Abstract

A two part paper has been written in order to summarize the results of a comparative study on the design of pallet racks upright frames subject to compression, according to the European provisions and the North-American ones.

In the first part of the paper, key features of the verifications were discussed, along with details about the experimental test, analyses and the software used. This companion paper presents all the calculus behind the results presented in the first paper and useful applications of the methods tested.
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1. THE RIGOROUS ANALYSIS METHOD

The Rigorous Analysis Method (RAM) considers the lack-of-verticality imperfections. When carrying out the checking of the structure, it shall be considered a non-sway frame and buckling lengths shall be put equal to the geometrical lengths. As my columns are fixed at the upper part, the model doesn't have imperfections due to the lack of verticality. Hence the model will not have any global or member imperfections and a linear first order analysis is performed.

Bending and axial compression with lateral torsional buckling

\[
\frac{N_{sd}}{X_{min} \cdot A_{eff} \cdot \frac{f_y}{\gamma_M}} + k_{LT} \cdot M_{y,SD} \cdot \frac{f_y}{\gamma_M} + k_z \cdot M_{z,SD} \cdot \frac{f_y}{\gamma_M} \leq 1
\]

(EN15512 form.35)

1.1 The Rigorous Analysis Method applied on the Column 2.99x3x0.07inch

\(y-y\) is the major axis

\[N_{Ed} := 85.4 \ kN\]
\[M_{Ed} := 0.1 \ kN \cdot m\]
\[N_{SD} := N_{Ed} = 85.4 \ kN\]
\[M_{y,SD} := 0 \ kN \cdot m\]
\[M_{z,SD} := M_{Ed} = 0.1 \ m \cdot kN\]

\(A_{eff} := 245.281 \ mm^2\) effective cross sectional area from the experiment
\(A_g := 445 \ mm^2\) gross cross sectional area from Consteel
\(W_{eff,y} := 12333.2 \ mm^3\) effective section modulus about y-y axis from Consteel
\(W_{eff,z} := 8384.5 \ mm^3\) effective section modulus about z-z axis from Consteel
\(I_y := 454667 \ mm^4\) moment of inertia about the y-y axis from Consteel
\(f_y := 430 \ MPa\) experimental yield stress
\(E := 210000 \ MPa\) modulus of elasticity
\(\gamma_M := 1\)
1.1.1 Applying the EU methods for distortional buckling

1. $\chi_{\text{min}}$

$\chi_{\text{min}}$ is the smallest of $\chi_{\text{db}}$, $\chi_{\text{y}}$, $\chi_{\text{z}}$ and the reduction factors corresponding to the flexural-torsional buckling modes.

1.a. $\chi_{xy}$ (EN15512 9.7.5.2)

\begin{align*}
i_y &= 31.9 \text{ mm} & \text{radius of giration of the gross cross-section} \\
i_z &= 29.2 \text{ mm} & \text{radius of giration of the gross cross-section} \\
y_0 &= 75.9 \text{ mm} & \text{distance along the y-axis from the shear center to the center of gravity of the gross cross-section from Consteel} \\
I_T &= 440 \text{ mm}^4 & \text{St Venant torsional constant of the gross cross-section from Consteel} \\
I_w &= 694532884 \text{ mm}^6 & \text{warping constant of the gross cross-section from Consteel} \\
E &= 210000 \text{ MPa} & \text{modulus of elasticity} \\
v &= 0.3 & \text{Poisson's ratio} \\
G &= \frac{E}{2 \cdot (1 + v)} = (8.077 \cdot 10^4) \text{ MPa} & \text{shear modulus} \\
L &= 1092 \text{ mm} & \text{distance between two bracing points} \\
\end{align*}

The end connection provides large warping restraint and torsional restraint.

\begin{align*}
L_{\text{eT}} &= 0.7 \cdot L = 764.4 \text{ mm} & \text{effective length of the member with respect to twisting} \\
i_0 &= \sqrt{i_y^2 + i_z^2 + y_0^2} = 87.356 \text{ mm} \\
\beta &= 1 - \left(\frac{y_0}{i_0}\right)^2 = 0.245 \\
L_y &= 1524 \text{ mm} & \text{buckling length between bracing points (EN15512 9.7.4.3 c)} \\
N_{\text{cr},y} &= \frac{\pi^2 \cdot E \cdot I_y}{L_y^2} = 405.735 \text{ kN} & \text{elastic critical load of the upright} \\
\end{align*}
\[ N_{\text{cr,T}} := \frac{1}{i_0} \left( G \cdot I_T + \frac{\pi^2 \cdot E \cdot I_w}{L_{eT}} \right) = 327.496 \text{ kN} \] critical force for torsional buckling (EN15512 9.7.5.2 form. 30)

\[ N_{\text{cr,FT}} := \frac{N_{\text{cr,T}}}{2 \cdot \beta} \left[ 1 + \frac{N_{\text{cr,T}}}{N_{\text{cr,y}}} - \sqrt{\left( 1 - \frac{N_{\text{cr,T}}}{N_{\text{cr,y}}} \right)^2 + 4 \cdot \left( \frac{y_0}{i_0} \right)^2 \cdot \frac{N_{\text{cr,T}}}{N_{\text{cr,y}}} } \right] = [193.771] \text{ kN} \]

\[ N_{\text{cr,FT}} \] critical force for flexural-torsional buckling (EN15512 9.7.5.2 form. 31)

\[ \lambda_{xy} := \sqrt{\frac{A_{\text{eff}} \cdot f_y}{N_{\text{cr,FT}}}} = [0.738] \] slenderness for flexural-torsional buckling

\[ \lambda_{xy} := 0.738 \]

buckling curve "b" (according to EN1993-1-3)

\[ \alpha := 0.34 \]

\[ \phi_{xy} := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda_{xy} - 0.2) + \lambda_{xy}^2 \right] = [0.864] \]

\[ \chi_{xy} := \frac{1}{\phi_{xy}^2 + \sqrt{\phi_{xy}^2 - \lambda_{xy}^2}} = [0.837] \]

1.b. \( \chi_y \) - reduction factor for flexural buckling about the y-y axis (EN15512 9.7.4). Although in this case, the buckling about y-y axis is not relevant.

\[ L_y := 1524 \text{ mm} \]

\[ i_y := 31.9 \text{ mm} \]

\[ \lambda_y := \frac{L_y}{i_y} = 47.774 \]

\[ \lambda_1 := \pi \cdot \sqrt{\frac{E}{f_y}} = 69.427 \]

\[ \beta_1 := \frac{A_{\text{eff}}}{A_g} = 0.551 \]

\[ \lambda'_y := \frac{\lambda_y}{\lambda_1} \cdot \sqrt{\beta_1} = 0.511 \]

for compression: buckling curve "b" (according to EN1993-1-3)

\[ \alpha := 0.34 \]

\[ \phi_y := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda'_y - 0.2) + \lambda'_y^2 \right] = [0.683] \]

\[ \chi_y := \frac{1}{\phi_y^2 + \sqrt{\phi_y^2 - \lambda'_y^2}} = [1.086] \]

\[ \chi_y := 1 \]
1.c. $\chi_z$ (EN15512 9.7.4)

\[ L_z := 1092 \text{ mm} \]
\[ i_z := 29.2 \text{ mm} \]
\[ \lambda_z := \frac{L_z}{i_z} = 37.397 \]
\[ E := 210000 \frac{N}{mm^2} = (2.1 \cdot 10^5) \text{ MPa} \]
\[ \lambda_1 := \pi \sqrt{\frac{E}{f_y}} = 69.427 \]
\[ \beta_1 := \frac{A_{\text{eff}}}{A_g} = 0.551 \]
\[ \lambda'_z := \frac{\lambda_z}{\lambda_1} \sqrt{\beta_1} = 0.4 \]

for compression: buckling curve "b" (according to EN1993-1-3)
\[ \alpha := 0.34 \]
\[ \phi_z := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda'_z - 0.2) + \lambda_z^2 \right] = [0.614] \]
\[ \chi_z := \frac{1}{\phi_z^2 + \sqrt{\phi_z^2 - \lambda_z^2}} = [1.187] \]
\[ \chi_z := 1 \]

1.d. $\chi_{db}$ (EN15512 9.7.2.c)

The value of the reduction factor $\chi_{db}$ is determined from a test performed in 2013: "Distortional buckling strength of American cold-formed rack columns". The calculation is done according to EN15512 part 9.7.6.3. The sections used in the experiment are similar to the ones used in this paper, from the experiment in 2015. The material properties, cross section properties, element lengths and aresulting internal forces are from the experiment.

1.d.1 $\chi_{z,\text{exp}}$

\[ f_{y,\text{exp}} := 471 \text{ MPa} \]
\[ L_{\text{exp}} := 1190 \text{ mm} \quad \text{specimen length} \]
\[ N_{\text{Ed,exp}} := 121730 \text{ N} \quad \text{Load from the stub-column test} \]
\[ A_{\text{eff,exp}} := \frac{N_{\text{Ed,exp}}}{f_{y,\text{exp}}} = 258.45 \text{ mm}^2 \quad \text{effective cross-section area from the stub column test} \]
\[ A_{g,\text{exp}} := 461 \text{ mm}^2 \quad \text{gross cross-section area} \]
\[ L_{z, \text{exp}} := 0.5 \cdot L_{\text{exp}} = 0.595 \, \text{m} \quad \text{specimen buckling length} \]

\[ i_z := 29.2 \, \text{mm} \]

\[ \lambda_{z, \text{exp}} := \frac{L_{z, \text{exp}}}{i_z} = 20.377 \]

\[ E := 210000 \frac{N}{\text{mm}^2} = (2.1 \cdot 10^7) \, \text{MPa} \]

\[ \lambda_1 := \pi \cdot \sqrt{\frac{E}{f_{y, \text{exp}}}} = 66.336 \]

\[ \beta_1 := \frac{A_{\text{eff,exp}}}{A_{\text{g,exp}}} = 0.561 \]

\[ \lambda'_{z, \text{exp}} := \frac{\lambda_{z, \text{exp}}}{\lambda_1} \cdot \sqrt{\beta_1} = 0.23 \]

for compression: buckling curve "b" (according to EN1993-1-3)

\[ \alpha := 0.34 \]

\[ \phi_{z, \text{exp}} := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda'_{z, \text{exp}} - 0.2) + \lambda'_{z, \text{exp}}^2 \right] = [0.532] \]

\[ X_{z, \text{exp}} := \frac{1}{\phi_{z, \text{exp}}^2 + \sqrt{\phi_{z, \text{exp}}^2 - \lambda'_{z, \text{exp}}^2}} = [1.313] \]

\[ X_{z, \text{exp}} := 1 \]

1.d.2 \( X_{xy, \text{exp}} \)

\[ i_y := 31.9 \, \text{mm} \quad \text{radius of giration of the gross cross-section about the y-y axis} \]

\[ i_z := 29.2 \, \text{mm} \quad \text{radius of giration of the gross cross-section about the z-z axis} \]

\[ y_0 := 75.9 \, \text{mm} \quad \text{distance along the y-axis from the shear center to the center of gravity of the gross cross-section} \]

\[ I_{y, \text{exp}} := 469052 \, \text{mm}^4 \quad \text{moment of inertia} \]

\[ I_{T, \text{exp}} := 486 \, \text{mm}^4 \quad \text{St Venant torsional constant of the gross cross-section} \]

\[ I_{w, \text{exp}} := 718317259 \, \text{mm}^6 \quad \text{warping constant of the gross cross-section} \]

\[ E := 210000 \, \text{MPa} \quad \text{modulus of elasticity} \]

\[ \nu := 0.3 \quad \text{Poisson's ratio} \]

\[ G := \frac{E}{2 \cdot (1 + \nu)} = (8.077 \cdot 10^4) \, \text{MPa} \quad \text{shear modulus} \]
**Distance between two bracing points**

\[ L_{\text{exp}} := 1190 \text{ mm} \]

**Effective length of the member**

\[ L_{\text{eT.exp}} := 0.5 \cdot L_{\text{exp}} = 595 \text{ mm} \]

**Effective length of the member**

\[ i_0 := \sqrt{i_y^2 + i_z^2 + y_0^2} = 87.356 \text{ mm} \]

**Effective length of the member**

\[ \beta := 1 - \left( \frac{y_0}{i_0} \right)^2 = 0.245 \]

**Buckling length between bracing points**

\[ L_{y,\text{exp}} := 0.5 \cdot L_{\text{exp}} = 595 \text{ mm} \]

**Elastic critical load**

\[ N_{\text{cr,y.exp}} := \frac{\pi^2 \cdot E \cdot I_{y,\text{exp}}}{L_{y,\text{exp}}} = \left(2.746 \cdot 10^3\right) kN \]

**Critical force for torsional buckling**

\[ N_{\text{cr,T.exp}} := \frac{1}{i_0^2} \left( G \cdot I_{T,\text{exp}} + \frac{\pi^2 \cdot E \cdot I_{w,\text{exp}}}{L_{\text{eT.exp}}} \right) = 556.226 kN \]

**Critical force for flexural-torsional buckling**

\[ N_{\text{cr,FT.exp}} := \frac{N_{\text{cr,y.exp}}}{2 \cdot \beta} \left[ 1 + \frac{N_{\text{cr,T.exp}}}{N_{\text{cr,y.exp}}} \sqrt{\left(1 - \frac{N_{\text{cr,T.exp}}}{N_{\text{cr,y.exp}}} \right)^2 + 4 \cdot \left( \frac{y_0}{i_0} \right)^2} \cdot \frac{N_{\text{cr,T.exp}}}{N_{\text{cr,y.exp}}} \right] = 479.608 kN \]

**Slenderness for flexural-torsional buckling**

\[ \lambda_{xy,\text{exp}} := \sqrt{\frac{A_{\text{eff,exp}} \cdot f_{y,\text{exp}}}{N_{\text{cr,FT.exp}}}} = [0.504] \]

**Buckling curve "b"**

\[ \alpha := 0.34 \]

\[ \phi_{xy,\text{exp}} := 0.5 \left[ 1 + \alpha \cdot (\lambda_{xy,\text{exp}} - 0.2) + \lambda_{xy,\text{exp}}^2 \right] = [0.679] \]

\[ X_{xy,\text{exp}} := \frac{1}{\phi_{xy,\text{exp}}^2 + \sqrt{\phi_{xy,\text{exp}}^2 - \lambda_{xy,\text{exp}}^2}} = [1.093] \]

**First I calculate the effective area from the stub-column test, from the same experiment. In the stub column test the distortional buckling is not relevant.**

\[ f_{y,\text{test}} := 471 \frac{N}{\text{mm}^2} \quad \text{yield stress from experiment} \]

\[ F_u := 121730 N \quad \text{ultimate load from the stub column test} \]
\[ A_{\text{eff.test}} := \frac{F_u}{f_{y\text{test}}} = 258.45 \text{ mm}^2 \] effective cross-sectional area from the experiment

\[ N_{\text{FT}} := X_z\text{.exp} \cdot X_{xy\text{.exp}} \cdot A_{\text{eff.test}} \cdot f_{y\text{test}} = (1.217 \cdot 10^5) \text{ N} \]

The reduction factor is calculated using the experimental buckling load, tested on a column with the length \( L_t = 1190 \text{ mm} \) (most similar with the length I have between two bracings).

\[ N_{\text{db.Rd}} := 114967 \text{ N} \]

\[ \chi_{\text{db}} := \frac{N_{\text{db.Rd}}}{N_{\text{FT}}} = 0.944 \]

The resulting value for \( \chi_{\text{min}} \) is the minimum between \( \chi_{xy}, \chi_y, \chi_z, \chi_{\text{db}} \)

\[ \chi_{\text{min}} := \min(\chi_{xy}, \chi_y, \chi_z, \chi_{\text{db}}) = 0.837 \]

2. \( kLT \) (EN15512 form. 36)

\[ \chi_{LT} \] (EN1993-1-3)

There is no lateral- torsional buckling on the major axis

\[ k_{LT} := 0 \]

\[ \chi_{LT} := 1 \]

3. \( k_z \) (EN15512 9.7.6.3)

The bending moments used in the verification are from the critical cross-section and they are different from the bending moments used for determining \( k_z \). Although the maximum bending moments are on another position on the column, in this case, the part of the column with the maximum length between two bracings is more sensitive to buckling. However, either segment is studied, for this model \( k_z \) will always be greater than 1.5, thus \( k_z = 1.5 \).

\[ M_1 := 0.05 \text{ kN} \cdot \text{m} \]

\[ M_2 := 0.04 \text{ kN} \cdot \text{m} \]

\[ \psi := \frac{M_1}{M_2} = 1.25 \]

\[ \beta_{M,z} := 1.8 - 0.7 \psi = 0.925 \]

\[ \mu_z := \lambda_{z}^* \cdot (2 \cdot \beta_{M,z} - 4) = -0.86 \]

\[ \mu_z \leq 0.9 = 1 \]
\[ k_z := 1 - \frac{\mu_z \cdot N_{sd}}{X_z \cdot A_{eff} \cdot f_y} = 1.696 \]
\[ k_z \leq 1.5 = 0 \]
\[ k_z := 1.5 \]

VERIFICATION WITH TAKING INTO ACCOUNT THE EFFECT OF DISTORTIONAL BUCKLING

The effect of distortional buckling is irrelevant because the distortional buckling reduction factor is higher than the flexural-torsional reduction factor. The verification doesn't modify.

\[ \chi_{\min} := \min (\chi_y, \chi_z, \chi_{xy}, \chi_{db}) = 0.837 \]

\[ \frac{N_{sd}}{X_{\min} \cdot A_{eff} \cdot f_y Y_M} + \frac{k_{LT} \cdot M_{y, sd}}{X_{LT} \cdot W_{eff.y} \cdot f_y Y_M} + \frac{k_z \cdot M_{z, sd}}{W_{eff.\ z} \cdot f_y Y_M} = 1.009 \]

VERIFICATION WITHOUT THE EFFECT OF DISTORTIONAL BUCKLING

\[ \chi_{\min} \] is determined without taking into consideration the effect of distortional buckling: \( \chi_{\text{db}} \)

\[ \chi_{\min} := \min (\chi_y, \chi_z, \chi_{xy}) = 0.837 \]

\[ \frac{N_{sd}}{X_{\min} \cdot A_{eff} \cdot f_y Y_M} + \frac{k_{LT} \cdot M_{y, sd}}{X_{LT} \cdot W_{eff.y} \cdot f_y Y_M} + \frac{k_z \cdot M_{z, sd}}{W_{eff.\ z} \cdot f_y Y_M} = 1.009 \]
1.1.2 Applying the US methods for distortional buckling

The RAM method will be applied, using the minimum effective cross-section area between the effective distortional buckling area $A_{\text{eff} D}$, calculated according to the North-American Standard AISI S100-07 Appendix 1 and the effective local buckling area from the 2015 Stub Column Test.

The effective area for distortional buckling

\[
A_{\text{net}, \text{min}} := 0.551 \text{ in}^2 = 355.483 \text{ mm}^2
\]

\[
A_g := A_{\text{net}, \text{min}} = 355.483 \text{ mm}^2
\]

net cross-section area from the experiment

\[
f_y := 430 \text{ MPa}
\]

yield strength

\[
P_{\text{crd}} := 97347.55 \text{ N}
\]

critical elastic distortional column buckling load determined by analysis in ANSYS

\[
P_y := A_g \cdot f_y = (1.529 \cdot 10^5) \text{ N}
\]

\[
\lambda_d := \sqrt{\frac{P_y}{P_{\text{crd}}}} = 1.253
\]

\[
\lambda_d > 0.561 = 1
\]

\[
P_{nd} := \left(1 - 0.25 \cdot \left(\frac{P_{\text{crd}}}{P_y}\right)^{0.6}\right) \cdot \left(\frac{P_{\text{crd}}}{P_y}\right)^{0.6} \cdot P_y = \left(9.437 \cdot 10^4\right) \text{ N}
\]
	nominal axial strength for distortional buckling (Eq. E4.1.-2)

\[
A_{\text{eff}, D} := \frac{P_{nd}}{f_y} = 219.456 \text{ mm}^2
\]

The effective area from the 2015 stub column test

\[
P_{ua, \text{stub}} := 23.75 \text{ kip} = (1.056 \cdot 10^5) \text{ N}
\]

\[
f_{y, \text{stub}} := 62454 \text{ psi} = 430.605 \text{ MPa}
\]

\[
A_{\text{eff}, L} := \frac{P_{ua, \text{stub}}}{f_{y, \text{stub}}} = 245.341 \text{ mm}^2
\]

\[
A_{\text{eff}, \text{min}} := \min (A_{\text{eff}, D}, A_{\text{eff}, L}) = 219.456 \text{ mm}^2
\]

1. $\chi_{\text{min}}$

$\chi_{\text{min}}$ is the smallest of $\chi_y$, $\chi_z$ and the reduction factors corresponding to the flexural-torsional buckling modes.
1.a. \( X_{xy} \) (EN15512 9.7.5.2)

- \( i_y := 31.9 \text{ mm} \) radius of giration of the gross cross-section about the y-y axis from Consteel
- \( i_z := 29.2 \text{ mm} \) radius of giration of the gross cross-section about the z-z axis from Consteel
- \( y_0 := 75.9 \text{ mm} \) distance along the y-axis from the shear center to the center of gravity of the gross cross-section from Consteel
- \( I_T := 440 \text{ mm}^4 \) St Venant torsional constant of the gross cross-section from Consteel
- \( I_w := 694532884 \text{ mm}^6 \) warping constant of the gross cross-section from Consteel
- \( E := 210000 \text{ MPa} \) modulus of elasticity
- \( v := 0.3 \) Poisson's ratio
- \( G := \frac{E}{2(1+v)} = (8.077 \cdot 10^4) \text{ MPa} \) shear modulus

- \( L := 1092 \text{ mm} \) distance between two bracing points

\[
\text{The end connection provides large warping restraint and torsional restraint.}
\]

- \( L_{et} := 0.7 \cdot L = 764.4 \text{ mm} \) effective length of the member with respect to twisting
- \( i_0 := \sqrt{i_y^2 + i_z^2 + y_0^2} = 87.356 \text{ mm} \)
- \( \beta := 1 - \left( \frac{y_0}{l_0} \right)^2 = 0.245 \)
- \( L_y := 1524 \text{ mm} \) buckling length between bracing points (EN15512 9.7.4.3 c)

- \( N_{cr,y} := \frac{\pi^2 \cdot E \cdot I_y}{L_y^2} = 405.735 \text{ kN} \) elastic critical load of the upright

- \( N_{cr,T} := \frac{1}{l_0^2} \left( G \cdot I_T + \frac{\pi^2 \cdot E \cdot I_w}{L_{et}^2} \right) = 327.496 \text{ kN} \) critical force for torsional buckling (EN15512 9.7.5.2 form. 30)

- \( N_{cr,FT} := \frac{N_{cr,y}}{2 \cdot \beta} \left[ 1 + \frac{N_{cr,T}}{N_{cr,y}} \sqrt{\left( 1 - \frac{N_{cr,T}}{N_{cr,y}} \right)^2 + 4 \cdot \left( \frac{y_0}{l_0} \right)^2 \cdot \frac{N_{cr,T}}{N_{cr,y}}} \right] = 193.771 \text{ kN} \) critical force for flexural-torsional buckling (EN15512 9.7.5.2 form. 31)
\[ \lambda_{xy} = \sqrt{\frac{A_{\text{eff.min}} \cdot f_y}{N_{\text{cr,FT}}}} = [0.698] \quad \text{slenderness for flexural-torsional buckling} \]

\[ \lambda_{xy} = 0.698 \]

buckling curve "b" (according to EN1993-1-3)

\[ \alpha := 0.34 \]

\[ \phi_{xy} := 0.5 \cdot [1 + \alpha \cdot (\lambda_{xy} - 0.2) + \lambda_{xy}^2] = [0.828] \]

\[ \chi_{xy} := \frac{1}{\phi_{xy}^2 + \sqrt{\phi_{xy}^2 - \lambda_{xy}^2}} = [0.883] \]

1.b. \( \chi_y \) - reduction factor for flexural buckling about the y-y axis (EN15512 9.7.4). Although in this case, the buckling about y-y axis is not relevant.

\[ L_y := 1524 \ mm \]

\[ i_y := 31.9 \ mm \]

\[ \lambda_y := \frac{L_y}{i_y} = 47.774 \]

\[ \lambda_1 := \pi \cdot \sqrt{\frac{E}{f_y}} = 69.427 \]

\[ \beta_1 := \frac{A_{\text{eff.min}}}{A_g} = 0.617 \]

\[ \lambda_y^* := \frac{\lambda_y}{\lambda_1} \cdot \sqrt{\beta_1} = 0.541 \]

for compression: buckling curve "b" (according to EN1993-1-3)

\[ \alpha := 0.34 \]

\[ \phi_y := 0.5 \cdot [1 + \alpha \cdot (\lambda_y^* - 0.2) + \lambda_y^*^2] = [0.704] \]

\[ \chi_y := \frac{1}{\phi_y^2 + \sqrt{\phi_y^2 - \lambda_y^*^2}} = [1.056] \]

\[ \chi_y := 1 \]

1.c. \( \chi_z \) (EN15512 9.7.4)

\[ L_z := 1092 \ mm \]

\[ i_z := 29.2 \ mm \]
\[
\lambda_z := \frac{L_z}{l_z} = 37.397
\]
\[
E := 210000 \frac{N}{mm^2} = (2.1 \cdot 10^5) \text{ MPa}
\]
\[
\lambda_1 := \pi \sqrt{\frac{E}{f_y}} = 69.427
\]
\[
\beta_1 := \frac{A_{\text{eff.min}}}{A_g} = 0.617
\]
\[
\lambda'_z := \frac{\lambda_z}{\lambda_1} \sqrt{\beta_1} = 0.423
\]

for compression: buckling curve "b" (according to EN1993-1-3)

\[
\alpha := 0.34
\]
\[
\phi_z := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda'_z - 0.2) + \lambda'_z^2 \right] = [0.628]
\]
\[
\chi_z := \frac{1}{\phi_z^2 + \sqrt{\phi_z^2 - \lambda'_z^2}} = [1.167]
\]
\[
\chi_z := 1
\]

The resulting value for \(\chi_{\min}\) is the minimum between \(X_{xy}, X_y, X_z\)

\[
\chi_{\min} := \min (X_{xy}, X_y, X_z) = 0.883
\]

2. \(kLT\) (EN15512 form. 36)
\(\chi_{LT}\) (EN1993-1-3)

I do not have lateral-torsional buckling on the major axis

\[
k_{LT} := 0
\]
\[
\chi_{LT} := 1
\]

3. \(kz\) (EN15512 9.7.6.3)

The bending moments used in the verification are from the critical cross-section and they are different from the bending moments used for determining \(kz\). Although the maximum bending moments are on another position on the column, in this case, the part of the column with the maximum length between two bracings is more sensitive to buckling. However, either segment is studied, for this model \(kz\) will always be greater than 1.5, thus \(kz = 1.5\).

\[
M_i := 0.05 \text{ kN} \cdot \text{m}
\]
\( M_2 := 0.04 \, kN \cdot m \)

\( \psi := \frac{M_1}{M_2} = 1.25 \)

\( \beta_{M_z} := 1.8 - 0.7 \psi = 0.925 \)

\( \mu_z := \lambda_z \cdot (2 \cdot \beta_{M_z} - 4) = -0.91 \)

\( \mu_z \leq 0.9 = 1 \)

\( k_z := 1 - \frac{\mu_z \cdot N_{Sd}}{X_z \cdot A_{eff.min} \cdot f_y} = 1.823 \)

\( k_z \leq 1.5 = 0 \)

\( k_z := 1.5 \)

**VERIFICATION WITH THE EFFECT OF DISTORTIONAL BUCKLING**

\[ A_{eff.min} = 219.456 \, mm^2 \]

\[ x_{min} := \min (X_y, X_z, X_{xy}) = 0.883 \]

\[ \frac{N_{Sd}}{X_{min} \cdot A_{eff.min} \cdot f_y} + \frac{k_{LT} \cdot M_{y,Sd}}{X_{LT} \cdot W_{eff,y} \cdot f_y} + \frac{k_z \cdot M_{z,Sd}}{W_{eff,z} \cdot f_y} = 1.066 \]

**VERIFICATION WITHOUT TAKING INTO ACCOUNT THE EFFECT OF DISTORTIONAL BUCKLING**

\( x_{min} \) will be calculated for \( A_{eff.min} := A_{eff,L} \) hence the result will be the same as for the EU method

\( x_{min} := 0.837 \)

\[ \frac{N_{Sd}}{X_{min} \cdot A_{eff.min} \cdot f_y} + \frac{k_{LT} \cdot M_{y,Sd}}{X_{LT} \cdot W_{eff,y} \cdot f_y} + \frac{k_z \cdot M_{z,Sd}}{W_{eff,z} \cdot f_y} = 1.009 \]
1.2 The Rigorous Analysis Method applied on the Column 2.99x3x0.105inch

\[ y - y \] is the major axis

\[ N_{Ed} = 131 \text{ kN} \]
\[ M_{Ed} = 0.3 \text{ kN} \cdot \text{m} \]
\[ N_{Sd} = N_{Ed} = 131 \text{ kN} \]
\[ M_{y,Sd} = 0 \text{ kN} \cdot \text{m} \]
\[ M_{z,Sd} = M_{Ed} = 0.3 \text{ m} \cdot \text{kN} \]

\[ A_{\text{eff}} = 547.589 \text{ mm}^2 \] effective cross sectional area from the experiment
\[ A_g = 695 \text{ mm}^2 \] gross cross sectional area from Consteel
\[ W_{\text{eff},y} = 18107 \text{ mm}^3 \] effective section modulus about y-y axis from Consteel
\[ W_{\text{eff},z} = 13445.7 \text{ mm}^3 \] effective section modulus about z-z axis from Consteel
\[ I_y = 687605 \text{ mm}^4 \] moment of inertia about the y-y axis from Consteel
\[ f_y = 370 \text{ MPa} \] experimental yield stress
\[ E = 210000 \text{ MPa} \] modulus of elasticity
\[ Y_M = 1 \]

1.2.1 Applying the EU methods for distortional buckling

1. \( \chi_{\text{min}} \)

\( \chi_{\text{min}} \) is the smallest of \( \chi_{db} , \chi_y , \chi_z \) and the reduction factors corresponding to the flexural-torsional buckling modes.

1.a. \( \chi_{xy} \) (EN15512 9.7.5.2)

\[ i_y = 31.5 \text{ mm} \] radius of giration of the gross cross-section about the y-y axis from Consteel
\[ i_z = 29.3 \text{ mm} \] radius of giration of the gross cross-section about the z-z axis from Consteel
\[ y_0 = 76.4 \text{ mm} \] distance along the y-axis from the shear center to the center of gravity of the gross cross-section from Consteel
\[ l_I = 1670 \text{ mm}^4 \] St Venant torsional constant of the gross cross-section from Consteel
\[ l_w = 1091949732 \text{ mm}^6 \] warping constant of the gross cross-section from Consteel
E := 210000 MPa  
modulus of elasticity

ν := 0.3  
Poisson's ratio

G := \frac{E}{2 \cdot (1 + \nu)} = (8.077 \cdot 10^4) \text{ MPa}  
shear modulus

L := 1092 mm  
distance between two bracing points

The end connection provides large warping restraint and torsional restraint.

L_{eT} := 0.7 \cdot L = 764.4 mm  
effective length of the member with respect to twisting

i_0 := \sqrt{i_y^2 + i_z^2 + y_0^2} = 87.68 mm

β := 1 - \left( \frac{y_0}{i_0} \right)^2 = 0.241

L_y := 1524 mm  
buckling length between bracing points (EN15512 9.7.4.3 c)

N_{cr,y} := \frac{\pi^2 \cdot E \cdot I_y}{L_y^2} = 613.604 kN  
elastic critical load of the upright

N_{cr,T} := \frac{1}{i_0^2} \left( G \cdot I_T + \frac{\pi^2 \cdot E \cdot I_w}{L_{eT}^2} \right) = 521.375 kN  
critical force for torsional buckling (EN15512 9.7.5.2 form. 30)

N_{cr,FT} := \frac{N_{cr,y}}{2 \cdot \beta} \left[ 1 + \frac{N_{cr,T}}{N_{cr,y}} \right] \left[ 1 - \left( \frac{N_{cr,T}}{N_{cr,y}} \right)^2 \right] + 4 \cdot \left( \frac{y_0}{i_0} \right)^2 \cdot \frac{N_{cr,T}}{N_{cr,y}} = [301.102] kN  
critical force for flexural-torsional buckling (EN15512 9.7.5.2 form. 31)

\lambda_{xy} = \sqrt{\frac{A_{eff} \cdot f_y}{N_{cr,FT}}} = [0.82]  
slenderness for flexural-torsional buckling

\lambda_{xy} := 0.82  
buckling curve "b" (according to EN1993-1-3)

α := 0.34

ϕ_{xy} := 0.5 \cdot [1 + α \cdot (\lambda_{xy} - 0.2) + \lambda_{xy}^2] = [0.942]
\[ \chi_{xy} := \frac{1}{\phi_{xy}^2 + \sqrt{\phi_{xy}^2 - \lambda_{xy}^2}} = [0.741] \]

1.b. \( \chi_y \) - reduction factor for flexural buckling about the y-y axis (EN15512 9.7.4). Although in this case, the buckling about y-y axis is not relevant.

\[ L_y := 1524 \text{ mm} \]
\[ i_y := 31.5 \text{ mm} \]
\[ \lambda_y := \frac{L_y}{i_y} = 48.381 \]
\[ \lambda := \pi \sqrt{\frac{E}{f_y}} = 74.844 \]
\[ \beta := \frac{A_{\text{eff}}}{A_g} = 0.788 \]
\[ \lambda' := \frac{\lambda_y}{\lambda} \cdot \sqrt{\beta} = 0.574 \]

for compression: buckling curve "b" (according to EN1993-1-3)

\[ \alpha := 0.34 \]
\[ \phi_y := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda' - 0.2) + \lambda'^2 \right] = [0.728] \]
\[ \chi_y := \frac{1}{\phi_y^2 + \sqrt{\phi_y^2 - \lambda'^2}} = [1.022] \]
\[ \chi_y := 1 \]

1.c. \( \chi_z \) (EN15512 9.7.4)

\[ L_z := 1092 \text{ mm} \]
\[ i_z := 29.3 \text{ mm} \]
\[ \lambda_z := \frac{L_z}{i_z} = 37.27 \]
\[ E := 210000 \text{ MPa} \]
\[ \lambda := \pi \sqrt{\frac{E}{f_y}} = 74.844 \]
\[ \beta := \frac{A_{\text{eff}}}{A_g} = 0.788 \]
\[
\lambda'_z := \frac{\lambda_z}{\lambda_1} \cdot \sqrt{\beta_1} = 0.442
\]

for compression: buckling curve "b" (according to EN1993-1-3)

\[
\alpha := 0.34
\]

\[
\phi_z := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda'_z - 0.2) + \lambda'_z^2 \right] = 0.639
\]

\[
\chi_z := \frac{1}{\phi_z^2 + \sqrt{\phi_z^2 - \lambda'_z^2}} = 1.15
\]

\[
\chi_z := 1
\]

1.d. \( \chi_{db} \) (EN15512 9.7.2.c)

The value of the reduction factor \( \chi_{db} \) is determined from a test performed in 2013: "Distortional buckling strength of American cold-formed rack columns". The calculation is done according to EN15512 part 9.7.6.3. The sections used in the experiment are similar to the ones used in this paper, from the experiment in 2015. The material properties, cross section properties, element lengths and resulting internal forces are from the experiment.

1.d.1 \( \chi_{z,exp} \)

\[
f_{y,exp} := 449 \text{ MPa}
\]

\[
L_{exp} := 1190 \text{ mm} \quad \text{specimen length}
\]

\[
N_{Ed,exp} := 215127 \text{ N} \quad \text{Load from the stub-column test}
\]

\[
A_{eff,exp} := \frac{N_{Ed,exp}}{f_{y,exp}} = 479.125 \text{ mm}^2 \quad \text{effective cross-section area from the stub column test}
\]

\[
A_{g,exp} := 702 \text{ mm}^2 \quad \text{gross cross-section area from the experiment}
\]

\[
l_z := 0.5 \cdot L_{exp} = 0.595 \text{ m} \quad \text{specimen buckling length}
\]

\[
i_z := 29.3 \text{ mm}
\]

\[
\lambda_z := \frac{l_z}{i_z} = 20.307
\]

\[
E := 210000 \frac{N}{\text{mm}^2} = (2.1 \cdot 10^5) \text{ MPa}
\]

\[
\lambda_1 := \pi \sqrt[3]{\frac{E}{f_{y,exp}}} = 67.942
\]

\[
\beta_1 := \frac{A_{eff,exp}}{A_{g,exp}} = 0.683
\]
\[
\lambda'_{z,\text{exp}} := \frac{\lambda_{z,\text{exp}}}{\lambda_1} \cdot \sqrt{\beta_1} = 0.247
\]

for compression: buckling curve "b" (according to EN1993-1-3)

\[
\alpha := 0.34
\]

\[
\phi_{z,\text{exp}} := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda'_{z,\text{exp}} - 0.2) + \lambda'_{z,\text{exp}}^2 \right] = 0.538
\]

\[
X_{z,\text{exp}} := \frac{1}{\phi_{z,\text{exp}}^2 + \sqrt{\phi_{z,\text{exp}}^2 - \lambda'_{z,\text{exp}}^2}} = 1.301
\]

\[
X_{z,\text{exp}} := 1
\]

1.d.2 \( X_{xy} \)

\[
i_y := 31.5 \text{ mm}
\]

radius of giration of the gross cross-section about the y-y axis

\[
i_z := 29.3 \text{ mm}
\]

radius of giration of the gross cross-section about the z-z axis

\[
y_0 := 76.4 \text{ mm}
\]

distance along the y-axis from the shear center to the center of gravity of the gross cross-section

\[
l_{y,\text{exp}} := 694572 \text{ mm}^4
\]

moment of inertia

\[
l_{T,\text{exp}} := 1726 \text{ mm}^4
\]

St Venant torsional constant of the gross cross-section

\[
l_{w,\text{exp}} := 1104345655 \text{ mm}^6
\]

warping constant of the gross cross-section

\[
E := 210000 \text{ MPa}
\]

modulus of elasticity

\[
\nu := 0.3
\]

Poisson's ratio

\[
G := \frac{E}{2 \cdot (1 + \nu)} = (8.077 \cdot 10^4) \text{ MPa}
\]

shear modulus

\[
l_{\text{exp}} := 1190 \text{ mm}
\]

distance between two bracing points

\[
l_{eT,\text{exp}} := 0.5 \cdot l_{\text{exp}} = 595 \text{ mm}
\]

effective length of the member

\[
i_0 := \sqrt{i_y^2 + i_z^2 + y_0^2} = 87.68 \text{ mm}
\]

\[
\beta := 1 - \left( \frac{y_0}{i_0} \right)^2 = 0.241
\]

\[
l_{y,\text{exp}} := 0.5 \cdot l_{\text{exp}} = 595 \text{ mm}
\]

buckling length between bracing points (EN15512 9.7.4.3 c)
\[ N_{\text{cr,y,exp}} := \frac{\pi^2 \cdot E \cdot I_{y,\text{exp}}}{L_{y,\text{exp}}} = \left(4.066 \cdot 10^3\right) \text{kN} \quad \text{elastic critical load} \]

\[ N_{\text{cr,T,exp}} := \frac{1}{2 \cdot i_0^2} \left(G \cdot I_{T,\text{exp}} + \frac{\pi^2 \cdot E \cdot I_{w,\text{exp}}}{L_{eT,\text{exp}}}\right) = 859.129 \text{kN} \quad \text{critical force for torsional buckling} \]

(EN15512 9.7.5.2 form. 30)

\[ N_{\text{cr,FT,exp}} := \frac{N_{\text{cr,y,exp}}}{2 \cdot \beta} \left[1 + \frac{N_{\text{cr,T,exp}}}{N_{\text{cr,y,exp}}} \sqrt{1 - \left(\frac{N_{\text{cr,T,exp}}}{N_{\text{cr,y,exp}}}\right)^2 + 4 \cdot \left(\frac{Y_0}{i_0}\right)^2 \cdot \frac{N_{\text{cr,T,exp}}}{N_{\text{cr,y,exp}}}}\right] = 735.732 \text{kN} \]

\[ N_{\text{cr,FT,exp}} \quad \text{critical force for flexural-torsional buckling (EN15512 9.7.5.2 form. 31)} \]

\[ \lambda_{xy,\exp} := \sqrt[2]{\frac{A_{\text{eff,exp}} \cdot f_{y,\exp}}{N_{\text{cr,FT,exp}}}} = 0.541 \quad \text{blenderness for flexural-torsional buckling} \]

\[ \lambda_{xy,\exp} := 0.541 \]

buckling curve "b" (according to EN1993-1-3)

\[ \alpha := 0.34 \]

\[ \phi_{xy,\exp} := 0.5 \cdot \left[1 + \alpha \cdot \left(\lambda_{xy,\exp} - 0.2\right) + \lambda_{xy,\exp}^2\right] = 0.704 \]

\[ X_{xy,\exp} := \frac{1}{\phi_{xy,\exp}^2 + \sqrt{\phi_{xy,\exp}^2 - \lambda_{xy,\exp}^2}} = 1.056 \]

\[ X_{xy,\exp} = 1.056 \]

\[ X_{xy,\exp} := 1 \]

First I calculate the effective area from the stub-colum test, from the same experiment. In the stub column test the distortional buckling in not relevant.

\[ f_{y,\text{test}} := 449 \frac{N}{\text{mm}^2} \quad \text{yield stress from experiment} \]

\[ F_u := 215127 \text{N} \quad \text{ultimate load from the stub column test} \]

\[ A_{\text{eff,test}} := \frac{F_u}{f_{y,\text{test}}} = 479.125 \text{mm}^2 \quad \text{effective cross-sectional area from the experiment} \]

\[ N_{FT} := X_{z,\exp} \cdot X_{xy,\exp} \cdot A_{\text{eff,test}} \cdot f_{y,\text{test}} = \left(2.151 \cdot 10^5\right) \text{N} \]

The reduction factor is calculated using the experimental buckling load, tested on a column with the length \(L_t=1190\text{mm}\) (most similar with the length I have between two bracings).
\[ N_{db, Rd} := 167223 \ N \]
\[ X_{db} := \frac{N_{db, Rd}}{N_{FT}} = 0.777 \]

**The resulting value for** \( x_{min} \) **is the minimum between** \( X_{xy}, X_y, X_z, X_{db} \)

\[ x_{min} := \min (X_{xy}, X_y, X_z, X_{db}) = 0.741 \]

2. \( kLT \) (EN15512 form. 36)
\( \chi_{LT} \) (EN1993-1-3)

I do not have lateral-torsional buckling on the major axis

\[ k_{LT} := 0 \]
\[ \chi_{LT} := 1 \]

3. \( kz \) (EN15512 9.7.6.3)

The bending moments used in the verification are from the critical cross-section and they are different from the bending moments used for determining \( kz \). Although the maximum bending moments are on another position on the column, in this case, the part of the column with the maximum length between two bracings is more sensitive to buckling. However, either segment is studied, for this model \( kz \) will always be greater than 1.5, thus \( kz = 1.5 \).

\[ M_1 := 0.15 \ \text{kN} \cdot \text{m} \]
\[ M_2 := 0.08 \ \text{kN} \cdot \text{m} \]
\[ \psi := \frac{M_1}{M_2} = 1.875 \]
\[ \beta_{M,z} := 1.8 - 0.7 \ \psi = 0.488 \]
\[ \mu_z := \lambda_z \cdot (2 \cdot \beta_{M,z} - 4) = -1.337 \]
\[ \mu_z \leq 0.9 = 1 \]
\[ k_z := 1 - \frac{\mu_z \cdot N_{sd}}{X_z \cdot A_{eff} \cdot f_y} = 1.865 \]
\[ k_z \leq 1.5 = 0 \]
\[ k_z := 1.5 \]
VERIFICATION WITH TAKING INTO ACCOUNT THE EFFECT OF DISTORTIONAL BUCKLING

\[ x_{min} := \min (x_y, x_z, x_{db}, x_{xy}) = 0.741 \]

\[ \frac{N_{SD}}{x_{min} \cdot A_{eff} \cdot f_y / Y_M} + \frac{k_{LT} \cdot M_{y,SD}}{X_{LT} \cdot W_{eff,y} \cdot f_y / Y_M} + \frac{k_z \cdot M_{z,SD}}{W_{eff,z} \cdot f_y / Y_M} = 0.963 \]

VERIFICATION WITHOUT TAKING INTO ACCOUNT THE EFFECT OF DISTORTIONAL BUCKLING

\[ x_{min} := \min (x_{xy}, x_y, x_z, x_{db}) = 0.741 \]

\[ \frac{N_{SD}}{x_{min} \cdot A_{eff} \cdot f_y / Y_M} + \frac{k_{LT} \cdot M_{y,SD}}{X_{LT} \cdot W_{eff,y} \cdot f_y / Y_M} + \frac{k_z \cdot M_{z,SD}}{W_{eff,z} \cdot f_y / Y_M} = 0.963 \]

1.2.2 Applying the US methods for distortional buckling

The RAM method will be applied, using the minimum effective cross-section area between the effective distortional buckling area \( A_{eff,D} \), calculated according to the North-American Standard AISI S100-07 Appendix 1 and the effective local buckling area from the 2015 Stub Column Test.

The effective area for distortional buckling

\[ A_{net,min} := 0.849 \text{ in}^2 = 547.741 \text{ mm}^2 \]

\[ A_{g,exp} := A_{net,min} = 547.741 \text{ mm}^2 \text{ net cross-section area from the experiment} \]

\[ f_{y,exp} := 370 \text{ MPa} \text{ experimental yield strength} \]

\[ P_{crit} := 241922.811 \text{ N} \text{ critical elastic distortional column buckling load determined by analysis in ANSYS} \]

\[ P_y := A_{g,exp} \cdot f_{y,exp} = (2.027 \cdot 10^5) \text{ N} \]  \hspace{1cm} (1.2.1-4)

\[ \lambda_d := \sqrt{\frac{P_y}{P_{crit}}} = 0.915 \]

\[ \lambda_d > 0.561 = 1 \]
\[ P_{nd} := \left( 1 - 0.25 \cdot \left( \frac{P_{crd}}{P_y} \right)^{0.6} \right) \left( \frac{P_{crd}}{P_y} \right)^{0.6} \cdot P_y = (1.627 \cdot 10^5) \, N \]

\[ A_{\text{eff},D} := \frac{P_{nd}}{f_{y,\text{exp}}} = 439.783 \, \text{mm}^2 \]

The effective area from the stub column test

\[ P_{\text{ua.stub}} := 45.59 \, \text{kip} = (2.028 \cdot 10^5) \, N \]

\[ f_{y,\text{stub}} := 53699 \, \text{psi} = 370.242 \, \text{MPa} \]

\[ A_{\text{eff},L} := \frac{P_{\text{ua.stub}}}{f_{y,\text{stub}}} = 547.735 \, \text{mm}^2 \]

\[ A_{\text{eff.min}} := \min(A_{\text{eff},D}, A_{\text{eff},L}) = 439.783 \, \text{mm}^2 \]

1. \( X_{\text{min}} \)

\( X_{\text{min}} \) is the smallest of \( X_y, X_z \) and the reduction factors corresponding to the flexural-torsional buckling modes.

