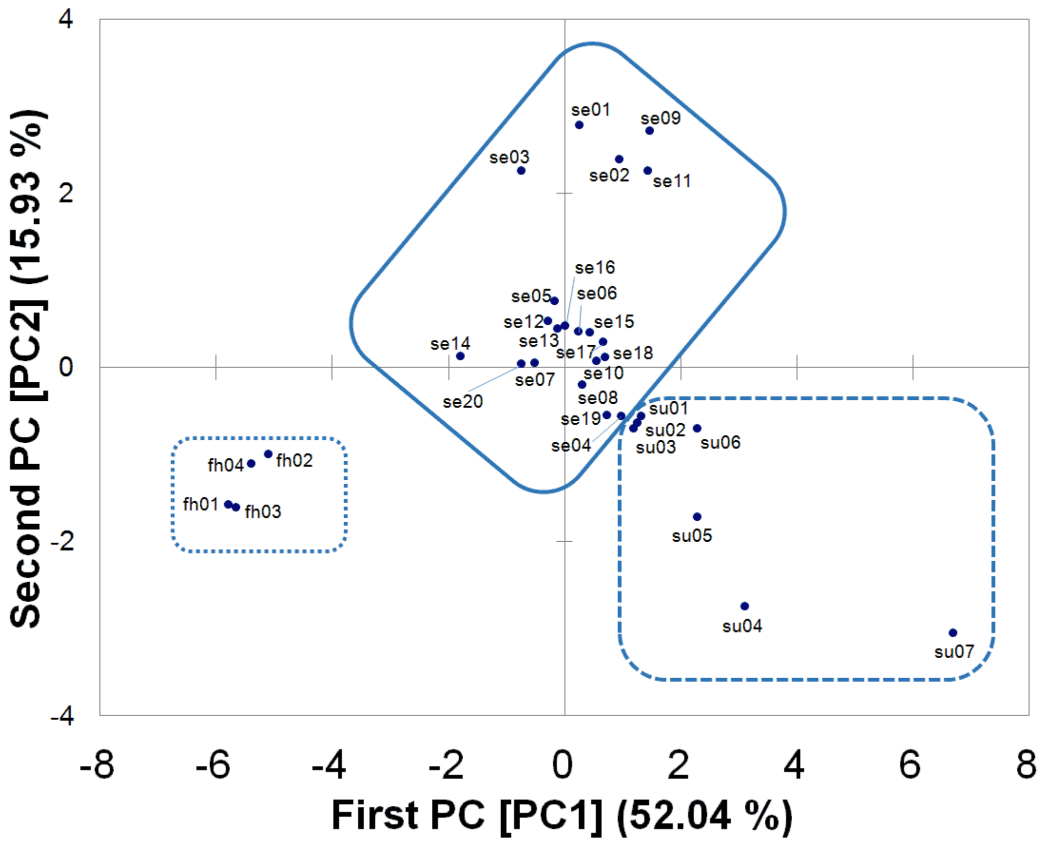

Figure 2: [fh] (left dotted-line frame), [se] (solid-line oblique frame) and [su] (dashed-line right frame) in collisions with pedestrians: the first Principal Component (PC1) separates correctly [fh] from pedestrian collisions ([se] & [su] categories). The combination of both first and second Principal Components separates all three categories. PC1 explains a 52.0% of the total variance and PC2 another 15.9% of the variance.

The PC_1 is higher for the SUV samples than for sedan samples, and the difference is statistically significant (p–value < 0.004, using an unilateral Mann–Whitney U test). Note that as it is shown in s 2 and 3, the first principal component (PC_1) is a measure positively correlated with all injury metrics for traumatic brain injury. This possibility will be further investigated in section 4.3 where a new metric, the *Combined Head Injury Criterion* (CHIC) is proposed as a predictor of PC_1.