1.a. \( X_{xy} \) (EN15512 9.7.5.2)

\[ i_y := 31.5 \, \text{mm} \] radius of giration of the gross cross-section about the y-y axis from Consteel

\[ i_z := 29.3 \, \text{mm} \] radius of giration of the gross cross-section about the z-z axis from Consteel

\[ y_0 := 76.4 \, \text{mm} \] distance along the y-axis from the shear center to the center of gravity of the gross cross-section from Consteel

\[ I_T := 1670 \, \text{mm}^4 \] St Venant torsional constant of the gross cross-section from Consteel

\[ I_w := 1091949732 \, \text{mm}^6 \] warping constant of the gross cross-section from Consteel

\[ E := 210000 \, \text{MPa} \] modulus of elasticity

\[ v := 0.3 \] Poisson's ratio

\[ G := \frac{E}{2 \cdot (1 + v)} = (8.077 \cdot 10^4) \, \text{MPa} \] shear modulus

\[ L := 1092 \, \text{mm} \] distance between two bracing points
The end connection provides large warping restraint and torsional restraint.

\[ L_{eT} := 0.7 \cdot L = 764.4 \text{ mm} \quad \text{effective length of the member with respect to twisting} \]

\[ i_0 := \sqrt{l_y^2 + l_z^2 + y_0^2} = 87.68 \text{ mm} \]

\[ \beta := 1 - \left( \frac{y_0}{i_0} \right)^2 = 0.241 \]

\[ L_y := 1524 \text{ mm} \quad \text{buckling length between bracing points (EN15512 9.7.4.3 c)} \]

\[ N_{cr,y} := \frac{\pi^2 \cdot E \cdot l_y}{L_y^2} = 613.604 \text{ kN} \quad \text{elastic critical load of the upright} \]

\[ N_{cr,T} := \frac{1}{i_0^2} \left[ \left( G \cdot I_T + \frac{\pi^2 \cdot E \cdot I_w}{L_e^2} \right) \right] = 521.375 \text{ kN} \quad \text{critical force for torsional buckling (EN15512 9.7.5.2 form. 30)} \]

\[ N_{cr,FT} := \frac{N_{cr,y}}{2 \cdot \beta} \left[ 1 + \frac{N_{cr,T}}{N_{cr,y}} - \sqrt{\left( 1 - \frac{N_{cr,T}}{N_{cr,y}} \right)^2 + 4 \cdot \left( \frac{y_0}{i_0} \right)^2 \cdot \frac{N_{cr,T}}{N_{cr,y}}} \right] = [301.102] \text{ kN} \]

\[ \lambda_{xy} := \sqrt{\frac{A_{\text{eff.min}} + f_y}{N_{cr,FT}}} = [0.735] \quad \text{slenderness for flexural-torsional buckling} \]

\[ \lambda_{xy} := 0.735 \]

buckling curve "b" (according to EN1993-1-3)

\[ \alpha := 0.34 \]

\[ \phi_{xy} := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda_{xy} - 0.2) + \lambda_{xy}^2 \right] = [0.861] \]

\[ X_{xy} := \frac{1}{\phi_{xy}^2 + \sqrt{\phi_{xy}^2 - \lambda_{xy}^2}} = [0.84] \]

\[ X_{xy} = [0.84] \]
1.b. \( \chi_y \) - reduction factor for flexural buckling about the y-y axis (EN15512 9.7.4). Although in this case, the buckling about y-y axis is not relevant.

\[
\begin{align*}
L_y &:= 1524 \text{ mm} \\
i_y &:= 31.5 \text{ mm} \\
\lambda_y &:= \frac{L_y}{i_y} = 48.381 \\
\lambda_1 &:= \pi \sqrt{\frac{E}{f_y}} = 74.844 \\
\beta_1 &:= \frac{A_{\text{eff.min}}}{A_g} = 0.633 \\
\lambda'_y &:= \frac{\lambda_y}{\lambda_1} \cdot \sqrt{\beta_1} = 0.514 \\
\end{align*}
\]

for compression: buckling curve "b" (according to EN1993-1-3)

\[
\begin{align*}
\alpha &:= 0.34 \\
\phi_y &:= 0.5 \left[ 1 + \alpha \left( \lambda'_y - 0.2 \right) + \lambda'_y^2 \right] = 0.686 \\
\chi_y &:= \frac{1}{\phi_y^2 + \sqrt{\phi_y^2 - \lambda'_y^2}} = 1.083 \\
\end{align*}
\]

\[
\chi_y := 1
\]

1.c. \( \chi_z \) (EN15512 9.7.4)

\[
\begin{align*}
L_z &:= 1092 \text{ mm} \\
i_z &:= 29.3 \text{ mm} \\
\lambda_z &:= \frac{L_z}{i_z} = 37.27 \\
E &:= 210000 \frac{N}{\text{mm}^2} = 2.1 \cdot 10^5 \text{ MPa} \\
\lambda_1 &:= \pi \sqrt{\frac{E}{f_y}} = 74.844 \\
\beta_1 &:= \frac{A_{\text{eff.min}}}{A_g} = 0.633 \\
\lambda'_z &:= \frac{\lambda_z}{\lambda_1} \cdot \sqrt{\beta_1} = 0.396 \\
\end{align*}
\]

for compression: buckling curve "b" (according to EN1993-1-3)
\[ \alpha := 0.34 \]
\[ \phi_z := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda'_z - 0.2) + \lambda''_z \right] = [0.612] \]
\[ X_z := \frac{1}{\phi_z^2 + \sqrt{\phi_z^2 - \lambda_z^2}} = [1.19] \]
\[ X_z := 1 \]

The resulting value for \( X_{\min} \) is the minimum between \( X_{xy} \), \( X_y \), and \( X_z \)

\[ X_{\min} := \min (X_{xy}, X_y, X_z) = 0.84 \]

2. \( kLT \) (EN15512 form. 36)
\( \chi_{LT} \) (EN1993-1-3)

I do not have lateral-torsional buckling on the major axis

\[ k_{LT} := 0 \]
\[ \chi_{LT} := 1 \]

3. \( k_z \) (EN15512 9.7.6.3)

The bending moments used in the verification are from the critical cross-section and they are different from the bending moments used for determining \( k_z \). Although the maximum bending moments are on another position on the column, in this case, the part of the column with the maximum length between two bracings is more sensitive to buckling. However, either segment is studied, for this model \( k_z \) will always be greater than 1.5, thus \( k_z = 1.5 \).

\[ M_1 := 0.15 \text{ kN} \cdot \text{m} \]
\[ M_2 := 0.08 \text{ kN} \cdot \text{m} \]
\[ \psi := \frac{M_1}{M_2} = 1.875 \]
\[ \beta_{M_z} := 1.8 - 0.7 \psi = 0.488 \]
\[ \mu_z := \lambda'_z \cdot (2 \cdot \beta_{M_z} - 4) = -1.198 \]
\[ \mu_z \leq 0.9 = 1 \]
\[ k_z := 1 - \frac{\mu_z \cdot N_{sd}}{X_z \cdot A_{eff.min} \cdot f_y} = 1.965 \]
\[ k_z \leq 1.5 = 0 \]
\[ k_z := 1.5 \]

**VERIFICATION TAKING INTO ACCOUNT THE EFFECT OF DISTORTIONAL BUCKLING ACCORDING TO THE AMERICAN PROCEDURE**

\[ \chi_{min} := \min \left( \chi_y, \chi_z, \chi_{xy} \right) = 0.84 \]

\[ \frac{N_{Sd}}{X_{min} \cdot A_{eff.min} \cdot \frac{f_y}{Y_M}} + \frac{k_{LT} \cdot M_{y.Sd}}{X_{LT} \cdot W_{eff.y} \cdot \frac{f_y}{Y_M}} + \frac{k_z \cdot M_{z.Sd}}{W_{eff.z} \cdot \frac{f_y}{Y_M}} = 1.048 \]

**VERIFICATION WITHOUT TAKING INTO ACCOUNT THE EFFECT OF DISTORTIONAL BUCKLING ACCORDING TO THE AMERICAN PROCEDURE**

\[ \chi_{min} \] will be calculated for \[ A_{eff.min} := A_{eff.L} \] hence the result will be the same as for the EU method

\[ \chi_{min} := 0.741 \]

\[ \frac{N_{Sd}}{X_{min} \cdot A_{eff.min} \cdot \frac{f_y}{Y_M}} + \frac{k_{LT} \cdot M_{y.Sd}}{X_{LT} \cdot W_{eff.y} \cdot \frac{f_y}{Y_M}} + \frac{k_z \cdot M_{z.Sd}}{W_{eff.z} \cdot \frac{f_y}{Y_M}} = 0.963 \]
2. EU - Direct Analysis Method

The Direct Analysis Method (DAM) is an advanced three-dimensional analysis described in EN15512. It takes into account both overall rack and member imperfections, and if necessary, joint eccentricities. I made two analyses in which I take into account the imperfections calculated by Consteel, and next, the imperfections calculated by hand.

2.1 EU - DAM for Column 2.99x3x0.07 inch

2.1.1 Software calculated imperfections

In the first analysis I introduced the global imperfection calculated by Consteel. The imperfections are according to the first buckling mode. The amplitude of the imperfections was calculated according to the maximum displacement (in column B4 - 11.5 mm). I will also add the effect of the distortional buckling, reducing the effective area of the column with the distortional buckling reduction factor $X_{db}$.

Results from the second order analysis

\[ N_{Ed} = 85.2 \text{ kN} \]
\[ M_{y,Ed} = 0.3 \text{ kN} \cdot \text{m} \]
\[ M_{z,Ed} = 0.1 \text{ kN} \cdot \text{m} \]
\[ A_{eff} = 245.281 \text{ mm}^2 \]
\[ W_{eff,y} = 10499.4 \text{ mm}^3 \]
\[ W_{eff,z} = 8308.1 \text{ mm}^3 \]
\[ f_y = 430 \text{ MPa} \]
\[ \gamma_{M1} = 1 \]
\[ \frac{N_{Ed}}{A_{eff} \cdot f_y} + \frac{M_{y,Ed}}{W_{eff,y} \cdot f_y} + \frac{M_{z,Ed}}{W_{eff,z} \cdot f_y} = 0.902 \]

2.1.1.a DISTORTIONAL BUCKLING According to the European Standards

The effects of the distortional buckling will be taken into account, using the reduction factor \( \chi_{db} \) from EU-RAM (EN15512 part 9.7.6). This term will be used in order to calculate a new effective area.

\[ A_{eff} = 245.281 \text{ mm}^2 \]

\[ \chi_{db} := 0.944 \quad \text{distortional buckling reduction factor} \]

\[ A_{eff,db} := \chi_{db} \cdot A_{eff} = 231.545 \text{ mm}^2 \]

\[ \frac{N_{Ed}}{A_{eff,db} \cdot f_y} + \frac{M_{y,Ed}}{W_{eff,y} \cdot f_y} + \frac{M_{z,Ed}}{W_{eff,z} \cdot f_y} = 0.95 \]

2.1.1.b DISTORTIONAL BUCKLING According to the North-American Standards

The effects of the distortional buckling will be taken into account, using the effective cross-section area \( A_{eff,D} \) calculated according to AISI S100-07 Appendix 1.

\[ A_{eff,D} := 219.456 \text{ mm}^2 \quad \text{effective cross-section area calculated with the American procedure (1.2.1.3)} \]

\[ \frac{N_{Ed}}{A_{eff,D} \cdot f_y} + \frac{M_{y,Ed}}{W_{eff,y} \cdot f_y} + \frac{M_{z,Ed}}{W_{eff,z} \cdot f_y} = 0.997 \]
2.1.2 Polynomial imperfections

The calculation is carried out according to EN1993-1-1 5.3.2 (11). The amplitude of the imperfection is calculated using a six degree polynomial. The six degree equation is \( \eta_{cr} \), the shape of the elastic critical buckling mode. The second order derivative of this equation will lead to the calculation of the \( EI\eta''_{cr} \), the bending moment due to \( \eta_{cr} \) at the critical cross section.

For calculating the imperfections, the results from the first order analysis will be used.

\[
N_{Ed} := 85.4 \text{ kN}
\]
\[
M_{Ed} := 0.1 \text{ kN} \cdot \text{m}
\]
\[
W_{eff,z.min} := 8384.5 \text{ mm}^3
\]
\[
A_{eff} := 245.281 \text{ mm}^2
\]
\[
l_y := 454148 \text{ mm}^4
\]
\[
f_y := 430 \text{ MPa}
\]
\[
E := 210000 \frac{N}{\text{mm}^2}
\]
\[
\nu_{M1} := 1
\]
\[
W_y := 11959 \text{ mm}^3
\]
\[
M_{R,k} := W_y \cdot f_y = 5.142 \text{ kN} \cdot \text{m}
\]
\[
N_{R,k} := A_{eff} \cdot f_y = 105.471 \text{ kN}
\]
\[
\alpha_{ult,k} := \frac{1}{N_{Ed}} = 1.235
\]
\[
\alpha_{cr,op} := 2.42
\]
\[
\lambda := \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} = 0.714 \quad \text{(EN1993-1-1 form. 5.11)}
\]

for compression: buckling curve "b" \( \text{(EN1993-1-3)} \)

\[
\sigma := 0.34
\]
\[
\phi := 0.5 \cdot [1 + \sigma \cdot (\lambda - 0.2) + \lambda^2] = 0.843
\]
\[
\chi := \frac{1}{\phi^2 + \sqrt{\phi^2 - \lambda^2}} = 0.864
\]
\[ e_0 := \frac{\alpha (\lambda - 0.2)}{\lambda^2} \cdot \frac{M_{R,k}}{N_{R,k}} \cdot \frac{1 - \chi \cdot \lambda^2}{1 - \chi \cdot \lambda} = 16.708 \text{ mm} \] (EN1993-1-1 form. 5.10)

**Column B4**: On the X axis - length, on the Y axis - displacement.

\[ x := 917 \] position of the maximum displacement

\[ \eta_{cr} := 9.12 \cdot 10^{-15} \cdot x^5 - 6.84 \cdot 10^{-11} \cdot x^4 + 1.728 \cdot 10^{-7} \cdot x^3 - 0.0001606 \cdot x^2 + 0.0369 \cdot x - 0.8003 = -11.216 \]

\[ \eta_{cr} := -11.216 \text{ mm} \]

\[ \eta''_{cr} := 5.4 \cdot 9.12 \cdot 10^{-15} \cdot x^3 - 4 \cdot 3 \cdot 6.84 \cdot 10^{-11} \cdot x^2 + 3 \cdot 2 \cdot 1.728 \cdot 10^{-7} \cdot x - 2 \cdot 0.0001606 = 7.999 \cdot 10^{-5} \]

\[ \eta''_{cr} := 7.999 \cdot 10^{-5} \cdot \frac{1}{\text{mm}} \]

It is an out-of-plane buckling shape, so it’s buckling about the z-z axis (z-z is the symmetry axis)

\[ \eta_{init} := \frac{e_0}{\lambda} \cdot \frac{N_{R,k}}{E \cdot I_y} \cdot \eta''_{cr} \cdot \eta_{cr} = -3.627 \text{ mm} \]

I will introduce \( \eta_{init} \) in Consteel, as a global imperfection. In the following results, from the second order analysis will be used.

\[ N_{Ed} := 85.4 \text{ kN} \]

\[ M_{y,Ed} := 0.1 \text{ kN} \cdot \text{m} \]

\[ M_{z,Ed} := 0.2 \text{ kN} \cdot \text{m} \]

\[ A_{eff} = 245.281 \text{ mm}^2 \]

\[ W_{eff,y} := 10674.9 \text{ mm}^3 \]
\[ W_{\text{eff},z} := 8402.5 \text{ mm}^3 \]

VERIFICATION:

\[ \frac{N_{\text{Ed}}}{A_{\text{eff}} \cdot f_y} + \frac{M_{y,\text{Ed}}}{W_{\text{eff},y} \cdot f_y} + \frac{M_{z,\text{Ed}}}{W_{\text{eff},z} \cdot f_y} = 0.887 \]

2.1.2.a DISTORTIONAL BUCKLING According to the European Standards

The effects of the distortional buckling will be taken into account, using the reduction factor \( \chi_{db} \) from EU-RAM (EN15512 part 9.7.6). This term will be used in order to calculate a new effective area.

\[ A_{\text{eff}} = 245.281 \text{ mm}^2 \]

\[ \chi_{db} := 0.944 \quad \text{distortional buckling reduction factor} \]

\[ A_{\text{eff},db} := \chi_{db} \cdot A_{\text{eff}} = 231.545 \text{ mm}^2 \]

VERIFICATION:

\[ \frac{N_{\text{Ed}}}{A_{\text{eff},db} \cdot f_y} + \frac{M_{y,\text{Ed}}}{W_{\text{eff},y} \cdot f_y} + \frac{M_{z,\text{Ed}}}{W_{\text{eff},z} \cdot f_y} = 0.935 \]

2.1.2.b DISTORTIONAL BUCKLING According to the North-American Standards

The effects of the distortional buckling will be taken into account, using the effective cross-section area \( A_{\text{eff},DAISI} \) calculated according to AISI S100-07 Appendix 1.

\[ A_{\text{eff},DAISI} := 219.456 \text{ mm}^2 \quad \text{effective cross-section area calculated with the American procedure (1.2.1.3)} \]

VERIFICATION:

\[ \frac{N_{\text{Ed}}}{A_{\text{eff},DAISI} \cdot f_y} + \frac{M_{y,\text{Ed}}}{W_{\text{eff},y} \cdot f_y} + \frac{M_{z,\text{Ed}}}{W_{\text{eff},z} \cdot f_y} = 0.9821 \]
2.2 EU - DAM for Column 2.99x3x0.105inch

2.2.1 Software calculated imperfections

In the first analysis I introduced the global imperfection calculated by Consteel. The imperfections are according to the first buckling mode. The amplitude of the imperfections was calculated according to the maximum displacement (in column B4 - 9.6mm). I will also add the effect of the distortional buckling, reducing the effective area of the column with the distortional buckling reduction factor $X_{db}$.

Results from the second order analysis

\[ N_{Ed} = 128.8 \text{ kN} \]
\[ M_{y,Ed} = 0.7 \text{ kN} \cdot \text{m} \]
\[ M_{z,Ed} = 0.1 \text{ kN} \cdot \text{m} \]
\[ A_{\text{eff}} = 547.589 \text{ mm}^2 \]
\[ W_{\text{eff},y} = 18119.4 \text{ mm}^3 \]
\[ W_{\text{eff},z} = 13445.7 \text{ mm}^3 \]
\[ f_y = 370 \text{ MPa} \]
\[ V_{Ml} = 1 \]

VERIFICATION:

\[ \frac{N_{Ed}}{A_{\text{eff}} \cdot f_y} + \frac{M_{y,Ed}}{W_{\text{eff},y} \cdot f_y} + \frac{M_{z,Ed}}{W_{\text{eff},z} \cdot f_y} = 0.76 \]
2.2.1.a DISTORTIONAL BUCKLING According to the European Standards

The effects of the distortional buckling will be taken into account, using the reduction factor $X_{db}$ from EU-RAM (EN15512 part 9.7.6). This term will be used in order to calculate a new effective area.

$$A_{eff} = 547.589 \text{ mm}^2$$

$$X_{db} = 0.777 \quad \text{distortional buckling reduction factor}$$

$$A_{eff,db} := X_{db} \cdot A_{eff} = 425.477 \text{ mm}^2$$

VERIFICATION:

$$\frac{N_{Ed}}{A_{eff,db} \cdot f_y} + \frac{M_{yEd}}{W_{eff,y} \cdot f_y} + \frac{M_{zEd}}{W_{eff,z} \cdot f_y} = 0.943$$

2.2.1.b DISTORTIONAL BUCKLING According to the North-American Standards

The effects of the distortional buckling will be taken into account, using the effective cross-section area $A_{eff,D}.\text{AISI}$ calculated according to AISI S100-07 Appendix 1.

$$A_{eff,D} := 439.783 \text{ mm}^2 \quad \text{effective cross-section area calculated with the American procedure (1.2.1.3)}$$

VERIFICATION:

$$\frac{N_{Ed}}{A_{eff,D} \cdot f_y} + \frac{M_{yEd}}{W_{eff,y} \cdot f_y} + \frac{M_{zEd}}{W_{eff,z} \cdot f_y} = 0.916$$

2.2.2 Polynomial imperfections

The calculation is carried out according to EN1993-1-1 5.3.2 (11). The amplitude of the imperfection is calculated using a six degree polynomial. The six degree equation is $\eta_{cr}$, the shape of the elastic critical buckling mode. The second order derivative of this equation will lead to the calculation of the EI $\eta''_{cr}$, the bending moment due to $\eta_{cr}$ at the critical cross section.

For calculating the imperfections, the results from the first order analysis will be used.

$$N_{Ed} := 131 \text{ kN}$$

$$M_{Ed} := 0.3 \text{ kN} \cdot \text{m}$$

$$W_{eff,x.min} := 13445.7 \text{ mm}^3$$

$$W_y := 18107 \text{ mm}^3$$

$$A_{eff} := 547.589 \text{ mm}^2$$

$$f_y := 370 \text{ MPa}$$

$$E := 210000 \frac{N}{\text{mm}^2}$$
\[ \begin{align*}
I_z &= 687605 \text{ mm}^4 \\
M_{R,k} &= W_y \cdot f_y = 6.7 \text{ kN} \cdot \text{m} \\
N_{R,k} &= A_{eff} \cdot f_y = 202.608 \text{ kN} \\
\alpha_{ult,k} &= \frac{1}{N_{Ed}} = 1.547 \\
\alpha_{cr,op} &= 2.38 \\
\lambda &= \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} = 0.806 \quad \text{ (EN1993-1-1 form. 5.11)}
\end{align*} \]

for compression: buckling curve "b" \quad \text{(EN1993-1-3)}

\[ \begin{align*}
\alpha &= 0.34 \\
\phi &= 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda - 0.2) + \lambda^2 \right] = 0.928 \\
\chi &= \frac{1}{\phi^2 + \sqrt{\phi^2 - \lambda^2}} = 0.757 \\
e_0 &= \frac{\alpha \cdot (\lambda - 0.2) \cdot M_{R,k} \cdot \frac{1 - \chi \cdot \lambda^2}{N_{R,k} \cdot \gamma_{MI}}}{\lambda^2} = 10.486 \text{ mm} \quad \text{(EN1993-1-1 form. 5.10)}
\end{align*} \]

**Column B4**: On the X axis - length, on the Y axis - displacement.

\[ x := 917 \quad \text{position of the maximum displacement} \]
\[ \eta_{cr} := 7.38665 \cdot 10^{-15} \cdot x^5 - 5.5228 \cdot 10^{-11} \cdot x^4 + 1.385 \cdot 10^{-7} \cdot x^3 - 0.0001266 \cdot x^2 + 0.0273 \cdot x - 0.594 = -9.482 \]

\[ \eta_{cr,max} := 5.4 \cdot 7.38665 \cdot 10^{-15} \cdot x^3 - 4.3 \cdot 5.5228 \cdot 10^{-11} \cdot x^2 + 3.2 \cdot 1.385 \cdot 10^{-7} \cdot x - 0.0001266 = 1.921 \cdot 10^{-4} \]

\[ \eta_{cr} := -9.482 \cdot \text{mm} \quad \eta_{cr,max} := 1.921 \cdot 10^{-4} \cdot \frac{1}{\text{mm}} \]

\[ \eta_{\text{init}} := \frac{e_0 \cdot N_{R,k}}{\lambda \cdot E \cdot I_y \cdot \eta_{cr,max}} \cdot \eta_{cr} = -1.364 \cdot \text{mm} \]

I will introduce \( \eta_{\text{init}} \) in Consteel, as a global imperfection. I will use the following results, from the second order analysis.

\[ N_{Ed} := 131 \, \text{kN} \]
\[ M_{y,Ed} := 0.1 \, \text{kN} \cdot \text{m} \]
\[ M_{z,Ed} := 0.3 \, \text{kN} \cdot \text{m} \]
\[ A_{eff} = 547.589 \, \text{mm}^2 \]
\[ W_{eff,y} = 18119.4 \, \text{mm}^3 \]
\[ W_{eff,z} = 13445.7 \, \text{mm}^3 \]

**VERIFICATION:**

\[ \frac{N_{Ed}}{A_{eff} \cdot f_y} + \frac{M_{y,Ed}}{W_{eff,y} \cdot f_y} + \frac{M_{z,Ed}}{W_{eff,z} \cdot f_y} = 0.722 \]

**2.2.2.a DISTORTIONAL BUCKLING** According to the European Standards

The effects of the distortional buckling will be taken into account, using the reduction factor \( \chi_{db} \) from EU-RAM (EN15512 part 9.7.6). This term will be used in order to calculate a new effective area.

\[ A_{eff} = 547.589 \, \text{mm}^2 \]
\[ \chi_{db} := 0.777 \quad \text{distortional buckling reduction factor} \]
\[ A_{eff,db} := \chi_{db} \cdot A_{eff} = 425.477 \, \text{mm}^2 \]

**VERIFICATION:**

\[ \frac{N_{Ed}}{A_{eff,db} \cdot f_y} + \frac{M_{y,Ed}}{W_{eff,y} \cdot f_y} + \frac{M_{z,Ed}}{W_{eff,z} \cdot f_y} = 0.907 \]
2.2.2.b DISTORTIONAL BUCKLING According to the North-American Standards

The effects of the distortional buckling will be taken into account, using the effective cross-section area $A_{\text{eff},\text{D.AISI}}$ calculated according to AISI S100-07 Appendix 1.

$$A_{\text{eff},\text{D.AISI}} := 439.783 \text{ mm}^2$$

effective cross-section area calculated with the American procedure (1.2.1.3)

**VERIFICATION:**

$$\frac{N_{\text{Ed}}}{A_{\text{eff},\text{D.AISI}} \cdot f_y} + \frac{M_{x,\text{Ed}}}{W_{\text{eff},y} \cdot f_y} + \frac{M_{z,\text{Ed}}}{W_{\text{eff},z} \cdot f_y} = 0.8803$$
3. EU - GENERAL METHOD

The General Method applies part 6.3.4 of EN1993-1-1. Consteel applies the same method, but due to the fact that the effective area of the cross section is different, and the software applies EN1993-1-1 when calculating the buckling curves, I will perform by hand the same calculations as Consteel does. Two analyses have been realised, one with a model without imperfections, and one model with member imperfections according to the first in-plane buckling mode. For both analyses the effect of distortional buckling is added, reducing the effective cross section area with a distortional buckling reduction factor.

3.1 EU - GEM applied on Column 0.07i

3.1.1. Model without imperfections

Results from the second order analysis.

\[ N_{Ed} = 85.4 \text{ kN} \]
\[ M_{z,Ed} = 0.1 \text{ kN} \cdot \text{m} \]
\[ W_{eff,z.min} = 8384.5 \text{ mm}^3 \]
\[ f_y = 430 \text{ MPa} \]
\[ V_{M0} = 1 \]
\[ A_{eff} = 245.281 \text{ mm}^2 \]
\[ V_{M1} = 1 \]

GLOBAL STABILITY RESISTANCE [EN1993-1-1 6.3.4 (2)-(3),(4)b formula (6.63-6.64,6.66)]

\[
\frac{N_{Ed}}{X \cdot N_{Rk}} + \frac{M_{z,Ed}}{X_L \cdot M_{z,Rk}} \leq 1 \\
Y_{M1} \quad Y_{M1}
\]

\[ N_{Rk} := A_{eff} \cdot f_y = 105.471 \text{ kN} \]

\[ M_{z,Rk} := W_{eff,z.min} \cdot f_y = 3.605 \text{ kN} \cdot \text{m} \]

\[ \alpha_{ult.k} := \frac{1}{N_{Ed} + M_{z,Ed}} = 1.194 \]

\[ \alpha_{cr.op} := 2.42 \]

\[ \lambda_{op} := \sqrt{\frac{\alpha_{ult.k}}{\alpha_{cr.op}}} = 0.702 \quad \text{out of plane slenderness} \]
for compression: buckling curve "b" (according to EN1993-1-3)

\[
\alpha := 0.34 \\
\phi := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = [0.832] \\
\chi := \frac{1}{\phi^2 + \sqrt{\phi^2 - \lambda_{op}^2}} = [0.878]
\]

for lateral-torsional: buckling curve "b" (according to EN1993-1-3)

\[
\alpha := 0.34 \\
\phi_{LT} := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = [0.832] \\
\chi_{LT} := \frac{1}{\phi_{LT}^2 + \sqrt{\phi_{LT}^2 - \lambda_{op}^2}} = [0.878]
\]

VERIFICATION:

\[
\frac{N_{Ed}}{X \cdot N_{Rk}} + \frac{M_{z,Ed}}{\chi_{LT} \cdot M_{z,Rk}} = [0.953]
\]

3.1.1.a DISTORTIONAL BUCKLING According to the European Standards

I will take into account the effects of the distortional buckling, using the reduction factor \( \chi_{min} \) from EU-RAM (EN15512 part 9.7.6). I will apply this term in order to calculate a new effective area.

\[
A_{eff} = 245.281 \text{ mm}^2 \quad \text{effective cross-section area from the stub-column test} \\
X_{db} := 0.944 \quad \text{distortional buckling reduction factor} \\
A_{eff,db} := X_{db} \cdot A_{eff} = 231.545 \text{ mm}^2 \\
N_{Rk} := A_{eff,db} \cdot f_y = 99.564 \text{ kN} \\
\alpha_{ult,k} := \frac{1}{N_{Ed} + M_{z,Ed}} = 1.129 \\
\alpha_{cr,op} := 2.42 \\
\lambda_{op} := \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} = 0.683 \quad \text{out of plane slenderness}
\]
for compression: buckling curve "b" (according to EN1993-1-3)

\[ \alpha := 0.34 \]

\[ \phi := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = [0.815] \]

\[ \chi := \frac{1}{\phi^2 + \sqrt{\phi^2 - \lambda_{op}^2}} = [0.901] \]

for lateral-torsional: buckling curve "b" (according to EN1993-1-3)

\[ \alpha := 0.34 \]

\[ \phi_{LT} := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = [0.815] \]

\[ \chi_{LT} := \frac{1}{\phi_{LT}^2 + \sqrt{\phi_{LT}^2 - \lambda_{op}^2}} = [0.901] \]

**VERIFICATION:**

\[ \frac{N_{Ed}}{X \cdot N_{Rk}} + \frac{M_{z,Ed}}{\chi_{LT} \cdot M_{z,Rk}} = [0.983] \]

\[ \gamma M_1 \]

### 3.1.1.b DISTORTIONAL BUCKLING According to the North-American Standards

I will take into account the effects of the distortional buckling, using the effective cross-section area \( A_{eff,D} \) calculated according to AISI S100-07 Appendix 1.

\[ A_{eff,D} := 219.456 \text{ mm}^2 \]

**effective cross-section area calculated with the American procedure (1.2.1.3)**

\[ N_{Rk,D} := A_{eff,D} \cdot f_y = 94.366 \text{ kN} \]

\[ \alpha_{ult,k} := \frac{1}{N_{Ed} + \frac{M_{z,Ed}}{M_{z,Rk}}} = 1.072 \]

\[ \alpha_{cr,op} := 2.42 \]

\[ \lambda_{op} := \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} = 0.666 \] out of plane slenderness

for compression: buckling curve "b" (according to EN1993-1-3)

\[ \alpha := 0.34 \]

\[ \phi := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = [0.801] \]
\[ \chi \equiv \frac{1}{\phi^2 + \sqrt{\phi^2 - \lambda_{\text{op}}^2}} = 0.921 \]

for lateral-torsional: buckling curve "b"  (according to EN1993-1-3)

\[ \alpha = 0.34 \]
\[ \phi_{LT} := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda_{\text{op}} - 0.2) + \lambda_{\text{op}}^2 \right] = 0.801 \]
\[ \chi_{LT} \equiv \frac{1}{\phi_{LT}^2 + \sqrt{\phi_{LT}^2 - \lambda_{\text{op}}^2}} = 0.921 \]

**VERIFICATION:**

\[
\frac{N_{Ed}}{\chi \cdot N_{Rk,D}} + \frac{M_{z,Ed}}{\chi_{LT} \cdot M_{z,Rk}} \gamma_{M1} = 1.013
\]

### 3.1.2. Model with member imperfections

I have applied member imperfections according to the first in-plane buckling mode (buckling mode 8 in this case).

Results from the second order analysis.

\[ N_{Ed} := 85.4 \text{ kN} \]
\[ M_{z,Ed} := 0.4 \text{ kN} \cdot \text{m} \]
(without considering eccentricities)
\[ W_{\text{eff,z,min}} := 8384.5 \text{ mm}^3 \]
\[ f_y := 430 \text{ MPa} \]
\[ Y_{M0} := 1 \]
\[ A_{\text{eff}} := 245.281 \text{ mm}^2 \] effective area from the stub column test
\[ Y_{M1} := 1 \]

GLOBAL STABILITY RESISTANCE [EN1993-1-1 6.3.4 (2)-(3),(4)b formula (6.63-6.64,6.66)]

\[
\frac{N_{Ed}}{X \cdot N_{Rk}} + \frac{M_{z,Ed}}{X_{LT} \cdot M_{z,Rk}} \leq 1
\]
\[ Y_{M1} \quad Y_{M1} \]

\[ N_{Rk} := A_{\text{eff}} \cdot f_y = 105.471 \text{ kN} \]

\[ M_{z,Rk} := W_{\text{eff},z.min} \cdot f_y = 3.605 \text{ kN.m} \]

\[ \alpha_{\text{ult},k} := \frac{1}{N_{Ed} + \frac{M_{z,Ed}}{M_{z,Rk}}} = 1.086 \]

\[ \alpha_{\text{cr},op} := 2.42 \]

\[ \lambda_{op} := \sqrt{\frac{\alpha_{\text{ult},k}}{\alpha_{\text{cr},op}}} = 0.67 \]

for compression: buckling curve "b" (according to EN1993-1-3)

\[ \alpha := 0.34 \]

\[ \phi := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = [0.804] \]

\[ X := \frac{1}{\phi^2 + \sqrt{\phi^2 - \lambda_{op}^2}} = [0.916] \]

for lateral-torsional: buckling curve "b" (according to EN1993-1-3)

\[ \alpha := 0.34 \]

\[ \phi_{LT} := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = [0.804] \]

\[ X_{LT} := \frac{1}{\phi_{LT}^2 + \sqrt{\phi_{LT}^2 - \lambda_{op}^2}} = [0.916] \]

VERIFICATION:

\[
\frac{N_{Ed}}{X \cdot N_{Rk}} + \frac{M_{z,Ed}}{X_{LT} \cdot M_{z,Rk}} \leq 1.005
\]
\[ Y_{M1} \quad Y_{M1} \]
3.1.2 a DISTORTIONAL BUCKLING According to the European Standards

I will take into account the effects of the distortional buckling, using the reduction factor $\chi_{\text{min}}$ from EU- RAM (EN15512 part 9.7.6). I will apply this term in order to calculate a new effective area.

\[ A_{\text{eff}} = 245.281 \text{ mm}^2 \]  

Effective cross-section area from the stub-colum test

\[ \chi_{db} := 0.944 \]  

Distortional buckling reduction factor

\[ A_{\text{eff,db}} := \chi_{db} \cdot A_{\text{eff}} = 231.545 \text{ mm}^2 \]

\[ N_{Rk} := A_{\text{eff,db}} \cdot f_y = 99.564 \text{ kN} \]

\[ a_{\text{ult,k}} := \frac{1}{N_{Ed} + M_{z,Ed}} = 1.032 \]

\[ N_{Rk} \quad M_{z,Rk} \]

\[ a_{\text{cr,op}} := 2.42 \]

\[ \lambda_{op} := \sqrt{\frac{a_{\text{ult,k}}}{a_{\text{cr,op}}}} = 0.653 \]

For compression: buckling curve "b"

\[ \alpha := 0.34 \]

\[ \phi := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = [0.79] \]

\[ \chi := \frac{1}{\phi^2 + \sqrt{\phi^2 - \lambda_{op}^2}} = [0.935] \]

For lateral-torsional: buckling curve "b"

\[ \alpha := 0.34 \]

\[ \phi_{LT} := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = [0.79] \]

\[ \chi_{LT} := \frac{1}{\phi_{LT}^2 + \sqrt{\phi_{LT}^2 - \lambda_{op}^2}} = [0.935] \]

\[ \frac{N_{Ed}}{\chi \cdot N_{Rk}} + \frac{M_{z,Ed}}{\chi_{LT} \cdot M_{z,Rk}} = 1.036 \]
3.1.2.b DISTORTIONAL BUCKLING According to the North-American Standards

I will take into account the effects of the distortional buckling, using the effective cross-section area $A_{\text{eff},D}$ calculated according to AISI S100-07 Appendix 1.

$$A_{\text{eff},D} := 219.456 \text{ mm}^2$$

effective cross-section area calculated with the American procedure (1.2.1.3)

$$N_{Rk,D} := A_{\text{eff},D} \cdot f_y = 94.366 \text{ kN}$$

$$\alpha_{\text{ult},k} := \frac{1}{N_{Ed} + \frac{M_{z,Ed}}{M_{z,Rk}}} = 0.984$$

$$\alpha_{\text{cr},op} := 2.42$$

$$\lambda_{op} := \sqrt{\frac{\alpha_{\text{ult},k}}{\alpha_{\text{cr},op}}} = 0.638$$

for compression: buckling curve "b"

$$\alpha := 0.34$$

$$\phi := 0.5 \left[ 1 + \alpha \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = 0.778$$

$$\chi := \frac{1}{\phi^2 + \sqrt{\phi^2 - \lambda_{op}^2}} = 0.952$$

for lateral-torsional: buckling curve "b"

$$\alpha := 0.34$$

$$\phi_{LT} := 0.5 \left[ 1 + \alpha \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = 0.778$$

$$\chi_{LT} := \frac{1}{\phi_{LT}^2 + \sqrt{\phi_{LT}^2 - \lambda_{op}^2}} = 0.952$$

**VERIFICATION:**

$$\frac{N_{Ed}}{\chi \cdot N_{Rk,D}} \cdot \frac{M_{z,Ed}}{Y_{M1}} + \frac{M_{z,Ed}}{X_{LT} \cdot M_{z,Rk}} \cdot \frac{Y_{M1}}{\chi_{LT} \cdot M_{z,Rk}} = 1.067$$
3.2 EU - GEM applied on Column 0.105in

3.2.1. Model without imperfections

Results from the second order analysis.

\[ N_{Ed} = 131 \text{ kN} \]

\[ M_{z,Ed} = 0.3 \text{ kN} \cdot \text{m} \]

\[ W_{eff,z,min} = 13445.7 \text{ mm}^3 \]

\[ f_y = 370 \text{ MPa} \]

\[ Y_{M0} = 1 \]

\[ Y_{M1} = 1 \]

\[ A_{eff} = 547.589 \text{ mm}^2 \]

effective cross-section area from the stub column test

GLOBAL STABILITY RESISTANCE [EN1993-1-1 6.3.4 (2)-(3),(4)b formula (6.63-6.64,6.66)]

\[
\frac{N_{Ed}}{X \cdot N_{Rk}} + \frac{M_{z,Ed}}{X_{LT} \cdot M_{z,Rk}} \leq 1
\]

\[ Y_{M1} \]

\[ N_{Rk} = A_{eff} \cdot f_y = 202.608 \text{ kN} \]

\[ M_{z,Rk} = W_{eff,z,min} \cdot f_y = 4.975 \text{ kN} \cdot \text{m} \]

\[ \alpha_{ult,k} = \frac{1}{N_{Ed} \cdot M_{z,Ed} + N_{Rk} \cdot M_{z,Rk}} = 1.415 \]

\[ \alpha_{cr,op} = 2.38 \]

\[ \lambda_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} = 0.771 \]

cross section slenderness

for compression: buckling curve "b" (according to EN1993-1-3)

\[ \sigma = 0.34 \]

\[ \phi = 0.5 \cdot \left[ 1 + \sigma \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = 0.894 \]
\[ X := \frac{1}{\phi^2 + \sqrt{\phi^2 - \lambda_{op}^2}} = [0.798] \]

for lateral-torsional: buckling curve "b" (according to EN1993-1-3)

\[ \alpha := 0.34 \]

\[ \phi_{LT} := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = [0.894] \]

\[ X_{LT} := \frac{1}{\phi_{LT}^2 + \sqrt{\phi_{LT}^2 - \lambda_{op}^2}} = [0.798] \]

\[ Y_{M1} := 1 \]

VERIFICATION:

\[ \frac{N_{Ed}}{X \cdot N_{Rk}} + \frac{M_{z,Ed}}{X_{LT} \cdot M_{z,Rk}} = [0.886] \]

3.2.1.a DISTORTIONAL BUCKLING

I will take into account the effects of the distortional buckling, using the reduction factor \( \chi_{\text{min}} \) from EURAM (EN15512 part 9.7.6). I will apply this term in order to calculate a new effective area.

\[ A_{eff} = 547.589 \text{ mm}^2 \]

effective cross-section are from the stub column test

\[ \chi_{db} := 0.777 \]

distortional buckling reduction factor

\[ A_{eff,db} := \chi_{db} \cdot A_{eff} = 425.477 \text{ mm}^2 \]

\[ N_{Rk} := A_{eff,db} \cdot f_y = 157.426 \text{ kN} \]

VERIFICATION:

\[ \alpha_{ult,k} := \frac{1}{N_{Ed} \frac{M_{z,Ed}}{N_{Rk} M_{z,Rk}}} = 1.121 \]

\[ \alpha_{cr,op} := 2.38 \]

\[ \lambda_{op} := \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} = 0.686 \]

out of plane slenderness

for compression: buckling curve "b" (according to EN1993-1-3)
$$\alpha := 0.34$$

$$\phi := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = [0.818]$$

$$\chi := \frac{1}{\phi^2 + \sqrt{\phi^2 - \lambda_{op}^2}} = [0.897]$$

for lateral-torsional: buckling curve "b" (according to EN1993-1-3)

$$\alpha := 0.34$$

$$\phi_{LT} := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = [0.818]$$

$$\chi_{LT} := \frac{1}{\phi_{LT}^2 + \sqrt{\phi_{LT}^2 - \lambda_{op}^2}} = [0.897]$$

$$\frac{N_{Ed}}{X \cdot N_{Rk} \cdot Y_{M1}} + \frac{M_{z,Ed}}{X_{LT} \cdot M_{z,Rk} \cdot Y_{M1}} = [0.995]$$

3.2.1.b DISTORTIONAL BUCKLING According to the North-American Standards

I will take into account the effects of the distortional buckling, using the effective cross-section area $A_{eff,D}$ calculated according to AISI S100-07 Appendix 1.

$$A_{eff,D} := 439.783 \text{ mm}^2$$ effective cross-section area calculated with the American procedure (1.2.1.3)

VERIFICATION:

$$N_{Rk,D} := A_{eff,D} \cdot f_y = 162.72 \text{ kN}$$

$$\alpha_{ult,k} := \frac{1}{N_{Ed} + M_{z,Ed} / M_{z,Rk}} = 1.156$$

$$\sigma_{cr,op} := 2.38$$

$$\lambda_{op} := \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} = 0.697$$ out of plane slenderness

for compression: buckling curve "b" (according to EN1993-1-3)

$$\alpha := 0.34$$

$$\phi := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = [0.827]$$

$$\chi := \frac{1}{\phi^2 + \sqrt{\phi^2 - \lambda_{op}^2}} = [0.885]$$
for lateral-torsional: buckling curve "b"  (according to EN1993-1-3)

\[ \alpha = 0.34 \]

\[ \phi_{LT} := 0.5 \cdot \left[1 + \alpha \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2\right] = 0.827 \]

\[ \chi_{LT} := \frac{1}{\phi_{LT}^2 + \sqrt{\phi_{LT}^2 - \lambda_{op}^2}} = 0.885 \]

\[ \frac{N_{Ed}}{\chi \cdot N_{Rk,D}} + \frac{M_{z,Ed}}{\chi_{LT} \cdot M_{z,Rk}} = 0.978 \]

3.2.2. Model with member imperfections

I have applied member imperfections according to the first in-plane buckling mode (buckling mode 9 in this case).

Results from the second order analysis.

\[ N_{Ed} = 129.6 \text{ kN} \]

\[ M_{z,Ed} = 1.3 \text{ kN} \cdot m \]

\[ W_{eff,z,min} = 13445.7 \text{ mm}^3 \]

\[ f_y = 370 \text{ MPa} \]

\[ \gamma_{M0} = 1 \]
\( A_{\text{eff}} = 547.589 \text{ mm}^2 \) effective cross-section area from the stub column test

GLOBAL STABILITY RESISTANCE [EN1993-1-1 6.3.4 (2)-(3),(4)b formula (6.63-6.64,6.66)]

\[
\frac{N_{Ed}}{\chi \cdot N_{Rk}} + \frac{M_{z,Ed}}{\chi_{LT} \cdot M_{z,Rk}} \leq 1
\]

\[ Y_{M1} \]

\( N_{Rk} := A_{\text{eff}} \cdot f_y = 202.608 \text{ kN} \)

\( M_{z,Rk} := W_{\text{eff},\text{min}} \cdot f_y = 4.975 \text{ kN} \cdot \text{m} \)

\[ a_{\text{ult,k}} := \frac{1}{N_{Ed} + M_{z,Ed}} = 1.11 \]

\[ N_{Rk} + M_{z,Rk} \]

\[ a_{\text{cr,op}} := 2.37 \]

\[ \lambda_{op} := \sqrt{\frac{a_{\text{ult,k}}}{a_{\text{cr,op}}} = 0.684} \]

for compression: buckling curve "b" (according to EN1993-1-3)

\[ \alpha := 0.34 \]

\[ \phi := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = [0.816] \]

\[ \chi := \frac{1}{\phi^2 + \sqrt{\phi^2 - \lambda_{op}^2}} = [0.899] \]

for lateral-torsional: buckling curve "b" (according to EN1993-1-3)

\[ \alpha := 0.34 \]

\[ \phi_{LT} := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = [0.816] \]

\[ \chi_{LT} := \frac{1}{\phi_{LT}^2 + \sqrt{\phi_{LT}^2 - \lambda_{op}^2}} = [0.899] \]

VERIFICATION:

\[
\frac{N_{Ed}}{\chi \cdot N_{Rk}} + \frac{M_{z,Ed}}{\chi_{LT} \cdot M_{z,Rk}} \leq 1.002
\]

\[ Y_{M1} \]

\[ Y_{M1} \]
3.2.2 a DISTORTIONAL BUCKLING USING the EU - RAM method

I will take into account the effects of the distortional buckling, using the reduction factor \( \chi_{\min} \) from EU-RAM (EN15512 part 9.7.6). I will apply this term in order to calculate a new effective area.

\[
A_{\text{eff}} = 547.589 \ mm^2
\]

Effective cross-section area calculated with the stub column test

\[
\chi_{db} := 0.777
\]

Distortional buckling reduction factor

\[
A_{\text{eff,db}} := \chi_{db} \cdot A_{\text{eff}} = 425.477 \ mm^2
\]

\[
N_{Rk} := A_{\text{eff,db}} \cdot f_y = 157.426 \ kN
\]

\[
a_{\text{ult,k}} := \frac{1}{N_{Rk} / M_{z,Ed} + M_{z,Rk} / N_{Ed}} = 0.922
\]

\[
a_{\text{cr,op}} := 2.37
\]

\[
\lambda_{op} := \sqrt{\frac{a_{\text{ult,k}}}{a_{cr,op}}} = 0.624
\]

for compression: buckling curve "b" (according to EN1993-1-3)

\[
a := 0.34
\]

\[
\phi := 0.5 \cdot \left[ 1 + a \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = [0.767]
\]

\[
\chi := \frac{1}{\phi^2 + \sqrt{\phi^2 - \lambda_{op}^2}} = [0.968]
\]

for lateral-torsional: buckling curve "b" (according to EN1993-1-3)

\[
a := 0.34
\]

\[
\phi_{LT} := 0.5 \cdot \left[ 1 + a \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = [0.767]
\]

\[
\chi_{LT} := \frac{1}{\phi_{LT}^2 + \sqrt{\phi_{LT}^2 - \lambda_{op}^2}} = [0.968]
\]

\[
\frac{N_{Ed}}{\chi \cdot N_{Rk}} + \frac{M_{z,Ed}}{X_{LT} \cdot M_{z,Rk}} = [1.121]
\]

\[
\frac{N_{Ed}}{Y_{M1}} + \frac{M_{z,Ed}}{Y_{M1}} = [1.121]
\]
3.2.2. b DISTORTIONAL BUCKLING According to the North-American Standards

I will take into account the effects of the distortional buckling, using the effective cross-section area $A_{eff,D,AISI}$ calculated according to AISI S100-07 Appendix 1.

$$A_{eff,D,AISI} = 439.783 \text{ mm}^2$$

effective cross-section area calculated with the American procedure (1.2.1.3)

**VERIFICATION:**

$$N_{Rk} := A_{eff,D,AISI} \cdot f_y = 162.72 \text{ kN}$$

$$a_{ult,k} := \frac{1}{\frac{N_{Ed}}{N_{Rk}} + \frac{M_{z,Ed}}{M_{z,Rk}}} = 0.945$$

$$a_{cr,op} := 2.37$$

$$\lambda_{op} := \sqrt{\frac{a_{ult,k}}{a_{cr,op}}} = 0.632$$

for compression: buckling curve "b" (according to EN1993-1-3)

$$\alpha := 0.34$$

$$\phi := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = [0.773]$$

$$\chi := \frac{1}{\phi^2 + \sqrt{\phi^2 - \lambda_{op}^2}} = [0.959]$$

for lateral-torsional: buckling curve "b" (according to EN1993-1-3)

$$\alpha := 0.34$$

$$\phi_{LT} := 0.5 \cdot \left[ 1 + \alpha \cdot (\lambda_{op} - 0.2) + \lambda_{op}^2 \right] = [0.773]$$

$$\chi_{LT} := \frac{1}{\phi_{LT}^2 + \sqrt{\phi_{LT}^2 - \lambda_{op}^2}} = [0.959]$$

$$\frac{N_{Ed}}{X \cdot N_{Rk} + \frac{M_{z,Ed}}{Y_{M1}}} \cdot \frac{M_{z,Ed}}{X_{LT} \cdot M_{z,Rk} + \frac{Y_{M1}}{Y_{M1}}} = [1.103]$$
4. US - DIRECT ANALYSIS METHOD

\[
\frac{P}{P_a} + \frac{M_{x}}{M_{ax}} + \frac{M_{y}}{M_{ay}} \leq 1.0 \quad \text{(Eq. H1.2-1)}
\]

Magnitude of imperfections

It is calculated according to AISI S100-16 subchapter C1.1.1.2 and RMI subchapter 1.4.11.2. The out-of-straight ratio is L/240 ("maximum horizontal distance from the centerline at any point on the column to a plumb line from any other point on the column divided by the vertical distance between the two points").

Column 2.99x3x0.07inch

\[
L = 917 \, \text{mm} \quad \text{location of the maximum displacement on the column}
\]

\[
\delta = \frac{L}{240} = 3.821 \, \text{mm} \quad \text{magnitude of imperfections}
\]

Column 2.99x3x0.105inch

\[
L = 917 \, \text{mm} \quad \text{location of the maximum displacement on the column}
\]

\[
\delta = \frac{L}{240} = 3.821 \, \text{mm} \quad \text{magnitude of imperfections}
\]

The imperfection is introduced in Consteel according to the first buckling mode, using \( \delta \) as the maximum imperfection. In Consteel, a stiffness reduction will be applied, by reducing the elastic modulus (0.9*E). The resulting internal force are taken from the second order analysis in Consteel.

All the buckling lengths used in the CUTWP analysis are taken from a report from August 2015 (T. Pekoz to RMI). These buckling lengths will be used for determining both Pcre and Mcre.

- Lx RMI determination of the flexural buckling parameter with taking the effective length as 0.7 times the flexurally unsupported length of the column (60”). For the frame tested Lx RMI = .7 * 60
  - buckling length along y-y (1-1) \( KL := 0.7 \cdot 1524 \, \text{mm} = (1.067 \cdot 10^3) \, \text{mm} \)
  - buckling length along z-z (2-2) \( KL := 1 \cdot 1092.2 \, \text{mm} = (1.092 \cdot 10^3) \, \text{mm} \)
- Lt RMI determination of the torsional buckling parameter with taking the effective length as 0.8 times the torsionally unsupported length of the column (between the braces, 43”). For the frame tested Lt RMI = .8 * 43
  - tors. buckling length along z-z (3-3) \( KL := 0.8 \cdot 1092.2 \, \text{mm} = 873.76 \, \text{mm} \)
4.1 US - DAM Column 2.99x3x0.07inch

Consteel has implemented only the European standards, hence the axes are named after Eurocode (y and z). For this procedure the axes will be changed according to the American standard, so x-x will be the symmetry axis and y-y the one perpendicular to x-x.

\[ P := 85.6 \, kN \] required compressive axial strength

\[ M_x := 0.1 \, kN \cdot m \] required flexural strength along the x-x axis (y-y for EU)

\[ M_y := 0.2 \, kN \cdot m \] required flexural strength along the y-y axis (z-z for EU)

\[ P_a \] available axial strength determined in accordance with Chapter E

\[ M_{ax}, M_{ny} \] available flexural strengths determined as required in Section F

4.1.1 Global, local+global, distortional buckling

a. Members in compression (chapter E)

The available axial strength \( P_a \) (factored resistance) shall be the smallest of the available axial strength \( P_{ne} \) for global buckling, nominal axial strength \( P_{nl} \) for local buckling interacting with global buckling, and the nominal axial strength \( P_{nd} \) for distortional buckling.

a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]

\[ A_g := 445 \, mm^2 \] gross cross-section area from Consteel

\[ F_y := 430 \, MPa \] yield stress

\[ P_{cre} := 148419 \, N \] minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP, for reduced thickness, using the non-reduced elastic modulus

\[ F_{cre} := \frac{P_{cre}}{A_g} = 333.526 \, MPa \] least of the applicable elastic global(flexural, torsional and flexural torsional) buckling stress

\[ \lambda_c := \sqrt{\frac{F_y}{F_{cre}}} = 1.135 \] slenderness \hspace{1cm} (Eq. E2-4)

\[ \lambda_c \leq 1.5 = 1 \quad F_{n} := \left(0.658 \lambda^3\right) \cdot F_y = 250.677 \, MPa \] \hspace{1cm} (Eq. E2-2)
\[ F_n \quad \text{Compressive stress} \]
\[ P_{ne} := A_g \cdot F_n = 111.551 \text{ kN} \quad \text{nominal axial strength} \quad \text{(Eq. E2-1)} \]

**a.2 Local Buckling Interacting with Yielding and Global Buckling [E3.2]**

\[ P_{nl} \quad \text{nominal axial strength} \]
\[ P_{ne} = 111.551 \text{ kN} \]
\[ F_n := \frac{P_{ne}}{A_g} = 250.677 \text{ MPa} \]
\[ P_{ua} = 23.75 \text{ kip} \left(1.056 \cdot 10^5\right) \text{ N} \quad \text{experimental ultimate compressive strength of the stub column test} \]
\[ A_{net.min} := 355.483 \text{ mm}^2 \quad \text{cross-section area from the experiment (stub column test)} \]
\[ Q := \frac{P_{ua}}{A_{net.min} \cdot F_y} = 0.691 \]

Using Q RMI (see report from T. Pekoz, August 2015)
\[ P_{nl} := F_n \cdot \left[ 1 - (1 - Q) \cdot \left( \frac{F_n}{F_y} \right)^Q \right] \cdot A_g = [87.823] \text{ kN} \]
\[ P_{y.net} := A_{net.min} \cdot F_y = 152.858 \text{ kN} \]
\[ P_{nl} \leq P_{y.net} = [1] \]

**a.3 Distortional Buckling [E4.2]**

\[ P_{nd} \quad \text{nominal axial strength for distortional buckling} \]
\[ F_y = 430 \text{ MPa} \]
\[ A_{net} := 355.483 \text{ mm}^2 \]
\[ A_g := 445 \text{ mm}^2 \]
\[ P_y := A_g \cdot F_y = 191.35 \text{ kN} \quad \text{(Eq. E4.2-7)} \]
\[ P_{y.net} := A_{net} \cdot F_y = 152.858 \text{ kN} \quad \text{(Eq. E4.2-8)} \]

Using \( P_{crd} \) calculated with CUFSM
\[ Load := 1000 \text{ N} \]
\[ load.factor := 99.447 \]
\[ P_{crd} := \text{Load} \cdot \text{load factor} = 99.447 \, kN \] 

Critical elastic distortional column buckling load determined by analysis in CUFSM, for reduced thickness, with non-reduced elastic modulus.

\[ \lambda_d := \sqrt{\frac{P_y}{P_{crd}}} = 1.387 \quad \text{slenderness (Eq. E4.2-3)} \]

\[ \lambda_{d2} := 0.561 \left( 14 \cdot \left( \frac{P_y}{P_{net}} \right)^{0.4} - 13 \right) = 1.299 \quad \text{(Eq. E4.2-5)} \]

if \( \lambda_d \leq \lambda_{d2} = 0 \) will use subchapter E4.1 (members without holes)

\[ P_{nd} := \left( 1 - 0.25 \cdot \left( \frac{P_{crd}}{P_y} \right)^{0.6} \right) \cdot \left( \frac{P_{crd}}{P_y} \right)^{0.6} \cdot P_y = 107.396 \, kN \quad \text{(Eq. E4.1-2)} \]

\[ P_a := \min (P_{ne}, P_{nl}, P_{nd}) = 87.823 \, kN \]

b. Members in flexure (chapter F)

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and global buckling \( M_{ne} \), the available flexural strength due to the interaction of the yielding or global buckling with local buckling \( M_{nl} \) and the nominal flexural strength for distortional buckling \( M_{nd} \).

b.1. Flexure about x-x (1-1) axis

2.1.1 Yielding and Global Buckling [F2]

\[ M_{crex} := 2.7913 \cdot 10^7 \, N \cdot mm \quad \text{critical elastic bending moment from CUTWP, reduced thickness, non-reduced elastic modulus} \]

\[ S_{fx} := 11959 \, mm^3 \quad \text{gross cross section modulus} \]

\[ F_{cre} := \frac{M_{crex}}{S_{fx}} = (2.334 \cdot 10^3) \, MPa \quad \text{critical elastic lateral-torsional buckling stress} \]

\[ F_{cre} \geq 2.78 \cdot F_y = 1 \quad \text{(Eq. F2.1-1)} \]

\[ F_n := F_y = 430 \, MPa \]

\[ M_{nex} := S_{fx} \cdot F_n = 5.142 \, kN \cdot m \quad \text{nominal axial strength for yielding and global buckling about the x-x (1-1) axis (Eq. F2.1-1)} \]

b.1.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

As the Q cannot be determined, subchapter F3.2.2 of AISI S-100 will be used.
\[ M_{nx} = 5.142 \, kN \cdot m \]

\[ M_{crlx} := 3980.1442 \cdot 1000 \, N \cdot mm = 3.98 \, kN \cdot m \]

Critical elastic local buckling moment from CUFSM, **reduced thickness** (the same as for the global buckling), **non-reduced elastic modulus**

\[ \lambda_l := \sqrt{\frac{M_{nx}}{M_{crlx}}} = 1.137 \quad \text{(Eq. F3.2.1-3)} \]

\[ \lambda_l > 0.776 = 1 \]

\[ M_{nlx} := \left( 1 - 0.15 \cdot \left( \frac{M_{crlx}}{M_{nx}} \right)^{0.4} \right) \cdot \left( \frac{M_{crlx}}{M_{nx}} \right)^{0.4} \cdot M_{nx} = 4.013 \, kN \cdot m \quad \text{(Eq. F3.2.1-2)} \]

\[ I_{xx} := 373134.5763 \, mm^4 \]

Moment of inertia about the x-x axis for the minimum net cross section

\[ y_g := 37.1196 \, mm \]

\[ S_{fnetx} := \frac{I_{xx}}{y_g} = (1.005 \cdot 10^4) \, mm^3 \]

Minimum net cross section modulus

\[ M_{ynetx} := S_{fnetx} \cdot F_y = 4.322 \, kN \cdot m \]

Yield moment of net cross-section

\[ M_{nlx} \leq M_{ynetx} = 1 \]

**b.1.3 Distortional Buckling [F4.2]**

\[ S_{fx} := 11959 \, mm^3 \]

Gross cross section modulus

\[ F_y = 430 \, MPa \]

Yield stress

\[ M_{yx} := S_{fx} \cdot F_y = 5.142 \, kN \cdot m \]

Yield moment

\[ M_{crdx} := 4031.4623 \cdot 1000 \, N \cdot mm = 4.031 \, kN \cdot m \]

Critical elastic distortional buckling moment about x-x axis from CUFSM, **reduced thickness**, **reduced elastic modulus**

\[ \lambda_d := \sqrt{\frac{M_{yx}}{M_{crdx}}} = 1.129 \]

Slenderness

\[ \lambda_{d2} := 0.673 \cdot \left( \frac{M_{yx}}{M_{ynetx}} \right)^{2.7} - 0.7 \]

\[ = 1.358 \quad \text{(Eq. F4.2-5)} \]

\[ \lambda_d \leq \lambda_{d2} = 1 \quad \text{will use subchapter E4.2 (members with holes)} \]

\[ \lambda_{d1} := 0.673 \cdot \left( \frac{M_{ynetx}}{M_{yx}} \right)^{3} = 0.4 \quad \text{(Eq. F4.2-4)} \]
\[\lambda_{d1} < \lambda_d \leq \lambda_{d2} = 1\]

\[M_{d2} := \left(1 - 0.22 \cdot \left(\frac{1}{\lambda_{d2}}\right)\right) \cdot \left(\frac{1}{\lambda_{d2}}\right) \cdot M_{yx} = 3.174 \text{ kN} \cdot \text{m}\]

\[M_{ndx} := M_{netz} - \left(\frac{M_{netz} - M_{d2}}{\lambda_{d2} - \lambda_{d1}}\right) \cdot (\lambda_d - \lambda_{d1}) = 3.448 \text{ kN} \cdot \text{m} \quad \text{(Eq. F4.2-2)}\]

\[M_{ndx} \quad \text{nominal flexural strength about the x-x (1-1) axis}\]

\[
\left(1 - 0.22 \cdot \left(\frac{M_{crdx}}{M_{yx}}\right)^{0.5}\right) \cdot \left(\frac{M_{crdx}}{M_{yx}}\right)^{0.5} \cdot M_{yx} = 3.666 \text{ kN} \cdot \text{m}\]

\[M_{ndx} \leq \left(1 - 0.22 \cdot \left(\frac{M_{crdx}}{M_{yx}}\right)^{0.5}\right) \cdot \left(\frac{M_{crdx}}{M_{yx}}\right)^{0.5} \cdot M_{yx} = 1\]

\[M_{ax} := \min (M_{nex}, M_{ndx}, M_{ndx}) = 3.448 \text{ kN} \cdot \text{m}\]

b.2. Flexure about y-y (2-2) axis

b.2.1 Yielding and Global Buckling [F2]

\[M_{crey} := 8.46718 \times 10^6 \text{ N} \cdot \text{mm} \quad \text{critical elastic bending moment from CUTWP, reduced thickness, non-reduced elastic modulus}\]

\[S_{fy} := 8650 \text{ mm}^3 \quad \text{gross cross section modulus}\]

\[F_{cre} := \frac{M_{crey}}{S_{fy}} = 978.865 \text{ MPa} \quad \text{critical elastic lateral-torsional buckling stress}\]

\[F_{cre} \geq 2.78 \cdot F_y = 0\]

\[2.78 \cdot F_y > F_{cre} > 0.56 \quad F_y = 1\]

\[F_n := \frac{10}{9} \cdot F_y \cdot \left(1 - \frac{10 \cdot F_y}{36 \cdot F_{cre}}\right) = 419.478 \text{ MPa}\]

\[M_{ney} := S_{fy} \cdot F_n = 3.628 \text{ kN} \cdot \text{m} \quad \text{nominal axial strength for yielding and global buckling (Eq. F2.1-1)}\]

about the y-y (2-2) axis

b.2.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

\[M_{ney} = 3.628 \text{ kN} \cdot \text{m}\]

\[M_{crly} := 1568.7918 \times 1000 \text{ N} \cdot \text{mm} = 1.569 \text{ kN} \cdot \text{m} \quad \text{critical elastic local buckling moment from CUFSM, reduced thickness, reduced elastic modulus}\]
\[ \lambda_l := \sqrt{\frac{M_{nly}}{M_{crly}}} = 1.521 \quad \text{(Eq. F3.2.1-3)} \]

\[ \lambda_l > 0.776 = 1 \quad \text{(Eq. F3.2.1-2)} \]

\[ M_{nly} := \left( 1 - 0.15 \cdot \left( \frac{M_{crly}}{M_{nly}} \right)^{0.4} \right) \cdot \left( \frac{M_{crly}}{M_{nly}} \right)^{0.4} \cdot M_{nly} = 2.316 \, kN \cdot m \]

\[ I_{yy} := 308988.0814 \, mm^4 \quad \text{moment of inertia about the y-y axis} \]

\[ x_g := 35.5852 \, mm \]

\[ S_{fnety} := \frac{I_{yy}}{x_g} = 8.683 \times 10^3 \, mm^3 \quad \text{net section modulus} \]

\[ M_{ynety} := S_{fnety} \cdot F_y = 3.734 \, kN \cdot m \quad \text{yield moment of net cross-section} \quad \text{(Eq. F3.2.2-2)} \]

\[ M_{nly} \leq M_{ynety} = 1 \]

b.2.3 Distortional Buckling [F4.2]

\[ S_{fy} = (8.65 \times 10^3) \, mm^3 \quad \text{gross cross section modulus} \]

\[ F_y = 430 \, MPa \quad \text{yield stress} \]

\[ M_{yy} := S_{fy} \cdot F_y = 3.72 \, kN \cdot m \quad \text{yield moment} \quad \text{(Eq. F4.1-4)} \]

\[ M_{crdy} := 2838.471 \times 1000 \, N \cdot mm = 2.838 \, kN \cdot m \quad \text{critical elastic distortional buckling moment about y-y axis from CUFSM, reduced thickness, non-reduced elastic modulus} \]

\[ \lambda_d := \sqrt{\frac{M_{yy}}{M_{crdy}}} = 1.129 \quad \text{slenderness} \quad \text{(Eq. F4.1-3)} \]

\[ M_{ynety} = 3.734 \, kN \cdot m \]

\[ \lambda_{d2} := 0.673 \cdot \left( 1.7 \cdot \left( \frac{M_{yy}}{M_{ynety}} \right)^{2.7} - 0.7 \right) = 0.661 \quad \text{(Eq. F4.2-5)} \]

\[ \lambda_d \leq \lambda_{d2} = 0 \quad \text{will use subchapter F4.1 (members without holes)} \]

\[ M_{ndy} := \left( 1 - 0.22 \cdot \left( \frac{M_{crdy}}{M_{yy}} \right)^{0.5} \right) \cdot \left( \frac{M_{crdy}}{M_{yy}} \right)^{0.5} \cdot M_{yy} = 2.625 \, kN \cdot m \quad \text{(Eq. F4.1-2)} \]

\[ M_{ndy} \quad \text{nominal flexural strength about the y-y (2-2) axis} \]

\[ M_{ay} := \min (M_{nely}, M_{nly}, M_{ndy}) = 2.316 \, kN \cdot m \]
Verification:

\[ P = 0.975 \]
\[ P_a = 0.975 \]
\[ M_x = 0.029 \]
\[ M_{ax} = 0.029 \]
\[ M_y = 0.086 \]
\[ M_{ay} = 0.086 \]
\[ \frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} = 1.09 \]

4.1.2 Global, local+global, distortional+global buckling

a. Members in compression (chapter E)

The available axial strength \( P_a \) (factored resistance) shall be the smallest of the available axial strength \( F_n \) for global buckling, nominal axial strength \( P_{nl} \) for local buckling interacting with global buckling, and the nominal axial strength \( P_{nd} \) for distortional buckling interacting with global buckling.

a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]

\[ A_g = 445 \, \text{mm}^2 \] gross cross-section area from Consteel
\[ F_y = 430 \, \text{MPa} \] yield stress
\[ P_{cre} = 148419 \, \text{N} \] minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP, for reduced thickness, using the non-reduced elastic modulus
\[ F_{cre} = \frac{P_{cre}}{A_g} = 333.526 \, \text{MPa} \] least of the applicable elastic global(flexural, torsional and flexural torsional) buckling stress
\[ \lambda_c = \sqrt{\frac{F_y}{F_{cre}}} = 1.135 \] slenderness \hspace{1cm} (Eq. E2-4)
\[ \lambda_c \leq 1.5 = 1 \] \[ F_n = (0.658 \lambda^3) \cdot F_y = 250.677 \, \text{MPa} \] \hspace{1cm} (Eq. E2-2)
\[ F_n \] Compressive stress
\[ P_{nc} = A_g \cdot F_n = 111.551 \, \text{kN} \] nominal axial strength \hspace{1cm} (Eq. E2-1)
a.2 Local Buckling Interacting with Yielding and Global Buckling [E3.2]

\[ P_{nl} \] nominal axial strength

\[ P_{ne} = 111.551 \text{ kN} \]

\[ F_n := \frac{P_{ne}}{A_g} = 250.677 \text{ MPa} \]

\[ P_{ua} := 23.75 \text{ kip} = (1.056 \times 10^5) \text{ N} \] experimental ultimate compressive strength of the stub column test

\[ A_{net,min} := 355.483 \text{ mm}^2 \] cross-section area from the experiment (stub column test)

\[ Q := \frac{P_{ua}}{A_{net,min} \cdot F_y} = 0.691 \]

Using Q RMI (see report from T. Pekoz, August 2015)

\[ P_{nl} := F_n \left[ 1 - (1 - Q) \left( \frac{F_n}{F_y} \right)^Q \right] \cdot A_g = [87.823] \text{ kN} \]

\[ P_{y.net} := A_{net.min} \cdot F_y = 152.858 \text{ kN} \]

\[ P_{nd} \leq P_{y.net} = [1] \]

a.3 Distortional Buckling interacting with Global Buckling [E4.2 and E2]

For simulating the interaction of distortional and global buckling, the nominal axial strength for global buckling will be introduced in the calculation formulas of the distortional buckling, namely, instead of using \( P_y \) and \( P_{y.net} \) calculated with the yield stress \( F_y \), they will be calculated with \( F_n \), the compressive stress for global buckling.

\[ P_{nd} \] nominal axial strength for distortional buckling

\[ F_y = 430 \text{ MPa} \]

\[ A_{net} := 355.483 \text{ mm}^2 \]

\[ A_g := 445 \text{ mm}^2 \]

\[ P_{ne} := A_g \cdot F_n = 111.551 \text{ kN} \] \hspace{1cm} (Eq. E4.2-7)

\[ P_{ne.net} := A_{net} \cdot F_n = 89.112 \text{ kN} \] \hspace{1cm} (Eq. E4.2-8)

Using \( P_{crd} \) calculated with CUFSM
\( Load := 1000 \, N \)

\( load.factor := 99.447 \)

\( P_{crd} := Load \cdot load.factor = 99.447 \, kN \) critical elastic distortalional column buckling load determined by analysis in CUFSM, for reduced thickness, with non-reduced elastic modulus

\( \lambda_d := \sqrt{\frac{P_{nc}}{P_{crd}}} = 1.059 \) slenderness (Eq. E4.2-3)

b. Members in flexure (chapter F)

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and global buckling, the available flexural strength due to the interaction of the yielding or global buckling with local buckling, and the nominal flexural strength for distortional buckling interacting with global buckling.

b.1. Flexure about \( x-x \) (1-1) axis

b.1.1 Yielding and Global Buckling [F2]

\( M_{crex} := \left(2.7913 \cdot 10^{-7}\right) N \cdot mm \) critical elastic bending moment from CUTWP, reduced thickness, non-reduced elastic modulus

\( S_{fx} := 11959 \, mm^3 \) gross cross section modulus

\( F_{cre} := \frac{M_{crex}}{S_{fx}} = \left(2.334 \cdot 10^3\right) \, MPa \) critical elastic lateral-torsional buckling stress

\( F_{cre} \geq 2.78 \cdot F_y = 1 \)

\( F_n := F_y = 430 \, MPa \)

\( M_{nex} := S_{fx} \cdot F_n = 5.142 \, kN \cdot m \) nominal axial strength for yielding and global buckling about the \( x-x \) (1-1) axis (Eq. F2.1-1)

b.1.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

As the \( Q \) cannot be determined, subchapter F3.2.2 of AISI S-100 will be used.
\( M_{nex} = 5.142 \, kN \cdot m \)

\( M_{crlx} := 3980.1442 \cdot 1000 \, N \cdot mm = 3.98 \, kN \cdot m \) \text{ critical elastic local buckling moment from CUFSM, reduced thickness (the same as for the global buckling), non-reduced elastic modulus}

\( \lambda_l := \sqrt{\frac{M_{nex}}{M_{crlx}}} = 1.137 \)

\( \lambda_l > 0.776 = 1 \)

\( M_{nlx} := \left( 1 - 0.15 \cdot \left( \frac{M_{crlx}}{M_{nex}} \right)^{0.4} \right) \cdot \left( \frac{M_{crlx}}{M_{nex}} \right)^{0.4} \cdot M_{nex} = 4.013 \, kN \cdot m \)

\( I_{xx} := 373134.5763 \, mm^4 \) \text{ moment of inertia about the x-x axis}

\( y_g := 37.1196 \, mm \)

\( S_{fnetx} := \frac{I_{xx}}{y_g} = (1.005 \cdot 10^4) \, mm^3 \) \text{ net section modulus}

\( M_{ynetx} := S_{fnetx} \cdot F_y = 4.322 \, kN \cdot m \) \text{ yield moment of net cross-section (Eq. F3.2.2-2)}

\( M_{nlx} \leq M_{ynetx} = 1 \)

b.1.3 Distortional Buckling interacting with Global Buckling [F4.2 and F2]

\( S_{fx} := 11959 \, mm^3 \) \text{ gross cross section modulus}

\( F_n = 430 \, MPa \) \text{ yield stress}

\( M_{nex} := S_{fx} \cdot F_n = 5.142 \, kN \cdot m \) \text{ yield moment (Eq. F4.1-4)}

\( M_{crdx} := 4031.4623 \cdot 1000 \, N \cdot mm = 4.031 \, kN \cdot m \) \text{ critical elastic distortional buckling moment about x-x axis from CUFSM, reduced thickness, reduced elastic modulus}

\( \lambda_d := \sqrt{\frac{M_{nex}}{M_{crdx}}} = 1.129 \) \text{ slenderness (Eq. F4.1-3)}

\( M_{ne.netx} := S_{fnetx} \cdot F_n = 4.322 \, kN \cdot m \)

\( \lambda_{d2} := 0.673 \cdot \left( 1.7 \cdot \left( \frac{M_{nex}}{M_{ne.netx}} \right)^{2.7} - 0.7 \right) = 1.358 \) \text{ (Eq. F4.2-5)}

\( \lambda_d \leq \lambda_{d2} = 1 \) \text{ will use subchapter F4.2}
\[
\lambda_{d1} := 0.673 \cdot \left( \frac{M_{ne,netz}}{M_{nex}} \right)^3 = 0.4 \quad \text{(Eq. F4.2-4)}
\]
\[
\lambda_{d1} \leq \lambda_d = 1
\]
\[
M_{d2} := \left( 1 - 0.22 \cdot \left( \frac{1}{\lambda_{d2}} \right) \right) \cdot \left( \frac{1}{\lambda_{d2}} \right) \cdot M_{nex} = 3.174 \, kN \cdot m
\]
\[
M_{ndx} := M_{ne,netz} - \left( \frac{M_{ne,netz} - M_{d2}}{\lambda_{d2} - \lambda_{d1}} \right) \cdot (\lambda_d - \lambda_{d1}) = 3.448 \, kN \cdot m \quad \text{(Eq. F4.2-2)}
\]

\[M_{ndx}\] nominal flexural strength about the x-x (1-1) axis

\[
\left( 1 - 0.22 \cdot \left( \frac{M_{erdx}}{M_{nex}} \right) ^{0.5} \right) \cdot \left( \frac{M_{erdx}}{M_{nex}} \right) ^{0.5} \cdot M_{nex} = 3.666 \, kN \cdot m
\]
\[
M_{ndx} \leq \left( 1 - 0.22 \cdot \left( \frac{M_{erdx}}{M_{nex}} \right) ^{0.5} \right) \cdot \left( \frac{M_{erdx}}{M_{nex}} \right) ^{0.5} \cdot M_{nex} = 1
\]
\[
M_{ax} := \min (M_{nex}, M_{nlx}, M_{ndx}) = 3.448 \, kN \cdot m
\]

b.2. Flexure about y-y (2-2) axis

b.2.1 Yielding and Global Buckling [F2]

\[
M_{crey} := (8.46718 \cdot 10^6) \, N \cdot mm \quad \text{critical elastic bending moment from CUTWP, reduced thickness, non-reduced elastic modulus}
\]
\[
S_{fy} := 8650 \, mm^3 \quad \text{gross cross section modulus}
\]
\[
F_{cre} := \frac{M_{crey}}{S_{fy}} = 978.865 \, MPa \quad \text{critical elastic lateral-torsional buckling stress}
\]
\[
F_{cre} \geq 2.78 \cdot F_y = 0
\]
\[
2.78 \cdot F_y > F_{cre} > 0.56 \ F_y = 1
\]
\[
F_a := \frac{10}{9} \cdot F_y \cdot \left( 1 - \frac{10 \cdot F_y}{36 \cdot F_{cre}} \right) = 419.478 \, MPa \quad \text{(Eq. F2.1-2)}
\]
\[
M_{ney} := S_{fy} \cdot F_a = 3.628 \, kN \cdot m \quad \text{nominal axial strength for yielding and global buckling about the y-y (2-2) axis (Eq. F2.1-1)}
\]
b.2.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

\[ M_{ne} = 3.628 \, kN \cdot m \]

\[ M_{crly} := 1568.7918 \cdot 1000 \, N \cdot mm = 1.569 \, kN \cdot m \]

critical elastic local buckling moment from CUFSM, reduced thickness, non-reduced elastic modulus

\[ \lambda_i := \sqrt{\frac{M_{ne}}{M_{crly}}} = 1.521 \]

\[ \lambda_i > 0.776 = 1 \]

\[ M_{nly} := \left(1 - 0.15 \cdot \left(\frac{M_{crly}}{M_{ne}}\right)^{0.4}\right) \cdot \left(\frac{M_{crly}}{M_{ne}}\right)^{0.4} \cdot M_{ne} = 2.316 \, kN \cdot m \]

\[ I_{yy} := 308988.0814 \, mm^4 \]

moment of inertia about the y-y axis

\[ x_g := 35.5852 \, mm \]

\[ S_{fnety} := \frac{I_{yy}}{x_g} = (8.683 \cdot 10^3) \, mm^3 \]

net section modulus

\[ M_{ynety} := S_{fnety} \cdot F_y = 3.734 \, kN \cdot m \]

yield moment of net cross-section (Eq. F3.2.2-2)

\[ M_{nly} \leq M_{ynety} = 1 \]

b.2.3 Distortional Buckling + Global Buckling [F4.2]

\[ S_{fy} = (8.65 \cdot 10^3) \, mm^3 \]

gross cross section modulus

\[ F_n = 419.478 \, MPa \]

yield stress

\[ M_{ne} := S_{fy} \cdot F_n = 3.628 \, kN \cdot m \]

yield moment (Eq. F4.1-4)

\[ M_{crdy} := 2838.471 \cdot 1000 \, N \cdot mm = 2.838 \, kN \cdot m \]

critical elastic distortional buckling moment about y-y axis from CUFSM, reduced thickness, non-reduced elastic modulus

\[ \lambda_d := \sqrt{\frac{M_{ne}}{M_{crdy}}} = 1.131 \]

slenderness (Eq. F4.1-3)

\[ M_{ne,nety} := S_{fnety} \cdot F_n = 3.642 \, kN \cdot m \]
\[ \lambda_{d2} := 0.673 \cdot \left( 1.7 \cdot \left( \frac{M_{ney}}{M_{ney,ney}} \right)^2 - 0.7 \right) = 0.661 \quad \text{(Eq. F4.2-5)} \]

\[ \lambda_d \leq \lambda_{d2} = 0 \quad \text{will use subchapter F4.1 (members without holes)} \]

\[ M_{ndy} := \left( 1 - 0.22 \cdot \left( \frac{M_{crdy}}{M_{ney}} \right)^{0.5} \right) \cdot \left( \frac{M_{crdy}}{M_{ney}} \right)^{0.5} \cdot M_{ney} = 2.585 \text{kN} \cdot \text{m} \]

\[ M_{ndy} \quad \text{nominal flexural strength about the y-y (2-2) axis} \]

\[ M_{ogy} := \min (M_{ney}, M_{ndy}, M_{ndy}) = 2.316 \text{kN} \cdot \text{m} \]

**Verification:**

\[ \frac{P}{P_a} = 1.072 \]

\[ \frac{M_x}{M_{ax}} = 0.029 \]

\[ \frac{M_y}{M_{ay}} = 0.086 \]

\[ \frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} = 1.188 \]

### 4.1.3 Local, distortional buckling

Assuming that the second order analysis already takes into account the effect of global buckling, in this model the **global buckling verification will be neglected.**

**a. Members in compression (chapter E)**

The available axial strength \( P_a \) (factored resistance) shall be the smallest of the available axial strength \( F_n \) for global buckling, nominal axial strength \( P_{nl} \) for local buckling interacting with global buckling, and the nominal axial strength \( P_{nd} \) for distortional buckling.

\[ A_g := 445 \text{ mm}^2 \quad \text{gross cross-section area from Consteel} \]

\[ F_y := 430 \text{ MPa} \quad \text{yield stress} \]
a.1 Local Buckling [E3.2]

For local buckling, the nominal axial strength is the axial strength from the stub column test

\[ P_{nl} \text{ nominal axial strength} \]

\[ P_{ua} := 23.75 \text{ kip} = (1.056 \cdot 10^5) \text{ N} \quad \text{experimental ultimate compressive strength of the stub column test} \]

\[ P_{nl} := P_{ua} = 105.645 \text{ kN} \]

\[ A_{net.min} := 355.483 \text{ mm}^2 \quad \text{cross-section area from the experiment (stub column test)} \]

\[ P_{y.net} := A_{net.min} \cdot F_y = 152.858 \text{ kN} \]

\[ Q := \frac{P_{ua}}{A_{net.min} \cdot F_y} = 0.691 \]

\[ P_{nl} \leq P_{y.net} = 1 \]

a.2 Distortional Buckling [E4.2]

\[ P_{nd} \text{ nominal axial strength for distortional buckling} \]

\[ F_y = 430 \text{ MPa} \]

\[ A_{net} := 355.483 \text{ mm}^2 \]

\[ A_y := 445 \text{ mm}^2 \]

\[ P_y := A_y \cdot F_y = 191.35 \text{ kN} \quad \text{(Eq. E4.2-7)} \]

\[ P_{y.net} := A_{net} \cdot F_y = 152.858 \text{ kN} \quad \text{(Eq. E4.2-8)} \]

Using \( P_{crd} \) calculated with CUFSM

\[ \text{Load} := 1000 \text{ N} \]

\[ \text{load.factor} := 99.447 \]

\[ P_{crd} := \text{Load} \cdot \text{load.factor} = 99.447 \text{ kN}\]

Critical elastic distortional column buckling load determined by analysis in CUFSM, for reduced thickness, with non-reduced elastic modulus

\[ \lambda_d := \sqrt{\frac{P_y}{P_{crd}}} = 1.387 \quad \text{slenderness} \quad \text{(Eq. E4.2-3)} \]
\[
\lambda_{d2} := 0.561 \cdot \left( 14 \cdot \left( \frac{P_y}{P_{y_{net}}} \right)^{0.4} - 13 \right) = 1.299 \quad \text{(Eq. E4.2-5)}
\]

if \( \lambda_d \leq \lambda_{d2} = 0 \) will use subchapter E4.1 (members without holes)

\[
P_{nd} := \left( 1 - 0.25 \cdot \left( \frac{P_{crd}}{P_y} \right)^{0.6} \right) \cdot \left( \frac{P_{crd}}{P_y} \right)^{0.6} \cdot P_y = 107.396 \text{ kN} \quad \text{(Eq. E4.1-2)}
\]

\[
P_a := \min (P_{nl}, P_{nd}) = 105.645 \text{ kN}
\]

b. Members in flexure (chapter F)

The available flexural strength shall be the smallest of the nominal flexural strength for the available flexural strength due local buckling \( M_{nl} \) and the nominal flexural strength for distortional buckling \( M_{nd} \).

b.1. Flexure about x-x (1-1) axis

\[
S_{fx} := 11959 \text{ mm}^3 \quad \text{gross cross section modulus}
\]

b.1.1 Local Buckling [F3.2.2]

As the \( Q \) cannot be determined I will use subchapter F3.2.2 of AISI S-100.

\[
M_{y_{x}} := S_{fx} \cdot F_y = 5.142 \text{ kN} \cdot \text{m}
\]

\[
M_{crIx} := 3980.1442 \cdot 1000 \text{ N} \cdot \text{mm} = 3.98 \text{ kN} \cdot \text{m} \quad \text{critical elastic local buckling moment from CUFSM, reduced thickness (the same as for the global buckling), non-reduced elastic modulus}
\]

\[
\lambda_l := \sqrt{\frac{M_{y_{x}}}{M_{crIx}}} = 1.137
\]

\[
\lambda_l > 0.776 = 1
\]

\[
M_{nlx} := \left( 1 - 0.15 \cdot \left( \frac{M_{crIx}}{M_{y_{x}}} \right)^{0.4} \right) \cdot \left( \frac{M_{crIx}}{M_{y_{x}}} \right)^{0.4} \cdot M_{y_{x}} = 4.013 \text{ kN} \cdot \text{m}
\]

\[
I_{xx} := 373134.5763 \text{ mm}^4 \quad \text{moment of inertia about the x-x axis}
\]

\[
y_{y} := 37.1196 \text{ mm}
\]
Design of pallet rack upright frames subject to compression

\[ S_{\text{netx}} := \frac{I_{xx}}{y_g} = \left(1.005 \times 10^4\right) \text{ mm}^3 \]  
net section modulus

\[ M_{\text{ynetx}} := S_{\text{netx}} \cdot F_y = 4.322 \text{ kN} \cdot \text{m} \quad \text{yield moment of net cross-section} \]  
(Eq. F3.2.2-2)

\[ M_{\text{nlx}} \leq M_{\text{ynetx}} = 1 \]

b.1.2 Distortional Buckling [F4.2]

\[ S_{fz} := 11959 \text{ mm}^3 \]  
gross cross section modulus

\[ F_y = 430 \text{ MPa} \]  
yield stress

\[ M_{yz} := S_{fz} \cdot F_y = 5.142 \text{ kN} \cdot \text{m} \quad \text{yield moment} \]  
(Eq. F4.1-4)

\[ M_{\text{crdx}} := 4031.4623 \times 1000 \text{ N} \cdot \text{mm} = 4.031 \text{ kN} \cdot \text{m} \quad \text{critical elastic distortional buckling moment about x-x axis from CUFSM, reduced thickness, reduced elastic modulus} \]

\[ \lambda_d := \sqrt{\frac{M_{yz}}{M_{\text{crdx}}}} = 1.129 \quad \text{slenderness} \]  
(Eq. F4.1-3)

\[ M_{\text{ynetx}} = 4.322 \text{ kN} \cdot \text{m} \]

\[ \lambda_{d2} := 0.673 \left( 1 + \frac{\left( M_{yz} \right)^{2.7}}{M_{\text{ynetx}}^2} - 0.7 \right) = 1.358 \]  
(Eq. F4.2-5)

\[ \lambda_d \leq \lambda_{d2} = 1 \quad \text{will use subchapter E4.2 (members with holes)} \]

\[ \lambda_{d1} := 0.673 \frac{M_{\text{ynetx}}}{M_{yz}}^3 = 0.4 \]  
(Eq. F4.2-4)

\[ \lambda_{d1} < \lambda_d \leq \lambda_{d2} = 1 \]

\[ M_{d2} := \left( 1 - 0.22 \frac{1}{\lambda_{d2}} \right) \frac{1}{\lambda_{d2}} M_{yz} = 3.174 \text{ kN} \cdot \text{m} \]

\[ M_{\text{ndx}} := M_{\text{ynetx}} \left( \frac{M_{\text{ynetx}} - M_{d2}}{\lambda_{d2} - \lambda_{d1}} \right) \cdot \left( \lambda_d - \lambda_{d1} \right) = 3.448 \text{ kN} \cdot \text{m} \]  
(Eq. F4.2-2)

\[ M_{\text{ndx}} \quad \text{nominal flexural strength about the x-x (1-1) axis} \]

\[ \left( 1 - 0.22 \frac{M_{\text{crdx}}}{M_{yz}} ^{0.5} \right) \left( \frac{M_{\text{crdx}}}{M_{yz}} ^{0.5} \right) M_{yz} = 3.666 \text{ kN} \cdot \text{m} \]

\[ M_{\text{ndx}} \leq \left( 1 - 0.22 \frac{M_{\text{crdx}}}{M_{yz}} ^{0.5} \right) \left( \frac{M_{\text{crdx}}}{M_{yz}} ^{0.5} \right) M_{yz} = 1 \]
\[ M_{ax} := \min (M_{nx}, M_{ndx}) = 3.448 \text{ kN} \cdot \text{m} \]  
\text{distortional buckling}

b.2. Flexure about y-y (2-2) axis

\[ S_{fy} := 8650 \text{ mm}^3 \]  
gross cross section modulus

b.2.1 Local Buckling [F3.2.2]

\[ M_{yy} := S_{fy} \cdot F_y = 3.72 \text{ kN} \cdot \text{m} \]

\[ M_{crly} := 1568.7918 \cdot 1000 \text{ N} \cdot \text{mm} = 1.569 \text{ kN} \cdot \text{m} \]  
critical elastic local buckling moment from CUFSM, reduced thickness, non-reduced elastic modulus

\[ \lambda_l := \sqrt{\frac{M_{yy}}{M_{crly}}} = 1.54 \]

\[ \lambda_l > 0.776 = 1 \]

\[ M_{nly} := \left( 1 - 0.15 \left( \frac{M_{crly}}{M_{yy}} \right)^{0.4} \right) \left( \frac{M_{crly}}{M_{yy}} \right)^{0.4} \cdot M_{yy} = 2.354 \text{ kN} \cdot \text{m} \]

\[ I_{yy} := 308988.0814 \text{ mm}^4 \]  
moment of inertia about the y-y axis

\[ x_g := 35.5852 \text{ mm} \]

\[ S_{fnety} := \frac{I_{yy}}{x_g} = (8.683 \cdot 10^3) \text{ mm}^3 \]  
net section modulus

\[ M_{ynety} := S_{fnety} \cdot F_y = 3.734 \text{ kN} \cdot \text{m} \]  
yield moment of net cross-section  
\text{(Eq. F3.2.2-2)}

\[ M_{nly} \leq M_{ynety} = 1 \]

b.2.2 Distortional Buckling [F4.2]

\[ S_{fy} = (8.65 \cdot 10^3) \text{ mm}^3 \]  
gross cross section modulus

\[ F_y = 430 \text{ MPa} \]  
yield stress

\[ M_{yy} := S_{fy} \cdot F_y = 3.72 \text{ kN} \cdot \text{m} \]  
yield moment  
\text{(Eq. F4.1-4)}

\[ M_{crdy} := 2838.471 \cdot 1000 \text{ N} \cdot \text{mm} = 2.838 \text{ kN} \cdot \text{m} \]  
critical elastic distortional buckling moment about y-y axis from CUFSM, reduced thickness, non-reduced elastic modulus

\[ \lambda_d := \sqrt{\frac{M_{yx}}{M_{crdx}}} = 1.129 \]  
slenderness  
\text{(Eq. F4.1-3)}
\[ M_{\text{ymenty}} = 3.734 \text{ kN} \cdot \text{m} \]
\[
\lambda_{d2} = 0.673 \cdot \left( 1.7 \cdot \left( \frac{M_{yy}}{M_{\text{ymenty}}} \right)^{2.7} - 0.7 \right) = 0.661 \quad \text{(Eq. F4.2-5)}
\]

\[ \lambda_d \leq \lambda_{d2} = 0 \quad \text{will use subchapter F4.1 (members without holes)} \]

\[ M_{ndy} := \left( 1 - 0.22 \cdot \left( \frac{M_{crdy}}{M_{yy}} \right)^{0.5} \right) \cdot \left( \frac{M_{crdy}}{M_{yy}} \right)^{0.5} \cdot M_{yy} = 2.625 \text{ kN} \cdot \text{m} \]

\[ M_{ndy} \quad \text{nominal flexural strength about the y-y (2-2) axis} \]

\[ M_{ay} := \min(M_{ny}, M_{ndy}) = 2.354 \text{ kN} \cdot \text{m} \]

**Verification:**

\[
\frac{P}{P_a} = 0.81
\]
\[
\frac{M_x}{M_{\text{ax}}} = 0.029
\]
\[
\frac{M_y}{M_{\text{ay}}} = 0.085
\]
\[
\frac{P}{P_a} + \frac{M_x}{M_{\text{ax}}} + \frac{M_y}{M_{\text{ay}}} = 0.924
\]

### 4.1.4 Global, local+global and distortional buckling, using only \( A_{\text{net.min}} \)

In this model only the **minimul cross section area** will be used.