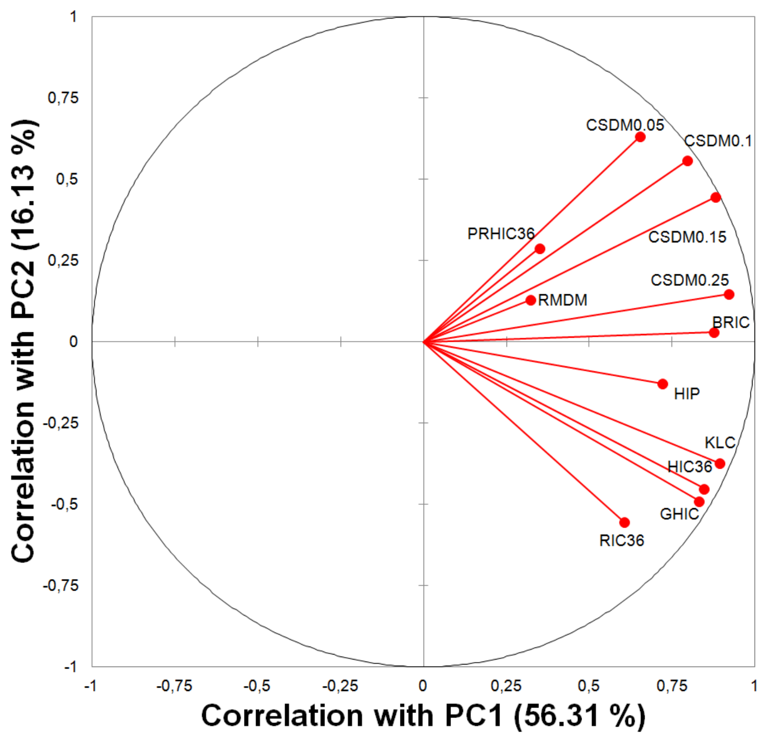

Figure 3: A plot showing the correlations of main metrics with PC_1 and PC_2, a high value of PC_1 is positively correlated with most of the Injury Metrics, thus PC_1 can be interpreted as a kind of "severity index". Because the angle among all arrows < 90° there is pair-consistency (any pair of metrics is always positively correlated).

4.3 Proposed metrics

Many different studies have pointed the importance of abrupt rotations of the head for predicting TBI [8, 10, 18, 20] and, for this reason some authors tried to generalize the functional form of HIC in order to incorporate the effect of rotation. Experimental data in this study showed that RIC or PRHIC are not adequate generalizations (see Fig. 3), in the sense that they are not well correlated with the other well-founded metrics (in particular PRHIC is mainly correlated with a third component factor, not related with PC_1 and PC_2). Instead, a more physical justified generalization shows better correlation with the first PC. This generalization uses not the *conventional acceleration* $\mathbf{a}$ of the head (with respect to the inertial reference frame associated to the ground), but the *"non-inertial" acceleration* $\mathbf{A}$ (with respect to a non-inertial reference frame associated with the skull), Newtonian mechanics allows us to relate both accelerations as:

$$\mathbf{A} = \mathbf{a} + \boldsymbol{\alpha} \times \mathbf{r}_0 + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_0) \tag{20}$$

Thus $\mathrm{GHIC}_\Delta(\mathbf{a}, \boldsymbol{\omega}) = \mathrm{HIC}_\Delta(\mathbf{a} + \boldsymbol{\alpha} \times \mathbf{r}_0 + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_0))$. This new generalization can be physically justified (probably for this reason it presents higher correlations with the rest of the injury metrics than than RIC or PRHIC which lack a direct physical interpretation). The GHIC_Δ is given by:

$$\mathrm{GHIC}_\Delta(\mathbf{a}, \boldsymbol{\omega}) = \max_{t_2 - t_1 \leq \Delta} \left\{ \left| \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \mathbf{a} + \boldsymbol{\alpha} \times \mathbf{r}_0 + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_0) \; d\tau \right|^{2.5} (t_2 - t_1) \right\} \tag{21}$$

This metric founded on physical arguments can be used, to investigate how to approximate the first principal component PC_1 so it can be expressed in terms of empirical metrics as:

$$\mathrm{PC}_1 \approx \frac{\mathrm{GHIC}_\Delta}{\mathrm{GHIC}_0} + \frac{\overline{\mathrm{HIP}}}{\overline{\mathrm{HIP}}_0} + \frac{\mathrm{RMDM}}{\mathrm{RMDM}_0} \tag{22}$$

The adjusted coefficients are $\mathrm{GHIC}_0 = 13610\ g^{2.5}\cdot\mathrm{s}$, $\overline{\mathrm{HIP}}_0 = 212250$ N·m/s, and $\mathrm{RMDM}_0 = -0{,}3029$. All three coefficients are significant (with p−value < 0.0015) and the correlation coefficient is $r = +0.8663$ (unfortunately, this metrics is not suitable because $\mathrm{RMDM}_0 < 0$). We can consider an alternative metric suitable:

$$\mathrm{CHIC} = \frac{\mathrm{GHIC}_\Delta}{\mathrm{GHIC}_{cr}} + \frac{\overline{\mathrm{HIP}}}{\overline{\mathrm{HIP}}_{cr}} \tag{23}$$