**a. Members in compression (chapter E)**

aThe available axial strength \( P_n \) (factored resistance) shall be the smallest of the available axial strength \( F_n \) for global buckling, nominal axial strength \( P_{nl} \) for local buckling interacting with global buckling, and the nominal axial strength \( P_{nd} \) for distortional buckling.

**a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]**

\[ A_{\text{net.min}} := 355.483 \text{ mm}^2 \quad \text{minimul net cross section area} \]
\[ F_y := 430 \text{ MPa} \quad \text{yield stress} \]
\[ P_{cre} := 148419 \text{ N} \] 
minimum of the critical elastic column buckling load in flexural, torsional 
or flexural-torsional buckling determined by analysis in CUTWP, for
\textbf{reduced thickness}

\[ F_{cre} := \frac{P_{cre}}{A_{net.min}} = 417.514 \text{ MPa} \] 
least of the applicable elastic global (flexural, torsional
and flexural torsional) buckling stress

\[ \lambda_c := \sqrt{\frac{F_y}{F_{cre}}} = 1.015 \] 
slenderness \hspace{1cm} (Eq. E2-4)

\[ \lambda_c \leq 1.5 = 1 \] 
\[ F_n := \left(0.658 \lambda^3 \right) \cdot F_y = 279.42 \text{ MPa} \] \hspace{1cm} (Eq. E2-2)

\[ F_n \] Compressive stress

\[ P_{ne} := A_{net.min} \cdot F_n = 99.329 \text{ kN} \] nominal axial strength \hspace{1cm} (Eq. E2-1)

\[ P_{nl} \] 
\[ P_{ne} = 99.329 \text{ kN} \]

\[ F_n := \frac{P_{ne}}{A_{net.min}} = 279.42 \text{ MPa} \]

\[ P_{ua} := 23.75 \text{ kip} = \left(1.056 \cdot 10^5\right) \text{ N} \] 
experimental ultimate compressive strength of the stub
column test

\[ A_{net.min} := 355.483 \text{ mm}^2 \] 
minimum net cross-section area

\[ Q := \frac{A_{net.min}}{P_{ua}} = 0.691 \]

Using Q RMI (see report from T. Pekoz, August 2015)

\[ P_{nl} := F_n \cdot \left[ 1 - (1 - Q) \cdot \left(\frac{F_n}{F_y}\right)^Q \right] \cdot A_{net.min} = [76.554] \text{ kN} \]

\[ a.3 \text{ Distortional Buckling [E4.2]} \]

\[ P_{nd} \] 
\[ P_{nd} \] nominal axial strength for distortional buckling

\[ F_y = 430 \text{ MPa} \]

\[ A_{net} := 355.483 \text{ mm}^2 \]

\[ P_y := A_{net.min} \cdot F_y = 152.858 \text{ kN} \] \hspace{1cm} (Eq. E4.2-7)
Using $P_{crd}$ calculated with CUFSM

$$\text{Load} := 1000 \; N$$

$$\text{load. factor} := 99.447$$

$$P_{crd} := \text{Load} \times \text{load. factor} = 99.447 \; kN$$

critical elastic distortional column buckling load determined by analysis in CUFSM, for reduced thickness

$$\lambda_d := \sqrt{\frac{P_y}{P_{crd}}} = 1.24$$

slenderness \hspace{1cm} (Eq. E4.2-3)

$$\lambda_d > 0.561 \; = 1$$

$$P_{nd} := \left(1 - 0.25 \left(\frac{P_{crd}}{P_y}\right)^{0.6}\right) \cdot \left(\frac{P_{crd}}{P_y}\right)^{0.6} \cdot P_y = 95.292 \; kN$$ \hspace{1cm} (Eq. E4.1-2)

$$P_a := \min(P_{ne}, P_{nl}, P_{nd}) = 76.554 \; kN$$

b. Members in flexure (chapter F)

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and global buckling $M_{ne}$, the available flexural strength due to the interaction of the yielding or global buckling with local buckling $M_{nl}$ and the nominal flexural strength for distortional buckling $M_{nd}$.

b.1. Flexure about x-x (1-1) axis

b.1.1 Yielding and Global Buckling [F2]

$$M_{crex} := \left(2.7913 \cdot 10^7\right) \; N \cdot mm$$

critical elastic bending moment from CUTWP, reduced thickness

$$I_{xx} := 373134.5763 \; mm^4$$

moment of inertia about the x-x axis

$$y_g := 37.1196 \; mm$$

$$S_{fretx} := \frac{I_{xx}}{y_g} = \left(1.005 \cdot 10^4\right) \; mm^3$$

net section modulus

$$F_{cre} := \frac{M_{crex}}{S_{fretx}} = \left(2.777 \cdot 10^3\right) \; MPa$$

critical elastic lateral-torsional buckling stress

$$F_{cre} \geq 2.78 \cdot F_y = 1$$

$$F_n := F_y = 430 \; MPa$$
\[
M_{nex} := S_{fueb} \cdot F_n = 4.322 \text{ kN} \cdot \text{m} \quad \text{nominal axial strength for yielding and global buckling about the x-x (1-1) axis (Eq. F2.1-1)}
\]

b.1.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

As the Q cannot be determined I will use subchapter F3.2.2 of AISI S-100

\[
M_{nex} = 4.322 \text{ kN} \cdot \text{m}
\]

\[
M_{crlz} := 3980.1442 \cdot 1000 \text{ N} \cdot \text{mm} = 3.98 \text{ kN} \cdot \text{m} \quad \text{critical elastic local buckling moment from CUFSM, reduced thickness (the same as for the global buckling)}
\]

\[
\lambda_l := \sqrt{\frac{M_{nex}}{M_{crlz}}} = 1.042
\]

\[
\lambda_l > 0.776 = 1
\]

\[
M_{nlx} := \left(1 - 0.15 \cdot \left(\frac{M_{crlz}}{M_{nex}}\right)^{0.4}\right) \cdot \left(\frac{M_{crlz}}{M_{nex}}\right)^{0.4} \cdot M_{nex} = 3.575 \text{ kN} \cdot \text{m}
\]

b.1.3 Distortional Buckling [F4.2]

\[
F_y = 430 \text{ MPa} \quad \text{yield stress}
\]

\[
M_{yx} := S_{fueb} \cdot F_y = 4.322 \text{ kN} \cdot \text{m} \quad \text{yield moment (Eq. F4.1-4)}
\]

\[
M_{crdx} := 4031.4623 \cdot 1000 \text{ N} \cdot \text{mm} = 4.031 \text{ kN} \cdot \text{m} \quad \text{critical elastic distortional buckling moment about x-x axis from CUFSM, reduced thickness (Eq. F4.1-3)}
\]

\[
\lambda_d := \sqrt{\frac{M_{yx}}{M_{crdx}}} = 1.035 \quad \text{slenderness}
\]

\[
\lambda_d > 0.673 = 1
\]

\[
M_{ndx} := \left(1 - 0.22 \cdot \left(\frac{M_{crdx}}{M_{yx}}\right)^{0.5}\right) \cdot \left(\frac{M_{crdx}}{M_{yx}}\right)^{0.5} \cdot M_{yx} = 3.288 \text{ kN} \cdot \text{m}
\]

\[
M_{ndx} \quad \text{nominal flexural strength about the x-x (1-1) axis}
\]

\[
M_{ax} := \min (M_{nex}, M_{nlx}, M_{ndx}) = 3.288 \text{ kN} \cdot \text{m}
\]
b.2. Flexure about y-y (2-2) axis

b.2.1 Yielding and Global Buckling [F2]

\[ M_{crey} := (8.46718 \cdot 10^6) \text{ N} \cdot \text{mm} \] critical elastic bending moment from CUTWP, reduced cross section

\[ I_{yy} := 308988.0814 \text{ mm}^4 \] moment of inertia about the y-y axis

\[ x_g := 35.5852 \text{ mm} \]

\[ S_{fnety} := \frac{I_{yy}}{x_g} (8.683 \cdot 10^3) \text{ mm}^3 \] net section modulus

\[ F_{cre} := \frac{M_{crey}}{S_{fnety}} = 975.139 \text{ MPa} \] critical elastic lateral-torsional buckling stress

\[ F_{cre} \geq 2.78 \cdot F_y = 0 \]

\[ 2.78 \cdot F_y > F_{cre} > 0.56 \cdot F_y = 1 \]

\[ F_n := \frac{10}{9} \cdot F_y \cdot \left(1 - \frac{10 \cdot F_y}{36 \cdot F_{cre}}\right) = 419.255 \text{ MPa} \] (Eq. F2.1-2)

\[ M_{ney} := S_{fnety} \cdot F_n = 3.64 \text{ kN} \cdot \text{m} \] nominal axial strength for yielding and global buckling about the y-y (2-2) axis (Eq. F2.1-1)

b.2.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

\[ M_{ney} = 3.64 \text{ kN} \cdot \text{m} \]

\[ M_{crly} := 1568.7918 \cdot 1000 \text{ N} \cdot \text{mm} = 1.569 \text{ kN} \cdot \text{m} \] critical elastic local buckling moment from CUFSM, reduced thickness

\[ \lambda_l := \sqrt{\frac{M_{ney}}{M_{crly}}} = 1.523 \]

\[ \lambda_l > 0.776 = 1 \]

\[ M_{nly} := \left(1 - 0.15 \cdot \left(\frac{M_{crly}}{M_{ney}}\right)^{0.4} \cdot \left(\frac{M_{crly}}{M_{ney}}\right)^{0.4}\right) \cdot M_{ney} = 2.321 \text{ kN} \cdot \text{m} \]

b.2.3 Distortional Buckling [F4.2]

\[ F_y = 430 \text{ MPa} \] yield stress

\[ M_{yy} := S_{fnety} \cdot F_y = 3.734 \text{ kN} \cdot \text{m} \] yield moment (Eq. F4.1-4)
\[ M_{crd}\gamma := 2838.471 \cdot 1000 \quad \text{N} \cdot \text{mm} = 2.838 \text{ kN} \cdot \text{m} \] critical elastic distortional buckling moment about x-x axis from CUFSM, reduced thickness

\[ \lambda_d := \sqrt{\frac{M_{yx}}{M_{crd\gamma}}} = 1.035 \quad \text{slenderness} \quad \text{(Eq. F4.1-3)} \]

\[ \lambda_d > 0.673 = 1 \]

\[ M_{nd\gamma} := \left(1 - 0.22 \cdot \left(\frac{M_{crd\gamma}}{M_{yy}}\right)^{0.5}\right) \cdot \left(\frac{M_{crd\gamma}}{M_{yy}}\right)^{0.5} \cdot M_{yy} = 2.631 \text{ kN} \cdot \text{m} \]

\[ M_{nd\gamma} \quad \text{nominal flexural strength about the y-y (2-2) axis} \]

\[ M_{ay} := \min(M_{ney}, M_{nd\gamma}, M_{nd\gamma}) = 2.321 \text{ kN} \cdot \text{m} \]

**Verification:**

\[ \frac{P}{P_a} = 1.118 \]

\[ \frac{M_x}{M_{ax}} = 0.03 \]

\[ \frac{M_y}{M_{ay}} = 0.086 \]

\[ \frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} = 1.235 \]

**4.1.5 Global, local+global buckling**

**a. Members in compression (chapter E)**

The available axial strength \( P_a \) (factored resistance) shall be the smallest of the available axial strength \( F_n \) for global buckling, nominal axial strength \( P_{nl} \) for local buckling interacting with global buckling.
a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]

\[ A_g = 445 \text{ mm}^2 \] gross cross-section area from Consteel

\[ F_y = 430 \text{ MPa} \] yield stress

\[ P_{cre} = 148419 \text{ N} \] minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTFP, for reduced thickness, using the non-reduced elastic modulus

\[ F_{cre} = \frac{P_{cre}}{A_g} = 333.526 \text{ MPa} \] least of the applicable elastic global (flexural, torsional and flexural torsional) buckling stress

\[ \lambda_c = \sqrt{\frac{F_y}{F_{cre}}} = 1.135 \quad \text{slenderness} \quad \text{(Eq. E2-4)} \]

\[ \lambda_c \leq 1.5 = 1 \quad F_n := (0.658^{\lambda_c}) \cdot F_y = 250.677 \text{ MPa} \quad \text{(Eq. E2-2)} \]

\[ F_n \quad \text{Compressive stress} \]

\[ P_{ne} := A_g \cdot F_n = 111.551 \text{ kN} \quad \text{nominal axial strength} \quad \text{(Eq. E2-1)} \]

a.2 Local Buckling Interacting with Yielding and Global Buckling [E3.2]

\[ P_{nl} \quad \text{nominal axial strength} \]

\[ P_{ne} = 111.551 \text{ kN} \]

\[ F_n := \frac{P_{ne}}{A_g} = 250.677 \text{ MPa} \]

\[ P_{ua} := 23.75 \text{ kip} = (1.056 \cdot 10^5) \text{ N} \] experimental ultimate compressive strength of the stub column test

\[ A_{net.min} := 355.483 \text{ mm}^2 \] cross-section area from the experiment (stub column test)

\[ Q := \frac{P_{ua}}{A_{net.min} \cdot F_y} = 0.691 \]

Using Q RMI (see report from T. Pekoz, August 2015)

\[ P_{nl} := F_n \cdot \left[ 1 - (1 - Q) \cdot \left( \frac{F_n}{F_y} \right)^Q \right] \cdot A_g = [87.823] \text{ kN} \]

\[ P_{ynet} := A_{net.min} \cdot F_y = 152.858 \text{ kN} \]
\[ P_{nl} \leq P_{yset} = [1] \]

\[ P_a := \min(P_{ne}, P_{nl}) = 87.823 \text{ kN} \]

b. Members in flexure (chapter F)

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and global buckling \( M_{ne} \), the available flexural strength due to the interaction of the yielding or global buckling with local buckling \( M_{nl} \).

b.1. Flexure about x-x (1-1) axis

b.1.1 Yielding and Global Buckling [F2]

\[ M_{crex} := 2.7913 \cdot 10^7 \text{ N} \cdot \text{mm} \]  \text{critical elastic bending moment from CUTWP, reduced thickness, non-reduced elastic modulus}

\[ S_{fx} := 11959 \text{ mm}^3 \]  \text{gross cross section modulus}

\[ F_{cre} := \frac{M_{crex}}{S_{fx}} = \left(2.33 \cdot 10^3\right) \text{ MPa} \]  \text{critical elastic lateral-torsional buckling stress}

\[ F_{cre} \geq 2.78 \cdot F_y = 1 \]

\[ F_n := F_y = 430 \text{ MPa} \]

\[ M_{nex} := S_{fx} \cdot F_n = 5.142 \text{ kN} \cdot \text{m} \]  \text{nominal axial strength for yielding and global buckling (Eq. F2.1-1) about the x-x (1-1) axis}

b.1.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

As the \( Q \) cannot be determined I will use subchapter F3.2.2 of AISI S-100

\[ M_{nex} = 5.142 \text{ kN} \cdot \text{m} \]

\[ M_{crlx} := 3980.1442 \cdot 1000 \text{ N} \cdot \text{mm} = 3.98 \text{ kN} \cdot \text{m} \]  \text{critical elastic local buckling moment from CUFSM, reduced thickness (the same as for the global buckling), non-reduced elastic modulus}

\[ \lambda_l := \sqrt{\frac{M_{nex}}{M_{crlx}}} = 1.137 \]

\[ \lambda_l > 0.776 = 1 \]
\[ M_{nlx} := \left(1 - 0.15 \cdot \left(\frac{M_{cre}}{M_{nex}}\right)^{0.4}\right) \cdot \left(\frac{M_{cre}}{M_{nex}}\right)^{0.4} \cdot M_{nex} = 4.013 \, kN \cdot m \]

\[ I_{xx} := \text{373134.5763 mm}^4 \]  
moment of inertia about the x-x axis

\[ y_g := \text{37.1196 mm} \]

\[ S_{fnetx} := \frac{I_{xx}}{y_g} = \left(1.005 \cdot 10^4\right) \, \text{mm}^3 \]  
net section modulus

\[ M_{ynetx} := S_{fnetx} \cdot F_y = 4.322 \, kN \cdot m \]  
yield moment of net cross-section  
(Eq. F3.2.2-2)

\[ M_{nlx} \leq M_{ynetx} = 1 \]

\[ M_{ax} := \min(M_{nex}, M_{nlx}) = 4.013 \, kN \cdot m \]

b.2. Flexure about y-y (2-2) axis

\section*{b.2.1 Yielding and Global Buckling [F2]}

\[ M_{crey} := \left(8.46718 \cdot 10^6\right) \, N \cdot mm \]  
critical elastic bending moment from CUTWP, reduced thickness, non-reduced elastic modulus

\[ S_{fy} := \text{8650 mm}^3 \]  
gross cross section modulus

\[ F_{cre} := \frac{M_{crey}}{S_{fy}} = 978.865 \, MPa \]  
critical elastic lateral-torsional buckling stress

\[ F_{cre} \geq 2.78 \cdot F_y = 0 \]

\[ 2.78 \cdot F_y > F_{cre} > 0.56 \, F_y = 1 \]

\[ F_n := \frac{10}{9} \cdot F_y \cdot \left(1 - \frac{10 \cdot F_y}{36 \cdot F_{cre}}\right) = 419.478 \, MPa \]  
(Eq. F2.1-2)

\[ M_{ney} := S_{fy} \cdot F_n = 3.628 \, kN \cdot m \]  
nominal axial strength for yielding and global buckling about the y-y (2-2) axis  
(Eq. F2.1-1)

b.2.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

\[ M_{ney} = 3.628 \, kN \cdot m \]  
- - -
\[ M_{\text{cly}} := 1568.7918 \times 1000 \, \text{N} \cdot \text{mm} = 1.569 \, \text{kN} \cdot \text{m} \]  

Critical elastic local buckling moment from CUFSM, reduced thickness, reduced elastic modulus

\[ \lambda_i := \sqrt{\frac{M_{\text{ney}}}{M_{\text{cly}}}} = 1.521 \]

\[ \lambda_i > 0.776 = 1 \]

\[ M_{\text{ny}} := \left( 1 - 0.15 \times \left( \frac{M_{\text{cly}}}{M_{\text{ney}}} \right)^{0.4} \right) \times \left( \frac{M_{\text{cly}}}{M_{\text{ney}}} \right)^{0.4} \times M_{\text{ney}} = 2.316 \, \text{kN} \cdot \text{m} \]

\[ I_{yy} := 308988.0814 \, \text{mm}^4 \]  

Moment of inertia about the y-y axis

\[ x_g := 35.5852 \, \text{mm} \]

\[ S_{\text{ney}} := \frac{I_{yy}}{x_g} = (8.683 \times 10^3) \, \text{mm}^3 \]  

Net section modulus

\[ M_{\text{ney}} := S_{\text{ney}} \cdot F_y = 3.734 \, \text{kN} \cdot \text{m} \]  

Yield moment of net cross-section  

(Eq. F3.2.2-2)

\[ M_{\text{ny}} \leq M_{\text{ney}} = 1 \]

\[ M_{\text{ay}} := \min (M_{\text{ney}}, M_{\text{ny}}) = 2.316 \, \text{kN} \cdot \text{m} \]

**Verification:**

\[ \frac{P}{P_a} = 0.975 \]

\[ \frac{M_x}{M_{ax}} = 0.025 \]

\[ \frac{M_y}{M_{ay}} = 0.086 \]

\[ \frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} = 1.086 \]
4.1.6 Global, local+global and distortional buckling, using the section without perforations

a. Members in compression (chapter E)

The available axial strength $P_a$ (factored resistance) shall be the smallest of the available axial strength $F_n$ for global buckling, nominal axial strength $P_{nl}$ for local buckling interacting with global buckling, and the nominal axial strength $P_{nd}$ for distortional buckling.

a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]

$$A_g := 445 \text{ mm}^2$$

$$F_y := 430 \text{ MPa}$$ yield stress

$$P_{cre} := 187745 \text{ N}$$ minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP, for gross cross section, using the non-reduced elastic modulus

$$F_{cre} = \frac{P_{cre}}{A_g} = 421.899 \text{ MPa}$$ least of the applicable elastic global(flexural, torsional and flexural torsional) buckling stress

$$\lambda_c := \sqrt{\frac{F_y}{F_{cre}}} = 1.01$$ slenderness (Eq. E2-4)

$$\lambda_c \leq 1.5 \Rightarrow F_n := (0.658 \lambda_c^3) \cdot F_y = 280.675 \text{ MPa}$$ (Eq. E2-2)

$$F_n$$ Compressive stress

$$P_{ne} := A_g \cdot F_n = 124.9 \text{ kN}$$ nominal axial strength (Eq. E2-1)

a.2 Local Buckling Interacting with Yielding and Global Buckling [E3.2.1]

For the model without holes, instead of applying the Q RMI method (which is suitable for models with holes only), I will apply the Direct Strength Method from AISI S-100. The value of $P_{nl}$ will be calculated using a critical elastic local buckling load, $P_{crl}$, calculated in CUFSM.

$$P_{nl}$$ nominal axial strength

$$P_{ne} = 124.9 \text{ kN}$$

$$Load := 1000 \text{ N}$$

$$load.factor := 182.4047$$

$$P_{crl} := Load \cdot load.factor = 182.405 \text{ kN}$$ critical elastic local buckling load determined by analysis in CUFSM, for the gross cross section, non-reduced stiffness
\[ \lambda_l := \sqrt{\frac{P_{ne}}{P_{cr}}} = 0.827 \]  

(Eq. E3.2.1-3)

\[ \lambda_l > 0.776 \rightarrow 1 \]

\[ P_{nl} := \left(1 - 0.15 \cdot \left(\frac{P_{cr}}{P_{ne}}\right)^{0.4}\right) \cdot \left(\frac{P_{cr}}{P_{ne}}\right)^{0.4} \cdot P_{ne} = 119.964 \text{ kN} \]

a.3 Distortional Buckling [E4.1]

\[ P_{nd} \]  

nominal axial strength for distortional buckling

\[ F_y = 430 \text{ MPa} \]

\[ P_y := A_y \cdot F_y = 191.35 \text{ kN} \]  

(Eq. E4.1-7)

Using \( P_{crd} \) calculated with CUFSM

\[ Load := 1000 \text{ N} \]

\[ load.factor := 157.1437 \]

\[ P_{crd} := Load \cdot load.factor = 157.144 \text{ kN} \]  

critical elastic distortional column buckling load  
determined by analysis in CUFSM, for gross cross section, with non-reduced elastic modulus

\[ \lambda_d := \sqrt{\frac{P_y}{P_{crd}}} = 1.103 \]  

slenderness  

(Eq. E4.1-3)

\[ \lambda_d > 0.561 \rightarrow 1 \]

\[ P_{nd} := \left(1 - 0.25 \cdot \left(\frac{P_{crd}}{P_y}\right)^{0.6}\right) \cdot \left(\frac{P_{crd}}{P_y}\right)^{0.6} \cdot P_y = 132.255 \text{ kN} \]  

(Eq. E4.1-2)

\[ P_a := \min(P_{ne}, P_{nl}, P_{nd}) = 119.964 \text{ kN} \]

b. Members in flexure (chapter F)

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and global buckling \( M_{ne} \), the available flexural strength due to the interaction of the yielding or global buckling with local buckling \( M_{nl} \) and the nominal flexural strength for distortional buckling \( M_{nd} \).
b.1. Flexure about x-x (1-1) axis

2.1.1 Yielding and Global Buckling [F2]

\[ M_{crex} := 3.1529 \times 10^7 \text{ N} \cdot \text{mm} \]  
\[ S_{fx} := 11959 \text{ mm}^3 \]

\[ F_{cre} := \frac{M_{crex}}{S_{fx}} = (2.636 \times 10^3) \text{ MPa} \]

\[ F_{cre} \geq 2.78 \times F_y = 1 \]

\[ F_n := F_y = 430 \text{ MPa} \]

\[ M_{nex} := S_{fx} \cdot F_n = 5.142 \text{ kN} \cdot \text{m} \]

nominal axial strength for yielding and global buckling about the x-x (1-1) axis

b.1.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

As the Q cannot be determined I will use subchapter F3.2.2 of AISI S-100

\[ M_{nex} = 5.142 \text{ kN} \cdot \text{m} \]

\[ M_{crlx} := 6163.7634 \times 1000 \text{ N} \cdot \text{mm} = 6.164 \text{ kN} \cdot \text{m} \]

\[ \lambda_l := \sqrt{\frac{M_{nex}}{M_{crlx}}} = 0.913 \]

\[ \lambda_l > 0.776 = 1 \]

\[ M_{nlx} := \left(1 - 0.15 \cdot \left(\frac{M_{crlx}}{M_{nex}}\right)^{0.4} + \left(\frac{M_{crlx}}{M_{nex}}\right)^{0.4} \cdot M_{nex}\right)^{0.4} = 4.637 \text{ kN} \cdot \text{m} \]

b.1.3 Distortional Buckling [F4.1]

\[ S_{fx} := 11959 \text{ mm}^3 \]
\[ F_y = 430 \text{ MPa} \]

\[ M_{yx} := S_{fx} \cdot F_y = 5.142 \text{ kN} \cdot \text{m} \]

yield moment (Eq. F4.1-4)
\[ M_{crdx} = 6038.9886 \cdot 1000 \, N \cdot mm = 6.039 \, kN \cdot m \]  

Critical elastic distorsional buckling moment about x-x axis from CUFSM, gross cross section, non-reduced elastic modulus

\[ \lambda_d := \sqrt{\frac{M_{yx}}{M_{crdx}}} = 0.923 \quad \text{slenderness} \quad (\text{Eq. F4.1-3}) \]

\[ \lambda_d > 0.673 = 1 \]

\[ M_{ndx} := \left(1 - 0.22 \cdot \left(\frac{M_{crdx}}{M_{yx}}\right)^{0.5} \right) \cdot \left(\frac{M_{crdx}}{M_{yx}}\right)^{0.5} \cdot M_{yx} = 4.244 \, kN \cdot m \]

\[ M_{ndx} \quad \text{nominal flexural strength about the x-x (1-1) axis} \]

\[ M_{ax} := \min (M_{nxx}, M_{nlx}, M_{ndx}) = 4.244 \, kN \cdot m \]

**b.2. Flexure about y-y (2-2) axis**

**b.2.1 Yielding and Global Buckling [F2]**

\[ M_{crey} := 9.54956 \cdot 10^6 \, N \cdot mm \]  

Critical elastic bending moment from CUTWP, gross section, non-reduced elastic modulus

\[ S_{fy} := 8650 \, mm^3 \]  

Gross section modulus

\[ F_{cre} := \frac{M_{crey}}{S_{fy}} = (1.104 \cdot 10^3) \, MPa \]  

Critical elastic lateral-torsional buckling stress

\[ F_{cre} \geq 2.78 \cdot F_y = 0 \]

\[ 2.78 \cdot F_y > F_{cre} > 0.56 \, F_y = 1 \]

\[ F_n := \frac{10}{9} \cdot F_y \cdot \left(1 - \frac{10 \cdot F_y}{36 \cdot F_{cre}}\right) = 426.086 \, MPa \quad (\text{Eq. F2.1-2}) \]

\[ M_{ney} := S_{fy} \cdot F_n = 3.686 \, kN \cdot m \]  

Nominal axial strength for yielding and global buckling about the y-y (2-2) axis  

\[ (\text{Eq. F2.1-1}) \]
b.2.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.1]

\[ M_{ney} = 3.686 \text{ kN} \cdot \text{m} \]

\[ M_{crly} = 6150.2983 \cdot 1000 \text{ N} \cdot \text{mm} = 6.15 \text{ kN} \cdot \text{m} \]

\[ \lambda_l := \sqrt{\frac{M_{ney}}{M_{crly}}} = 0.774 \]

\[ \lambda_l > 0.776 = 0 \]

\[ M_{nly} := M_{ney} = 3.686 \text{ kN} \cdot \text{m} \]

b.2.3 Distortional Buckling [F4.1]

\[ F_y = 430 \text{ MPa} \]

\[ M_{yy} := S_f y \cdot F_y = 3.72 \text{ kN} \cdot \text{m} \]

\[ M_{crdy} = 4083.3801 \cdot 1000 \text{ N} \cdot \text{mm} = 4.083 \text{ kN} \cdot \text{m} \]

\[ \lambda_d := \sqrt{\frac{M_{yy}}{M_{crdx}}} = 0.923 \]

\[ \lambda_d > 0.673 = 1 \]

\[ M_{ndy} := \left(1 - 0.22 \cdot \left(\frac{M_{crdy}}{M_{yy}}\right)^{0.5}\right) \cdot \left(\frac{M_{crdy}}{M_{yy}}\right)^{0.5} \cdot M_{yy} = 2.999 \text{ kN} \cdot \text{m} \]

\[ M_{ndy} \text{ nominal flexural strength about the y-y (2-2) axis} \]

\[ M_{ay} := \min(M_{ney}, M_{nly}, M_{ndy}) = 2.999 \text{ kN} \cdot \text{m} \]
Verification:

\[
P = 0.714
\]
\[
\frac{P_a}{M_{ax}} = 0.024
\]
\[
\frac{M_y}{M_{ay}} = 0.067
\]
\[
\frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} = 0.804
\]

4.1.7 Global, local+global and distortional buckling, using notional load imperfections

The consideration of initial imperfections is considered through application of notional loads (AISI S100-16 Subchapter C1.1.1.2 b). Notional loads are used to represent the effects of initial imperfections. The notional loads are applied in Consteel. They are applied as lateral loads at all levels (in the present case, at the levels with the horizontal bracing bracings).

The value of the notional loads:

\[Y_i := 83.54 \text{ kN}\] gravity load applied on level "i"
\[\alpha := 1\]
\[N_i := \frac{1}{240} \cdot \alpha \cdot Y_i = 0.348 \text{ kN}\] notional load applied at level "i"
Consteel has implemented only the European standards, hence the axes are named after Eurocode (y and z). For this procedure the axes will be changed according to the American standard, so x-x will be the symmetry axis and y-y the one perpendicular to x-x.

For determining the critical axial load in CUTWP the buckling lengths are reduced with a K factor, as in the 1st American procedure. For determining the critical bending moments in CUTWP the buckling length is equal to the unity.

The models in CUTWP and CUFSM are with the reduced thickness sections.

\[ P := 85.6 \text{ kN} \]

required compressive axial strength

\[ M_x := 0.1 \text{ kN} \cdot \text{m} \]

required flexural strength along the x-x axis (y-y for EU)

\[ M_y := 0.2 \text{ kN} \cdot \text{m} \]

required flexural strength along the y-y axis (z-z for EU)

\[ P_a \]

available axial strength determined in accordance with Chapter E

\[ M_{ax}, M_{ay} \]

available flexural strengths determined as required in Section F

**a. Members in compression (chapter E)**

The available axial strength \( P_a \) (factored resistance) shall be the smallest of the available axial strength \( P_{ne} \) for global buckling, nominal axial strength \( P_{nl} \) for local buckling interacting with global buckling, and the nominal axial strength \( P_{nd} \) for distortional buckling.

**a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]**

\[ A_g := 445 \text{ mm}^2 \]

gross cross-section area from Consteel

\[ F_y := 430 \text{ MPa} \]

yield stress

\[ P_{cre} := 148419 \text{ N} \]

minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP, for reduced thickness, using the non-reduced elastic modulus

\[ \frac{F_{cre}}{A_g} = 333.526 \text{ MPa} \]

least of the applicable elastic global(flexural, torsional and flexural torsional) buckling stress

\[ \lambda_c := \sqrt{\frac{F_y}{F_{cre}}} = 1.135 \]

slenderness \hspace{1cm} \text{(Eq. E2-4)}

\[ \lambda_c \leq 1.5 = 1 \]

\[ F_n := (0.658 \lambda_c^3) \cdot F_y = 250.677 \text{ MPa} \]

\hspace{1cm} \text{(Eq. E2-2)}

\[ F_n \]

Compressive stress

\[ P_{ne} := A_g \cdot F_n = 111.551 \text{ kN} \]

nominal axial strength \hspace{1cm} \text{(Eq. E2-1)}
### a.2 Local Buckling Interacting with Yielding and Global Buckling [E3.2]

\[ P_{nl} \quad \text{nominal axial strength} \]

\[ P_{ne} = 111.551 \text{ kN} \]

\[ F_n := \frac{P_{ne}}{A_y} = 250.677 \text{ MPa} \]

\[ P_{ua} := 23.75 \text{ kip} = \left(1.056 \cdot 10^5\right) \text{ N} \quad \text{experimental ultimate compressive strength of the stub column test} \]

\[ A_{net,min} := 355.483 \text{ mm}^2 \quad \text{cross-section area from the experiment (stub column test)} \]

\[ Q := \frac{P_{ua}}{A_{net,min} \cdot F_y} = 0.691 \]

Using Q RMI (see report from T. Pekoz, August 2015)

\[ P_{nl} := F_n \cdot \left[ 1 - (1 - Q) \cdot \left(\frac{F_n}{F_y}\right)^Q \right] \cdot A_y = [87.823] \text{ kN} \]

\[ P_{y,net} := A_{net,min} \cdot F_y = 152.858 \text{ kN} \]

\[ P_{nl} \leq P_{y,net} = [1] \]

### a.3 Distortional Buckling [E4.2]

\[ P_{nd} \quad \text{nominal axial strength for distortional buckling} \]

\[ F_y = 430 \text{ MPa} \]

\[ A_{net} := 355.483 \text{ mm}^2 \]

\[ A_y := 445 \text{ mm}^2 \]

\[ P_y := A_y \cdot F_y = 191.35 \text{ kN} \quad \text{(Eq. E4.2-7)} \]

\[ P_{y,net} := A_{net} \cdot F_y = 152.858 \text{ kN} \quad \text{(Eq. E4.2-8)} \]

Using \( P_{crd} \) calculated with CUFSM

\[ Load := 1000 \text{ N} \]

\[ load.factor := 99.447 \]

\[ P_{crd} := Load \cdot load.factor = 99.447 \text{ kN} \quad \text{critical elastic distortional column buckling load determined by analysis in CUFSM, for reduced thickness, with non-reduced elastic modulus} \]

\[ \lambda_d := \frac{P_y}{P_{crd}} = 1.387 \quad \text{slenderness} \quad \text{(Eq. E4.2-3)} \]
\[ \lambda_{d2} := 0.561 \left( 14 \cdot \left( \frac{P_y}{P_{ynet}} \right)^{0.4} - 13 \right) = 1.299 \]  \hspace{1cm} (Eq. E4.2-5)

If \( \lambda_d \leq \lambda_{d2} = 0 \) will use subchapter E4.1 (members without holes)

\[ P_{nd} := \left( 1 - 0.25 \cdot \left( \frac{P_{crd}}{P_y} \right)^{0.6} \right) \cdot \left( \frac{P_{crd}}{P_y} \right)^{0.6} \cdot P_y = 107.396 \text{ kN} \]  \hspace{1cm} (Eq. E4.1-2)

\[ P_a := \min(P_{ne}, P_{nl}, P_{nd}) = 87.823 \text{ kN} \]  \hspace{1cm} from local+global

b. Members in flexure (chapter F)

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and global buckling \( M_{ne} \), the available flexural strength due to the interaction of the yielding or global buckling with local buckling \( M_{nl} \) and the nominal flexural strength for distortional buckling \( M_{nd} \).

b.1. Flexure about x-x (1-1) axis

b.1.1 Yielding and Global Buckling [F2]

\[ M_{crex} := 2.7913 \cdot 10^7 \text{ N} \cdot \text{mm} \]  \hspace{1cm} critical elastic bending moment from CUTWP, reduced thickness, non-reduced elastic modulus

\[ S_{fx} := 11959 \text{ mm}^3 \]  \hspace{1cm} gross cross section modulus

\[ F_{cre} := \frac{M_{crex}}{S_{fx}} = \left( 2.334 \cdot 10^3 \right) \text{ MPa} \]  \hspace{1cm} critical elastic lateral-torsional buckling stress

\[ F_{cre} \geq 2.78 \cdot F_y = 1 \]

\[ F_n := F_y = 430 \text{ MPa} \]  \hspace{1cm} (Eq. F2.1-1)

\[ M_{nex} := S_{fx} \cdot F_n = 5.142 \text{ kN} \cdot \text{m} \]  \hspace{1cm} nominal axial strength for yielding and global buckling about the x-x (1-1) axis

b.1.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

As the Q cannot be determined I will use subchapter F3.2.2 of AISI S-100

\[ M_{nex} = 5.142 \text{ kN} \cdot \text{m} \]  \hspace{1cm} critical elastic local buckling moment from CUFSM, reduced thickness (the same

\[ M_{crlx} := 3980.1442 \cdot 1000 \text{ N} \cdot \text{mm} = 3.98 \text{ kN} \cdot \text{m} \]  \hspace{1cm} critical elastic local buckling moment from CUFSM, reduced thickness (the same
as for the global buckling, **non-reduced elastic modulus**

\[
\lambda_l := \sqrt{\frac{M_{nex}}{M_{cr lx}}} = 1.137
\]  

(Eq. F3.2.1-3)

\[
\lambda_l > 0.776 = 1
\]

\[
M_{nl x} := \left(1 - 0.15 \left(\frac{M_{cr lx}}{M_{nex}}\right)^{0.4}\right) \cdot \left(\frac{M_{cr lx}}{M_{nex}}\right)^{0.4} \cdot M_{nex} = 4.013 \text{ kN} \cdot \text{m}
\]  

(Eq. F3.2.1-2)

\[
I_{xx} := 373134.5763 \text{ mm}^4
\]

moment of inertia about the x-x axis for the minimum net cross section

\[
y_g := 37.1196 \text{ mm}
\]

\[
S_{f net x} := \frac{I_{xx}}{y_g} = (1.005 \cdot 10^4) \text{ mm}^3
\]

minimum net cross section modulus

\[
M_{y net x} := S_{f net x} \cdot F_y = 4.322 \text{ kN} \cdot \text{m}
\]  

yield moment of net cross-section  

(Eq. F3.2.2-2)

\[
M_{nl x} \leq M_{y net x} = 1
\]

**b.1.3 Distortional Buckling [F4.2]**

\[
S_{f x} := 11959 \text{ mm}^3
\]

gross cross section modulus

\[
F_y = 430 \text{ MPa}
\]

yield stress

\[
M_{yx} := S_{f x} \cdot F_y = 5.142 \text{ kN} \cdot \text{m}
\]  

yield moment  

(Eq. F4.1-4)

\[
M_{cr dx} := 4031.4623 \cdot 1000 \text{ N} \cdot \text{mm} = 4.031 \text{ kN} \cdot \text{m}
\]

critical elastic distortional buckling moment about x-x axis from CUFSM, **reduced thickness, reduced elastic modulus**

\[
\lambda_d := \sqrt{\frac{M_{yx}}{M_{cr dx}}} = 1.129
\]

slenderness  

(Eq. F4.1-3)

\[
M_{y net x} = 4.322 \text{ kN} \cdot \text{m}
\]

\[
\lambda_{d2} := 0.673 \cdot \left(1.7 \cdot \left(\frac{M_{yx}}{M_{y net x}}\right)^{2.7} - 0.7\right) = 1.358
\]  

(Eq. F4.2-5)

\[
\lambda_d \leq \lambda_{d2} = 1
\]

will use subchapter E4.2 (members with holes)

\[
\lambda_{d1} := 0.673 \cdot \left(\frac{M_{y net x}}{M_{yx}}\right)^{3} = 0.4
\]  

(Eq. F4.2-4)
\[ \lambda_{d1} < \lambda_d \leq \lambda_{d2} = 1 \]

\[ M_{d2} := \left(1 - 0.22 \cdot \left(\frac{1}{\lambda_{d2}}\right)\right) \cdot \left(\frac{1}{\lambda_{d2}}\right) \cdot M_{yx} = 3.174 \text{ kN} \cdot \text{m} \]

\[ M_{ndx} := M_{y\text{net}} - \left(\frac{M_{y\text{net}} - M_{d2}}{\lambda_{d2} - \lambda_{d1}}\right) \cdot (\lambda_d - \lambda_{d1}) = 3.448 \text{ kN} \cdot \text{m} \quad \text{(Eq. F4.2-2)} \]

\[ M_{ndx} \quad \text{nominal flexural strength about the x-x (1-1) axis} \]

\[ \left(1 - 0.22 \cdot \left(\frac{M_{crdx}}{M_{yx}}\right)^{0.5}\right) \cdot \left(\frac{M_{crdx}}{M_{yx}}\right)^{0.5} \cdot M_{yx} = 3.666 \text{ kN} \cdot \text{m} \]

\[ M_{nx} \leq \left(1 - 0.22 \cdot \left(\frac{M_{crdx}}{M_{yx}}\right)^{0.5}\right) \cdot \left(\frac{M_{crdx}}{M_{yx}}\right)^{0.5} \cdot M_{yx} = 1 \]

\[ M_{ax} := \min \{M_{nex}, M_{nlx}, M_{ndx}\} = 3.448 \text{ kN} \cdot \text{m} \]

**b.2. Flexure about y-y (2-2) axis**

**b.2.1 Yielding and Global Buckling [F2]**

\[ M_{crey} := 8.46718 \cdot 10^6 \text{ N} \cdot \text{mm} \quad \text{critical elastic bending moment from CUTWP, reduced thickness, non-reduced elastic modulus} \]

\[ S_{fy} := 8650 \text{ mm}^3 \quad \text{gross cross section modulus} \]

\[ F_{cre} := \frac{M_{crey}}{S_{fy}} = 978.865 \text{ MPa} \quad \text{critical elastic lateral-torsional buckling stress} \]

\[ F_{cre} \geq 2.78 \cdot F_y = 0 \]

\[ 2.78 \cdot F_y > F_{cre} > 0.56 \quad F_y = 1 \quad \text{(Eq. F2.1-2)} \]

\[ F_n := \frac{10}{9} \cdot F_y \cdot \left(1 - \frac{10 \cdot F_y}{36 \cdot F_{cre}}\right) = 419.478 \text{ MPa} \]

\[ M_{ney} := S_{fy} \cdot F_n = 3.628 \text{ kN} \cdot \text{m} \quad \text{nominal axial strength for yielding and global buckling about the y-y (2-2) axis (Eq. F2.1-1)} \]
b.2.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

\[ M_{\text{neq}} = 3.628 \text{ kN} \cdot \text{m} \]

\[ M_{\text{cly}} := 1568.7918 \cdot 1000 \text{ N} \cdot \text{mm} = 1.569 \text{ kN} \cdot \text{m} \]

critical elastic local buckling moment from CUFSM, reduced thickness, reduced elastic modulus

\[ \lambda_i := \sqrt{\frac{M_{\text{neq}}}{M_{\text{cly}}}} = 1.521 \]  
(Eq. F3.2.1-3)

\[ \lambda_i > 0.776 = 1 \]  
(Eq. F3.2.1-2)

\[ M_{\text{nty}} := \left(1 - 0.15 \cdot \left(\frac{M_{\text{cly}}}{M_{\text{neq}}}\right)^{0.4}\right) \cdot \left(\frac{M_{\text{cly}}}{M_{\text{neq}}}\right)^{0.4} \cdot M_{\text{neq}} = 2.316 \text{ kN} \cdot \text{m} \]

\[ I_{yy} := 308988.0814 \text{ mm}^4 \]  
moment of inertia about the y-y axis

\[ x_g := 35.5852 \text{ mm} \]

\[ S_{\text{nty}} = \frac{I_{yy}}{x_g} \left(8.683 \cdot 10^3\right) \text{ mm}^3 \]  
net section modulus

\[ M_{\text{nty}} := S_{\text{nty}} \cdot F_y = 3.734 \text{ kN} \cdot \text{m} \]  
yield moment of net cross-section  
(Eq. F3.2.2-2)

\[ M_{\text{nty}} \leq M_{\text{nty}} = 1 \]

b.2.3 Distortional Buckling [F4.2]

\[ S_fy = \left(8.65 \cdot 10^3\right) \text{ mm}^3 \]  
gross cross section modulus

\[ F_y = 430 \text{ MPa} \]  
yield stress

\[ M_{yy} := S_fy \cdot F_y = 3.72 \text{ kN} \cdot \text{m} \]  
yield moment  
(Eq. F4.1-4)

\[ M_{\text{crdy}} := 2838.471 \cdot 1000 \text{ N} \cdot \text{mm} = 2.838 \text{ kN} \cdot \text{m} \]  
critical elastic distortional buckling moment about y-y axis from CUFSM, reduced thickness, non-reduced elastic modulus

\[ \lambda_d := \sqrt{\frac{M_{yy}}{M_{\text{crdx}}}} = 1.129 \]  
slenderness  
(Eq. F4.1-3)

\[ M_{\text{nty}} = 3.734 \text{ kN} \cdot \text{m} \]

\[ \lambda_{d2} := 0.673 \cdot \left(1.7 \cdot \left(\frac{M_{yy}}{M_{\text{nty}}\right)^{2.7} - 0.7\right) = 0.661 \]  
(Eq. F4.2-5)
\[ \lambda_d \leq \lambda_{d2} = 0 \] will use subchapter F4.1 (members without holes)

\[ M_{ndy} := \left( 1 - 0.22 \cdot \left( \frac{M_{crdy}}{M_{yy}} \right)^{0.5} \right) \cdot \left( \frac{M_{crdy}}{M_{yy}} \right)^{0.5} \cdot M_{yy} = 2.625 \, kN \cdot m \] (Eq. F4.1-2)

\[ M_{ndy} \] nominal flexural strength about the y-y (2-2) axis

\[ M_{ay} := \min (M_{ney}, M_{nly}, M_{ndy}) = 2.316 \, kN \cdot m \]

**Verification:**

\[ \frac{P}{P_a} = 0.975 \]

\[ \frac{M_x}{M_{ax}} = 0.029 \]

\[ \frac{M_y}{M_{ay}} = 0.086 \]

\[ \frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} = 1.09 \]
4.2 US - DAM applied for Column 2.99x3x0.105inch

Consteel has implemented only the European standards, hence the axes are named after Eurocode (y and z). For this procedure the axes will be changed according to the American standard, so x-x will be the symmetry axis and y-y the one perpendicular to x-x.

\[ P := 131.2 \text{ kN} \] required compressive axial strength

\[ M_x := 0.1 \text{ kN} \cdot \text{m} \] required flexural strength along the x-x axis (y-y for EU)

\[ M_y := 0.3 \text{ kN} \cdot \text{m} \] required flexural strength along the y-y axis (z-z for EU)

\[ P_a \] available axial strength determined in accordance with Chapter E

\[ M_{ax}, M_{ay} \] available flexural strengths determined as required in Section F

4.2.1 Global, local+global, distortional buckling

a. Members in compression (chapter E)

The available axial strength \( P_a \) (factored resistance) shall be the smallest of the available axial strength \( F_n \) for global buckling, nominal axial strength \( P_{nl} \) for local buckling interacting with global buckling, and the nominal axial strength \( P_{nd} \) for distortional buckling.

a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]

\[ A_g := 694 \text{ mm}^2 \] gross cross-section area from Consteel

\[ F_y := 370 \text{ MPa} \] yield stress

\[ P_{cre} := 236979 \text{ N} \] minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP, for reduced thickness, using the non-reduced elastic modulus
\[ F_{cre} := \frac{P_{cre}}{A_g} = 341.468 \text{ MPa} \quad \text{least of the applicable elastic global (flexural, torsional and flexural torsional) buckling stress} \]

\[ \lambda_c := \sqrt{\frac{F_y}{F_{cre}}} = 1.041 \quad \text{slenderness} \quad \text{(Eq. E2-4)} \]

\[ \lambda_c \leq 1.5 = 1 \quad F_n := \left(0.658^{\frac{1}{2}}\right) \cdot F_y = 235.093 \text{ MPa} \quad \text{(Eq. E2-2)} \]

\[ F_n \quad \text{Compressive stress} \]

\[ P_{ne} := A_g \cdot F_n = 163.154 \text{ kN} \quad \text{nominal axial strength} \quad \text{(Eq. E2-1)} \]

\textbf{a.2 Local Buckling Interacting with Yielding and Global Buckling [E3.2]}

\[ P_{nl} \quad \text{nominal axial strength} \]

\[ P_{nl} = 163.154 \text{ kN} \]

\[ A_{net.min} := 547.741 \text{ mm}^2 \quad \text{minimum net cross section area} \]

\[ F_n := \frac{P_{ne}}{A_g} = 235.093 \text{ MPa} \]

\[ P_{ua} := 45.59 \text{ kip} = (2.028 \cdot 10^5) \text{ N} \quad \text{experimental ultimate compressive strength of the stub column test} \]

\[ Q := \frac{P_{ua}}{A_{net.min} \cdot F_y} = 1.001 \]

Using Q RMI (see report from T. Pekoz, August 2015)

\[ Q := 1 \]

\[ P_{nl} := F_n \cdot \left[ 1 - (1 - Q) \cdot \left(\frac{F_n}{F_y}\right)^Q \right] \cdot A_g = [163.154] \text{ kN} \]

\[ P_{y.net} := A_{net.min} \cdot F_y = 202.664 \text{ kN} \]

\[ P_{nl} \leq P_{y.net} = [1] \]

\textbf{a.3 Distortional Buckling [E4.2]}

\[ P_{nd} \quad \text{nominal axial strength for distortional buckling} \]

\[ F_y = 370 \text{ MPa} \]

\[ A_{net} := 547.741 \text{ mm}^2 \]

\[ A_g = 694 \text{ mm}^2 \]

\[ P_y := A_g \cdot F_y = 256.78 \text{ kN} \quad \text{(Eq. E4.2-7)} \]

\[ P_{y.net} := A_{net} \cdot F_y = 202.664 \text{ kN} \quad \text{(Eq. E4.2-8)} \]
Using $P_{crd}$ calculated with CUFSM

\[ \text{Load} := 1000 \text{ N} \]

\[ \text{load.factor} := 258.5945 \]

\[ P_{crd} := \text{Load} \cdot \text{load.factor} = 258.595 \text{kN} \]

Critical elastic distortional column buckling load determined by analysis in CUFSM, for reduced thickness, with non-reduced elastic modulus

\[ \lambda_d := \sqrt{\frac{P_y}{P_{crd}}} = 0.996 \quad \text{slenderness} \quad (\text{Eq. E4.2-3}) \]

\[ \lambda_{d2} := 0.561 \cdot \left( 14 \cdot \left( \frac{P_y}{P_{y\text{net}}} \right)^{0.4} - 13 \right) = 1.341 \quad (\text{Eq. E4.2-5}) \]

\[ \lambda_d \leq \lambda_{d2} = 1 \]

will use subchapter E4.2 (members with holes)

\[ P_{y\text{net}} := A_{\text{net}} \cdot F_y = 202.664 \text{kN} \quad (\text{Eq. E4.2-8}) \]

\[ \lambda_{d1} := 0.561 \cdot \left( \frac{P_{y\text{net}}}{P_y} \right) = 0.443 \quad (\text{Eq. E4.2-4}) \]

\[ P_{d2} := \left( 1 - 0.25 \cdot \left( \frac{1}{\lambda_{d2}} \right)^{1.2} \right) \cdot \left( \frac{1}{\lambda_{d2}} \right)^{1.2} \cdot P_y = 148.841 \text{kN} \quad (\text{Eq. E4.2-6}) \]

\[ \lambda_{d1} < \lambda_d \leq \lambda_{d2} = 1 \quad (\text{Eq. E4.2-2}) \]

\[ P_{nd} := P_{y\text{net}} - \left( \frac{P_{y\text{net}} - P_{d2}}{\lambda_{d2} - \lambda_{d1}} \right) \cdot (\lambda_d - \lambda_{d1}) = 169.479 \text{kN} \]

\[ P_a := \min (P_{ne}, P_{nl}, P_{nd}) = 163.154 \text{kN} \]

b. Members in flexure (chapter F)

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and global buckling $M_{ne}$, the available flexural strength due to the interaction of the yielding or global buckling with local buckling $M_{nl}$ and the nominal flexural strength for distortional buckling $M_{nd}$.

b.1. Flexure about x-x (1-1) axis

b.1.1 Yielding and Global Buckling [F2]

\[ M_{crex} := 4.42029 \cdot 10^7 \text{ N}\cdot\text{mm} \quad \text{critical elastic bending moment from CUTWP, reduced thickness, non-reduced elastic modulus} \]

\[ S_{fx} := 18107 \text{ mm}^3 \quad \text{gross cross section modulus} \]

\[ F_{cre} := \frac{M_{crex}}{S_{fx}} = \left( 2.441 \cdot 10^3 \right) \text{MPa} \quad \text{critical elastic lateral-torsional buckling stress} \]
\[ F_{cre} \geq 2.78 \cdot F_y = 1 \]  
(Eq. F2.1-1)

\[ F_n := F_y = 370 \text{ MPa} \]

\[ M_{nex} := S_{fx} \cdot F_n = 6.7 \text{ kN} \cdot \text{m} \]

nominal axial strength for yielding and global buckling about the x-x (1-1) axis  
(Eq. F2.1-1)

b.1.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

As the Q cannot be determined I will use subchapter F3.2.2 of AISI S-100

\[ M_{nex} = 6.7 \text{ kN} \cdot \text{m} \]

\[ M_{crux} := 14869.8255 \cdot 1000 \text{ N} \cdot \text{mm} = 14.87 \text{ kN} \cdot \text{m} \]

critical elastic local buckling moment from CUFSM, reduced thickness (the same as for the global buckling), non-reduced elastic modulus

\[ \lambda = \sqrt{\frac{M_{nex}}{M_{crux}}} = 0.671 \]  
(Eq. F3.2.1-3)

\[ \lambda < 0.776 = 1 \]  
(Eq. F3.2.1-1)

\[ M_{nlx} := M_{nex} = 6.7 \text{ kN} \cdot \text{m} \]

\[ I_{xx} := 583565.0981 \text{ mm}^4 \]

moment of inertia about the x-x axis

\[ y_g := 37.1196 \text{ mm} \]

\[ S_{fnetx} := \frac{I_{xx}}{y_g} (1.572 \cdot 10^4) \text{ mm}^3 \]

net section modulus

\[ M_{ynetx} := S_{fnetx} \cdot F_y = 5.817 \text{ kN} \cdot \text{m} \]  
(Eq. F3.2.2-2)

\[ M_{nlx} \leq M_{ynetx} = 0 \]

so \[ M_{nlx} := M_{ynetx} = 5.817 \text{ kN} \cdot \text{m} \]

b.1.3 Distortional Buckling [F4.2]

\[ S_{fx} := 18107 \text{ mm}^3 \]

gross cross section modulus

\[ F_y = 370 \text{ MPa} \]

yield stress

\[ M_{yx} := S_{fx} \cdot F_y = 6.7 \text{ kN} \cdot \text{m} \]  
yield moment  
(Eq. F4.1-4)

\[ M_{crdx} := 10338.2261 \cdot 1000 \text{ N} \cdot \text{mm} = 10.338 \text{ kN} \cdot \text{m} \]

critical elastic distortional buckling moment about x-x axis from CUFSM, reduced thickness, reduced elastic modulus
\[ \lambda_d := \sqrt{\frac{M_{yx}}{M_{crdx}}} = 0.805 \quad \text{slenderness} \quad \text{(Eq. F4.1-3)} \]

\[ M_{nytx} = 5.817 \ \text{kN} \cdot \text{m} \]

\[ \lambda_{d_2} := 0.673 \cdot \left( 1.7 \cdot \left( \frac{M_{yx}}{M_{nytx}} \right)^{2.7} - 0.7 \right) = 1.204 \quad \text{(Eq. F4.2-5)} \]

\[ \lambda_d \leq \lambda_{d_2} = 1 \quad \text{will use subchapter F4.2 (members with holes)} \]

\[ \lambda_{d_1} := 0.673 \cdot \left( \frac{M_{nytx}}{M_{yx}} \right)^3 = 0.44 \quad \text{(Eq. F4.2-4)} \]

\[ \lambda_{d_1} < \lambda_d \leq \lambda_{d_2} = 1 \]

\[ M_{d_2} := \left( 1 - 0.22 \cdot \left( \frac{1}{\lambda_{d_2}} \right) \right) \cdot \left( \frac{1}{\lambda_{d_2}} \right) \cdot M_{yx} = 4.547 \ \text{kN} \cdot \text{m} \]

\[ M_{ndx} := M_{nytx} \cdot \left( \frac{M_{nytx} - M_{d_2}}{\lambda_{d_2} - \lambda_{d_1}} \right) \cdot (\lambda_d - \lambda_{d_1}) = 5.211 \ \text{kN} \cdot \text{m} \quad \text{(Eq. F4.2-2)} \]

\[ M_{ndx} \quad \text{nominal flexural strength about the x-x (1-1) axis} \quad \text{(Eq. F4.1-2)} \]

\[ M_{ax} := \min(M_{nex}, M_{nlex}, M_{ndx}) = 5.211 \ \text{kN} \cdot \text{m} \]

**b.2. Flexure about y-y (2-2) axis**

**b.2.1 Yielding and Global Buckling [F2]**

\[ M_{crey} := 1.35583 \cdot 10^7 \ \text{N} \cdot \text{mm} \quad \text{critical elastic bending moment from CUTWP, reduced thickness, non-reduced elastic modulus} \]

\[ S_{fy} := 13429 \ \text{mm}^3 \quad \text{gross cross section modulus} \]

\[ F_{cre} := \frac{M_{crey}}{S_{fy}} = 1.01 \cdot 10^3 \ \text{MPa} \quad \text{critical elastic lateral-torsional buckling stress} \]

\[ F_{cre} \geq 2.78 \cdot F_y = 0 \quad \text{(Eq. F2.1-1)} \]

\[ 2.78 \cdot F_y > F_{cre} \geq 0.56 \cdot F_y = 1 \]

\[ F_n := \frac{10}{9} \cdot F_y \cdot \left( 1 - \frac{10 \cdot F_y}{36 \cdot F_{cre}} \right) = 369.261 \ \text{MPa} \]

\[ M_{ney} := S_{fy} \cdot F_n = 4.959 \ \text{kN} \cdot \text{m} \quad \text{nominal axial strength for yielding and global buckling about the y-y (2-2) axis} \quad \text{(Eq. F2.1-1)} \]
b.2.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

\[ M_{ney} = 4.959 \text{ kN} \cdot \text{m} \]

\[ M_{crly} = 6000.4396 \cdot 1000 \text{ N} \cdot \text{mm} = 6 \text{ kN} \cdot \text{m} \]

Critical elastic local buckling moment from CUFSM, reduced thickness, reduced elastic modulus

\[ \lambda_t = \sqrt{\frac{M_{ney}}{M_{crly}}} = 0.909 \]

\[ \lambda_t > 0.776 = 1 \]

\[ M_{nly} := \left(1 - 0.15 \cdot \left(\frac{M_{crly}}{M_{ney}}\right)^0.4\right) \cdot \left(\frac{M_{crly}}{M_{ney}}\right)^0.4 \cdot M_{ney} = 4.485 \text{ kN} \cdot \text{m} \quad \text{(Eq. F3.21-1)} \]

\[ I_{yy} := 483242.9716 \text{ mm}^4 \]

Moment of inertia about the y-y axis

\[ x_g := 35.5852 \text{ mm} \]

\[ S_{fnetly} := \frac{I_{yy}}{x_g} = (1.358 \cdot 10^4) \text{ mm}^3 \]

Net section modulus

\[ M_{ynety} := S_{fnetly} \cdot F_y = 5.025 \text{ kN} \cdot \text{m} \quad \text{yield moment of net cross-section} \quad \text{(Eq. F3.2.2-2)} \]

\[ M_{nly} \leq M_{ynety} = 1 \]

b.2.3 Distortional Buckling [F4.2]

\[ S_{fy} = (1.343 \cdot 10^4) \text{ mm}^3 \]

Gross cross section modulus

\[ F_y = 370 \text{ MPa} \]

Yield stress

\[ M_{yy} := S_{fy} \cdot F_y = 4.969 \text{ kN} \cdot \text{m} \quad \text{yield moment} \quad \text{(Eq. F4.1-4)} \]

\[ M_{crdy} := 7399.1076 \cdot 1000 \text{ N} \cdot \text{mm} = 7.399 \text{ kN} \cdot \text{m} \]

Critical elastic distortional buckling moment about y-y axis from CUFSM, reduced thickness, non-reduced elastic modulus

\[ \lambda_d := \sqrt{\frac{M_{yy}}{M_{crdy}}} = 0.805 \quad \text{slenderness} \quad \text{(Eq. F4.1-3)} \]

\[ M_{ynely} = 5.025 \text{ kN} \cdot \text{m} \]

\[ \lambda_{d2} := 0.673 \cdot \left(1.7 \cdot \left(\frac{M_{yy}}{M_{ynely}}\right)^{2.7} - 0.7\right) = 0.639 \quad \text{(Eq. F4.2-5)} \]
\[ \lambda_d \leq d_2 = 0 \] will use subchapter F4.1 (members without holes)

\[ M_{ndy} := \left( 1 - 0.22 \cdot \left( \frac{M_{crdy}}{M_{yy}} \right)^{0.5} \right) \cdot \left( \frac{M_{crdy}}{M_{yy}} \right)^{0.5} \cdot M_{yy} = 4.436 \text{ kN} \cdot \text{m} \] (Eq. F4.2-4)

\[ M_{ndy} \] nominal flexural strength about the y-y (2-2) axis (Eq. F4.2-2)

\[ M_{ay} := \min (M_{ney}, M_{nly}, M_{ndy}) = 4.436 \text{ kN} \cdot \text{m} \]

**Verification:**

\[
\frac{P}{P_a} = 0.804
\]

\[
\frac{M_x}{M_{ax}} = 0.019
\]

\[
\frac{M_y}{M_{ay}} = 0.068
\]

\[
\frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} = 0.891
\]

### 4.2.2 Global, local+global, distortional + global buckling

**a. Members in compression (chapter E)**

The available axial strength \( P_a \) (factored resistance) shall be the smallest of the available axial strength \( F_n \) for global buckling, nominal axial strength \( P_{ni} \) for local buckling interacting with global buckling, and the nominal axial strength \( P_{nd} \) for distortional buckling.

**a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]**

\[ A_g := 694 \text{ mm}^2 \] gross cross-section area from Consteel

\[ F_y := 370 \text{ MPa} \] yield stress

\[ P_{cre} := 236979 \text{ N} \] minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP, for **reduced thickness**, using the **non-reduced elastic modulus**

\[ F_{cre} := \frac{P_{cre}}{A_g} = 341.468 \text{ MPa} \] least of the applicable elastic global(flexural, torsional and flexural torsional) buckling stress
\[
\lambda_c := \sqrt{\frac{F_y}{F_{cre}}} = 1.041 \quad \text{slenderness} \quad \text{(Eq. E2-4)}
\]
\[
\lambda_c \leq 1.5 = 1 \quad F_n := (0.658 \lambda_c^2) \cdot F_y = 235.093 \text{ MPa} \quad \text{(Eq. E2-2)}
\]
\[
F_n \quad \text{Compressive stress}
\]
\[
P_{ne} := A_g \cdot F_n = 163.154 \text{ kN} \quad \text{nominal axial strength} \quad \text{(Eq. E2-1)}
\]

**a.2 Local Buckling Interacting with Yielding and Global Buckling [E3.2]**

\[
P_{nl} \quad \text{nominal axial strength}
\]
\[
P_{ne} = 163.154 \text{ kN}
\]
\[
A_{net.min} := 547.741 \text{ mm}^2 \quad \text{minimum net cross section area}
\]
\[
F_n := \frac{P_{ne}}{A_g} = 235.093 \text{ MPa}
\]
\[
P_{ua} := 45.59 \text{ kip} = (2.028 \cdot 10^5) \text{ N} \quad \text{experimental ultimate compressive strength of the stub column test}
\]
\[
Q := \frac{P_{ua}}{A_{net.min} \cdot F_y} = 1.001
\]

Using Q RMI (see report from T. Pekoz, August 2015)

\[
Q := 1
\]
\[
P_{nl} := F_n \cdot \left[ 1 - (1 - Q) \cdot \left( \frac{F_n}{F_y} \right)^0 \right] \cdot A_g = \left[ 163.154 \right] \text{ kN}
\]

**a.3 Distortional Buckling interacting with Global Buckling [E4.2 and E2]**

For simulating the interaction of distortional and global buckling, the nominal axial strength for global buckling will be introduced in the calculation formulas of the distortional buckling, namely, instead of using \( P_y \) and \( P_{ynet} \) calculated with the yield stress \( F_y \), they will be calculated with \( F_n \), the compressive stress for global buckling.

\[
P_{nd} \quad \text{nominal axial strength for distortional buckling}
\]
\[
F_n = 235.093 \text{ MPa}
\]
\[
A_{net} := 547.741 \text{ mm}^2
\]
\[
A_g = 694 \text{ mm}^2
\]
\( P_{ne} := A_p \cdot F_n = 163.154 \text{ kN} \)  
(Eq. E4.2-7)  

\( P_{ne.net} := A_{net} \cdot F_n = 128.77 \text{ kN} \)  
(Eq. E4.2-8)  

Using \( P_{crd} \) calculated with CUFSM  

\( \text{Load} := 1000 \text{ N} \)  

\( \text{load.factor} := 258.595 \)  

\( P_{crd} := \text{Load} \cdot \text{load.factor} = 258.595 \text{ kN} \) critical elastic distortional column buckling load determined by analysis in CUFSM, for reduced thickness, with non-reduced elastic modulus  

\( \lambda_y := \sqrt{\frac{P_{ne}}{P_{crd}}} = 0.794 \) slenderness  
(Eq. E4.2-3)  

subchapter E4.1 will be used, for members without holes  

\( P_{nd} := \left(1 - 0.25 \cdot \left(\frac{P_{crd}}{P_{ne}}\right)^{0.6}\right) \cdot \left(\frac{P_{crd}}{P_{ne}}\right)^{0.6} \cdot P_{ne} = 144.199 \text{ kN} \)  

\( P_a := \min(P_{ne}, P_{nl}, P_{nd}) = 144.199 \text{ kN} \)  

**b. Members in flexure (chapter F)**  

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and global buckling \( M_{ne} \), the available flexural strength due to the interaction of the yielding or global buckling with local buckling \( M_{nl} \) and the nominal flexural strength for distortional buckling \( M_{nd} \).  

**b.1. Flexure about x-x (1-1) axis**  

**b.1.1 Yielding and Global Buckling [F2]**  

\( M_{crex} := 4.42029 \cdot 10^7 \text{ N} \cdot \text{mm} \) critical elastic bending moment from CUTWP, reduced thickness, non-reduced elastic modulus  

\( S_{fx} := 18107 \text{ mm}^3 \) gross cross section modulus  

\( F_{cre} := \frac{M_{crex}}{S_{fx}} = (2.441 \cdot 10^3) \text{ MPa} \) critical elastic lateral-torsional buckling stress
\[ F_{cre} \geq 2.78 \cdot F_y = 1 \]  
(Eq. F2.1-1)

\[ F_n := F_y = 370 \text{ MPa} \]

\[ M_{nex} := S_{fx} \cdot F_n = 6.7 \text{ kN} \cdot \text{m} \]
nominal axial strength for yielding and global buckling about the x-x (1-1) axis  
(Eq. F2.1-1)

b.1.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

As the Q cannot be determined I will use subchapter F3.2.2 of AISI S-100

\[ M_{nex} = 6.7 \text{ kN} \cdot \text{m} \]

\[ M_{crx} := 14869.8255 \cdot 1000 \text{ N} \cdot \text{mm} = 14.87 \text{ kN} \cdot \text{m} \]
critical elastic local buckling moment from CUFSM, reduced thickness (the same as for the global buckling), non-reduced elastic modulus

\[ \lambda_j := \sqrt{\frac{M_{nex}}{M_{crx}}} = 0.671 \]  
(Eq. F3.2.1-3)

\[ \lambda_j < 0.776 = 1 \]  
(Eq. F3.2.1-1)

\[ M_{nx} := M_{nex} = 6.7 \text{ kN} \cdot \text{m} \]

\[ l_{xx} := 583565.0981 \text{ mm}^4 \]  
moment of inertia about the x-x axis

\[ y_g := 37.1196 \text{ mm} \]

\[ S_{netx} := \frac{l_{xx}}{y_g} = (1.572 \cdot 10^4) \text{ mm}^3 \]
net section modulus

\[ M_{yntx} := S_{netx} \cdot F_y = 5.817 \text{ kN} \cdot \text{m} \]
yield moment of net cross-section  
(Eq. F3.2.2-2)

\[ M_{nx} \leq M_{yntx} = 0 \]
so

\[ M_{nx} := M_{yntx} = 5.817 \text{ kN} \cdot \text{m} \]

b.1.3 Distortional Buckling interacting with Global Buckling [F4.2 + F2]

\[ S_{fx} := 18107 \text{ mm}^3 \]  
gross cross section modulus

\[ F_n = 370 \text{ MPa} \]
yield stress

\[ M_{nex} := S_{fx} \cdot F_n = 6.7 \text{ kN} \cdot \text{m} \]
yield moment  
(Eq. F4.1-4)

\[ M_{crdx} := 10338.2261 \cdot 1000 \text{ N} \cdot \text{mm} = 10.338 \text{ kN} \cdot \text{m} \]
critical elastic distortional buckling moment about x-x axis from CUFSM, reduced thickness, non-reduced elastic modulus
\[ \lambda_d := \sqrt{\frac{M_{\text{nex}}}{M_{\text{crdx}}}} = 0.805 \quad \text{slenderness} \quad (\text{Eq. F4.1-3}) \]

\[ M_{\text{ne.netx}} := S_{\text{netx}} \cdot F_n = 5.817 \, kN \cdot m \]

\[ \lambda_{d2} := 0.673 \cdot \left( 1.7 \cdot \left( \frac{M_{\text{nex}}}{M_{\text{ne.netx}}} \right)^{2.7} - 0.7 \right) = 1.204 \quad (\text{Eq. F4.2-5}) \]

\[ \lambda_d \leq \lambda_{d2} = 1 \]

\[ \lambda_{d1} := 0.673 \cdot \left( \frac{M_{\text{ne.netx}}}{M_{\text{nex}}} \right)^3 = 0.44 \]

\[ \lambda_{d1} < \lambda_d \leq \lambda_{d2} = 1 \]

\[ M_{d2} := \left( 1 - 0.22 \cdot \left( \frac{1}{\lambda_{d2}} \right) \right) \cdot \left( \frac{1}{\lambda_{d2}} \right) \cdot M_{\text{nex}} = 4.547 \, m \cdot kN \]

\[ M_{ndx} := M_{\text{ne.netx}} - \left( \frac{M_{\text{ne.netx}} - M_{d2}}{\lambda_{d2} - \lambda_{d1}} \right) \cdot \left( \lambda_d - \lambda_{d1} \right) = 5.211 \, kN \cdot m \]

\[ M_{ndx} \leq \left( 1 - 0.22 \cdot \left( \frac{M_{\text{crdx}}}{M_{\text{nex}}} \right)^{0.5} \right) \cdot \left( \frac{M_{\text{crdx}}}{M_{\text{nex}}} \right)^{0.5} \cdot M_{\text{nex}} = 1 \]

\[ M_{\text{nex}} \quad \text{nominal flexural strength about the x-x (1-1) axis} \quad (\text{Eq. F4.1-2}) \]

\[ M_{\text{ax}} := \min \left( M_{\text{nex}}, M_{\text{nlx}}, M_{\text{ndx}} \right) = 5.211 \, kN \cdot m \]

b.2. Flexure about y-y (2-2) axis

b.2.1 Yielding and Global Buckling [F2]

\[ M_{\text{crey}} := 1.35583 \cdot 10^7 \, N \cdot mm \quad \text{critical elastic bending moment from CUTWP, reduced thickness, non-reduced elastic modulus} \]

\[ S_{fy} := 13429 \, mm^3 \quad \text{gross cross section modulus} \]

\[ F_{\text{cre}} := \frac{M_{\text{crey}}}{S_{fy}} = (1.01 \cdot 10^3) \, MPa \quad \text{critical elastic lateral-torsional buckling stress} \]

\[ F_{\text{cre}} \geq 2.78 \cdot F_y = 0 \quad (\text{Eq. F2.1-1}) \]

\[ 2.78 \cdot F_y > F_{\text{cre}} \geq 0.56 \cdot F_y = 1 \]

\[ F_n := \frac{10}{9} \cdot F_y \cdot \left( 1 - \frac{10 \cdot F_y}{36 \cdot F_{\text{cre}}} \right) = 369.261 \, MPa \]
\[ M_{\text{ney}} := S_{y} \cdot F_{n} = 4.959 \, kN \cdot m \]  
nominal axial strength for yielding and global buckling about the y-y (2-2) axis  
(Eq. F2.1-1)

b.2.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

\[ M_{\text{ney}} = 4.959 \, kN \cdot m \]

\[ M_{\text{cly}} := 6000.4396 \cdot 1000 \, N \cdot mm = 6 \, kN \cdot m \]  
critical elastic local buckling moment from CUFSM, reduced thickness, reduced elastic modulus

\[ \lambda_{l} := \sqrt{\frac{M_{\text{ney}}}{M_{\text{cly}}}} = 0.909 \]

\[ \lambda_{l} > 0.776 = 1 \]

\[ M_{\text{ny}} := \left(1 - 0.15 \cdot \left(\frac{M_{\text{cly}}}{M_{\text{ney}}}\right)^{0.4}\right) \cdot \left(\frac{M_{\text{cly}}}{M_{\text{ney}}}\right)^{0.4} \cdot M_{\text{ney}} = 4.485 \, kN \cdot m \]  
(Eq. F3.21-1)

\[ I_{yy} := 483242.9716 \, mm^4 \]  
moment of inertia about the y-y axis

\[ x_{g} := 35.5852 \, mm \]

\[ S_{\text{fnety}} := \frac{I_{yy}}{x_{g}} = (1.358 \cdot 10^4) \, mm^3 \]  
net section modulus

\[ M_{\text{fnety}} := S_{\text{fnety}} \cdot F_{y} = 5.025 \, kN \cdot m \]  
yield moment of net cross-section  
(Eq. F3.2.2-2)

\[ M_{\text{fny}} \leq M_{\text{fnety}} = 1 \]

b.2.3 Distortional Buckling + Global Buckling[F4.2]

\[ S_{fy} = (1.343 \cdot 10^4) \, mm^3 \]  
gross cross section modulus

\[ F_{n} = 369.261 \, MPa \]  
yield stress

\[ M_{\text{ney}} := S_{fy} \cdot F_{n} = 4.959 \, kN \cdot m \]  
yield moment  
(Eq. F4.1-4)

\[ M_{\text{crdx}} := 7399.1076 \cdot 1000 \, N \cdot mm = 7.399 \, kN \cdot m \]  
critical elastic distortional buckling moment about y-y axis from CUFSM, reduced thickness, non-reduced elastic modulus

\[ \lambda_{d} := \sqrt{\frac{M_{\text{ney}}}{M_{\text{crdx}}}} = 0.693 \]  
slenderness  
(Eq. F4.1-3)

\[ M_{\text{ne.nety}} := S_{\text{fnety}} \cdot F_{n} = 5.015 \, kN \cdot m \]
\[ \lambda_{d2} := 0.673 \cdot \left( 1.7 \cdot \left( \frac{M_{ney}}{M_{ne,nety}} \right)^{2.7} - 0.7 \right) = 0.639 \]  
(Eq. F4.2-5)

\[ \lambda_d \leq \lambda_{d2} = 0 \]  
will use subchapter F4.1 (members without holes)

\[ M_{ndy} := \left( 1 - 0.22 \cdot \left( \frac{M_{crdy}}{M_{rey}} \right)^{0.5} \right) \cdot \left( \frac{M_{crdy}}{M_{rey}} \right)^{0.5} \cdot M_{rey} = 4.429 \text{ kN \cdot m} \]  
(Eq. F4.2-4)

\[ M_{ndy} \]  
nominal flexural strength about the y-y (2-2) axis  
(Eq. F4.2-2)

\[ M_{sy} := \min (M_{rey}, M_{ply}, M_{ndy}) = 4.429 \text{ kN \cdot m} \]

Verification:

\[ \frac{P}{P_a} = 0.91 \]

\[ \frac{M_x}{M_{ax}} = 0.019 \]

\[ \frac{M_y}{M_{ay}} = 0.068 \]

\[ \frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} = 0.997 \]

4.2.3 Local, distortional buckling

a. Members in compression (chapter E)

The available axial strength \( P_a \) (factored resistance) shall be the smallest of nominal axial strength \( P_{nl} \) for local buckling and the nominal axial strength \( P_{nd} \) for distortional buckling.

\[ A_g := 694 \text{ mm}^2 \]  
gross cross-section area from Consteel

\[ F_y := 370 \text{ MPa} \]  
yield stress

a.1 Local Buckling [E3.2]

For local buckling, the nominal axial strength is the axial strength from the stub column test.

\[ P_{nl} \]  
nominal axial strength
\[ P_{ua} = 45.59 \text{ kip} = (2.028 \cdot 10^5) \text{ N} \] experimental ultimate compressive strength of the stub column test

\[ P_{nl} = P_{ua} = 202.794 \text{ kN} \]

\[ A_{net.min} = 547.741 \text{ mm}^2 \] minimum net cross section area

\[ P_{ynet} = A_{net.min} \cdot F_y = 202.664 \text{ kN} \]

\[ P_{nl} < P_{ynet} = 0 \quad \text{Eq. 3.2.2-1) } \]

\[ P_{nl} = P_{ynet} = 202.664 \text{ kN} \]

### b.2 Distortional Buckling [E4.2]

- **Nominal Axial Strength for Distortional Buckling**
  
  \[ F_y = 370 \text{ MPa} \]

- **Minimum Net Cross Section Area**
  
  \[ A_{net} = 547.741 \text{ mm}^2 \]

- **Axial Cross Section Area**
  
  \[ A_g = 694 \text{ mm}^2 \]

- **Axial Force**
  
  \[ P_y = A_g \cdot F_y = 256.78 \text{ kN} \quad \text{(Eq. E4.2-7)} \]

- **Minimum Net Axial Force**
  
  \[ P_{ynet} = A_{net} \cdot F_y = 202.664 \text{ kN} \quad \text{(Eq. E4.2-8)} \]

Using \( P_{crd} \) calculated with CUFSM

- **Load**
  
  \[ Load = 1000 \text{ N} \]

- **Load Factor**
  
  \[ load.factor = 258.5945 \]

\[ P_{crd} = Load \cdot load.factor = 258.595 \text{ kN} \] critical elastic distortional column buckling load determined by analysis in CUFSM, for reduced thickness, with non-reduced elastic modulus

\[ \lambda_d = \sqrt{\frac{P_y}{P_{crd}}} = 0.996 \] slenderness \quad \text{(Eq. E4.2-3)}

\[ \lambda_{d2} = 0.561 \cdot \left(14 \cdot \left( \frac{P_y}{P_{ynet}} \right)^{0.4} - 13 \right) = 1.341 \] \quad \text{(Eq. E4.2-5)}

\[ \lambda_d \leq \lambda_{d2} = 1 \] will use subchapter E4.2 (members with holes)

\[ P_{ynet} = A_{net} \cdot F_y = 202.664 \text{ kN} \quad \text{(Eq. E4.2-8)} \]

\[ \lambda_{d1} = 0.561 \cdot \left( \frac{P_{ynet}}{P_y} \right) = 0.443 \] \quad \text{(Eq. E4.2-4)}
\[
P_{d2} = \left( 1 - 0.25 \cdot \left( \frac{1}{\lambda_{d2}} \right)^{1.2} \right) \cdot \left( \frac{1}{\lambda_{d2}} \right)^{1.2} \cdot P_y = 148.841 \text{kN} \quad \text{(Eq. E4.2-6)}
\]

\[\lambda_{d1} < \lambda_d \leq \lambda_{d2} = 1\]

\[
P_{nd} = P_{y\text{net}} - \left( \frac{P_{y\text{net}} - P_{d2}}{\lambda_{d2} - \lambda_{d1}} \right) \cdot (\lambda_d - \lambda_{d1}) = 169.479 \text{kN} \quad \text{(Eq. E4.2-2)}
\]

\[
P_a = \min (P_{nl}, P_{nd}) = 169.479 \text{kN}
\]

**b. Members in flexure (chapter F)**

The available flexural strength shall be the smallest of the nominal flexural strength due to the local buckling \(M_{nl}\) and the nominal flexural strength for distortional buckling \(M_{nd}\).

**b.1. Flexure about x-x (1-1) axis**

\[
S_{fx} = 18107 \text{mm}^3 \quad \text{gross cross section modulus}
\]

**b.1.1 Local Buckling [F3.2.2]**

As the \(Q\) cannot be determined I will use subchapter F3.2.2 of AISI S-100

\[
M_{yx} = S_{fx} \cdot F_y = 6.7 \text{kN} \cdot \text{m}
\]

\[
M_{crux} = 14869.8255 \cdot 1000 \text{N} \cdot \text{mm} = 14.87 \text{kN} \cdot \text{m} \quad \text{critical elastic local buckling moment from CUFSM, reduced thickness, non-reduced elastic modulus}
\]

\[\lambda_i = \sqrt{ \frac{M_{yx}}{M_{crux}} } = 0.671 \quad \text{(Eq. F3.2.1-3)}
\]

\[\lambda_i < 0.776 = 1 \quad \text{(Eq. F3.2.1-1)}
\]

\[
M_{nlx} = M_{yx} = 6.7 \text{kN} \cdot \text{m}
\]

\[
l_{xx} = 583565.0981 \text{mm}^4 \quad \text{moment of inertia about the x-x axis}
\]

\[y_g = 37.1196 \text{mm}
\]

\[
S_{fnex} = \frac{l_{xx}}{y_g} = (1.572 \cdot 10^4) \text{mm}^3 \quad \text{net section modulus}
\]

\[
M_{y\text{netx}} = S_{fnex} \cdot F_y = 5.817 \text{kN} \cdot \text{m} \quad \text{yield moment of net cross-section (Eq. F3.2.2-2)}
\]
b.1.2 Distortional Buckling [F4.2]

\[ S_{fx} := 18107 \text{ mm}^3 \quad \text{gross cross section modulus} \]
\[ F_y = 370 \text{ MPa} \quad \text{yield stress} \]
\[ M_{yx} := S_{fx} \cdot F_y = 6.7 \text{ kN} \cdot \text{m} \quad \text{yield moment} \quad \text{(Eq. F4.1-4)} \]
\[ M_{crdx} := 10338.2261 \cdot 1000 \text{ N} \cdot \text{mm} = 10.338 \text{ kN} \cdot \text{m} \quad \text{critical elastic distortional buckling moment about x-x axis from CUFSM, reduced thickness, reduced elastic modulus} \]
\[ \lambda_d := \sqrt{\frac{M_{yx}}{M_{crdx}}} = 0.805 \quad \text{slenderness} \quad \text{(Eq. F4.1-3)} \]
\[ M_{ynetx} = 5.817 \text{ kN} \cdot \text{m} \]
\[ \lambda_{d2} := 0.673 \cdot \left(1.7 \cdot \left(\frac{M_{yx}}{M_{ynetx}}\right)^{2.7} - 0.7 \right) = 1.204 \quad \text{(Eq. F4.2-5)} \]
\[ \lambda_d \leq \lambda_{d2} = 1 \quad \text{will use subchapter F4.2 (members with holes)} \]
\[ \lambda_{d1} := 0.673 \cdot \left(\frac{M_{ynetx}}{M_{yx}}\right)^3 = 0.44 \quad \text{(Eq. F4.2-4)} \]
\[ \lambda_{d1} < \lambda_d \leq \lambda_{d2} = 1 \]
\[ M_{d2} := \left(1 - 0.22 \cdot \left(\frac{1}{\lambda_{d2}}\right)\right) \cdot \left(\frac{1}{\lambda_{d2}}\right) \cdot M_{yx} = 4.547 \text{ kN} \cdot \text{m} \]
\[ M_{ndx} := M_{ynetx} - \left(\frac{M_{ynetx} - M_{d2}}{\lambda_{d2} - \lambda_{d1}}\right) \cdot (\lambda_d - \lambda_{d1}) = 5.211 \text{ kN} \cdot \text{m} \quad \text{(Eq. F4.2-2)} \]
\[ M_{ndx} \quad \text{nominal flexural strength about the x-x (1-1) axis} \quad \text{(Eq. F4.1-2)} \]
\[ M_{ax} := \min (M_{nx}, M_{ndx}) = 5.211 \text{ kN} \cdot \text{m} \]
b.2. Flexure about y-y (2-2) axis

\[ S_{fy} := 13429 \text{ mm}^3 \] gross cross section modulus

b.2.1 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

\[ M_{yy} := S_{fy} \cdot F_y = 4.969 \text{ kN} \cdot \text{m} \]

\[ M_{cly} := 6000.4396 \cdot 1000 \text{ N} \cdot \text{mm} = 6 \text{ kN} \cdot \text{m} \] critical elastic local buckling moment from CUFSM, reduced thickness, reduced elastic modulus

\[ \lambda_l := \sqrt{\frac{M_{yy}}{M_{cly}}} = 0.91 \]

\[ \lambda_l > 0.776 = 1 \]

\[ M_{nly} := \left( 1 - 0.15 \cdot \left( \frac{M_{cly}}{M_{yy}} \right)^{0.4} \right) \cdot \left( \frac{M_{cly}}{M_{yy}} \right)^{0.4} \cdot M_{yy} = 4.491 \text{ kN} \cdot \text{m} \] (Eq. F3.21-1)

\[ l_{yy} := 483242.9716 \text{ mm}^4 \] moment of inertia about the y-y axis

\[ x_g := 35.5852 \text{ mm} \]

\[ S_{fny} := \frac{l_{yy}}{x_g} = \left( 1.358 \cdot 10^4 \right) \text{ mm}^3 \] net section modulus

\[ M_{nly} := S_{fny} \cdot F_y = 5.025 \text{ kN} \cdot \text{m} \] yield moment of net cross-section (Eq. F3.2.2-2)

\[ M_{nly} \leq M_{nly} = 1 \]

b.2.2 Distortional Buckling [F4.2]

\[ S_{fy} = \left( 1.343 \cdot 10^4 \right) \text{ mm}^3 \] gross cross section modulus

\[ F_y = 370 \text{ MPa} \] yield stress

\[ M_{yy} := S_{fy} \cdot F_y = 4.969 \text{ kN} \cdot \text{m} \] yield moment (Eq. F4.1-4)

\[ M_{cly} := 7399.1076 \cdot 1000 \text{ N} \cdot \text{mm} = 7.399 \text{ kN} \cdot \text{m} \] critical elastic distortional buckling moment about y-y axis from CUFSM, reduced thickness, non-reduced elastic modulus

\[ \lambda_d := \sqrt[4]{\frac{M_{yx}}{M_{cly}}} = 0.805 \] slenderness (Eq. F4.1-3)
\[ M_{\text{yndy}} = 5.025 \, kN \cdot m \]

\[ \lambda_{d2} := 0.673 \cdot \left( 1.7 \cdot \left( \frac{M_{yy}}{M_{\text{yndy}}} \right)^{2.7} - 0.7 \right) = 0.639 \quad \text{(Eq. F4.2-5)} \]

\[ \lambda_d \leq \lambda_{d2} = 0 \quad \text{will use subchapter F4.1 (members without holes)} \]

\[ M_{\text{ndy}} := \left( 1 - 0.22 \cdot \left( \frac{M_{\text{crdy}}}{M_{yy}} \right)^{0.5} \right) \cdot \left( \frac{M_{\text{crdy}}}{M_{yy}} \right)^{0.5} \cdot M_{yy} = 4.436 \, kN \cdot m \quad \text{(Eq. F4.2-4)} \]

\[ M_{\text{ndy}} \quad \text{nominal flexural strength about the y-y (2-2) axis} \quad \text{(Eq. F4.2-2)} \]

\[ M_{ay} := \min (M_{\text{ply}}, M_{\text{ndy}}) = 4.436 \, kN \cdot m \]

**Verification:**

\[ \frac{P}{P_a} = 0.774 \]

\[ \frac{M_x}{M_{ax}} = 0.019 \]

\[ \frac{M_y}{M_{ay}} = 0.068 \]

\[ \frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} = 0.861 \]
4.2.4 Global, local+global, distortional buckling, using only Anet.min

a. Members in compression (chapter E)

The available axial strength $P_a$ (factored resistance) shall be the smallest of the available axial strength $F_n$ for global buckling, nominal axial strength $P_{nl}$ for local buckling interacting with global buckling, and the nominal axial strength $P_{nd}$ for distortional buckling.

a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]

$$ A_{net\text{.min}} := 547.741 \text{ mm}^2 $$
minimum net cross section area

$$ F_y := 370 \text{ MPa} $$
yield stress

$$ P_{cre} := 236979 \text{ N} $$
minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP, for reduced thickness, using the non-reduced elastic modulus

$$ F_{cre} := \frac{P_{cre}}{A_{net\text{.min}}} = 432.648 \text{ MPa} $$
least of the applicable elastic global(flexural, torsional and flexural torsional) buckling stress

$$ \lambda_c := \sqrt{\frac{F_y}{F_{cre}}} = 0.925 $$
slenderness (Eq. E2-4)

$$ \lambda_c \leq 1.5 = 1 $$

$$ F_n := \left(0.658\lambda_c^2\right) \cdot F_y = 258.672 \text{ MPa} $$ (Eq. E2-2)

$$ F_n $$
Compressive stress

$$ P_{ne} := A_{net\text{.min}} \cdot F_n = 141.685 \text{ kN} $$
nominal axial strength (Eq. E2-1)

b.2 Local Buckling Interacting with Yielding and Global Buckling [E3.2]

$$ P_{nl} $$
nominal axial strength

$$ P_{ne} = 141.685 \text{ kN} $$

$$ F_n := \frac{P_{ne}}{A_{net\text{.min}}} = 258.672 \text{ MPa} $$

$$ P_{ua} := 45.59 \text{ kip} = \left(2.028 \cdot 10^5\right) \text{ N} $$
experimental ultimate compressive strength of the stub column test

$$ Q := \frac{P_{ua}}{A_{net\text{.min}} \cdot F_y} = 1.001 $$

Using Q RMI (see report from T. Pekoz, August 2015)

$$ Q := 1 $$
\[ P_{nl} := F_n \left[ 1 - (1 - Q) \cdot \left( \frac{F_n}{F_y} \right)^n \right] \cdot A_{net, min} = [141.685] \text{ kN} \]

b.3 Distortional Buckling [E4.2]

\[ P_{nd} \] nominal axial strength for distortional buckling

\[ F_y = 370 \text{ MPa} \]

\[ A_{net, min} := 547.741 \text{ mm}^2 \]

\[ P_y := A_{net, min} \cdot F_y = 202.664 \text{ kN} \quad \text{(Eq. E4.2-7)} \]

Using \( P_{crd} \) calculated with CUFSM

\[ \text{Load} := 1000 \text{ N} \]

\[ \text{load.factor} := 258.5945 \]

\[ P_{crd} := \text{Load} \cdot \text{load.factor} = 258.595 \text{ kN} \] critical elastic distortional column buckling load determined by analysis in CUFSM, for reduced thickness, with non-reduced elastic modulus

\[ \lambda_d := \sqrt{\frac{P_y}{P_{crd}}} = 0.885 \] slenderness \quad \text{(Eq. E4.2-3)}

\[ \lambda_d > 0.561 = 1 \]

\[ P_{nd} := \left( 1 - 0.25 \cdot \left( \frac{P_{crd}}{P_y} \right)^{0.6} \right) \cdot \left( \frac{P_{crd}}{P_y} \right)^{0.6} \cdot P_y = 166.698 \text{ kN} \quad \text{(Eq. E4.1-2)} \]

\[ P_a := \min (P_{ne}, P_{nl}, P_{nd}) = 141.685 \text{ kN} \]

b. Members in flexure (chapter F)

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and global buckling \( M_{ne} \), the available flexural strength due to the interaction of the yielding or global buckling with local buckling \( M_{nl} \) and the nominal flexural strength for distortional buckling \( M_{nd} \).