Where CHIC is an acronym for *Combined Head Injury Criterion*, with $\mathrm{GHIC}_{cr} = 25 \cdot 10^3\ g^{2.5}\cdot\mathrm{s}$, $\overline{\mathrm{HIP}}_{cr} = 142 \cdot 10^3$ N·m/s. This new metric being close to the PC_1 is highly consistent with all the other metrics, indeed is a good predictor for all other metrics. Another injury metric that has been found accurate for predicting the CSDM for vehicle-pedestrian collisions is generalized BrIC or GBrIC:

$$\mathrm{GBrIC}(\mathbf{a}, \boldsymbol{\omega}) = \sum_i \frac{|a_{i,\max} + a_{i,0}|^2}{a_{i,cr}^2} + \sum_i \frac{|\omega_{i,\max} + \omega_{i,0}|^2}{\omega_{i,cr}^2} \tag{24}$$

The estimated coefficients $a_{i,0}, a_{i,\max}, \omega_{i,0}, \omega_{i,\max}$ are given in table 3 ($r = 0.74$).

5 Discussion and Conclusions

We have found that most Injury Metrics used to assess traumatic brain injury (TBI) have a set of mathematical properties in common. In particular, many of these metrics are scalable, continuous and convex (the technical term *suitable* is introduced for referring to a mathematical functional which is scalable, continuous and convex). In addition, with minor changes all the commonly used non-suitable (but "near-suitable") metrics can be turned suitable. This is the first study showing in detail the mathematical arguments of suitability for most popular Injury Metrics related to TBI [in some cases introducing minor modifications in their definitions]. This fact is important because these mathematical properties precisely ensure the existence of minimal-injury conditions for each of the metrics, and the existence of these minimal-injury conditions can be used to assess the design of restraint systems by imposing numerical constrains to the values of some magnitudes related to mechanisms that can produce TBI.

After analysis of the predictions of many metrics for the same set of data, we found there is a great consistency in the predictions (there are positive correlations among all the metrics, thus in general terms, there is a positive correlation among the predictions of injury probabilities). This is an expected result according to some comparisons reported in the literature [19, 20].

After this comparative study of the metrics, we consider recommendable for any new metric to be suitable and to have consistency with other relevant metrics, in order to be usable for damage minimization and comparability with the proposal of other authors. In addition, among all the metrics satisfying suitability and consistency, we recommend using metrics highly correlated to Principal Factors, and when it is possible, use metrics clearly related to injury mechanisms. The satisfaction of all these properties seems to be a good guide for selecting injury metrics. In a previous work [3], we suggested constructing a set of metrics identifying independent injury mechanism for representing the damage risk, and considering two of the proposed metrics GHIC and CHIC as good candidates for measuring the severity of pedestrian-vehicle collision. In addition, a quadratic modification of BrIC, namely GBrIC, seem to improve the ability to predict the value of CSDM, that is a good indicator of the risk of *diffuse axional injury*.

6 Appendix

This appendix contains the mathematical proofs of some of the claims of the paper. We begin with some elementary properties on convex functions:

1. (Definition) A function H defined on a convex subset U of a vector space is *convex* on U if for all $u, v \in U$ and $\mu \in [0,1]$ we have $H(\mu u + (1-\mu)v) \leq \mu H(u) + (1-\mu)H(v)$ [the function is *strictly convex* if $H(\mu u + (1-\mu)v) < \mu H(u) + (1-\mu)H(v)$].

2. (Theorem) A differentiable function H defined of a convex subset U is convex if and only if $H(v) \geq H(u) + H'(u)(v-u)$ [and strictly convex iff. $H(v) > H(u) + H'(u)(v-u)$].

3. (Theorem) A twice differentiable function H defined of a convex subset U is convex if and only if $H''(u)(v-u, v-u) \geq 0$ [and strictly convex iff. $H''(u)(v-u, v-u) > 0$].

4. (Theorem) For a convex function $H : U \to \mathbb{R}$ any local minimum is a global minimum. If H is strictly convex, it has at most one minimum in U, and it is a strict minimum. If H is differentiable a $u \in U$ is a minimum then $H'(u)(v-u) \geq 0$. If U is an open set, a point $u \in U$ is a minimum of H iff. $H'(u) = 0$.