b.1. Flexure about x-x (1-1) axis

b.1.1 Yielding and Global Buckling [F2]

\[ M_{crex} := (4.42029 \cdot 10^7) \text{ N-mm} \] critical elastic bending moment from CUTWP, reduced thickness, non-reduced elastic modulus

\[ I_{xx} := 583565.0981 \text{ mm}^4 \] moment of inertia about the x-x axis

\[ y_g := 37.1196 \text{ mm} \]
\[ S_{\text{netx}} = \frac{l_{xx}}{y_g} = (1.572 \cdot 10^4) \text{ mm}^3 \]

net section modulus

\[ F_{\text{cre}} := \frac{M_{\text{crex}}}{S_{\text{netx}}} = (2.812 \cdot 10^3) \text{ MPa} \]
critical elastic lateral-torsional buckling stress

\[ F_{\text{cre}} \geq 2.78 \cdot F_y = 1 \]  
(Eq. F2.1-1)

\[ F_n := F_y = 370 \text{ MPa} \]

nominal axial strength for yielding

\[ M_{\text{nex}} := S_{\text{netx}} \cdot F_n = 5.817 \text{ kN \cdot m} \]

and global buckling about the x-x (1-1) axis  
(Eq. F2.1-1)

b.1.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

As the Q cannot be determined I will use subchapter F3.2.2 of AISI S-100

\[ M_{\text{nex}} = 5.817 \text{ kN \cdot m} \]

\[ M_{\text{crdx}} := 14869.8255 \cdot 1000 \text{ N \cdot mm} = 14.87 \text{ kN \cdot m} \]
critical elastic local buckling moment from CUFSM, reduced thickness (the same as for the global buckling), non-reduced elastic modulus

\[ \lambda_i := \sqrt{\frac{M_{\text{nex}}}{M_{\text{crdx}}}} = 0.625 \]  
(Eq. F3.2.1-3)

\[ \lambda_i < 0.776 = 1 \]  
(Eq. F3.2.1-1)

\[ M_{\text{ndx}} := M_{\text{nex}} = 5.817 \text{ kN \cdot m} \]

b.1.3 Distortional Buckling [F4.2]

\[ F_y = 370 \text{ MPa} \]
yield stress

\[ M_{\text{yx}} := S_{\text{netx}} \cdot F_y = 5.817 \text{ kN \cdot m} \]
yield moment  
(Eq. F4.1-4)

\[ M_{\text{crdx}} := 10338.2261 \cdot 1000 \text{ N \cdot mm} = 10.338 \text{ kN \cdot m} \]
critical elastic distortional buckling moment about x-x axis from CUFSM, reduced thickness, reduced elastic modulus

\[ \lambda_d := \sqrt{\frac{M_{\text{yx}}}{M_{\text{crdx}}}} = 0.75 \]

slenderness  
(Eq. F4.1-3)

\[ \lambda_d > 0.673 = 1 \]

\[ M_{\text{ndx}} := \left(1 - 0.22 \cdot \left(\frac{M_{\text{crdx}}}{M_{\text{yx}}}\right)^{0.5}\right) \cdot \left(\frac{M_{\text{crdx}}}{M_{\text{yx}}}\right)^{0.5} \cdot M_{\text{yx}} = 5.48 \text{ kN \cdot m} \]
**M_{ndx}**: nominal flexural strength about the x-x (1-1) axis \(\text{(Eq. F4.1-2)}\)

\[ M_{ax} := \min (M_{nex}, M_{nlx}, M_{ndx}) = 5.48 \text{ kN} \cdot \text{m} \]

**b.2. Flexure about y-y (2-2) axis**

**b.2.1 Yielding and Global Buckling [F2]**

\[ M_{crey} := (1.35583 \cdot 10^7) \text{ N} \cdot \text{mm} \]

Critical elastic bending moment from CUTWP, reduced thickness, non-reduced elastic modulus

\[ I_{yy} := 483242.9716 \text{ mm}^4 \]

Moment of inertia about the y-y axis

\[ x_g := 35.5852 \text{ mm} \]

\[ S_{fnty} := \frac{I_{yy}}{x_g} = (1.358 \cdot 10^4) \text{ mm}^3 \]

Net section modulus

\[ F_{cre} := \frac{M_{crey}}{S_{fnty}} = 998.41 \text{ MPa} \]

Critical elastic lateral-torsional buckling stress

\[ F_{cre} \geq 2.78 \cdot F_y = 0 \]

(Eq. F2.1-1)

\[ 2.78 \cdot F_y > F_{cre} \geq 0.56 \cdot F_y = 1 \]

\[ F_n := \frac{10}{9} \cdot F_y \cdot \left( 1 - \frac{10 \cdot F_y}{36 \cdot F_{cre}} \right) = 368.791 \text{ MPa} \]

\[ M_{neg} := S_{fnty} \cdot F_n = 5.008 \text{ kN} \cdot \text{m} \]

Nominal axial strength for yielding and global buckling about the y-y (2-2) axis \(\text{(Eq. F2.1-1)}\)

**b.2.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]**

\[ M_{neg} = 5.008 \text{ kN} \cdot \text{m} \]

Critical elastic local buckling moment from CUFSM, reduced thickness, non-reduced elastic modulus

\[ \lambda := \sqrt{\frac{M_{neg}}{M_{cly}}} = 0.914 \]

\[ \lambda > 0.776 = 1 \]

\[ A_{cly} := \left( 1 - 0.15 \cdot \left( \frac{M_{cly}}{M_{neg}} \right)^{0.4} \right) \cdot \left( \frac{M_{cly}}{M_{neg}} \right)^{0.4} \cdot M_{neg} = 4.516 \text{ kN} \cdot \text{m} \]

(Eq. F3.21-1)
\[ M_{\text{nety}} := S_{\text{nety}} \cdot F_Y = 5.025 \, \text{kN} \cdot \text{m} \] (Eq. F3.2.2-2)

\[ M_{\text{nety}} \leq M_{\text{nety}} = 1 \]

**b.2.3 Distortional Buckling [F4.2]**

\[ F_Y = 370 \, \text{MPa} \]

\[ M_{yy} := S_{\text{nety}} \cdot F_Y = 5.025 \, \text{kN} \cdot \text{m} \] (Eq. F4.1-4)

\[ M_{\text{crdy}} := 7399.1076 \times 1000 \, \text{N} \cdot \text{mm} = 7.399 \, \text{kN} \cdot \text{m} \]

Critical elastic distortional buckling moment about y-y axis from CUFSM, reduced thickness, non-reduced elastic modulus

\[ \lambda_d := \sqrt{\frac{M_{yx}}{M_{\text{crdx}}}} = 0.75 \] slenderness (Eq. F4.1-3)

\[ \lambda_d > 0.673 = 1 \]

\[ M_{ndy} := \left(1 - 0.22 \cdot \left(\frac{M_{\text{crdy}}}{M_{yy}}\right)^{0.5}\right) \cdot \left(\frac{M_{\text{crdy}}}{M_{yy}}\right)^{0.5} \cdot M_{yy} = 4.47 \, \text{kN} \cdot \text{m} \] (Eq. F4.2-4)

\[ M_{\text{ndy}} \] nominal flexural strength about the y-y (2-2) axis (Eq. F4.2-2)

\[ M_{ay} := \min \{ M_{\text{neq}}, M_{\text{nly}}, M_{\text{ndy}}\} = 4.47 \, \text{kN} \cdot \text{m} \]

**Verification:**

\[ \frac{P}{P_a} = 0.926 \]

\[ \frac{M_x}{M_{ax}} = 0.018 \]

\[ \frac{M_y}{M_{ay}} = 0.067 \]

\[ \frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} = 1.011 \]
4.2.5 Global, local+global buckling

a. Members in compression (chapter E)

The available axial strength $P_a$ (factored resistance) shall be the smallest of the available axial strength $F_n$ for global buckling, nominal axial strength $P_{nl}$ for local buckling interacting with global buckling.

a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]

$$A_g := 694 \text{ mm}^2$$  gross cross-section area from Consteel

$$F_y := 370 \text{ MPa}$$  yield stress

$$P_{cre} := 236979 \text{ N}$$  minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP, for reduced thickness, using the non-reduced elastic modulus

$$F_{cre} := \frac{P_{cre}}{A_g} = 341.468 \text{ MPa}$$  least of the applicable elastic global(flexural, torsional and flexural-torsional) buckling stress

$$\lambda_c := \sqrt{\frac{F_y}{F_{cre}}} = 1.041$$  slenderness  

(Eq. E2-4)

$$\lambda_c \leq 1.5 = 1 \quad F_n := (0.658 \cdot \lambda_c^2) \cdot F_y = 235.093 \text{ MPa} \quad \text{(Eq. E2-2)}$$

$F_n$  Compressive stress

$$P_{ne} := A_g \cdot F_n = 163.154 \text{ kN}$$  nominal axial strength  

(Eq. E2-1)

a.2 Local Buckling Interacting with Yielding and Global Buckling [E3.2]

$$P_{nl}$$  nominal axial strength

$$P_{ne} = 163.154 \text{ kN}$$

$$A_{net.min} := 547.741 \text{ mm}^2$$  minimum net cross section area

$$F_n := \frac{P_{ne}}{A_g} = 235.093 \text{ MPa}$$

$$P_{ua} := 45.59 \text{ kip} = (2.028 \cdot 10^5) \text{ N}$$  experimental ultimate compressive strength of the stub column test

$$Q := \frac{P_{ua}}{A_{net.min} \cdot F_y} = 1.001$$

Using Q RMI (see report from T. Pekoz, August 2015)

$$Q := 1$$
\[ P_{nl} := F_n \left[ 1 - (1 - Q) \cdot \left( \frac{F_n}{F_y} \right)^Q \right] \cdot A_g = [163.154] \, kN \]

\[ P_a := \min(P_{ne}, P_{nl}) = 163.154 \, kN \]

b. Members in flexure (chapter F)

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and global buckling, the available flexural strength due to the interaction of the yielding or global buckling with local buckling \( M_{nl} \).

b.1. Flexure about x-x (1-1) axis

b.1.1 Yielding and Global Buckling [F2]

\[ M_{crex} := 4.42029 \cdot 10^7 \, N \cdot mm \] critical elastic bending moment from CUTWP, reduced thickness, non-reduced elastic modulus

\[ S_{fx} := 18107 \, mm^3 \] gross cross section modulus

\[ F_{cre} := \frac{M_{crex}}{S_{fx}} = (2.441 \cdot 10^3) \, MPa \] critical elastic lateral-torsional buckling stress

\[ F_{cre} \geq 2.78 \cdot F_y = 1 \] (Eq. F2.1-1)

\[ F_n := F_y = 370 \, MPa \]

\[ M_{nex} := S_{fx} \cdot F_n = 6.7 \, kN \cdot m \] nominal axial strength for yielding and global buckling about the x-x (1-1) axis (Eq. F2.1-1)

b.1.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

As the Q cannot be determined I will use subchapter F3.2.2 of AISI S-100

\[ M_{nex} = 6.7 \, kN \cdot m \]

\[ M_{crfx} := 14869.8255 \cdot 1000 \, N \cdot mm = 14.87 \, kN \cdot m \] critical elastic local buckling moment from CUFSM, reduced thickness (the same as for the global buckling), non-reduced elastic modulus

\[ \lambda_l := \sqrt{\frac{M_{nex}}{M_{crfx}}} = 0.671 \] (Eq. F3.2.1-3)

\[ \lambda_l < 0.776 = 1 \]
\[
M_{nlx} := M_{nex} = 6.7 \text{ kN} \cdot \text{m} \quad \text{(Eq. F3.2.1-1)}
\]

\[I_{xx} := 583565.0981 \text{ mm}^4\] moment of inertia about the x-x axis

\[y_g := 37.1196 \text{ mm}\]

\[S_{fnetx} := \frac{I_{xx}}{y_g} = (1.572 \cdot 10^4) \text{ mm}^3\] net section modulus

\[M_{ynetx} := S_{fnetx} \cdot F_y = 5.817 \text{ kN} \cdot \text{m} \quad \text{(Eq. F3.2.2-2)}\]

\[M_{nlx} \leq M_{ynetx} = 0 \quad \text{so} \quad M_{nlx} := M_{ynetx} = 5.817 \text{ kN} \cdot \text{m}\]

\[M_{ax} := \min (M_{nx}, M_{nlx}) = 5.817 \text{ kN} \cdot \text{m}\]

**b.2. Flexure about y-y (2-2) axis**

**b.2.1 Yielding and Global Buckling [F2]**

\[M_{crey} := 1.35583 \cdot 10^7 \text{ N} \cdot \text{mm} \quad \text{critical elastic bending moment from CUTWP, reduced thickness, non-reduced elastic modulus}\]

\[S_{fy} := 13429 \text{ mm}^3\] gross cross section modulus

\[F_{cre} := \frac{M_{crey}}{S_{fy}} = (1.01 \cdot 10^3) \text{ MPa} \quad \text{critical elastic lateral-torsional buckling stress}\]

\[F_{cre} \geq 2.78 \cdot F_y = 0\] \quad \text{(Eq. F2.1-1)}

\[2.78 \cdot F_y > F_{cre} \geq 0.56 \cdot F_y = 1\]

\[F_n := \frac{10}{9} \cdot F_y \cdot \left(1 - \frac{10 \cdot F_y}{36 \cdot F_{cre}}\right) = 369.261 \text{ MPa}\]

\[M_{ney} := S_{fy} \cdot F_n = 4.959 \text{ kN} \cdot \text{m} \quad \text{nominal axial strength for yielding and global buckling about the y-y (2-2) axis}\] \quad \text{(Eq. F2.1-1)}

**b.2.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]**

\[M_{ney} = 4.959 \text{ kN} \cdot \text{m}\]

\[M_{cly} := 6000.4396 \cdot 1000 \text{ N} \cdot \text{mm} = 6 \text{ kN} \cdot \text{m}\] \quad \text{critical elastic local buckling moment from CUFSM, reduced thickness, reduced elastic modulus}

\[
\lambda_i := \sqrt{\frac{M_{rey}}{M_{cly}}} = 0.909
\]
\[ \lambda > 0.776 = 1 \]

\[ M_{nly} := \left(1 - 0.15 \cdot \left( \frac{M_{cly}}{M_{ney}} \right)^{0.4} \right) \cdot \left( \frac{M_{cly}}{M_{ney}} \right)^{0.4} \cdot M_{ney} = 4.485 \text{ kN} \cdot \text{m} \quad (\text{Eq. F3.21-1}) \]

\[ l_{yy} = 483242.9716 \text{ mm}^4 \quad \text{moment of inertia about the y-y axis} \]

\[ x_g = 35.5852 \text{ mm} \]

\[ S_{nety} := \frac{l_{yy}}{x_g} = \left(1.358 \cdot 10^4 \right) \text{ mm}^3 \quad \text{net section modulus} \]

\[ M_{ynety} := S_{nety} \cdot F_y = 5.025 \text{ kN} \cdot \text{m} \quad \text{yield moment of net cross-section} \quad (\text{Eq. F3.2.2-2}) \]

\[ M_{nly} \leq M_{ynety} = 1 \]

\[ M_{ay} := \min(M_{ney}, M_{nly}) = 4.485 \text{ kN} \cdot \text{m} \]

**Verification:**

\[ \frac{P}{P_a} = 0.804 \]

\[ \frac{M_x}{M_{ax}} = 0.017 \]

\[ \frac{M_y}{M_{ay}} = 0.067 \]

\[ \frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} = 0.888 \]

**4.2.6 Global, local+global, distortional buckling, using the section without perforations**

**a. Members in compression (chapter E)**

The available axial strength \( P_a \) (factored resistance) shall be the smallest of the available axial strength \( F_n \) for global buckling, nominal axial strength \( P_{nf} \) for local buckling interacting with global buckling, and the nominal axial strength \( P_{nd} \) for distortional buckling.
a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]

\[ A_g := 694 \text{ mm}^2 \]

\[ F_y := 370 \text{ MPa} \quad \text{yield stress} \]

\[ P_{cre} := 300891 \text{ N} \quad \text{minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP, for gross cross section, using the non-reduced elastic modulus} \]

\[ F_{cre} := \frac{P_{cre}}{A_g} = 433.561 \text{ MPa} \quad \text{least of the applicable elastic global(flexural, torsional and flexural torsional) buckling stress} \]

\[ \lambda_c := \sqrt{\frac{F_y}{F_{cre}}} = 0.924 \quad \text{slenderness} \quad \text{(Eq. E2-4)} \]

\[ \lambda_c \leq 1.5 = 1 \quad F_n := \left(0.658 \lambda_c^2\right) \cdot F_y = 258.867 \text{ MPa} \quad \text{(Eq. E2-2)} \]

\[ F_n \quad \text{Compressive stress} \]

\[ P_{ne} := A_g \cdot F_n = 179.653 \text{ kN} \quad \text{nominal axial strength} \quad \text{(Eq. E2-1)} \]

a.2 Local Buckling Interacting with Yielding and Global Buckling [E3.2.1]

For the model without holes, instead of applying the Q RMI method (which is suitable for models with holes only), I will apply the Direct Strength Method from AISI S-100. The value of Pnl will be calculated using a critical elastic local buckling load, Pcrl, calculated in CUFSM.

\[ P_{nl} \quad \text{nominal axial strength} \]

\[ P_{ne} = 179.653 \text{ kN} \]

\[ \text{Load} := 1000 \text{ N} \]

\[ \text{load.factor} := 685.1192 \]

\[ P_{crl} := \text{Load} \cdot \text{load.factor} = 685.119 \text{ kN} \quad \text{critical elastic local buckling load determined by analysis in CUFSM, for the gross cross section, non-reduced stiffness} \]

\[ \lambda_l := \sqrt{\frac{P_{ne}}{P_{crl}}} = 0.512 \quad \text{(Eq. E3.2.1-3)} \]

\[ \lambda_l \leq 0.776 = 1 \]

\[ P_{nl} := P_{ne} = 179.653 \text{ kN} \quad \text{(Eq. E3.2.1-1)} \]
**b.3 Distortional Buckling [E4.1]**

\[ P_{nd} \] nominal axial strength for distortional buckling

\[ F_y = 370 \text{ MPa} \]

\[ P_y := A_g \cdot F_y = 256.78 \text{ kN} \] (Eq. E4.1-7)

Using \( P_{crd} \) calculated with CUFSM

\[ \text{Load} := 1000 \text{ N} \]

\[ \text{load.factor} := 404.3171 \]

\[ P_{crd} := \text{Load} \cdot \text{load.factor} = 404.317 \text{ kN} \] critical elastic distortional column buckling load determined by analysis in CUFSM, for gross cross section, with non-reduced elastic modulus

\[ \lambda_d := \sqrt{\frac{P_y}{P_{crd}}} = 0.797 \] slenderness (Eq. E4.1-3)

\[ \lambda_d > 0.561 = 1 \]

\[ P_{nd} := \left(1 - 0.25 \cdot \left(\frac{P_{crd}}{P_y}\right)^{0.6}\right) \cdot \left(\frac{P_{crd}}{P_y}\right)^{0.6} \cdot P_y = 226.49 \text{ kN} \] (Eq. E4.1-2)

\[ P_a := \min(P_{ne}, P_{nl}, P_{nd}) = 179.653 \text{ kN} \]

**b. Members in flexure (chapter F)**

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and global buckling \( M_{ne} \), the available flexural strength due to the interaction of the yielding or global buckling with local buckling \( M_{nl} \) and the nominal flexural strength for distortional buckling \( M_{nd} \).

**b.1. Flexure about x-x (1-1) axis**

**b.1.1 Yielding and Global Buckling [F2]**

\[ M_{crex} := 5.00633 \cdot 10^7 \text{ N-mm} \] critical elastic bending moment from CUTWP, gross section, non-reduced elastic modulus
\[ S_{fx} := 18107 \text{ mm}^3 \]

\[ F_{cre} := \frac{M_{crex}}{S_{fx}} = (2.765 \cdot 10^3) \text{ MPa} \]

Critical elastic lateral-torsional buckling stress

\[ F_{cre} \geq 2.78 \cdot F_y = 1 \]

\[ F_n := F_y = 370 \text{ MPa} \]

\[ M_{nex} := S_{fx} \cdot F_n = 6.7 \text{ kN} \cdot \text{m} \]

Nominal axial strength for yielding and global buckling about the x-x (1-1) axis (Eq. F2.1-1)

b.1.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

As the Q cannot be determined I will use subchapter F3.2.2 of AISI S-100

\[ M_{nex} = 6.7 \text{ kN} \cdot \text{m} \]

\[ M_{crdx} := 23121.2747 \cdot 1000 \text{ N} \cdot \text{mm} = 23.121 \text{ kN} \cdot \text{m} \]

Critical elastic local buckling moment from CUFSM, gross cross section, non-reduced elastic modulus

\[ \lambda_f := \sqrt{\frac{M_{nex}}{M_{crdx}}} = 0.538 \]

\[ \lambda_f < 0.776 = 1 \]

\[ M_{nlx} := M_{nex} = 6.7 \text{ kN} \cdot \text{m} \]

b.1.3 Distortional Buckling [F4.1]

\[ S_{fx} := 11959 \text{ mm}^3 \]

Gross cross section modulus

\[ F_y = 370 \text{ MPa} \]

Yield stress

\[ M_{yx} := S_{fx} \cdot F_y = 4.425 \text{ kN} \cdot \text{m} \]

Yield moment (Eq. F4.1-4)

\[ M_{crdx} := 15307.6696 \cdot 1000 \text{ N} \cdot \text{mm} = 15.308 \text{ kN} \cdot \text{m} \]

Critical elastic distortional buckling moment about x-x axis from CUFSM, gross cross section, non-reduced elastic modulus

\[ \lambda_d := \sqrt{\frac{M_{yx}}{M_{crdx}}} = 0.538 \]

Slenderness (Eq. F4.1-3)

\[ \lambda_d < 0.673 = 1 \] (Eq. F4.1-1)
\[ M_{ndx} := M_{yx} = 4.425 \text{ } kN \cdot m \]

\[ M_{ndx} \] nominal flexural strength about the x-x (1-1) axis

\[ M_{ax} := \min (M_{nex}, M_{nlx}, M_{ndx}) = 4.425 \text{ } kN \cdot m \] distortional buckling

**b.2. Flexure about y-y (2-2) axis**

**b.2.1 Yielding and Global Buckling [F2]**

\[ M_{crey} := 1.53646 \cdot 10^7 \text{ N} \cdot \text{mm} \] critical elastic bending moment from CUTWP, gross section, non-reduced elastic modulus

\[ S_{fy} := 13429 \text{ mm}^3 \] net section modulus

\[ F_{cre} := \frac{M_{crey}}{S_{fy}} = (1.144 \cdot 10^3) \text{ MPa} \] critical elastic lateral-torsional buckling stress

\[ F_{cre} \geq 2.78 \cdot F_y = 1 \]

\[ F_n := F_y = 370 \text{ MPa} \] (Eq. F2.1-2)

\[ M_{ney} := S_{fy} \cdot F_n = 4.969 \text{ kN} \cdot \text{m} \] nominal axial strength for yielding and global buckling about the y-y (2-2) axis (Eq. F2.1-1)

**b.2.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.1]**

\[ M_{ney} = 4.969 \text{ kN} \cdot \text{m} \]

\[ M_{crly} := 40572.5838 \cdot 1000 \text{ N} \cdot \text{mm} = 40.573 \text{ kN} \cdot \text{m} \] critical elastic local buckling moment from CUFSM, gross cross section, non-reduced elastic modulus

\[ \lambda_l := \sqrt{\frac{M_{ney}}{M_{crly}}} = 0.35 \]

\[ \lambda l \leq 0.776 = 1 \] (Eq. F3.1-1)

\[ M_{nly} := M_{ney} = 4.969 \text{ kN} \cdot \text{m} \]
b.2.3 Distortional Buckling [F4.1]

\[ F_y = 370 \text{ MPa} \]

yield stress

\[ M_{yy} := S_y \cdot F_y = 4.969 \text{ kN} \cdot \text{m} \]

yield moment \hspace{1cm} (Eq. F4.1-4)

\[ M_{crdy} := 10525.145 \cdot 1000 \text{ N} \cdot \text{mm} = 10.525 \text{ kN} \cdot \text{m} \]
critical elastic distortional buckling moment about y-y axis from CUFSM, gross cross section, non-reduced elastic modulus

\[ \lambda_d := \sqrt{\frac{M_{yx}}{M_{crdx}}} = 0.538 \]

slenderness \hspace{1cm} (Eq. F4.1-3)

\[ \lambda_d \leq 0.673 = 1 \]

(Eq. F4.1-1)

\[ M_{ndy} := M_{yy} = 4.969 \text{ m} \cdot \text{kN} \]

nominal flexural strength about the y-y (2-2) axis

\[ M_{sy} := \min (M_{ney}, M_{nlx}, M_{ndy}) = 4.969 \text{ kN} \cdot \text{m} \]

local+global buckling

Verification:

\[ P = 0.73 \]
\[ P_a = \frac{M_x}{M_{ax}} = 0.023 \]
\[ M_y = \frac{M_{sy}}{M_{ay}} = 0.06 \]
\[ P + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} = 0.813 \]

4.2.7 Global, local+global, distortional buckling, using imperfections through notional loads

The consideration of initial imperfections is considered through application of notional loads (AISI S100-16 Subchapter C1.1.1.2 b). Notional loads are used to represent the effects of initial imperfections. The notional loads are applied in Consteel. They are applied as lateral loads at all levels (in the present case, at the levels with the horizontal bracing bracings).

The value of the notional loads:

\[ Y_i := 127.4 \text{ kN} \]

gravity load applied on level "i"

\[ a := 1 \]
\[ N_i := \frac{1}{240} \cdot \sigma \cdot Y_i = 0.531 \text{ kN} \]

Notional load applied at level "i".

\[ P := 131.3 \text{ kN} \]

Required compressive axial strength.

\[ M_x := 0.1 \text{ kN} \cdot m \]

Required flexural strength along the x-x axis (y-y for EU).

\[ M_y := 0.3 \text{ kN} \cdot m \]

Required flexural strength along the y-y axis (z-z for EU).

\[ P_a \]

Available axial strength determined in accordance with Chapter E.

\[ M_{ax}, M_{ay} \]

Available flexural strengths determined as required in Section F.

### a. Members in compression (chapter E)

The available axial strength \( P_a \) (factored resistance) shall be the smallest of the available axial strength \( F_n \) for global buckling, nominal axial strength \( P_n \) for local buckling interacting with global buckling, and the nominal axial strength \( P_{nd} \) for distortional buckling.

### a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]

\[ A_g := 694 \text{ mm}^2 \]

Gross cross-section area from Consteel.

\[ F_y := 370 \text{ MPa} \]

Yield stress.

\[ P_{cre} := 236979 \text{ N} \]

Minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP, for reduced thickness, using the non-reduced elastic modulus.

\[ F_{cre} := \frac{P_{cre}}{A_g} = 341.468 \text{ MPa} \]

Least of the applicable elastic global (flexural, torsional and flexural torsional) buckling stress.

\[ \lambda_c := \sqrt{\frac{F_y}{F_{cre}}} = 1.041 \]

Slenderness (Eq. E2-4).
\[ \lambda_c \leq 1.5 = 1 \quad F_n := (0.658^{\lambda_c^2}) \cdot F_y = 235.093 \text{ MPa} \quad \text{(Eq. E2-2)} \]

\( F_n \) Compressive stress

\[ P_{ne} := A_g \cdot F_n = 163.154 \text{ kN} \quad \text{nominal axial strength} \quad \text{(Eq. E2-1)} \]

### a.2 Local Buckling Interacting with Yielding and Global Buckling [E3.2]

\( P_{nl} \) nominal axial strength

\[ P_{ne} = 163.154 \text{ kN} \]

\( A_{\text{net.min}} := 547.741 \text{ mm}^2 \) minimum net cross section area

\[ F_n := \frac{P_{ne}}{A_g} = 235.093 \text{ MPa} \]

\( P_{ua} := 45.59 \text{ kip} \cdot (2.028 \cdot 10^5) \text{ N} \) experimental ultimate compressive strength of the stub column test

\[ Q := \frac{P_{ua}}{A_{\text{net.min}} \cdot F_y} = 1.001 \]

Using Q RMI (see report from T. Pekoz, August 2015)

\[ Q := 1 \]

\[ P_{nl} := F_n \cdot \left[1 - (1 - Q) \cdot \left(\frac{F_n}{F_y}\right)^0\right] \cdot A_g = [163.154] \text{ kN} \]

\[ P_{y.net} := A_{\text{net.min}} \cdot F_y = 202.664 \text{ kN} \]

\[ P_{nl} \leq P_{y.net} = [1] \]

### a.3 Distortional Buckling [E4.2]

\( P_{nd} \) nominal axial strength for distortional buckling

\[ F_y = 370 \text{ MPa} \]

\[ A_{\text{net}} := 547.741 \text{ mm}^2 \]

\[ A_g = 694 \text{ mm}^2 \]

\[ P_y := A_g \cdot F_y = 256.78 \text{ kN} \quad \text{(Eq. E4.2-7)} \]

\[ P_{y.net} := A_{\text{net}} \cdot F_y = 202.664 \text{ kN} \quad \text{(Eq. E4.2-8)} \]
Using \( P_{\text{crd}} \) calculated with CUFSM

\[
\text{Load} := 1000 \ N
\]

\[
\text{load.factor} := 258.5945
\]

\[
P_{\text{crd}} := \text{Load} \cdot \text{load.factor} = 258.595 \ kN
\]
critical elastic distortional column buckling load determined by analysis in CUFSM, for reduced thickness, with non-reduced elastic modulus

\[
\lambda_d := \sqrt{\frac{P_y}{P_{\text{crd}}}} = 0.996
\]
slenderness  \( \text{Eq. E4.2-3} \)

\[
\lambda_{d2} := 0.561 \cdot \left(14 \cdot \left(\frac{P_y}{P_{\text{y.net}}}\right)^{0.4} - 13\right) = 1.341
\]
\( \text{Eq. E4.2-5} \)

\( \lambda_d \leq \lambda_{d2} = 1 \) will use subchapter E4.2 (members with holes)

\[
P_{\text{y.net}} := A_{\text{net}} \cdot F_y = 202.664 \ kN
\]
\( \text{Eq. E4.2-8} \)

\[
\lambda_{d1} := 0.561 \cdot \left(\frac{P_{\text{y.net}}}{P_y}\right) = 0.443
\]
\( \text{Eq. E4.2-4} \)

\[
P_{d2} := \left(1 - 0.25 \cdot \left(\frac{1}{\lambda_{d2}}\right)^{1.2}\right) \cdot \left(\frac{1}{\lambda_{d2}}\right)^{1.2} \cdot P_y = 148.841 \ kN
\]
\( \text{Eq. E4.2-6} \)

\( \lambda_{d1} \leq \lambda_d \leq \lambda_{d2} = 1 \)
\( \text{Eq. E4.2-2} \)

\[
P_{n_d} := P_{\text{y.net}} - \frac{P_{\text{y.net}} - P_{d2}}{\lambda_{d2} - \lambda_{d1}} \cdot (\lambda_d - \lambda_{d1}) = 169.479 \ kN
\]

\[
P_a := \min\left(P_{\text{ne}}, P_{nl}, P_{n_d}\right) = 163.154 \ kN
\]

**b. Members in flexure (chapter F)**

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and global buckling \( M_{ne} \), the available flexural strength due to the interaction of the yielding or global buckling with local buckling \( M_{nl} \) and the nominal flexural strength for distortional buckling \( M_{nd} \).

**b.1. Flexure about x-x (1-1) axis**

**b.1.1 Yielding and Global Buckling [F2]**

\[
M_{\text{crex}} := 4.42029 \times 10^7 \ N \cdot mm
\]
critical elastic bending moment from CUTWP, reduced thickness, non-reduced elastic modulus

\[
S_{fx} := 18107 \ mm^3
\]
gross cross section modulus
$$F_{\text{cre}} := \frac{M_{\text{cre}}}{S_{f}} = \left(2.441 \cdot 10^{3}\right) \text{ MPa}$$

Critical elastic lateral-torsional buckling stress

$$F_{\text{cre}} \geq 2.78 \cdot F_{y} = 1$$  \hspace{1cm} \text{(Eq. F2.1-1)}

$$F_{n} := F_{y} = 370 \text{ MPa}$$

$$M_{\text{nex}} := S_{f} \cdot F_{n} = 6.7 \text{ kN-m}$$

Nominal axial strength for yielding and global buckling about the x-x (1-1) axis

b.1.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

As the Q cannot be determined I will use subchapter F3.2.2 of AISI S-100

$$M_{\text{nex}} = 6.7 \text{ kN-m}$$

$$M_{\text{crlx}} := 14869.8255 \times 1000 \text{ N-mm} = 14.87 \text{ kN-m}$$

Critical elastic local buckling moment from CUFSM, reduced thickness (the same as for the global buckling), non-reduced elastic modulus

$$\lambda_{j} := \sqrt{\frac{M_{\text{nex}}}{M_{\text{crlx}}}} = 0.671$$  \hspace{1cm} \text{(Eq. F3.2.1-3)}

$$\lambda_{j} < 0.776 = 1$$  \hspace{1cm} \text{(Eq. F3.2.1-1)}

$$M_{nlx} := M_{\text{nex}} = 6.7 \text{ kN-m}$$

$$I_{xx} := 583565.0981 \text{ mm}^{4}$$

Moment of inertia about the x-x axis

$$y_{g} := 37.1196 \text{ mm}$$

$$S_{\text{netx}} := \frac{I_{xx}}{y_{g}} = \left(1.572 \cdot 10^{4}\right) \text{ mm}^{3}$$

Net section modulus

$$M_{\text{yntx}} := S_{\text{netx}} \cdot F_{y} = 5.817 \text{ kN-m}$$

Yield moment of net cross-section  \hspace{1cm} \text{(Eq. F3.2.2-2)}

$$M_{nlx} \leq M_{\text{yntx}} = 0$$

So

$$M_{nlx} := M_{\text{yntx}} = 5.817 \text{ kN-m}$$

b.1.3 Distortional Buckling [F4.2]

$$S_{f} := 18107 \text{ mm}^{3}$$

Gross cross section modulus

$$F_{y} = 370 \text{ MPa}$$

Yield stress

$$M_{yx} := S_{f} \cdot F_{y} = 6.7 \text{ kN-m}$$

Yield moment  \hspace{1cm} \text{(Eq. F4.1-4)}
\[ M_{crdx} := 10338.2261 \cdot 1000 \frac{N \cdot mm}{10.338 \cdot kN \cdot m} \]  
\( = \text{critical elastic distortional buckling moment about x-x axis from CUFSM, reduced thickness, reduced elastic modulus} \)

\[ \lambda_d := \sqrt{\frac{M_{yx}}{M_{crdx}}} = 0.805 \]  
\( = \text{slenderness} \)  
(Eq. F4.1-3)

\[ M_{yntx} = 5.817 \frac{kN \cdot m}{5.211} \]  
\( = \text{critical elastic bending moment from CUTWP, reduced thickness, non-reduced elastic modulus} \)

\[ \lambda_{d2} := 0.673 \cdot \left( 1.7 \cdot \left( \frac{M_{yx}}{M_{yntx}} \right)^{2.7} \right) = 1.204 \]  
(Eq. F4.2-5)

\[ \lambda_d \leq \lambda_{d2} = 1 \]  
will use subchapter F4.2 (members with holes)

\[ \lambda_{d1} := 0.673 \cdot \left( \frac{M_{yntx}}{M_{yx}} \right)^3 = 0.44 \]  
(Eq. F4.2-4)

\[ \lambda_{d1} < \lambda_d \leq \lambda_{d2} = 1 \]

\[ M_{d2} := \left( 1 - 0.22 \cdot \left( \frac{1}{\lambda_{d2}} \right) \cdot \left( \frac{1}{\lambda_{d2}} \right) \cdot M_{yx} \right) = 4.547 \frac{kN \cdot m}{5.211} \]  

\[ M_{ndx} := M_{yntx} \cdot \frac{M_{yntx} - M_{d2}}{\lambda_{d2} - \lambda_{d1}} \cdot \left( \lambda_d - \lambda_{d1} \right) = 5.211 \frac{kN \cdot m}{\lambda_{d1}} \]  
(Eq. F4.2-2)

\[ M_{ndx} = \text{nominal flexural strength about the x-x (1-1) axis} \]  
(Eq. F4.1-2)

\[ M_{ax} := \min \left( M_{nex}, M_{nix}, M_{ndx} \right) = 5.211 \frac{kN \cdot m}{\lambda_{d1}} \]

b.2. Flexure about y-y (2-2) axis

b.2.1 Yielding and Global Buckling [F2]

\[ M_{crey} := 1.35583 \cdot 10^7 \frac{N \cdot mm}{13429 \ mm^3} \]  
\( = \text{critical elastic bending moment from CUTWP, reduced thickness, non-reduced elastic modulus} \)

\[ S_{fy} := 13429 \ mm^3 \]  
\( = \text{gross cross section modulus} \)

\[ F_{cre} := \frac{M_{crey}}{S_{fy}} = \left( 1.01 \cdot 10^3 \right) \frac{MPa}{3427} \]  
\( = \text{critical elastic lateral-torsional buckling stress} \)

\[ F_{cre} \geq 2.78 \cdot F_y = 0 \]  
\( \geq 2.78 \cdot F_y \geq 0.56 \cdot F_y = 1 \)  
(Eq. F2.1-1)

\[ 2.78 \cdot F_y > F_{cre} \geq 0.56 \cdot F_y = 1 \]
\[ F_n := \frac{10}{9} \cdot F_y \cdot \left( 1 - \frac{10 \cdot F_y}{36 \cdot F_{cre}} \right) = 369.261 \text{ MPa} \]

\[ M_{ney} := S_y \cdot F_n = 4.959 \text{ kN} \cdot \text{m} \quad \text{nominal axial strength for yielding and global buckling about the y-y (2-2) axis} \quad (\text{Eq. F2.1-1}) \]

### b.2.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

\[ M_{ney} = 4.959 \text{ kN} \cdot \text{m} \]

\[ M_{cly} := 6000.4396 \cdot 1000 \text{ N-mm} = 6 \text{ kN-m} \quad \text{critical elastic local buckling moment from CUFSM, reduced thickness, reduced elastic modulus} \]

\[ \lambda_i := \sqrt{\frac{M_{ney}}{M_{cly}}} = 0.909 \]

\[ \lambda_i > 0.776 \equiv 1 \]

\[ M_{nly} := \left( 1 - 0.15 \cdot \left( \frac{M_{cly}}{M_{ney}} \right)^{0.4} \right) \cdot \left( \frac{M_{cly}}{M_{ney}} \right)^{0.4} \cdot M_{ney} = 4.485 \text{ kN} \cdot \text{m} \quad (\text{Eq. F3.21-1}) \]

\[ I_{yy} := 483242.9716 \text{ mm}^4 \quad \text{moment of inertia about the y-y axis} \]

\[ x_g := 35.5852 \text{ mm} \]

\[ S_{fnety} := \frac{I_{yy}}{x_g} = \left( 1.358 \cdot 10^4 \right) \text{ mm}^3 \quad \text{net section modulus} \]

\[ M_{ynety} := S_{fnety} \cdot F_y = 5.025 \text{ kN-m} \quad \text{yield moment of net cross-section} \quad (\text{Eq. F3.2.2-2}) \]

\[ M_{nly} \leq M_{ynety} = 1 \]

### b.2.3 Distortional Buckling [F4.2]

\[ S_{fy} = \left( 1.343 \cdot 10^4 \right) \text{ mm}^3 \quad \text{gross cross section modulus} \]

\[ F_y = 370 \text{ MPa} \quad \text{yield stress} \]

\[ M_{yy} := S_{fy} \cdot F_y = 4.969 \text{ kN-m} \quad \text{yield moment} \quad (\text{Eq. F4.1-4}) \]

\[ M_{cly} := 7399.1076 \cdot 1000 \text{ N-mm} = 7.399 \text{ kN-m} \quad \text{critical elastic distortional buckling moment about y-y axis from CUFSM, reduced thickness, non-reduced elastic modulus} \]

\[ \lambda_d := \sqrt{\frac{M_{yx}}{M_{crdy}}} = 0.805 \quad \text{slenderness} \quad (\text{Eq. F4.1-3}) \]
\[
M_{\text{unet}} = 5.025 \text{ kN} \cdot \text{m}
\]

\[
\lambda_{d2} := 0.673 \cdot \left( 1.7 \cdot \left( \frac{M_{yy}}{M_{\text{unet}}} \right)^{2.7} - 0.7 \right) = 0.639 \quad \text{(Eq. F4.2-5)}
\]

\[
\lambda_d \leq \lambda_{d2} = 0 \quad \text{will use subchapter F4.1 (members without holes)}
\]

\[
M_{\text{ndy}} := \left( 1 - 0.22 \cdot \left( \frac{M_{\text{crdy}}}{M_{yy}} \right)^{0.5} \right) \cdot \left( \frac{M_{\text{crdy}}}{M_{yy}} \right)^{0.5} \cdot M_{yy} = 4.436 \text{ kN} \cdot \text{m} \quad \text{(Eq. F4.2-4)}
\]

\[M_{\text{ndy}}\] nominal flexural strength about the y-y (2-2) axis \quad \text{(Eq. F4.2-2)}

\[
M_{\text{ay}} := \min (M_{\text{ney}}, M_{\text{poly}}, M_{\text{ndy}}) = 4.436 \text{ kN} \cdot \text{m}
\]

**Verification:**

\[
\frac{P}{P_a} = 0.805
\]

\[
\frac{M_x}{M_{ax}} = 0.019
\]

\[
\frac{M_y}{M_{ay}} = 0.068
\]

\[
\frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} = 0.892
\]
5. US - EFFECTIVE LENGTH METHOD

\[ \frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} \leq 1.0 \quad \text{(Eq. H1.2-1)} \]

The consideration of initial imperfections is considered through application of notional loads (AISI S100-16 Subchapter C1.1.1.2 b). Notional loads are used to represent the effects of initial imperfections. The notional loads are applied in Consteel. They are applied as lateral loads at all levels (in the present case, at the levels with the horizontal bracing bracings).

All the buckling lengths used in the CUTWP analysis are taken from a report from August 2015 (T. Pekoz to RMI). These buckling lengths will be used for determining both Pcre and Mcre.

- Lx RMI determination of the flexural buckling parameter with taking the effective length as 0.7 times the flexurally unsupported length of the column (60\(^\circ\)). For the frame tested Lx RMI = .7 \* 60
  
  buckling length along y-y (1-1) \[ KL := 0.7 \times 1524 \text{ mm} = (1.067 \times 10^3) \text{ mm} \]

- Lx RMI determination of the flexural buckling parameter with taking the effective length as 1 times the flexurally unsupported length of the column.
  
  buckling length along z-z (2-2) \[ KL := 1 \times 1092.2 \text{ mm} = (1.092 \times 10^3) \text{ mm} \]

- Lt RMI determination of the torsional buckling parameter with taking the effective length as 0.8 times the torsionally unsupported length of the column (between the braces, 43\(^\circ\)). For the frame tested Lt RMI = .8 \* 43\(^\circ\)
  
  tors. buckling length along z-z (3-3) axis \[ KL := 0.8 \times 1092.2 \text{ mm} = 873.76 \text{ mm} \]

5.1. US - ELM for Column 2.99x3x0.07inch

The value of the notional loads:

\[ Y_i := 83.54 \text{ kN} \]

gravity load applied on level "i"

\[ \alpha := 1 \]

\[ N_i := \frac{1}{240} \times \alpha \times Y_i = 0.348 \text{ kN} \]

notional load applied at level "i"
Consteel has implemented only the European standards, hence the axes are named after Eurocode (y and z). For this procedure the axes will be changed according to the American standard, so x-x will be the symmetry axis and y-y the one perpendicular to x-x.

- $P_{nt} := 85.5 \text{ kN}$ axial force from first-order analysis, with the structure restrained against lateral translation
- $M_{nt,x} := 0.1 \text{ kN} \cdot \text{m}$ moment from first-order elastic analysis, with the structure restrained against lateral translation the x-x axis (y-y for EU)
- $M_{nt,y} := 0.2 \text{ kN} \cdot \text{m}$ required flexural strength along the y-y axis (z-z for EU)

**Required strengths**

**a. Required axial strength**

$P$ required second-order axial strength. It is permitted to be taken as $P_{nt} + P_{lt}$

$P_{lt} := 0$ axial force from first order analysis, with the structure restrained against lateral translation

$P := P_{nt} + P_{lt} = 85.5 \text{ kN}$ (Eq. C1.2.1.1-2)

$\alpha := 1$

$P_{el} := 512591 \text{ N}$ elastic critical buckling strength of the member in the plane of bending, calculated based on the assumption of no lateral translation at member ends

**b. Required flexural strength about the x-x axis**

- $M_1 := -0.03 \text{ kN} \cdot \text{m}$ smaller and larger moments, respectively, at the ends of that portion of the member unbraced in the plane of bending under consideration. $M_1$ and $M_2$ are calculated from a first-order elastic analysis. $M_1/M_2$ is positive when the member is bent in reverse curvature, negative when bent in single curvature.

- $M_2 := 0.07 \text{ kN} \cdot \text{m}$
\[
\frac{M_1}{M_2} = -0.429
\]

c_m \quad \text{coefficient assuming no lateral translation of the frame}

\[
c_m := 0.6 - 0.4 \cdot \left(\frac{M_1}{M_2}\right) = 0.771 \quad \text{(Eq. C1.2.1-4)}
\]

\[
B_1 := \frac{c_m}{1 - \alpha \cdot \frac{P}{P_{e1}}} = 0.926 \quad \text{(Eq. C1.2.1-3)}
\]

\[
B_1 \geq 1 = 0
\]

\[
B_1 := 1
\]

\[
M_{lt} := 0 \cdot kN \cdot m \quad \text{moment from first-order analysis due to lateral translation of the structure only}
\]

\[
B_2 := 0 \quad \text{multiplier to account for P- \(\Delta\) effects, determined for each story of the \(\Delta\) structure and each direction of lateral translation of the story}
\]

\[
M_x := B_1 \cdot M_{nt,x} + B_2 \cdot M_{lt} = 0.1 \ kN \cdot m \quad \text{(Eq. C1.2.1-1)}
\]

\[
M_x \quad \text{required flexural strength about the x-x axis}
\]

c. Required flexural strength about the y-y axis

\[
M_1 := 0.03 \ kN \cdot m \quad \text{smaller and larger moments, respectively, at the ends of that portion of the member unbraced in the plane of bending under consideration.} \ M_1 \ \text{and} \ M_2 \ \text{are calculated from a first-order elastic analysis.} \ M_1/M_2 \ \text{is positive when the member is bent in reverse curvature, negative when bent in single curvature.}
\]

\[
\frac{M_1}{M_2} = 0.429
\]

c_m \quad \text{coefficient assuming no lateral translation of the frame}

\[
c_m := 0.6 - 0.4 \cdot \left(\frac{M_1}{M_2}\right) = 0.429 \quad \text{(Eq. C1.2.1-4)}
\]

\[
B_1 := \frac{c_m}{1 - \alpha \cdot \frac{P}{P_{e1}}} = 0.514 \quad \text{(Eq. C1.2.1-3)}
\]

\[
B_1 \geq 1 = 0
\]

\[
B_1 := 1
\]
\[ M_{lt} := 0 \cdot kN \cdot m \] moment from first-order analysis due to lateral translation of the structure only

\[ B_2 := 0 \] multiplier to account for P-Δ effects, determined for each story of the structure and each direction of lateral translation of the story

\[ M_y = B_1 \cdot M_{nt,y} + B_2 \cdot M_{lt} = 0.2 \ kN \cdot m \] (Eq. C1.2.1.1-1)

\[ M_y \] required flexural strength about the y-y axis

\[ P_a \] available axial strength determined in accordance with Chapter E

\[ M_{ax}, \quad M_{ay} \] available flexural strengths determined as required in Section F

5.1.1 Global, local+global and distortional buckling

a. Members in compression (chapter E)

The available axial strength \( P_a \) (factored resistance) shall be the smallest of the available axial strength \( P_{nc} \) for global buckling, nominal axial strength \( P_{nl} \) for local buckling interacting with global buckling, and the nominal axial strength \( P_{nd} \) for distortional buckling.

a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]

\[ A_g := 445 \ mm^2 \] gross cross-section area from Consteel

\[ F_y := 430 \ MPa \] yield stress

\[ P_{cre} := 148419 \ N \] minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP, for reduced thickness

\[ F_{cre} := \frac{P_{cre}}{A_g} = 333.526 \ MPa \] least of the applicable elastic global(flexural, torsional and flexural torsional) buckling stress

\[ \lambda_c := \sqrt{\frac{F_y}{F_{cre}}} = 1.135 \] slenderness (Eq. E2-4)

\[ \lambda_c \leq 1.5 = 1 \quad F_n := \left(0.658 \lambda^3\right) \cdot F_y = 250.677 \ MPa \] (Eq. E2-2)

\[ F_n \] Compressive stress

\[ P_{nc} := A_g \cdot F_n = 111.551 \ kN \] nominal axial strength (Eq. E2-1)

a.2 Local Buckling Interacting with Yielding and Global Buckling [E3.2]

\[ P_{nl} \] nominal axial strength
\( P_{ne} = 111.551 \text{ kN} \)

\( F_n := \frac{P_{ne}}{A_g} = 250.677 \text{ MPa} \)

\( P_{ua} := 23.75 \text{ kip} = (1.056 \cdot 10^5) \text{ N} \)  
Experimental ultimate compressive strength of the stub column test

\( A_{net.min} := 355.483 \text{ mm}^2 \)  
Minimum net cross section area

\[ Q := \frac{P_{ua}}{A_{net.min} \cdot F_y} = 0.691 \]

Using \( Q \) RMI (see report from T. Pekoz, August 2015)

\[ P_{nd} := F_n \cdot \left[ 1 - (1 - Q) \cdot \left( \frac{F_n}{F_y} \right)^Q \right] \cdot A_g = [87.823] \text{ kN} \]

\[ P_{ynet} := A_{net.min} \cdot F_y = 152.858 \text{ kN} \]

\[ P_{nd} \leq P_{ynet} = [1] \]

**a.3 Distortional Buckling [E4.2]**

\( F_y = 430 \text{ MPa} \)

\( A_{net} := 355.483 \text{ mm}^2 \)

\( A_g := 445 \text{ mm}^2 \)

\[ P_y := A_g \cdot F_y = 191.35 \text{ kN} \quad \text{(Eq. E4.2-7)} \]

\[ P_{ynet} := A_{net} \cdot F_y = 152.858 \text{ kN} \quad \text{(Eq. E4.2-8)} \]

Using \( P_{crd} \) calculated with CUFSM

\[ Load := 1000 \text{ N} \]

\[ load.factor := 99.447 \]

\[ P_{crd} := Load \cdot load.factor = 99.447 \text{ kN} \]  
Critical elastic distortional column buckling load determined by analysis in CUFSM, for **reduced thickness**

\[ \lambda_d := \sqrt{\frac{P_y}{P_{crd}}} = 1.387 \]  
Slenderness  
\text{(Eq. E4.2-3)}

\[ \lambda_{d2} := 0.561 \left( 14 \left( \frac{P_y}{P_{ynet}} \right)^{0.4} - 13 \right) = 1.299 \]  
\text{(Eq. E4.2-5)}
if
\[ \lambda_d \leq \lambda_{d2} = 0 \]
will use subchapter E4.1 (members without holes)
\[
P_{nd} := \left( 1 - 0.25 \left( \frac{P_{crd}}{P_y} \right)^{0.6} \right) \cdot \left( \frac{P_{crd}}{P_y} \right)^{0.6} \cdot P_y = 107.396 \text{ kN}
\]
\[
P_a := \min(P_{ne}, P_{nl}, P_{nd}) = 87.823 \text{ kN}
\]

b. Members in flexure (chapter F)

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and global buckling \(M_{ne}\), the available flexural strength due to the interaction of the yielding or global buckling with local buckling \(M_{nl}\) and the nominal flexural strength for distortional buckling \(M_{nd}\).

b.1. Flexure about x-x (1-1) axis

b.1.1 Yielding and Global Buckling [F2]

\[
M_{crex} := 2.7913 \times 10^7 \text{ N} \cdot \text{mm} \quad \text{critical elastic bending moment from CUTWP, reduced thickness}
\]
\[
S_{fx} := 11959 \text{ mm}^3 \quad \text{gross cross section modulus}
\]
\[
F_{cre} := \frac{M_{crex}}{S_{fx}} = (2.334 \times 10^3) \text{ MPa} \quad \text{critical elastic lateral-torsional buckling stress}
\]
\[
F_{cre} \geq 2.78 \cdot F_y = 1
\]
\[
F_n := F_y = 430 \text{ MPa} \quad \text{(Eq. F2.1-1)}
\]
\[
M_{nex} := S_{fx} \cdot F_n = 5.142 \text{ kN} \cdot \text{m} \quad \text{nominal axial strength for yielding and global buckling about the x-x (1-1) axis} \quad \text{(Eq. F2.1-1)}
\]

b.1.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

As the \(Q\) cannot be determined I will use subchapter F3.2.2 of AISI S-100

\[
M_{nex} = 5.142 \text{ kN} \cdot \text{m}
\]
\[
M_{crel} := 3980.1442 \times 1000 \text{ N} \cdot \text{mm} = 3.98 \text{ kN} \cdot \text{m} \quad \text{critical elastic local buckling moment from CUFSM, reduced thickness} \text{ (the same as for the global buckling)}
\]
\[
\lambda_d := \sqrt{\frac{M_{nex}}{M_{crel}}} = 1.137 \quad \text{(Eq. F3.2.1-3)}
\]
\[ \lambda_l > 0.776 = 1 \]

\[ M_{nlx} := \left(1 - 0.15 \cdot \left(\frac{M_{crlx}}{M_{nex}}\right)^{0.4}\right) \cdot \left(\frac{M_{crlx}}{M_{nex}}\right)^{0.4} \cdot M_{nex} = 4.013 \text{ kN} \cdot \text{m} \]  
(Eq. F3.2.1-2)

\[ I_{xx} := 373134.5763 \text{ mm}^4 \quad \text{moment of inertia about the x-x axis for the minimum net cross section} \]

\[ y_g := 37.1196 \text{ mm} \]

\[ S_{fnetx} := \frac{I_{xx}}{y_g} = \left(1.005 \cdot 10^4\right) \text{ mm}^3 \quad \text{minimum net cross section modulus} \]

\[ M_{ynetx} := S_{fnetx} \cdot F_y = 4.322 \text{ kN} \cdot \text{m} \quad \text{yield moment of net cross-section} \]

\[ M_{nlx} \leq M_{ynetx} = 1 \]

b.1.3 Distortional Buckling [F4.2]

\[ S_{fx} := 11959 \text{ mm}^3 \quad \text{gross cross section modulus} \]

\[ F_y = 430 \text{ MPa} \quad \text{yield stress} \]

\[ M_{yx} := S_{fx} \cdot F_y = 5.142 \text{ kN} \cdot \text{m} \quad \text{yield moment} \]  
(Eq. F4.1-4)

\[ M_{crdx} := 4031.4623 \cdot 1000 \text{ N} \cdot \text{mm} = 4.031 \text{ kN} \cdot \text{m} \quad \text{critical elastic distortional buckling moment about x-x axis from CUFSM, reduced thickness} \]

\[ \lambda_d := \sqrt{\frac{M_{yx}}{M_{crdx}}} = 1.129 \quad \text{slenderness} \]  
(Eq. F4.1-3)

\[ M_{ynetx} = 4.322 \text{ kN} \cdot \text{m} \]

\[ \lambda_{d2} := 0.673 \cdot \left(1.7 \cdot \left(\frac{M_{yx}}{M_{ynetx}}\right)^{2.7} - 0.7\right) = 1.358 \]  
(Eq. F4.2-5)

\[ \lambda_{d} \leq \lambda_{d2} = 1 \quad \text{will use subchapter E4.2 (members with holes)} \]

\[ \lambda_{d1} := 0.673 \cdot \left(\frac{M_{ynetx}}{M_{yx}}\right)^{3} = 0.4 \]  
(Eq. F4.2-4)

\[ \lambda_{d1} < \lambda_{d} \leq \lambda_{d2} = 1 \]

\[ M_{d2} := \left(1 - 0.22 \cdot \left(\frac{1}{\lambda_{d2}}\right)\right) \cdot \left(\frac{1}{\lambda_{d2}}\right) \cdot M_{yx} = 3.174 \text{ kN} \cdot \text{m} \]

\[ M_{ndx} := M_{ynetx} - \left(\frac{M_{ynetx} - M_{d2}}{\lambda_{d2} - \lambda_{d1}}\right) \cdot (\lambda_d - \lambda_{d1}) = 3.448 \text{ kN} \cdot \text{m} \]  
(Eq. F4.2-2)
\[ M_{ndx} \text{ nominal flexural strength about the x-x (1-1) axis} \]

\[
\left( 1 - 0.22 \cdot \left( \frac{M_{crdx}}{M_{yx}} \right)^{0.5} \right) \cdot \left( \frac{M_{crdx}}{M_{yx}} \right)^{0.5} \cdot M_{yx} = 3.666 \text{ kN} \cdot \text{m} 
\]

\[ M_{ndx} \leq \left( 1 - 0.22 \cdot \left( \frac{M_{crdx}}{M_{yx}} \right)^{0.5} \right) \cdot \left( \frac{M_{crdx}}{M_{yx}} \right)^{0.5} \cdot M_{yx} = 1 
\]

\[ M_{ax} := \min (M_{nex}, M_{nlx}, M_{ndx}) = 3.448 \text{ kN} \cdot \text{m} \]

b.2. Flexure about y-y (2-2) axis

b.2.1 Yielding and Global Buckling [F2]

\[ M_{crey} := 8.46718 \cdot 10^6 \text{ N} \cdot \text{mm} \quad \text{critical elastic bending moment from CUTWP, reduced thickness} \]

\[ S_{fy} := 8650 \text{ mm}^3 \quad \text{gross cross section modulus} \]

\[ F_{cre} := \frac{M_{crey}}{S_{fy}} = 978.865 \text{ MPa} \quad \text{critical elastic lateral-torsional buckling stress} \]

\[ F_{cre} \geq 2.78 \cdot F_y = 0 
\]

\[ 2.78 \cdot F_y > F_{cre} > 0.56 \quad F_y = 1 \]

\[ (\text{Eq. F2.1-2}) \]

\[ F_n := \frac{10}{9} \cdot F_y \cdot \left( 1 - \frac{10 \cdot F_y}{36 \cdot F_{cre}} \right) = 419.478 \text{ MPa} \]

\[ M_{ney} := S_{fy} \cdot F_n = 3.628 \text{ kN} \cdot \text{m} \quad \text{nominal axial strength for yielding and global buckling about the y-y (2-2) axis} \quad (\text{Eq. F2.1-1}) \]

b.2.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

\[ M_{ney} = 3.628 \text{ kN} \cdot \text{m} \]

\[ M_{crly} := 1568.7918 \cdot 1000 \text{ N} \cdot \text{mm} = 1.569 \text{ kN} \cdot \text{m} \quad \text{critical elastic local buckling moment from CUFSM, reduced thickness} \]

\[ \lambda := \sqrt{\frac{M_{ney}}{M_{crly}}} = 1.521 \quad (\text{Eq. F3.2.1-3}) \]
\lambda_1 > 0.776 = 1 \quad \text{(Eq. F3.2.1-2)}

\[ M_{nly} := \left(1 - 0.15 \cdot \left(\frac{M_{cly}}{M_{ney}}\right)^{0.4}\right) \cdot \left(\frac{M_{cly}}{M_{ney}}\right)^{0.4} \cdot M_{ney} = 2.316 \, kN \cdot m \]

\[ I_{yy} := 308988.0814 \, mm^4 \quad \text{moment of inertia about the y-y axis} \]

\[ x_g := 35.5852 \, mm \]

\[ S_{fnety} = \frac{I_{yy}}{x_g} = (8.683 \cdot 10^3) \, mm^3 \quad \text{net section modulus} \]

\[ M_{ynety} := S_{fnety} \cdot F_y = 3.734 \, kN \cdot m \quad \text{yield moment of net cross-section} \quad \text{(Eq. F3.2.2-2)} \]

\[ M_{nly} \leq M_{ynety} = 1 \]

b.2.3 Distortional Buckling [F4.2]

\[ S_{fy} = (8.65 \cdot 10^3) \, mm^3 \quad \text{gross cross section modulus} \]

\[ F_y = 430 \, MPa \quad \text{yield stress} \]

\[ M_{yy} := S_{fy} \cdot F_y = 3.72 \, kN \cdot m \quad \text{yield moment} \quad \text{(Eq. F4.1-4)} \]

\[ M_{cly} := 2838.471 \cdot 1000 \, N \cdot mm = 2.838 \, kN \cdot m \quad \text{critical elastic distortional buckling moment about x-x axis from CUFSM, reduced thickness} \]

\[ \lambda_d := \sqrt{\frac{M_{yy}}{M_{cly}}} = 1.129 \quad \text{slenderness} \quad \text{(Eq. F4.1-3)} \]

\[ M_{ynety} = 3.734 \, kN \cdot m \quad \text{(Eq. F4.2-5)} \]

\[ \lambda_{d2} := 0.673 \cdot \left(1.7 \cdot \left(\frac{M_{yy}}{M_{ynety}}\right)^{2.7} - 0.7\right) = 0.661 \]

\[ \lambda_d \leq \lambda_{d2} = 0 \quad \text{will use subchapter F4.1 (members without holes)} \]

\[ M_{ndy} := \left(1 - 0.22 \cdot \left(\frac{M_{cly}}{M_{yy}}\right)^{0.5}\right) \cdot \left(\frac{M_{cly}}{M_{yy}}\right)^{0.5} \cdot M_{yy} = 2.625 \, kN \cdot m \quad \text{(Eq. F4.1-2)} \]

\[ M_{ndy} \quad \text{nominal flexural strength about the y-y (2-2) axis} \]

\[ M_{ny} := \min \left( M_{ney}, M_{ndy}, M_{ndy} \right) = 2.316 \, kN \cdot m \]
5.1.2 Global, local+global and distortional+global buckling

a. Members in compression (chapter E)

The available axial strength $P_a$ (factored resistance) shall be the smallest of the available axial strength $F_n$ for global buckling, nominal axial strength $P_{nl}$ for local buckling interacting with global buckling, and the nominal axial strength $P_{nd}$ for distortional buckling.

a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]

\[ A_g := 445 \, \text{mm}^2 \]  

\[ F_y := 430 \, \text{MPa} \]  

\[ P_{cre} := 148419 \, \text{N} \]  

minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP, for reduced thickness

\[ F_{cre} := \frac{P_{cre}}{A_g} = 333.526 \, \text{MPa} \]  

least of the applicable elastic global(flexural, torsional and flexural torsional) buckling stress

\[ \lambda_c := \sqrt{\frac{F_y}{F_{cre}}} = 1.135 \]  

slenderness  \quad (\text{Eq. E2-4})

\[ \lambda_c \leq 1.5 = 1 \quad F_n := \left(0.658 \lambda_c^2\right) \cdot F_y = 250.677 \, \text{MPa} \]  

\quad (\text{Eq. E2-2})

\[ F_n \]  

Compressive stress

\[ P_{ne} := A_g \cdot F_n = 111.551 \, \text{kN} \]  

nominal axial strength  \quad (\text{Eq. E2-1})
a.2 Local Buckling Interacting with Yielding and Global Buckling [E3.2]

\[ P_{nl} \quad \text{nominal axial strength} \]

\[ P_{ne} = 111.551 \text{ kN} \]

\[ F_n := \frac{P_{ne}}{A_g} = 250.677 \text{ MPa} \]

\[ P_{ua} := 23.75 \text{ kip} = (1.056 \cdot 10^5) \text{ N} \quad \text{experimental ultimate compressive strength of the stub column test} \]

\[ A_{net.min} := 355.483 \text{ mm}^2 \quad \text{cross-section area from the experiment (stub column test)} \]

\[ Q := \frac{P_{ua}}{A_{net.min} \cdot F_y} = 0.691 \]

Using Q RMI (see report from T. Pekoz, August 2015)

\[ P_{nl} := F_n \cdot \left[ 1 - (1 - Q) \cdot \left( \frac{F_n}{F_y} \right)^Q \right] \cdot A_g = [87.823] \text{ kN} \]

\[ P_{y.net} := A_{net.min} \cdot F_y = 152.858 \text{ kN} \]

\[ P_{nl} \leq P_{y.net} = [1] \]

a.3 Distortional Buckling interacting with Global Buckling [E4.2 and E2]

For simulating the interaction of distortional and global buckling, the nominal axial strength for global buckling will be introduced in the calculation formulas of the distortional buckling, namely, instead of using \( P_y \) and \( P_{y.net} \) calculated with the yield stress \( F_y \), they will be calculated with \( F_n \), the compressive stress for global buckling.

\[ P_{nd} \quad \text{nominal axial strength for distortional buckling} \]

\[ F_y = 430 \text{ MPa} \]

\[ A_{net} := 355.483 \text{ mm}^2 \]

\[ A_g := 445 \text{ mm}^2 \]

\[ P_{ne} := A_g \cdot F_n = 111.551 \text{ kN} \quad \text{(Eq. E4.2-7)} \]

\[ P_{ne.net} := A_{net} \cdot F_n = 89.112 \text{ kN} \quad \text{(Eq. E4.2-8)} \]

Using \( P_{crd} \) calculated with CUFSM
Load := 1000 N

load_factor := 99.447

\[ P_{crd} := Load \cdot load\_factor = 99.447 \text{ kN} \] 

- Critical elastic distortional column buckling load determined by analysis in CUFSM, for reduced thickness.

\[ \lambda_d := \sqrt{\frac{P_{ne}}{P_{crd}}} = 1.059 \] 

- Slenderness

(Eq. E4.2-3)

\[ P_{nd} := \left(1 - 0.25 \cdot \left(\frac{P_{crd}}{P_{ne}}\right)^{0.6}\right) \cdot \left(\frac{P_{crd}}{P_{ne}}\right)^{0.6} \cdot P_{ne} = 79.825 \text{ kN} \]

\[ P_a := \min(P_{ne}, P_{nd}, P_{nd}) = 79.825 \text{ kN} \]

b. Members in flexure (chapter F)

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and global buckling \( M_{ne} \), the available flexural strength due to the interaction of the yielding or global buckling with local buckling \( M_{nl} \) and the nominal flexural strength for distortional buckling \( M_{nd} \).

b.1. Flexure about x-x (1-1) axis

b.1.1 Yielding and Global Buckling [F2]

\[ M_{crex} := \left(2.7913 \cdot 10^7\right) \text{ N} \cdot \text{mm} \] 

- Critical elastic bending moment from CUTWP, reduced thickness

\[ S_{fx} := 11959 \text{ mm}^3 \] 

- Gross cross section modulus

\[ F_{cre} := \frac{M_{crex}}{S_{fx}} = \left(2.334 \cdot 10^3\right) \text{ MPa} \] 

- Critical elastic lateral-torsional buckling stress

\[ F_n := F_y = 430 \text{ MPa} \]

\[ M_{nex} := S_{fx} \cdot F_n = 5.142 \text{ kN} \cdot \text{m} \] 

- Nominal axial strength for yielding and global buckling about the x-x (1-1) axis

(Eq. F2.1-1)
b.1.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

As the \( Q \) cannot be determined, subchapter F3.2.2 will be used

\[
M_{nex} = 5.142 \text{ kN}\cdot \text{m}
\]

\( M_{cr lx} := 3980.1442 \cdot 1000 \text{ N}\cdot \text{mm} = 3.98 \text{ kN}\cdot \text{m} \)  

Critical elastic local buckling moment from CUFSM, reduced thickness (the same as for the global buckling)

\[
\lambda_l := \sqrt{\frac{M_{nex}}{M_{cr lx}}} = 1.137
\]

\( \lambda_l > 0.776 = 1 \)

\[
M_{nlx} := \left(1 - 0.15 \cdot \left(\frac{M_{cr lx}}{M_{nex}}\right)^{0.4}\right) \cdot \left(\frac{M_{cr lx}}{M_{nex}}\right)^{0.4} \cdot M_{nex} = 4.013 \text{ kN}\cdot \text{m}
\]

\( I_{xx} := 373134.5763 \text{ mm}^4 \)  

Moment of inertia about the x-x axis

\[
y_g := 37.1196 \text{ mm}
\]

\[
S_{fnetx} := \frac{I_{xx}}{y_g} = \left(1.005 \cdot 10^4\right) \text{ mm}^3 \text{ net section modulus}
\]

\[
M_{gnetx} := S_{fnetx} \cdot F_y = 4.322 \text{ kN}\cdot \text{m} \text{ yield moment of net cross-section}
\]

\[M_{nlx} \leq M_{gnetx} = 1\]

b.1.3 Distortional Buckling interacting with Global Buckling [F4.2 and F2]

\[
S_{fx} := 11959 \text{ mm}^3 \text{ gross cross section modulus}
\]

\[
F_n = 430 \text{ MPa} \text{ yield stress}
\]

\[
M_{nex} := S_{fx} \cdot F_n = 5.142 \text{ kN}\cdot \text{m} \text{ yield moment (Eq. F4.1-4)}
\]

\( M_{cr dx} := 4031.4623 \cdot 1000 \text{ N}\cdot \text{mm} = 4.031 \text{ kN}\cdot \text{m} \)  

Critical elastic distortional buckling moment about x-x axis from CUFSM, reduced thickness,

\[
\lambda_d := \sqrt{\frac{M_{nex}}{M_{cr dx}}} = 1.129 \text{ slenderness (Eq. F4.1-3)}
\]

\[
M_{ne.netx} := S_{fnetx} \cdot F_n = 4.322 \text{ kN}\cdot \text{m}
\]

\[
\lambda_{d2} := 0.673 \cdot \left(1.7 \cdot \left(\frac{M_{nex}}{M_{ne.netx}}\right)^{2.7} - 0.7\right)^{0.7} = 1.358 \text{ (Eq. F4.2-5)}
\]
\[ \lambda_d \leq \lambda_{d2} = 1 \]  
will use subchapter F4.2

\[ \lambda_{d1} := 0.673 \cdot \left( \frac{M_{nc.netx}}{M_{nex}} \right)^3 = 0.4 \]  
(Eq. F4.2-4)

\[ \lambda_{d1} < \lambda_d \leq \lambda_{d2} = 1 \]

\[ M_{d2} := \left( 1 - 0.22 \cdot \left( \frac{1}{\lambda_{d2}} \right) \right) \cdot \left( \frac{1}{\lambda_{d2}} \right) \cdot M_{nex} = 3.174 \, kN \cdot m \]

\[ M_{ndx} := M_{nc.netx} - \left( \frac{M_{nc.netx} - M_{d2}}{\lambda_{d2} - \lambda_{d1}} \right) \cdot (\lambda_d - \lambda_{d1}) = 3.448 \, kN \cdot m \]  
(Eq. F4.2-2)

\[ M_{ndx} \]  
nominal flexural strength about the x-x (1-1) axis

\[ \left( 1 - 0.22 \cdot \left( \frac{M_{crdx}}{M_{nex}} \right)^{0.5} \right) \cdot \left( \frac{M_{crdx}}{M_{nex}} \right)^{0.5} \cdot M_{nex} = 3.666 \, kN \cdot m \]

\[ M_{ndx} \leq \left( 1 - 0.22 \cdot \left( \frac{M_{crdx}}{M_{nex}} \right)^{0.5} \right) \cdot \left( \frac{M_{crdx}}{M_{nex}} \right)^{0.5} \cdot M_{nex} = 1 \]

\[ M_{ax} := \min (M_{nex}, M_{nlx}, M_{ndx}) = 3.448 \, kN \cdot m \]

b.2. Flexure about y-y (2-2) axis

b.2.1 Yielding and Global Buckling [F2]

\[ M_{crey} := \left( 8.46718 \cdot 10^6 \right) N \cdot \text{mm} \]  
critical elastic bending moment from CUTWP, gross section

\[ S_{fy} = 8650 \, \text{mm}^3 \]  
gross cross section modulus

\[ F_{cre} := \frac{M_{crey}}{S_{fy}} = 978.865 \, \text{MPa} \]  
critical elastic lateral-torsional buckling stress

\[ F_{cre} \geq 2.78 \cdot F_y = 0 \]

\[ 2.78 \cdot F_y > F_{cre} > 0.56 \cdot F_y = 1 \]

\[ F_n := \frac{10}{9} \cdot F_y \cdot \left( 1 - \frac{10 \cdot F_y}{36 \cdot F_{cre}} \right) = 419.478 \, \text{MPa} \]  
(Eq. F2.1-2)

\[ M_{ney} := S_{fy} \cdot F_n = 3.628 \, kN \cdot m \]  
nominal axial strength for yielding and global buckling about the y-y (2-2) axis  
(Eq. F2.1-1)
b.2.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

\[ M_{ney} = 3.628 \text{ kN} \cdot \text{m} \]

\[ M_{crly} := 1568.7918 \cdot 1000 \text{ N} \cdot \text{mm} = 1.569 \text{ kN} \cdot \text{m} \quad \text{critical elastic local buckling moment from CUFSM, reduced thickness} \]

\[ \lambda_l := \sqrt{\frac{M_{crly}}{M_{ney}}} = 1.521 \]

\[ \lambda_l > 0.776 = 1 \]

\[ M_{nly} := \left(1 - 0.15 \cdot \left(\frac{M_{crly}}{M_{ney}}\right)^{0.4}\right) \cdot \left(\frac{M_{crly}}{M_{ney}}\right)^{0.4} \cdot M_{ney} = 2.316 \text{ kN} \cdot \text{m} \]

\[ I_{yy} := 308988.0814 \text{ mm}^4 \quad \text{moment of inertia about the y-y axis} \]

\[ x_g := 35.5852 \text{ mm} \]

\[ S_{fnety} = \frac{I_{yy}}{x_g} = \left(8.683 \cdot 10^3\right) \text{ mm}^3 \quad \text{net section modulus} \]

\[ M_{ynty} := S_{fnety} \cdot F_y = 3.734 \text{ kN} \cdot \text{m} \quad \text{yield moment of net cross-section} \quad (\text{Eq. F3.2.2-2}) \]

\[ M_{nly} \leq M_{ynty} = 1 \]

b.2.3 Distortional Buckling + Global Buckling [F4.2]

\[ S_{fy} = \left(8.65 \cdot 10^3\right) \text{ mm}^3 \quad \text{gross cross section modulus} \]

\[ F_n = 419.478 \text{ MPa} \quad \text{yield stress} \]

\[ M_{ney} := S_{fy} \cdot F_n = 3.628 \text{ kN} \cdot \text{m} \quad \text{yield moment} \quad (\text{Eq. F4.1-4}) \]

\[ M_{crdy} := 2838.471 \cdot 1000 \text{ N} \cdot \text{mm} = 2.838 \text{ kN} \cdot \text{m} \quad \text{critical elastic distortional buckling moment about x-x axis from CUFSM, reduced thickness} \]

\[ \lambda_d := \sqrt{\frac{M_{ney}}{M_{crdy}}} = 1.131 \quad \text{slenderness} \quad (\text{Eq. F4.1-3}) \]

\[ M_{ne.nety} := S_{fnety} \cdot F_n = 3.642 \text{ kN} \cdot \text{m} \]

\[ \lambda_{d2} := 0.673 \cdot \left(1.7 \cdot \left(\frac{M_{ney}}{M_{ne.nety}}\right)^{2.7} - 0.7\right) = 0.661 \quad (\text{Eq. F4.2-5}) \]
\[ \lambda_d \leq \lambda_{d2} = 0 \quad \text{will use subchapter F4.1 (members without holes)} \]

\[ M_{ndy} := \left( 1 - 0.22 \cdot \left( \frac{M_{crdy}}{M_{ney}} \right)^{0.5} \right) \cdot \left( \frac{M_{crdy}}{M_{ney}} \right)^{0.5} \cdot M_{ney} = 2.585 \text{ kN m} \]

\[ M_{ndy} \quad \text{nominal flexural strength about the y-y (2-2) axis} \]

\[ M_{agy} := \min (M_{ney}, M_{ndy}, M_{ndg}) = 2.316 \text{ kN m} \]

**Verification:**

\[ P = 1.071 \]

\[ \frac{M_x}{M_{ax}} = 0.029 \]

\[ \frac{M_y}{M_{ay}} = 0.086 \]

\[ \frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} = 1.186 \]

5.1.3 **Global, local+global and distortional buckling, using only \( A_{net.min} \)**

In this model only the **minimal cross section area** will be used.

**a. Members in compression (chapter E)**

The available axial strength \( P_a \) (factored resistance) shall be the smallest of the available axial strength \( P_n \) for global buckling, nominal axial strength \( P_{nl} \) for local buckling interacting with global buckling, and the nominal axial strength \( P_{nd} \) for distortional buckling.

**a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]**

\[ A_{net.min} := 355.483 \text{ mm}^2 \quad \text{minimal net cross section area} \]

\[ F_y := 430 \text{ MPa} \quad \text{yield stress} \]

\[ P_{cre} := 148419 \text{ N} \quad \text{minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP, for reduced thickness} \]
\[ F_{\text{cre}} := \frac{P_{\text{cre}}}{A_{\text{net.min}}} = 417.514 \text{ MPa} \]

\[ \lambda_c := \sqrt{\frac{F_y}{F_{\text{cre}}}} = 1.015 \quad \text{slenderness} \quad \text{(Eq. E2-4)} \]

\[ \lambda_c \leq 1.5 \quad F_n := (0.658 \lambda_c^2) \cdot F_y = 279.42 \text{ MPa} \quad \text{(Eq. E2-2)} \]

\[ P_{\text{ne}} := A_{\text{net.min}} \cdot F_n = 99.329 \text{ kN} \quad \text{nominal axial strength} \quad \text{(Eq. E2-1)} \]

b.2 Local Buckling Interacting with Yielding and Global Buckling [E3.2]

\[ P_{\text{nl}} \quad \text{nominal axial strength} \]

\[ P_{\text{ne}} = 99.329 \text{ kN} \]

\[ F_n := \frac{P_{\text{ne}}}{A_{\text{net.min}}} = 279.42 \text{ MPa} \]

\[ P_{\text{ua}} = 23.75 \text{ kip} = (1.056 \cdot 10^5) \text{ N} \quad \text{experimental ultimate compressive strength of the stub column test} \]

\[ A_{\text{net.min}} = 355.483 \text{ mm}^2 \quad \text{minimum net cross-section area} \]

\[ Q := \frac{P_{\text{ua}}}{A_{\text{net.min}} \cdot F_y} = 0.691 \]

Using Q RMI (see report from T. Pekoz, August 2015)

\[ P_{\text{nl}} := F_n \left[ 1 - (1 - Q) \cdot \left( \frac{F_n}{F_y} \right)^Q \right] \cdot A_{\text{net.min}} = [76.554] \text{ kN} \]

a.3 Distortional Buckling [E4.2]

\[ P_{\text{nd}} \quad \text{nominal axial strength for distortional buckling} \]

\[ F_y = 430 \text{ MPa} \]

\[ A_{\text{net}} = 355.483 \text{ mm}^2 \]

\[ P_y := A_{\text{net.min}} \cdot F_y = 152.858 \text{ kN} \quad \text{(Eq. E4.2-7)} \]

Using \( P_{\text{crd}} \) calculated with CUFSM
Load := 1000 N
load.factor := 99.447

\[ P_{crd} := \text{Load} \cdot \text{load.factor} = 99.447 \text{ kN} \]

Critical elastic distortional column buckling load determined by analysis in CUFSM, for reduced thickness

\[ \lambda_d := \sqrt{\frac{P_y}{P_{crd}}} = 1.24 \text{ slenderness} \quad (\text{Eq. E4.2-3}) \]

\[ \lambda_d > 0.561 = 1 \]

\[ P_{nd} := \left(1 - 0.25 \cdot \left(\frac{P_{crd}}{P_y}\right)^{0.6}\right) \cdot \left(\frac{P_{crd}}{P_y}\right)^{0.6} \cdot P_y = 95.292 \text{ kN} \quad (\text{Eq. E4.1-2}) \]

\[ P_a := \min(P_{ne}, P_{nl}, P_{nd}) = 76.554 \text{ kN} \]

**b. Members in flexure (chapter F)***

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and global buckling \(M_{ne}\), the available flexural strength due to the interaction of the yielding or global buckling with local buckling \(M_{nl}\) and the nominal flexural strength for distortional buckling \(M_{nd}\).

**b.1. Flexure about x-x (1-1) axis**

**b.1.1 Yielding and Global Buckling [F2]**

\[ M_{crex} := \left(2.7913 \cdot 10^7\right) \text{ N} \cdot \text{mm} \]

Critical elastic bending moment from CUTWP, reduced thickness

\[ I_{xx} := 373134.5763 \text{ mm}^4 \]

Moment of inertia about the x-x axis

\[ y_g := 37.1196 \text{ mm} \]

\[ S_{fnetx} := \frac{I_{xx}}{y_g} = \left(1.005 \cdot 10^4\right) \text{ mm}^3 \]

Net section modulus

\[ F_{cre} := \frac{M_{crex}}{S_{fnetx}} = \left(2.777 \cdot 10^3\right) \text{ MPa} \]

Critical elastic lateral-torsional buckling stress

\[ F_{cre} \geq 2.78 \cdot F_y = 1 \]

\[ F_n := F_y = 430 \text{ MPa} \]

\[ M_{nex} := S_{fnetx} \cdot F_n = 4.322 \text{ kN} \cdot \text{m} \]

Nominal axial strength for yielding and global buckling about the x-x (1-1) axis \( (\text{Eq. F2.1-1}) \)
b.1.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

As the Q cannot be determined I will use subchapter F3.2.2 of AISI S-100

\[ M_{n_e} = 4.322 \, kN \cdot m \]

\[ M_{cr_{lx}} := 3980.1442 \cdot 1000 \, N \cdot mm = 3.98 \, kN \cdot m \]

\[ \lambda_i := \sqrt{\frac{M_{n_e}}{M_{cr_{lx}}}} = 1.042 \]

\[ \lambda_i > 0.776 = 1 \]

\[ M_{nl_{x}} := \left( 1 - 0.15 \cdot \left( \frac{M_{cr_{lx}}}{M_{n_e}} \right)^{0.4} \right) \cdot \left( \frac{M_{cr_{lx}}}{M_{n_e}} \right)^{0.4} \cdot M_{n_e} = 3.575 \, kN \cdot m \]

b.1.3 Distortional Buckling [F4.2]

\[ F_y = 430 \, MPa \]

\[ M_{yx} := S_{f_{ux}} \cdot F_y = 4.322 \, kN \cdot m \]

\[ M_{cr_{dx}} := 4031.4623 \cdot 1000 \, N \cdot mm = 4.031 \, kN \cdot m \]

\[ \lambda_d := \sqrt{\frac{M_{yx}}{M_{cr_{dx}}}} = 1.035 \]

\[ \lambda_d > 0.673 = 1 \]

\[ M_{nd_{x}} := \left( 1 - 0.22 \cdot \left( \frac{M_{cr_{dx}}}{M_{yx}} \right)^{0.5} \right) \cdot \left( \frac{M_{cr_{dx}}}{M_{yx}} \right)^{0.5} \cdot M_{yx} = 3.288 \, kN \cdot m \]

\[ M_{nd_{x}} \]

nominal flexural strength about the x-x (1-1) axis

\[ M_{ax} := \min \left( M_{n_e}, M_{nl_{x}}, M_{nd_{x}} \right) = 3.288 \, kN \cdot m \]
b.2. Flexure about y-y (2-2) axis

b.2.1 Yielding and Global Buckling [F2]

\[ M_{crey} := (8.46718 \cdot 10^6) \text{ N} \cdot \text{mm} \]  

Critical elastic bending moment from CUTWP, reduced cross section

\[ I_{yy} := 308988.0814 \text{ mm}^4 \]  

Moment of inertia about the y-y axis

\[ x_g := 35.5852 \text{ mm} \]  

\[ S_{fnety} := \frac{I_{yy}}{x_g} = (8.683 \cdot 10^3) \text{ mm}^3 \]  

Net section modulus

\[ F_{cre} := \frac{M_{cre}}{S_{fnety}} = 975.139 \text{ MPa} \]  

Critical elastic lateral-torsional buckling stress

\[ F_{cre} \geq 2.78 \cdot F_y = 0 \]

\[ 2.78 \cdot F_y > F_{cre} > 0.56 \cdot F_y = 1 \]

\[ F_n := \frac{10}{9} \cdot F_y \cdot \left(1 - \frac{10 \cdot F_y}{36 \cdot F_{cre}}\right) = 419.255 \text{ MPa} \]  

(Eq. F2.1-2)

\[ M_{ney} := S_{fnety} \cdot F_n = 3.64 \text{ kN} \cdot \text{m} \]

b.2.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

\[ M_{ney} = 3.64 \text{ kN} \cdot \text{m} \]

\[ M_{crly} := 1568.7918 \cdot 1000 \text{ N} \cdot \text{mm} = 1.569 \text{ kN} \cdot \text{m} \]  

Critical elastic local buckling moment from CUFSM, reduced thickness

\[ \lambda_l := \sqrt{\frac{M_{ney}}{M_{crly}}} = 1.523 \]

\[ \lambda_l > 0.776 = 1 \]

\[ M_{nlg} := \left(1 - 0.15 \cdot \left(\frac{M_{crly}}{M_{ney}}\right)^{0.4}\right) \cdot \left(\frac{M_{crly}}{M_{ney}}\right)^{0.4} \cdot M_{ney} = 2.321 \text{ kN} \cdot \text{m} \]

b.2.3 Distortional Buckling [F4.2]

\[ F_y = 430 \text{ MPa} \]  

Yield stress

\[ M_{yy} := S_{fnety} \cdot F_y = 3.734 \text{ kN} \cdot \text{m} \]  

Yield moment  

(Eq. F4.1-4)
The critical elastic distortional buckling moment about x-x axis from CUFSM, reduced thickness is:

\[ M_{crdx} := 2838.471 \times 1000 \text{ kN} \cdot \text{m} \]

The slenderness is:

\[ \lambda_d := \sqrt{\frac{M_{crdx}}{M_{crdx}}} = 1.035 \]

The slenderness \( \lambda_d \) is greater than 0.673, thus:

\[ \lambda_d > 0.673 = 1 \]

The nominal flexural strength about the y-y (2-2) axis is:

\[ M_{ndy} := \left( 1 - 0.22 \left( \frac{M_{crdy}}{M_{yy}} \right)^{0.5} \right) \left( \frac{M_{crdy}}{M_{yy}} \right)^{0.5} \times M_{yy} = 2.631 \text{ kN} \cdot \text{m} \]

Verification:

\[ \frac{P}{P_a} = 1.117 \]

\[ \frac{M_x}{M_{ax}} = 0.03 \]

\[ \frac{M_y}{M_{ay}} = 0.086 \]

\[ \frac{P}{P_a} \left( \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} \right) = 1.233 \]

### 5.1.4 Global, local+global buckling

This model won't take into account the effect of distortional buckling.

**a. Members in compression (chapter E)**

The available axial strength \( P_a \) (factored resistance) shall be the smallest of the available axial strength \( F_n \) for global buckling, nominal axial strength \( P_{nl} \) for local buckling interacting with global buckling.

**a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]**

\[ A_g := 445 \text{ mm}^2 \]

\[ F_y := 430 \text{ MPa} \]
minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP, for **reduced thickness**

\[
P_{cre} := 148419 \text{ N}
\]

least of the applicable elastic global(flexural, torsional and flexural torsional) buckling stress

\[
F_{cre} := \frac{P_{cre}}{A_g} = 333.526 \text{ MPa}
\]

\[
\lambda_c := \sqrt{\frac{F_y}{F_{cre}}} = 1.135 \quad \text{slenderness (Eq. E2-4)}
\]

\[
\lambda_c \leq 1.5 = 1 \quad F_n := \left(0.658 \lambda_c^2\right) \cdot F_y = 250.677 \text{ MPa (Eq. E2-2)}
\]

Compressive stress

\[
P_{ne} := A_g \cdot F_n = 111.551 \text{ kN} \quad \text{nominal axial strength (Eq. E2-1)}
\]

### a.2 Local Buckling Interacting with Yielding and Global Buckling [E3.2]

nominal axial strength

\[
P_{nl} = 111.551 \text{ kN}
\]

\[
F_n := \frac{P_{ne}}{A_g} = 250.677 \text{ MPa}
\]

\[
P_{ua} = 23.75 \text{ kip} = (1.056 \cdot 10^5) \text{ N} \quad \text{experimental ultimate compressive strength of the stub column test}
\]

\[
A_{net.min} = 355.483 \text{ mm}^2 \quad \text{minimum net cross-section area}
\]

\[
Q := \frac{P_{ua}}{A_{net.min} \cdot F_y} = 0.691
\]

Using Q RMI (see report from T. Pekoz, August 2015)

\[
P_{nl} := F_n \cdot \left(1 - (1 - Q) \cdot \left(\frac{F_n}{F_y}\right)^Q \right) \cdot A_g = [87.823] \text{ kN}
\]

\[
P_{y.net} := A_{net.min} \cdot F_y = 152.858 \text{ kN}
\]

\[
P_{nl} \leq P_{y.net} = [1]
\]

\[
P_a := \min (P_{ne}, P_{nl}) = 87.823 \text{ kN}
\]
b. Members in flexure (chapter F)

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and global buckling $M_{ne}$, the available flexural strength due to the interaction of the yielding or global buckling with local buckling $M_{nl}$.

b.1. Flexure about x-x (1-1) axis

b.1.1 Yielding and Global Buckling [F2]

$M_{crex} := 2.7913 \cdot 10^7 \ N \cdot mm$  

critical elastic bending moment from CUTWP, reduced thickness

$S_{fx} := 11959 \ mm^3$  

gross cross section modulus

$F_{cre} := \frac{M_{crex}}{S_{fx}} = (2.334 \cdot 10^3) \ MPa$  

critical elastic lateral-torsional buckling stress

$F_{cre} \geq 2.78 \cdot F_y = 1$

$F_n := F_y = 430 \ MPa$

$M_{nex} := S_{fx} \cdot F_n = 5.142 \ kN \cdot m$  

nominal axial strength for yielding and global buckling about the x-x (1-1) axis  

(Eq. F2.1-1)

b.1.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

As the Q cannot be determined, subchapter F3.2.2 of AISI S-100 will be used.