5. (Theorem) The functions $f_i : \mathbb{R} \to \mathbb{R}$ given by $f_1(x) = ax + b, f_2(x) = |x|^p$ $(p \geq 1)$ are convex. For a vector space V, the functions $F_i : V \to \mathbb{R}$ given by $F_1(x) = L(x) + b$ (with L linear and $b \in \mathbb{R}$), $F_2 = f(F_1(x))$ (with f convex and increasing) are convex. In addition, for a collection of convex functions $\{\Phi_i : V \to \mathbb{R}\}$ the functions $\Phi_{\text{sum}}(x) = \sum_{k=1}^{n} \alpha_k \Phi_k(x)$ $(\alpha_k \geq 0)$ and $\Phi_{\max}(x) = \max\{\Phi_k(x)\}$ are convex.

6. (Theorem) If $f : V \times W \to \mathbb{R}$ is convex in $x \in V$ for all $y \in W$ then $g(x) = \sup_{y \in W} f(x,y)$ is always convex and $h(x) = \inf_{y \in W} f(x,y)$ is convex if W is convex.

6.1 Suitability of HIC

In this section, we prove that HIC_Δ is *scalable, differentiable* and *convex*, thus it is *suitable*. First, from definition (6), we clearly have $\text{HIC}_\Delta(\lambda \mathbf{a}) =$

$\lambda^{2.5}\text{HIC}_\Delta(\mathbf{a}) > \text{HIC}_\Delta(\mathbf{a})$ (for $\lambda > 1$) [thus the functional is suitable]. Second, for differentiability and convexity, we write in the one-dimensional case:

$$\text{HIC}_\Delta(\mathbf{a}) = \max_{t_1,t_2,t_2-t_1\leq\Delta}\{H_{t_1,t_2}(\mathbf{a})(t_2-t_1)\} \tag{25}$$

$$\begin{cases} H_{t_1,t_2} := f \circ L_{t_1,t_2}(\mathbf{a}) \\ L_{t_1,t_2}(\mathbf{a}) := \dfrac{1}{(t_2-t_1)}\int_{t_1}^{t_2} \hat{\mathbf{u}}\cdot\mathbf{a}(\tau)\ d\tau, \quad f(s) := |s|^{2.5} \end{cases} \tag{26}$$

Being $f \in \mathcal{C}^2$ and $L_{t_1,t_2} \in \mathcal{C}^\infty$ for $t_2 > t_1$, we have $H_{t_1,t_2} \in \mathcal{C}^2$ [thus the function is differentiable]. The second derivative of the functional H_{t_1,t_2} is $H_{t_1,t_2} : L^2(\mathbb{R}^3)\times L^2(\mathbb{R}^3) \to \mathbb{R}$ given by:

$$H_{t_1,t_2}(\mathbf{a})(\boldsymbol{\alpha}_1,\boldsymbol{\alpha}_2) = f''(L_{t_1,t_2}(\mathbf{a}))\langle L'_{t_1,t_2}(\mathbf{a}),\boldsymbol{\alpha}_1\rangle\langle L'_{t_1,t_2}(\mathbf{a}),\boldsymbol{\alpha}_2\rangle \tag{27}$$

The last term is always positive if $\boldsymbol{\alpha}_1 = \boldsymbol{\alpha}_2 = \boldsymbol{\alpha}$, then we have that $H_{t_1,t_2}(\mathbf{a})(\boldsymbol{\alpha},\boldsymbol{\alpha}) > 0$ and using theorem 3, we conclude that H_{t_1,t_2} is a convex functional [it can be derived from theorem 5]. Finally using theorem 6 we have that HIC_Δ is convex, and thus *suitable*. For the general three dimensional case we need to define $L_{t_1,t_2}(\mathbf{a}) \in \mathbb{R}^3$ and to replace $f(s) = |s|^{2.5}$ for $\tilde{f}(\mathbf{s}) = \|\mathbf{s}\|^{2.5}$ to achieve the same conclusions.

Formally, RIC_Δ has the same functional form and domain that HIC_Δ, so it is suitable. On the other hand HIC_Δ is a linear combination of two convex functions (namely, HIC_Δ and $\omega_{\max} = \max_t \omega(t)$ being scalable, continuous and convex) so it is also suitable.

6.2 Suitability of HIP

First, we show that $\overline{\text{HIP}}$ is scalable because $\text{HIP}^\pm_{c,t}(\lambda a_i) = \lambda^2\text{HIP}^\pm_{c,t}(a_i) \geq \text{HIP}^\pm_{c,t}(a_i)$ for $\lambda \geq 1$, then it follows that $\text{HIP}_{c,t}, \overline{\text{HIP}}_t$ and $\overline{\text{HIP}}$ are scalable. Because of the continuity of the ramp function $\langle\cdot\rangle$, it is straightforward to see that $\text{HIP}^\pm_{c,t}, \text{HIP}_{c,t}, \overline{\text{HIP}}_t$ and $\overline{\text{HIP}}$ are continuous functionals. Finally for the convexity we have:

$$\begin{aligned}
&\langle\lambda a_1 + (1-\lambda)a_2\rangle^\pm \leq \lambda\langle a_1\rangle^\pm + (1-\lambda)\langle a_2\rangle^\pm && \Rightarrow \\
&\langle\lambda a_1 + (1-\lambda)a_2\rangle^\pm \int_0^t\langle\lambda a_1 + (1-\lambda)a_2\rangle^\pm d\tau && \leq \\
&\leq \lambda^2\langle a_1\rangle^\pm \int_0^t\langle a_1\rangle^\pm d\tau + (1-\lambda)^2\langle a_2\rangle^\pm \int_0^t\langle a_2\rangle^\pm d\tau + \dots \\
&\dots + \lambda(1-\lambda)(\langle a_1\rangle^\pm \int_0^t\langle a_2\rangle^\pm d\tau + \langle a_1\rangle^\pm \int_0^t\langle a_2\rangle^\pm d\tau) && \leq \\
&\leq \lambda\langle a_1\rangle^\pm \int_0^t\langle a_1\rangle^\pm d\tau + (1-\lambda)\langle a_2\rangle^\pm \int_0^t\langle a_2\rangle^\pm d\tau
\end{aligned}$$

This implies that $\mathrm{HIP}^{\pm}_{c,t}$ is convex, and then so are $\mathrm{HIP}_{c,t}, \overline{\mathrm{HIP}}_t$ (by theorem 6). Finally $\overline{\mathrm{PRCHIC}}_{\Delta} = \mathrm{HIC}_{\Delta}(\overline{\mathrm{HIP}}_t)$ and thus by theorem 5 is convex (and, trivially, scalable and continuous).

6.3 Scalability and continuity of CSDM

The forces per unit of volume depend on accelerations and the angular velocity $\mathbf{b}(\mathbf{a},\boldsymbol{\alpha},\boldsymbol{\omega})$, if accelerations are scaled by a factor $\lambda > 1$ then the forces of volume become $\mathbf{b} = \mathbf{b}_0 + \mathbf{b}_1\lambda + \mathbf{b}_2\lambda^2$, where $\mathbf{b}_0$ represents all the terms independent of acceleration (basically weight), $\mathbf{b}_1$ depends on the linear acceleration of the center of mass, the Euler acceleration, the Coriolis acceleration, and $\mathbf{b}_2$ depends on the centripetal acceleration. Being the equilibrium equation linear:

$$\mathbf{b} + \mathrm{div}\boldsymbol{\sigma} = \rho\frac{\partial\mathbf{v}}{\partial t}$$

Using the scaled accelerations, we have $\boldsymbol{\sigma} = \boldsymbol{\sigma}_0 + \boldsymbol{\sigma}_1\lambda + \boldsymbol{\sigma}_2\lambda^2$. Then for a linear elastic or viscoelastic material we have $\boldsymbol{\varepsilon} = \boldsymbol{\epsilon}_0 + \boldsymbol{\epsilon}_1\lambda + \boldsymbol{\epsilon}_2\lambda^2$. Then using a result of Weyl related to the *Horn's conjecture* [21], and assuming that $\varepsilon_{1,I} > 0, \varepsilon_{2,I} > 0$, we have $\phi^{\varepsilon}_{\mathbf{a}}(\varepsilon_0, t) \le \phi^{\varepsilon}_{\lambda\mathbf{a}}(\varepsilon_0, t)$. With respect to continuity, the functional CSDM is not continuous because the presence of the Heaviside step functions (which only takes the values 0 and 1). Replacing in the definition the Heaviside step function H for a continuous function $\tilde{H} : \mathbb{R} \to [0,1]$ the resulting functional is continuous for example:

$$\tilde{H}_m(x) = \frac{1}{2}(1 + \tanh(mx)), \quad \lim_{m\to\infty} \tilde{H}_m(x) = H(x)$$

where $m > 0$ needs to be a constant with a large value for approximating H.

6.4 Suitability of BrIC and GBrIC

The norm of a vector $\boldsymbol{\omega}(t) \mapsto \left(\omega_x^2 + \omega_y^2 + \omega_z^2\right)^{1/2} = \|\boldsymbol{\omega}(t)\|$ is a scalable, continuous and convex function. By theorem 5, the functionals $\omega_{\max} \mapsto \max_t \|\boldsymbol{\omega}(t)\|$ and $\alpha_{\max} \mapsto \max_t \|\boldsymbol{\alpha}(t)\|$ are convex, and so is any linear combination of them. T $\mathrm{BrIC} = \omega_{\max}/\omega_{cr} + \alpha_{\max}/\alpha_{cr}$ is a scalable, continuous and convex functional and, thus, it is a suitable metric (indeed $\mathrm{BrIC}(\lambda\boldsymbol{\omega}, \lambda\boldsymbol{\alpha}) = |\lambda|\mathrm{BrIC}(\boldsymbol{\omega}, \boldsymbol{\alpha})$). For GBRIC, we have that the functionals $f_i(t) \mapsto f_{i,\max} = \max_t f_i(t)$ (for

$i \in \{x, y, z\}, f \in \{a, \omega\}$) are convex, and GBrIC is a linear combinations of terms, being each term a composition of convex functions, then using theorem 5 the whole sum is a convex function.

6.5 Suitability of RMDM

We define $f(x, y) = \langle x \rangle^{+} / \varepsilon_u(y)$ where ε_u is given in equation (19), then we have $\mathrm{RMDM}_t = f(\varepsilon, \dot{\varepsilon})$. The function f is continuous (and even differentiable [in the classical sense] for $x > 0$). This function is convex in $D = \{(\varepsilon, \dot{\varepsilon}) | \dot{\varepsilon} \leq 2\}$ because $f''_{yy}(x, y) > 0$ for $y \leq 2$. For showing that $\mathrm{RMDM}_t(\mathbf{a}, \boldsymbol{\alpha})$ is suitable we need to relate accelerations and strains. We assume that the material is a linear viscoelastic. Experimentally we know that in the veins the stress-strain relation is given by a convex, continuous and monotonous function $\epsilon = h_1(\sigma)$ and we know from the equilibrium equation that stress is a linear function of accelerations $\sigma_t = h_2(\mathbf{a}_t, \boldsymbol{\alpha}_t)$, so $\varepsilon_t = h_1 \circ h_2(\mathbf{a}_t, \boldsymbol{\alpha}_t)$ is also convex (by theorem 5). The function $g(\varepsilon_t) = (\varepsilon_t, \dot{\varepsilon}_t)$ is linear, then $\mathrm{RMDM}_t = (f \circ g \circ h_1 \circ h_2)(\varepsilon, \dot{\varepsilon})$ is convex (by theorem 5 again). In addition, RMDM_t is continuous being a composition of continuous functions. For seeing that RMDM_t is scalable we compute:

$$\frac{\mathrm{RMDM}_t(\lambda\mathbf{a}, \lambda\boldsymbol{\alpha})}{\mathrm{RMDM}_t(\mathbf{a}, \boldsymbol{\alpha})} = \lambda \frac{0.0608\dot{\varepsilon}^2 - 0.4414\dot{\varepsilon} + 0.9872}{0.0608\dot{\varepsilon}^2\lambda^2 - 0.4414\dot{\varepsilon}\lambda + 0.9872} \geq 1$$

A direct computation shows that the function only has local maxima, and that the global minimum is achieved for $\lambda = 1$.

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Table 1: Computed Injury Metrics for the 31 cases (part 1).

Case	HIC_{36}	GHIC	RIC_{36}	HIP	$PRHIC_{36}$	KLC
fh01	1626	1629	0.82	45543	0	0.378
fh02	1626	1670	1.59	45645	0	0.387
fh03	1230	1241	0.82	36445	0	0.289
fh04	1230	1258	0.94	36550	0	0.298
se01	2784	10758	230	115677	1679	0.882
se02	2524	9776	228	305132	14980	0.872
se03	2180	7230	247	191802	1418	0.700
se04	8040	43470	482	372547	8524	2.179
se05	3878	25602	392	114854	2034	1.188
se06	5343	27070	203	190430	3681	1.492
se07	3537	9152	162	439720	19423	1.050
se08	7012	31300	262	342053	6187	1.851
se09	3350	19648	223	230266	2321	1.051
se10	5313	23098	165	383230	3690	1.469
se11	5234	16643	60	205894	1936	1.523
se12	4187	14420	369	168845	1845	1.202
se13	3640	12463	342	378449	15647	1.126
se14	2478	7545	337	200726	1028	0.760
se15	6449	21946	50	217532	3231	1.834
se16	5549	17062	66	175305	1893	1.597
se17	5970	18293	73	350890	4362	1.715
se18	4446	16843	47	301124	2483	1.360
se19	5968	22679	904	247580	791	1.603
se20	2712	7514	333	316599	5703	0.869
su01	4347	24631	1174	116444	1990	1.279
su02	6090	36740	2148	156911	2001	1.705
su03	5712	41285	2231	137982	2340	1.636
su04	6518	34315	3039	572061	4138	1.784
su05	7595	44898	2311	426191	3404	2.072
su06	6630	33985	817	332762	2686	1.846
su07	12507	54647	3465	600744	8236	3.176

Table 2: Computed Injury Metrics for the 31 cases (part 2).

Curve	$CSDM_{0.05}$	$CSDM_{0.1}$	$CSDM_{0.15}$	$CSDM_{0.25}$	BRIC	RMDM
fh01	0.5218	0.2818	0.0055	0.0299	0.065	0.1915
fh02	0.5294	0.2805	0.1439	0.0340	0.106	0.7444
fh03	0.4835	0.2403	0.1111	0.0211	0.064	0.1915
fh04	0.4923	0.2456	0.1201	0.0227	0.105	0.7444
se01	0.9924	0.9903	0.9347	0.5520	2.066	0.5203
se02	0.9924	0.9923	0.9865	0.7387	3.076	0.5520
se03	0.9923	0.9496	0.7115	0.2391	1.864	0.4196
se04	0.7193	0.7192	0.7187	0.6884	2.496	0.8372
se05	0.7193	0.7188	0.7103	0.5380	1.745	0.6719
se06	0.7193	0.7192	0.7180	0.6386	1.770	0.6582
se07	0.7193	0.7161	0.5918	0.1989	4.147	0.7606
se08	0.7193	0.7180	0.6841	0.4826	3.051	0.6812
se09	0.9924	0.9923	0.9889	0.8463	2.028	0.7530
se10	0.7193	0.7176	0.6909	0.4846	3.408	0.8222
se11	0.9924	0.9919	0.9675	0.6703	2.270	0.8155
se12	0.7193	0.7185	0.6846	0.4215	6.022	0.8630
se13	0.7193	0.7192	0.7170	0.5567	10.591	0.7663
se14	0.7193	0.6970	0.5234	0.1710	6.805	0.4072
se15	0.7192	0.7192	0.7153	0.5890	2.329	0.8135
se16	0.7192	0.7192	0.7120	0.5545	2.108	0.6467
se17	0.7192	0.7192	0.7169	0.6482	2.619	0.7407
se18	0.7192	0.7177	0.6982	0.4831	2.411	0.6493
se19	0.7193	0.7192	0.7078	0.5201	8.274	0.6732
se20	0.7192	0.7170	0.6407	0.3946	10.201	0.8263
su01	0.7192	0.7192	0.7145	0.6318	1.658	0.6581
su02	0.7192	0.7191	0.7145	0.5736	1.761	0.6450
su03	0.7192	0.7191	0.7118	0.5422	1.826	0.8583
su04	0.7193	0.7192	0.7172	0.5911	18.620	0.6616
su05	0.7192	0.7191	0.7122	0.5625	2.210	0.6844
su06	0.7192	0.7192	0.7170	0.6144	2.650	0.6398
su07	0.9923	0.9923	0.9922	0.9637	17.494	0.3580

Table 3: Coefficients for the computation of GBrIC.

i	$a_{i,0}$ [m/s^2]	$a_{\max,0}$ [m/s^2]	$\omega_{i,0}$ [1/s]	$\omega_{\max,0}$ [1/s]
x	—	—	24.363	152.39
y	74670	127450	—	—
z	0.0000	978510	136455	6004.0