$M_{nex} = 5.142 \ kN \cdot m$

$M_{crex} := 3980.1442 \cdot 1000 \ N \cdot mm = 3.98 \ kN \cdot m$  

critical elastic local buckling moment from CUFSM, reduced thickness (the same as for the global buckling)

$\lambda_l := \sqrt{\frac{M_{nex}}{M_{crex}}} = 1.137$

$\lambda_l > 0.776 = 1$

$M_{nlx} := \left( 1 - 0.15 \left( \frac{M_{crex}}{M_{nex}} \right)^{0.4} \right) \left( \frac{M_{crex}}{M_{nex}} \right)^{0.4} \cdot M_{nex} = 4.013 \ kN \cdot m$

$I_{xx} := 373134.5763 \ mm^4$  

moment of inertia about the x-x axis

$y_g := 37.1196 \ mm$
\[ S_{f_{netx}} := \frac{I_{xx}}{y_g} = (1.005 \cdot 10^4) \text{ mm}^3 \] net section modulus

\[ M_{y_{netx}} := S_{f_{netx}} \cdot F_y = 4.322 \text{ kN} \cdot \text{m} \] yield moment of net cross-section \hspace{1cm} (Eq. F3.2.2-2)

\[ M_{nlx} \leq M_{y_{netx}} = 1 \]

\[ M_{ax} := \min(M_{nex}, M_{nlx}) = 4.013 \text{ kN} \cdot \text{m} \]

b.2. Flexure about y-y (2-2) axis

b.2.1 Yielding and Global Buckling [F2]

\[ M_{crey} := (8.46718 \cdot 10^6) \text{ N} \cdot \text{mm} \] critical elastic bending moment from CUTWP, reduced thickness

\[ S_{fy} := 8650 \text{ mm}^3 \] gross cross section modulus

\[ F_{cre} := \frac{M_{crey}}{S_{fy}} = 978.865 \text{ MPa} \] critical elastic lateral-torsional buckling stress

\[ F_{cre} \geq 2.78 \cdot F_y = 0 \]

\[ 2.78 \cdot F_y > F_{cre} > 0.56 \cdot F_y = 1 \]

\[ F_n := \frac{10}{9} \cdot F_y \left(1 - \frac{10 \cdot F_y}{36 \cdot F_{cre}}\right) = 419.478 \text{ MPa} \] (Eq. F2.1-2)

\[ M_{ney} := S_{fy} \cdot F_n = 3.628 \text{ kN} \cdot \text{m} \] nominal axial strength for yielding and global buckling about the y-y (2-2) axis

b.2.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

\[ M_{ney} = 3.628 \text{ kN} \cdot \text{m} \]

\[ M_{crly} := 1568.7918 \cdot 1000 \text{ N} \cdot \text{mm} = 1.569 \text{ kN} \cdot \text{m} \] critical elastic local buckling moment from CUFSM, reduced thickness

\[ \lambda_i := \sqrt{\frac{M_{ney}}{M_{crly}}} = 1.521 \]
\[ \lambda_l > 0.776 = 1 \]

\[
M_{nly} := \left( 1 - 0.15 \left( \frac{M_{crl}}{M_{ney}} \right)^{0.4} \right) \cdot \left( \frac{M_{crl}}{M_{ney}} \right)^{0.4} \cdot M_{ney} = 2.316 \text{ kN} \cdot \text{m}
\]

\[ I_{yy} = 308988.0814 \text{ mm}^4 \]  

\[ x_g = 35.5852 \text{ mm} \]

\[ S_{nety} = \frac{I_{yy}}{x_g} = (8.683 \cdot 10^3) \text{ mm}^3 \]  

\[ M_{ynety} = S_{nety} \cdot F_y = 3.734 \text{ kN} \cdot \text{m} \]  

\[ M_{nly} \leq M_{ynety} = 1 \]

\[ M_{ag} := \min(M_{ney}, M_{nly}) = 2.316 \text{ kN} \cdot \text{m} \]

**Verification:**

\[ \frac{P}{P_a} = 0.974 \]

\[ \frac{M_x}{M_{ax}} = 0.025 \]

\[ \frac{M_y}{M_{ay}} = 0.086 \]

\[ \frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} = 1.085 \]

**5.1.5 Global, local+global and distortional buckling, using the section without perforations**

This a model for the **section without perforations**.

\[ P_{nt} = 85.5 \text{ kN} \]  

\[ M_{nt,x} = 0.1 \text{ kN} \cdot \text{m} \]  

axial force from first-order analysis, with the structure restrained against lateral translation

moment from first-order elastic analysis, with the structure restrained against lateral translation the x-x axis (y-y for EU)
M_{nt,y} := 0.2 \ kN \cdot m \quad \text{required flexural strength along the y-y axis (z-z for EU)}

a. Required axial strength

\( P \) \quad \text{required second-order axial strength. It is permitted to be taken as } P_{nt} + P_{lt} \\
\( P_{lt} := 0 \) \quad \text{axial force from first order analysis, with the structure restrained against lateral translation}

\( P := P_{nt} + P_{lt} = 85.5 \ \text{kN} \) \quad \text{(Eq. C1.2.1.1-2)}

\[ \alpha := 1 \]

\( P_{e1} := 610460 \ \text{N} \) \quad \text{elastic critical buckling strength of the member in the plane of bending, calculated based on the assumption of no lateral translation at member ends}

b. Required flexural strength about the x-x axis

\( M_1 := -0.03 \ \text{kN} \cdot \text{m} \) \quad \text{smaller and larger moments, respectively, at the ends of that portion of the member unbraced in the plane of bending under consideration. } M_1 \text{ and } M_2 \text{ are calculated from a first-order elastic analysis. } M_1/M_2 \text{ is positive when the member is bent in reverse curvature, negative when bent in single curvature.}

\[ \frac{M_1}{M_2} = -0.429 \]

\[ c_m := \text{coefficient assuming no lateral translation of the frame} \]

\[ c_m := 0.6 - 0.4 \cdot \left( \frac{M_1}{M_2} \right) = 0.771 \] \quad \text{(Eq. C1.2.1.1-4)}
\[
B_1 := \frac{c_m}{1 - \alpha \cdot \frac{P}{P_{c1}}} = 0.897 \quad \text{(Eq. C1.2.1.1-3)}
\]

\[
B_1 \geq 1 = 0
\]

\[
B_1 := 1
\]

\[
M_{lt} := 0 \cdot kN \cdot m \quad \text{moment from first-order analysis due to lateral translation of the structure only}
\]

\[
B_2 := 0 \quad \text{multiplier to account for P-Δ effects, determined for each story of the structure and each direction of lateral translation of the story}
\]

\[
M_x := B_1 \cdot M_{nt,x} + B_2 \cdot M_{lt} = 0.1 \quad kN \cdot m \quad \text{(Eq. C1.2.1.1-1)}
\]

\[
M_x \quad \text{required flexural strength about the x-x axis}
\]

c. Required flexural strength about the y-y axis

\[
M_1 := 0.03 \quad kN \cdot m \quad \text{smaller and larger moments, respectively, at the ends of that portion of the member unbraced in the plane of bending under consideration. M1 and M2 are calculated from a first-order elastic analysis. M1/M2 is positive when the member is bent in reverse curvature, negative when bent in single curvature.}
\]

\[
M_2 := 0.07 \quad kN \cdot m
\]

\[
\frac{M_1}{M_2} = 0.429
\]

\[
c_m := 0.6 - 0.4 \cdot \left( \frac{M_1}{M_2} \right) = 0.429 \quad \text{(Eq. C1.2.1.1-4)}
\]

\[
B_1 := \frac{c_m}{1 - \alpha \cdot \frac{P}{P_{c1}}} = 0.498 \quad \text{(Eq. C1.2.1.1-3)}
\]

\[
B_1 \geq 1 = 0
\]

\[
B_1 := 1
\]

\[
M_{lt} := 0 \cdot kN \cdot m \quad \text{moment from first-order analysis due to lateral translation of the structure only}
\]

\[
B_2 := 0 \quad \text{multiplier to account for P-Δ effects, determined for each story of the structure and each direction of lateral translation of the story}
\]

\[
M_y := B_1 \cdot M_{nt,y} + B_2 \cdot M_{lt} = 0.2 \quad kN \cdot m \quad \text{(Eq. C1.2.1.1-1)}
\]

\[
M_y \quad \text{required flexural strength about the y-y axis}
\]
The available axial strength $P_a$ (factored resistance) shall be the smallest of the available axial strength $F_n$ for global buckling, nominal axial strength $P_{nl}$ for local buckling interacting with global buckling, and the nominal axial strength $P_{nd}$ for distortional buckling.

### a. Members in compression (chapter E)

The available flexural strengths determined as required in Section F.

#### a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]

- $A_y := 445 \text{ mm}^2$
- $F_y := 430 \text{ MPa}$ yield stress
- $P_{cre} := 187745 \text{ N}$ minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP, for gross cross section

$$F_{cre} = \frac{P_{cre}}{A_y} = 421.899 \text{ MPa}$$ least of the applicable elastic global(flexural, torsional and flexural torsional) buckling stress

$$\lambda_c := \sqrt{\frac{F_y}{F_{cre}}} = 1.01$$ slenderness (Eq. E2-4)

$$\lambda_c \leq 1.5 = 1 \quad F_n := \left(0.658 \lambda_c^2\right) F_y = 280.675 \text{ MPa} \quad (\text{Eq. E2-2})$$

$$P_{ne} := A_y F_n = 124.9 \text{ kN}$$ nominal axial strength (Eq. E2-1)

#### a.2 Local Buckling Interacting with Yielding and Global Buckling [E3.2.1]

For the model without holes, instead of applying the Q RMI method (which is suitable for models with holes only), I will apply the Direct Strength Method from AISI S-100. The value of $P_{nl}$ will be calculated using a critical elastic local buckling load, $P_{crl}$, calculated in CUFSM.

- $P_{nl}$ nominal axial strength
- $P_{ne} = 124.9 \text{ kN}$
- $Load := 1000 \text{ N}$
- $load.factor := 182.4047$

$$P_{crl} := Load \cdot load.factor = 182.405 \text{ kN}$$ critical elastic local buckling load determined by analysis in CUFSM, for the gross cross section
\[
\lambda_l := \sqrt{\frac{P_{ne}}{P_{crl}}} = 0.827 \quad \text{(Eq. E3.2.1-3)}
\]

\[
\lambda_l > 0.776 = 1
\]

\[
P_{nl} := \left(1 - 0.15 \cdot \left(\frac{P_{crl}}{P_{ne}}\right)^{0.4}\right) \cdot \left(\frac{P_{crl}}{P_{ne}}\right)^{0.4} \cdot P_{ne} = 119.964 \text{ kN}
\]

### a.3 Distortional Buckling [E4.1]

\[P_{nd} \quad \text{nominal axial strength for distortional buckling}\]

\[F_y = 430 \text{ MPa}\]

\[P_y := A_y \cdot F_y = 191.35 \text{ kN} \quad \text{(Eq. E4.1-7)}\]

Using \(P_{crd}\) calculated with CUFSM

\[Load := 1000 \text{ N}\]

\[load.factor := 157.1437\]

\[P_{crd} := \text{Load} \cdot \text{load.factor} = 157.144 \text{ kN} \quad \text{critical elastic distortional column buckling load determined by analysis in CUFSM, for gross cross section}\]

\[
\lambda_d := \sqrt{\frac{P_{crd}}{P_y}} = 1.103 \quad \text{slenderness} \quad \text{(Eq. E4.1-3)}
\]

\[
\lambda_d > 0.561 = 1 \quad \text{(Eq. E4.1-2)}
\]

\[
P_{nd} := \left(1 - 0.25 \cdot \left(\frac{P_{crd}}{P_y}\right)^{0.6}\right) \cdot \left(\frac{P_{crd}}{P_y}\right)^{0.6} \cdot P_y = 132.255 \text{ kN}
\]

\[P_a := \min(P_{ne}, P_{nl}, P_{nd}) = 119.964 \text{ kN}\]

### b. Members in flexure (chapter F)

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and global buckling \(M_{ne}\), the available flexural strength due to the interaction of the yielding or global buckling with local buckling \(M_{nl}\) and the nominal flexural strength for distortional buckling \(M_{nd}\).
b.1. Flexure about x-x (1-1) axis

b.1.1 Yielding and Global Buckling [F2]

$$M_{crex} := 3.1529 \cdot 10^7 \ N \cdot mm$$ critical elastic bending moment from CUTWP, gross section

$$S_{fx} := 11959 \ mm^3$$ gross cross section modulus

$$F_{cre} := \frac{M_{crex}}{S_{fx}} = (2.636 \cdot 10^3) \ MPa$$ critical elastic lateral-torsional buckling stress

$$F_{cre} \geq 2.78 \cdot F_y = 1$$

$$F_n := F_y = 430 \ MPa$$

$$M_{nex} := S_{fx} \cdot F_n = 5.142 \ kN \cdot m$$ nominal axial strength for yielding and global buckling about the x-x (1-1) axis (Eq. F2.1-1)

b.1.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

As the Q cannot be determined I will use subchapter F3.2.2 of AISI S-100

$$M_{nex} = 5.142 \ kN \cdot m$$

$$M_{crlx} := 6163.7634 \cdot 1000 \ N \cdot mm = 6.164 \ kN \cdot m$$ critical elastic local buckling moment from CUFSM, gross cross section

$$\lambda_l := \sqrt{\frac{M_{nex}}{M_{crlx}}} = 0.913$$

$$\lambda_l > 0.776 = 1$$

$$M_{nlx} := \left(1 - 0.15 \cdot \left(\frac{M_{crlx}}{M_{nex}}\right)^{0.4} \cdot \left(\frac{M_{crlx}}{M_{nex}}\right)^{0.4} \cdot M_{nex} = 4.637 \ kN \cdot m$$

b.1.3 Distortional Buckling [F4.1]

$$S_{fx} := 11959 \ mm^3$$ gross cross section modulus

$$F_y = 430 \ MPa$$ yield stress
\[ M_{yz} := S_{fy} \cdot F_y = 5.142 \text{ kN} \cdot \text{m} \] yield moment \hspace{1cm} \text{(Eq. F4.1-4)}

\[ M_{crdx} := 6038.9886 \cdot 1000 \text{ N} \cdot \text{mm} = 6.039 \text{ kN} \cdot \text{m} \] critical elastic distrotional buckling moment about x-x axis from CUFSM, gross cross section

\[ \lambda_d := \sqrt{\frac{M_{yz}}{M_{crdx}}} = 0.923 \] slenderness \hspace{1cm} \text{(Eq. F4.1-3)}

\[ \lambda_d > 0.673 = 1 \]

\[ M_{ndx} := \left( 1 - 0.22 \cdot \left( \frac{M_{crdx}}{M_{yz}} \right)^{0.5} \right) \cdot \left( \frac{M_{crdx}}{M_{yz}} \right)^{0.5} \cdot M_{yz} = 4.244 \text{ kN} \cdot \text{m} \]

\[ M_{ndx} \] nominal flexural strength about the x-x (1-1) axis

\[ M_{ax} := \min (M_{nex}, M_{nlx}, M_{ndx}) = 4.244 \text{ kN} \cdot \text{m} \]

b.2. Flexure about y-y (2-2) axis

b.2.1 Yielding and Global Buckling [F2]

\[ M_{crey} := 9.54956 \cdot 10^6 \text{ N} \cdot \text{mm} \] critical elastic bending moment from CUTWP, gross section

\[ S_{fy} := 8650 \text{ mm}^3 \] gross section modulus

\[ F_{cre} := \frac{M_{crey}}{S_{fy}} = (1.104 \cdot 10^3) \text{ MPa} \] critical elastic lateral-torsional buckling stress

\[ F_{cre} \geq 2.78 \cdot F_y = 0 \]

\[ 2.78 \cdot F_y > F_{cre} > 0.56 \cdot F_y = 1 \]

\[ F_n := \frac{10}{9} \cdot F_y \left( 1 - \frac{10 \cdot F_y}{36 \cdot F_{cre}} \right) = 426.086 \text{ MPa} \] \hspace{1cm} \text{(Eq. F2.1-2)}

\[ M_{ney} := S_{fy} \cdot F_n = 3.686 \text{ kN} \cdot \text{m} \] nominal axial strength for yielding and global buckling about the y-y (2-2) axis \hspace{1cm} \text{(Eq. F2.1-1)}
b.2.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.1]

\[ M_{ney} = 3.686 \text{ kN} \cdot \text{m} \]

\[ M_{crly} := 6150.2983 \cdot 1000 \text{ N} \cdot \text{mm} = 6.15 \text{ kN} \cdot \text{m} \]

critical elastic local buckling moment from CUFSM, gross cross section

\[ \lambda_l := \sqrt{\frac{M_{ney}}{M_{crly}}} = 0.774 \]

\[ \lambda_l > 0.776 = 0 \]

\[ M_{nly} := M_{ney} = 3.686 \text{ kN} \cdot \text{m} \]

b.2.3 Distortional Buckling [F4.1]

\[ F_y = 430 \text{ MPa} \]

yield stress

\[ M_{yy} := S_{fy} \cdot F_y = 3.72 \text{ kN} \cdot \text{m} \]

yield moment \hspace{1cm} (Eq. F4.1-4)

\[ M_{crdy} := 4083.3801 \cdot 1000 \text{ N} \cdot \text{mm} = 4.083 \text{ kN} \cdot \text{m} \]

critical elastic distortional buckling moment about y-y axis from CUFSM, gross cross section

\[ \lambda_d := \sqrt{\frac{M_{yx}}{M_{crdx}}} = 0.923 \]

slenderness \hspace{1cm} (Eq. F4.1-3)

\[ \lambda_d > 0.673 = 1 \]

\[ M_{ndy} := \left( 1 - 0.22 \left( \frac{M_{crdy}}{M_{yy}} \right)^{0.5} \right) \left( \frac{M_{crdy}}{M_{yy}} \right)^{0.5} \cdot M_{yy} = 2.999 \text{ kN} \cdot \text{m} \]

\[ M_{ndy} \]

nominal flexural strength about the y-y (2-2) axis

\[ M_{ay} := \min{(M_{ney}, M_{nly}, M_{ndy})} = 2.999 \text{ kN} \cdot \text{m} \]

Verification:

\[ \frac{P}{P_a} = 0.713 \]

\[ \frac{M_y}{M_{ay}} = 0.067 \]

\[ \frac{M_x}{M_{ax}} = 0.024 \]

\[ \frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} = 0.803 \]
5.2. US - ELM for Column 2.99x3x0.105inch

The value of the notional loads:

\[ Y_i := 127.4 \text{ kN} \]

\[ \alpha := 1 \]

\[ N_i := \frac{1}{240} \cdot \alpha \cdot Y_i = 0.531 \text{ kN} \]

Consteel has implemented only the European standards, hence the axes are named after Eurocode (y and z). For this procedure the axes will be changed according to the American standard, so x-x will be the symmetry axis and y-y the one perpendicular to x-x.

Results from the first-order analysis

\[ P_{nt} := 131 \text{ kN} \]

axial force from first-order analysis, with the structure restrained against lateral translation

\[ M_{nt,x} := 0.1 \text{ kN} \cdot \text{m} \]

moment from first-order elastic analysis, with the structure restrained against lateral translation the x-x axis (y-y for EU)

\[ M_{nt,y} := 0.3 \text{ kN} \cdot \text{m} \]

required flexural strength along the y-y axis (z-z for EU)

\[ a. \text{ Required axial strength} \]

\[ P \]

required second-order axial strength. It is permitted to be taken as \( P_{nt} + P_{lt} \)

\[ P_{lt} := 0 \]

axial force from first order analysis, with the structure restrained against lateral translation

\[ P := P_{nt} + P_{lt} = 131 \text{ kN} \]

(Eq. C1.2.1.1-2)
\( \alpha := 1 \)

\( P_{e1} := 801648 \text{ N} \) elastic critical buckling strength of the member in the plane of bending, calculated based on the assumption of no lateral translation at member ends

b. Required flexural strength about the x-x axis

\( M_1 := -0.04 \text{ kN} \cdot \text{m} \)

\( M_2 := 0.011 \text{ kN} \cdot \text{m} \)

\( \frac{M_1}{M_2} = -3.636 \)

\( c_m \) coefficient assuming no lateral translation of the frame

\( c_m := 0.6 - 0.4 \cdot \left( \frac{M_1}{M_2} \right) = 2.055 \) \hspace{1cm} (Eq. C1.2.1.1-4)

\( B_1 := \frac{c_m}{1 - \alpha \cdot \frac{P}{P_{e1}}} = 2.456 \) \hspace{1cm} (Eq. C1.2.1.1-3)

\( B_1 \geq 1 = 1 \)

\( B_1 = 2.456 \)

\( M_l := 0 \text{ kN} \cdot \text{m} \) moment from first-order analysis due to lateral translation of the structure only

\( B_2 := 0 \) multiplier to account for P-\( \Delta \) effects, determined for each story of the structure and each direction of lateral translation of the story

\( M_x := B_1 \cdot M_{nl,x} + B_2 \cdot M_l = 0.246 \text{ kN} \cdot \text{m} \) \hspace{1cm} (Eq. C1.2.1.1-1)

\( M_x \) required flexural strength about the x-x axis
c. Required flexural strength about the y-y axis

\[ M_1 := 0.17 \text{ kN} \cdot \text{m} \]
\[ M_2 := 0.33 \text{ kN} \cdot \text{m} \]

smaller and larger moments, respectively, at the ends of that portion of the member unbraced in the plane of bending under consideration. \( M_1 \) and \( M_2 \) are calculated from a first-order elastic analysis. \( M_1/M_2 \) is positive when the member is bent in reverse curvature, negative when bent in single curvature.

\[ \frac{M_1}{M_2} = 0.515 \]

\( c_m \) coefficient assuming no lateral translation of the frame

\[ c_m := 0.6 - 0.4 \cdot \left( \frac{M_1}{M_2} \right) = 0.394 \]  \( \text{Eq. C1.2.1.1-4} \)

\[ B_1 := \frac{c_m}{1 - \frac{\alpha \cdot P}{P_{e1}}} = 0.471 \]  \( \text{Eq. C1.2.1.1-3} \)

\[ B_1 \geq 1 = 0 \]

\[ B_1 := 1 \]

\[ M_l := 0.3 \text{ kN} \cdot \text{m} \]

moment from first-order analysis due to lateral translation of the structure only

\[ B_2 := 0 \]

multiplier to account for \( P-\Delta \) effects, determined for each story of the structure and each direction of lateral translation of the story

\[ M_y := B_1 \cdot M_{nt.y} + B_2 \cdot M_l = 0.3 \text{ kN} \cdot \text{m} \]  \( \text{Eq. C1.2.1.1-1} \)

\( M_y \) required flexural strength about the y-y axis

\( P_a \) available axial strength determined in accordance with Chapter E

\( M_{ax}, M_{ay} \) available flexural strengths determined as required in Section F

5.2.1 Global, local+global and distortional buckling

a. Members in compression (chapter E)

The available axial strength \( P_a \) (factored resistance) shall be the smallest of the available axial strength \( F_n \) for global buckling, nominal axial strength \( P_{nl} \) for local buckling interacting with global buckling, and the nominal axial strength \( P_{nd} \) for distortional buckling.

a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]

\[ A_g := 694 \text{ mm}^2 \]

gross cross-section area from Consteel

\[ F_y := 370 \text{ MPa} \]

yield stress

\[ P_{cre} := 236979 \text{ N} \]

minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP, for reduced thickness
\[ F_{\text{cre}} := \frac{P_{\text{cre}}}{A_g} = 341.468 \text{ MPa} \]

least of the applicable elastic global (flexural, torsional and flexural torsional) buckling stress

\[ \lambda_c := \sqrt{\frac{F_y}{F_{\text{cre}}}} = 1.041 \]  

slenderness  

(Eq. E2-4)

\[ \lambda_c \leq 1.5 = 1 \quad F_n := (0.658 \lambda_c^2) \cdot F_y = 235.093 \text{ MPa} \]  

(Eq. E2-2)

\[ F_n \]  

Compressive stress

\[ P_{\text{ne}} := A_g \cdot F_n = 163.154 \text{ kN} \]  

nominal axial strength  

(Eq. E2-1)

### a.2 Local Buckling Interacting with Yielding and Global Buckling [E3.2]

\[ P_{nl} \]  

nominal axial strength

\[ P_{\text{ne}} = 163.154 \text{ kN} \]

\[ A_{\text{net.min}} := 547.741 \text{ mm}^2 \]  

cross-section area from the experiment (stub column test)

\[ F_n := \frac{P_{\text{ne}}}{A_g} = 235.093 \text{ MPa} \]

\[ P_{\text{ua}} := 45.59 \text{ kip} = (2.028 \cdot 10^5) \text{ N} \]  

experimental ultimate compressive strength of the stub column test

\[ Q := \frac{P_{\text{ua}}}{A_{\text{net.min}} \cdot F_y} = 1.001 \]

Using Q RMI (see report from T. Pekoz, August 2015)

\[ Q := 1 \]

\[ P_{nl} := F_n \left[ 1 - (1 - Q) \cdot \left( \frac{F_n}{F_y} \right)^Q \right] \cdot A_g = [163.154] \text{ kN} \]

\[ P_{\text{ynet}} := A_{\text{net.min}} \cdot F_y = 202.664 \text{ kN} \]

\[ P_{nl} \leq P_{\text{ynet}} = [1] \]

### a.3 Distortional Buckling [E4.2]

\[ P_{nd} \]  

nominal axial strength for distortional buckling

\[ F_y = 370 \text{ MPa} \]

\[ A_{\text{net}} := 547.741 \text{ mm}^2 \]

\[ A_g = 694 \text{ mm}^2 \]

\[ P_y := A_g \cdot F_y = 256.78 \text{ kN} \]  

(Eq. E4.2-7)
\[ P_{\text{ynet}} = A_{\text{net}} \cdot F_y = 202.664 \text{ kN} \quad \text{(Eq. E4.2-8)} \]

Using \( P_{\text{crd}} \) calculated with CUFSM

\[ \text{Load} := 1000 \text{ N} \]
\[ \text{load.factor} := 258.5945 \]

\[ P_{\text{crd}} := \text{Load} \cdot \text{load.factor} = 258.595 \text{ kN} \text{ critical elastic distortional column buckling load determined by analysis in CUFSM, for reduced thickness} \]

\[ \lambda_d := \sqrt{\frac{P_y}{P_{\text{crd}}}} = 0.996 \quad \text{slenderness} \quad \text{(Eq. E4.2-3)} \]

\[ \lambda_{d2} := 0.561 \cdot \left( 14 \cdot \left( \frac{P_y}{P_{\text{ynet}}} \right)^{0.4} - 13 \right) = 1.341 \quad \text{(Eq. E4.2-5)} \]

\( \lambda_d \leq \lambda_{d2} = 1 \) will use subchapter E4.2 (members with holes)

\[ P_{\text{ynet}} = A_{\text{net}} \cdot F_y = 202.664 \text{ kN} \quad \text{(Eq. E4.2-8)} \]

\[ \lambda_{d1} := 0.561 \cdot \left( \frac{P_{\text{ynet}}}{P_y} \right) = 0.443 \quad \text{(Eq. E4.2-4)} \]

\[ P_{d2} := \left( 1 - 0.25 \cdot \left( \frac{1}{\lambda_{d2}} \right)^{1.2} \right) \cdot \left( \frac{1}{\lambda_{d2}} \right)^{1.2} \cdot P_y = 148.841 \text{ kN} \quad \text{(Eq. E4.2-6)} \]

\( \lambda_{d1} < \lambda_d \leq \lambda_{d2} = 1 \)

\[ P_{\text{nd}} := P_{\text{ynet}} \cdot \left( \frac{P_{\text{ynet}} - P_{d2}}{\lambda_{d2} - \lambda_{d1}} \right) \cdot (\lambda_d - \lambda_{d1}) = 169.479 \text{ kN} \quad \text{(Eq. E4.2-2)} \]

\[ P_a := \min (P_{\text{ne}}, P_{\text{nl}}, P_{\text{nd}}) = 163.154 \text{ kN} \]

b. Members in flexure (chapter F)

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and global buckling \( M_{\text{ne}} \), the available flexural strength due to the interaction of the yielding or global buckling with local buckling \( M_{\text{nl}} \) and the nominal flexural strength for distortional buckling \( M_{\text{nd}} \).

b.1. Flexure about x-x (1-1) axis

b.1.1 Yielding and Global Buckling [F2]

\[ M_{\text{crex}} := 4.42029 \cdot 10^7 \text{ N} \cdot \text{mm} \quad \text{critical elastic bending moment from CUTWP, reduced thickness} \]

\[ S_{tx} := 18107 \text{ mm}^3 \quad \text{gross cross section modulus} \]
\[ F_{cre} := \frac{M_{cre}}{S_{fx}} = (2.441 \cdot 10^3) \text{ MPa} \] critical elastic lateral-torsional buckling stress

\[ F_{cre} \geq 2.78 \cdot F_y = 1 \] (Eq. F2.1-1)

\[ F_n := F_y = 370 \text{ MPa} \]

\[ M_{nex} := S_{fx} \cdot F_n = 6.7 \text{ kN \cdot m} \] nominal axial strength for yielding and global buckling about the x-x (1-1) axis (Eq. F2.1-1)

b.1.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

As the Q cannot be determined, subchapter F3.2.2 will be used

\[ M_{nex} = 6.7 \text{ kN \cdot m} \]

\[ M_{crk} := 14869.8255 \cdot 1000 \text{ N \cdot mm} = 14.87 \text{ kN \cdot m} \] critical elastic local buckling moment from CUFSM, reduced thickness (the same as for the global buckling)

\[ \lambda_l := \sqrt{\frac{M_{nex}}{M_{crk}}} = 0.671 \] (Eq. F3.2.1-3)

\[ \lambda_l < 0.776 = 1 \] (Eq. F3.2.1-1)

\[ M_{nlx} := M_{nex} = 6.7 \text{ kN \cdot m} \]

\[ I_{xx} := 583565.0981 \text{ mm}^4 \] moment of inertia about the x-x axis

\[ y_g := 37.1196 \text{ mm} \]

\[ S_{fnetx} := \frac{I_{xx}}{y_g} = (1.572 \cdot 10^4) \text{ mm}^3 \] net section modulus

\[ M_{ynetx} := S_{fnetx} \cdot F_y = 5.817 \text{ kN \cdot m} \] yield moment of net cross-section (Eq. F3.2.2-2)

\[ M_{nlx} \leq M_{ynetx} = 0 \] so \[ M_{nlx} := M_{ynetx} = 5.817 \text{ kN \cdot m} \]

b.1.3 Distortional Buckling [F4.2]

\[ S_{fx} := 18107 \text{ mm}^3 \] gross cross section modulus

\[ F_y = 370 \text{ MPa} \] yield stress

\[ M_{yx} := S_{fx} \cdot F_y = 6.7 \text{ kN \cdot m} \] yield moment (Eq. F4.1-4)

\[ M_{crk} := 10338.2261 \cdot 1000 \text{ N \cdot mm} = 10.338 \text{ kN \cdot m} \] critical elastic distortional buckling moment about x-x axis from CUFSM, reduced thickness
\[ \lambda_d := \sqrt{\frac{M_{yx}}{M_{crx}}} = 0.805 \text{ slenderness (Eq. F4.1-3)} \]

\[ M_{ynetx} = 5.817 \text{ kN} \cdot \text{m} \]

\[ \lambda_{d2} := 0.673 \cdot \left( 1.7 \cdot \left( \frac{M_{yx}}{M_{ynetx}} \right)^{2.7} - 0.7 \right) = 1.204 \text{ (Eq. F4.2-5)} \]

\[ \lambda_d \leq \lambda_{d2} = 1 \text{ will use subchapter F4.2 (members with holes)} \]

\[ \lambda_{d1} := 0.673 \cdot \left( \frac{M_{ynetx}}{M_{yx}} \right)^3 = 0.44 \text{ (Eq. F4.2-4)} \]

\[ \lambda_{d1} < \lambda_d \leq \lambda_{d2} = 1 \]

\[ M_{d2} := \left( 1 - 0.22 \cdot \left( \frac{1}{\lambda_{d2}} \right) \right) \cdot \left( \frac{1}{\lambda_{d2}} \right) \cdot M_{yx} = 4.547 \text{ kN} \cdot \text{m} \]

\[ M_{ndx} := M_{ynetx} - \frac{M_{ynetx} - M_{d2}}{\lambda_{d2} - \lambda_{d1}} \cdot (\lambda_d - \lambda_{d1}) = 5.211 \text{ kN} \cdot \text{m} \text{ (Eq. F4.2-2)} \]

\[ M_{ndx} \text{ nominal flexural strength about the x-x (1-1) axis (Eq. F4.1-2)} \]

\[ M_{ax} := \min \left( M_{nex}, M_{nlx}, M_{ndx} \right) = 5.211 \text{ kN} \cdot \text{m} \]

**b.2. Flexure about y-y (2-2) axis**

**b.2.1 Yielding and Global Buckling [F2]**

\[ M_{crey} := 1.35583 \cdot 10^7 \text{ N} \cdot \text{mm} \text{ critical elastic bending moment from CUTWP, reduced thickness} \]

\[ S_{fy} := 13429 \text{ mm}^3 \text{ gross cross section modulus} \]

\[ F_{cre} := \frac{M_{crey}}{S_{fy}} = (1.01 \cdot 10^3) \text{ MPa} \text{ critical elastic lateral-torsional buckling stress} \]

\[ F_{cre} \geq 2.78 \cdot F_y = 0 \text{ (Eq. F2.1-1)} \]

\[ 2.78 \cdot F_y \geq F_{cre} \geq 0.56 \cdot F_y = 1 \]

\[ F_n := \frac{10}{9} \cdot F_y \cdot \left( 1 - \frac{10 \cdot F_y}{36 \cdot F_{cre}} \right) = 369.261 \text{ MPa} \]

\[ M_{ney} := S_{fy} \cdot F_n = 4.959 \text{ kN} \cdot \text{m} \text{ nominal axial strength for yielding and global buckling about the y-y (2-2) axis (Eq. F2.1-1)} \]
b.2.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

\[ M_{ney} = 4.959 \text{ kN} \cdot \text{m} \]

\[ M_{cly} := 6000.4396 \cdot 1000 \text{ N} \cdot \text{mm} = 6 \text{ kN} \cdot \text{m} \]

critical elastic local buckling moment from CUFSM, reduced thickness

\[ \lambda_1 := \sqrt{\frac{M_{ney}}{M_{cly}}} = 0.909 \]

\[ \lambda_1 > 0.776 = 1 \]

\[ M_{ney} := \left(1 - 0.15 \cdot \left(\frac{M_{cly}}{M_{ney}}\right)^{0.4}\right) \cdot \left(\frac{M_{cly}}{M_{ney}}\right)^{0.4} \cdot M_{ney} = 4.485 \text{ kN} \cdot \text{m} \] (Eq. F3.21-1)

\[ I_{yy} := 483242.9716 \text{ mm}^4 \]

moment of inertia about the y-y axis

\[ x_g := 35.5852 \text{ mm} \]

\[ S_{fney} := \frac{I_{yy}}{x_g} = \left(1.358 \cdot 10^4\right) \text{ mm}^3 \]

net section modulus

\[ M_{ynety} := S_{fney} \cdot F_y = 5.025 \text{ kN} \cdot \text{m} \]

yield moment of net cross-section (Eq. F3.2.2-2)

\[ M_{nly} \leq M_{ynety} = 1 \]

b.2.3 Distortional Buckling [F4.2]

\[ S_fy = \left(1.343 \cdot 10^4\right) \text{ mm}^3 \]

gross cross section modulus

\[ F_y = 370 \text{ MPa} \]

yield stress

\[ M_{fy} := S_fy \cdot F_y = 4.969 \text{ kN} \cdot \text{m} \]

yield moment (Eq. F4.1-4)

\[ M_{cly} := 7399.1076 \cdot 1000 \text{ N} \cdot \text{mm} = 7.399 \text{ kN} \cdot \text{m} \]

critical elastic distortional buckling moment about y-y axis from CUFSM, reduced thickness

\[ \lambda_d := \sqrt{\frac{M_{fy}}{M_{cly}}} = 0.805 \]

slenderness (Eq. F4.1-3)

\[ M_{ynety} = 5.025 \text{ kN} \cdot \text{m} \]
\[ \lambda_{d2} = 0.673 \cdot \left( 1.7 \cdot \left( \frac{M_{yy}}{M_{y_{net}}} \right)^{2.7} - 0.7 \right) = 0.639 \]  
(Eq. 1-4.2-5)

\[ \lambda_d \leq \lambda_{d2} = 0 \quad \Rightarrow \quad \text{will use subchapter F4.1 (members without holes)} \]

\[ M_{ndy} = \left( 1 - 0.22 \cdot \left( \frac{M_{crdy}}{M_{yy}} \right)^{0.5} \right) \cdot \left( \frac{M_{crdy}}{M_{yy}} \right)^{0.5} \cdot M_{yy} = 4.436 \, kN \cdot m \]  
(Eq. F4.2-4)

\[ M_{ndy} \quad \text{nominal flexural strength about the y-y (2-2) axis} \]  
(Eq. F4.2-2)

\[ M_{ay} := \min (M_{ny}, M_{ny}, M_{ndy}) = 4.436 \, kN \cdot m \]

**Verification:**

\[ \frac{P}{P_a} = 0.803 \]

\[ \frac{M_x}{M_{ax}} = 0.047 \]

\[ \frac{M_y}{M_{ay}} = 0.068 \]

\[ \frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} = 0.918 \]

### 5.2.2 Global, local+global and distortional+global buckling

**a. Members in compression (chapter E)**

The available axial strength \( P_a \) (factored resistance) shall be the smallest of the available axial strength \( F_n \) for global buckling, nominal axial strength \( P_{nl} \) for local buckling interacting with global buckling, and the nominal axial strength \( P_{nd} \) for the interaction of distortional and global buckling.

**a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]**

\[ A_g := 694 \, \text{mm}^2 \quad \text{gross cross-section area from Consteel} \]

\[ F_y := 370 \, \text{MPa} \quad \text{yield stress} \]

\[ P_{cre} := 236979 \, \text{N} \quad \text{minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP, for reduced thickness} \]
\[ F_{cre} := \frac{P_{cre}}{A_g} = 341.468 \text{ MPa} \]  
least of the applicable elastic global (flexural, torsional and flexural torsional) buckling stress

\[ \lambda_c := \sqrt{\frac{F_y}{F_{cre}}} = 1.041 \]  
slenderness  
(Eq. E2-4)

\[ \lambda_c \leq 1.5 = 1 \quad F_n := (0.658 \lambda_c^2) \cdot F_y = 235.093 \text{ MPa} \]  
(Eq. E2-2)

\[ F_n \quad \text{Compressive stress} \]

\[ P_{ne} := A_g \cdot F_n = 163.154 \text{ kN} \]  
nominal axial strength  
(Eq. E2-1)

### a.2 Local Buckling Interacting with Yielding and Global Buckling [E3.2]

\[ P_{nl} \quad \text{nominal axial strength} \]

\[ P_{ne} = 163.154 \text{ kN} \]

\[ A_{net.min} := 547.741 \text{ mm}^2 \]  
minimum net cross-section area

\[ F_n := \frac{P_{ne}}{A_g} = 235.093 \text{ MPa} \]

\[ P_{ua} := 45.59 \text{ kip} = (2.028 \cdot 10^5) \text{ N} \]  
experimental ultimate compressive strength of the stub column test

\[ Q := \frac{P_{ua}}{A_{net.min} \cdot F_y} = 1.001 \]

Using Q RMI (see report from T. Pekoz, August 2015)

\[ Q := 1 \]

\[ P_{nl} := F_n \left[ 1 - (1 - Q) \cdot \left( \frac{F_n}{F_y} \right)^2 \right] \cdot A_g = [163.154] \text{ kN} \]

### a.3 Distortional Buckling interacting with Global Buckling [E4.2 and E2]

For simulating the interaction of distortional and global buckling, the nominal axial strength for global buckling will be introduced in the calculation formulas of the distortional buckling, namely, instead of using \( P_y \) and \( P_{ynet} \) calculated with the yield stress \( F_y \), they will be calculated with \( F_n \), the compressive stress for global buckling.

\[ P_{nd} \quad \text{nominal axial strength for distortional buckling} \]

\[ F_n = 235.093 \text{ MPa} \]
\[ A_{\text{net}} = 547.741 \text{ mm}^2 \]
\[ A_g = 694 \text{ mm}^2 \]
\[ P_{ne} = A_g \cdot F_n = 163.154 \text{ kN} \quad (\text{Eq. E4.2-7}) \]
\[ P_{ne,\text{net}} = A_{\text{net}} \cdot F_n = 128.77 \text{ kN} \quad (\text{Eq. E4.2-8}) \]

Using \( P_{\text{crd}} \) calculated with CUFSM
\[ \text{Load} := 1000 \text{ N} \]
\[ \text{load.factor} := 258.5945 \]
\[ P_{\text{crd}} := \text{Load} \cdot \text{load.factor} = 258.595 \text{ kN} \]

Critical elastic distortional column buckling load determined by analysis in CUFSM, for reduced thickness
\[ \lambda_d := \sqrt{\frac{P_{ne}}{P_{\text{crd}}}} = 0.794 \quad \text{slenderness} \quad (\text{Eq. E4.2-3}) \]

Subchapter E4.1 will be used, for members without holes
\[ \lambda_d > 0.561 = 1 \]
\[ P_{nd} := \left( 1 - 0.25 \cdot \left( \frac{P_{\text{crd}}}{P_{\text{ne}}} \right)^{0.6} \right) \cdot \left( \frac{P_{\text{crd}}}{P_{\text{ne}}} \right)^{0.6} \cdot P_{\text{ne}} = 144.199 \text{ kN} \]
\[ P_a := \min (P_{\text{ne}}, P_{nl}, P_{nd}) = 144.199 \text{ kN} \]

b. Members in flexure (chapter F)

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and global buckling \( M_{ne} \), the available flexural strength due to the interaction of the yielding or global buckling with local buckling \( M_{nl} \), and the nominal flexural strength for distortional buckling \( M_{nd} \).

b.1. Flexure about x-x (1-1) axis

b.1.1 Yielding and Global Buckling [F2]

\[ M_{crex} := 4.42029 \cdot 10^7 \text{ N} \cdot \text{mm} \quad \text{critical elastic bending moment from CUTWP, reduced thickness} \]
\[ S_{fx} := 18107 \text{ mm}^3 \quad \text{gross cross section modulus} \]
\[ F_{cre} := \frac{M_{crex}}{S_{fx}} = (2.441 \cdot 10^3) \text{ MPa} \quad \text{critical elastic lateral-torsional buckling stress} \]
\[ F_{cre} \geq 2.78 \cdot F_y = 1 \quad (\text{Eq. F2.1-1}) \]
\[ F_n := F_y = 370 \text{ MPa} \]

\[ M_{nex} := S_{fx} \cdot F_n = 6.7 \text{ kN} \cdot \text{m} \]

nominal axial strength for yielding and global buckling about the x-x (1-1) axis (Eq. F2.1-1)

b.1.2 Local Buckling Intersecting with Yielding and Global Buckling [F3.2.2]

As the Q cannot be determined I will use subchapter F3.2.2 of AISI S-100

\[ M_{nex} = 6.7 \text{ kN} \cdot \text{m} \]

\[ M_{crlx} := 14869.8255 \cdot 1000 \text{ N} \cdot \text{mm} = 14.87 \text{ kN} \cdot \text{m} \]

critical elastic local buckling moment from CUFSM, reduced thickness (the same as for the global buckling)

\[ \lambda := \frac{M_{nex}}{M_{crlx}} = 0.671 \] (Eq. F3.2.1-3)

\[ \lambda < 0.776 = 1 \] (Eq. F3.2.1-1)

\[ M_{nlx} := M_{nex} = 6.7 \text{ kN} \cdot \text{m} \]

\[ I_{xx} := 583565.0981 \text{ mm}^4 \]

moment of inertia about the x-x axis

\[ y_g := 37.1196 \text{ mm} \]

\[ S_{netlx} := \frac{I_{xx}}{y_g} = (1.572 \cdot 10^4) \text{ mm}^3 \]

net section modulus

\[ M_{ynetlx} := S_{netlx} \cdot F_y = 5.817 \text{ kN} \cdot \text{m} \]

yield moment of net cross-section (Eq. F3.2.2-2)

\[ M_{nlx} \leq M_{ynetlx} = 0 \]

so

\[ M_{nlx} := M_{ynetlx} = 5.817 \text{ kN} \cdot \text{m} \]

b.1.3 Distortional Buckling intersecting with Global Buckling [F4.2 + F2]

\[ S_{fx} := 18107 \text{ mm}^3 \]

gross cross section modulus

\[ F_n = 370 \text{ MPa} \]

yield stress

\[ M_{nex} := S_{fx} \cdot F_n = 6.7 \text{ kN} \cdot \text{m} \]

yield moment (Eq. F4.1-4)

\[ M_{crdx} := 10338.2261 \cdot 1000 \text{ N} \cdot \text{mm} = 10.338 \text{ kN} \cdot \text{m} \]

critical elastic distortional buckling moment about x-x axis from CUFSM, reduced thickness
\[ \lambda_d = \sqrt{\frac{M_{nex}}{M_{crdx}}} = 0.805 \quad \text{slenderness} \quad \text{(Eq. F4.1-3)} \]

\[ M_{ne.netx} = S_{netx} \cdot F_n = 5.817 \text{ kN} \cdot \text{m} \]

\[ \lambda_{d2} = 0.673 \cdot \left( \frac{1.7 \cdot M_{nex}}{M_{ne.netx}} \right)^{2.7} - 0.7 = 1.204 \quad \text{(Eq. F4.2-5)} \]

\[ \lambda_d \leq \lambda_{d2} = 1 \]

\[ \lambda_{d1} = 0.673 \cdot \left( \frac{M_{ne.netx}}{M_{nex}} \right)^3 = 0.44 \]

\[ \lambda_{d1} < \lambda_d \leq \lambda_{d2} = 1 \]

\[ M_{d2} = \left( 1 - 0.22 \cdot \left( \frac{1}{\lambda_{d2}} \right) \right) \cdot \left( \frac{1}{\lambda_{d2}} \right) \cdot M_{nex} = 4.547 \text{ m} \cdot \text{kN} \]

\[ M_{ndx} = M_{ne.netx} - \left( \frac{M_{ne.netx} - M_{d2}}{\lambda_{d2} - \lambda_{d1}} \right) \cdot (\lambda_d - \lambda_{d1}) = 5.211 \text{ kN} \cdot \text{m} \quad \text{(Eq. F4.2-2)} \]

\[ M_{ndx} \leq \left( 1 - 0.22 \cdot \left( \frac{M_{crdx}}{M_{nex}} \right)^{0.5} \right) \cdot \left( \frac{M_{crdx}}{M_{nex}} \right)^{0.5} \cdot M_{nex} = 1 \]

\[ M_{ax} = \min (M_{nex}, M_{nlx}, M_{ndx}) = 5.211 \text{ kN} \cdot \text{m} \]

b.2. Flexure about y-y (2-2) axis

b.2.1 Yielding and Global Buckling [F2]

\[ M_{crey} = 1.35583 \cdot 10^7 \text{ N} \cdot \text{mm} \quad \text{critical elastic bending moment from CUTWP, reduced thickness} \]

\[ S_{fy} = 13429 \text{ mm}^3 \quad \text{gross cross section modulus} \]

\[ F_{cre} = \frac{M_{crey}}{S_{fy}} = \left( 1.01 \cdot 10^3 \right) \text{ MPa} \quad \text{critical elastic lateral-torsional buckling stress} \]

\[ F_{cre} \geq 2.78 \cdot F_y = 0 \quad \text{(Eq. F2.1-1)} \]

\[ 2.78 \cdot F_y > F_{cre} \geq 0.56 \cdot F_y = 1 \]

\[ F_n = \frac{10}{9} \cdot F_y \left( 1 - \frac{10 \cdot F_y}{36 \cdot F_{cre}} \right) = 369.261 \text{ MPa} \]
\[ M_{\text{ney}} := S_{fy} \cdot F_n = 4.959 \text{ kN} \cdot \text{m} \]

nominal axial strength for yielding and global buckling about the y-y (2-2) axis

(Eq. F2.1-1)

b.2.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

\[ M_{\text{ney}} = 4.959 \text{ kN} \cdot \text{m} \]

critical elastic local buckling moment from CUFSM, reduced thickness

\[ \lambda_l := \sqrt{\frac{M_{\text{ney}}}{M_{\text{crly}}}} = 0.909 \]

\[ \lambda_l > 0.776 \Rightarrow 1 \]

\[ M_{\text{crly}} := \left(1 - 0.15 \cdot \left(\frac{M_{\text{crly}}}{M_{\text{ney}}}\right)^{0.4}\right) \cdot \left(M_{\text{crly}} \right)^{0.4} \cdot M_{\text{ney}} = 4.485 \text{ kN} \cdot \text{m} \]

(Eq. F3.21-1)

\[ I_{yy} := 483242.9716 \text{ mm}^4 \]

moment of inertia about the y-y axis

\[ x_g := 35.5852 \text{ mm} \]

\[ S_{\text{fnety}} := \frac{I_{yy}}{x_g} = (1.358 \cdot 10^4) \text{ mm}^3 \]

net section modulus

\[ M_{\text{ynety}} := S_{\text{fnety}} \cdot F_y = 5.025 \text{ kN} \cdot \text{m} \]

yield moment of net cross-section

(Eq. F3.2.2-2)

\[ M_{\text{ly}} \leq M_{\text{ynety}} = 1 \]

b.2.3 Distortional Buckling + Global Buckling [F4.2]

\[ S_{fy} = (1.343 \cdot 10^4) \text{ mm}^3 \]

gross cross section modulus

\[ F_n = 369.261 \text{ MPa} \]

yield stress

\[ M_{\text{ney}} := S_{fy} \cdot F_n = 4.959 \text{ kN} \cdot \text{m} \]

yield moment

(Eq. F4.1-4)

\[ M_{\text{crdx}} := 7399.1076 \cdot 1000 \text{ N} \cdot \text{mm} = 7.399 \text{ kN} \cdot \text{m} \]

critical elastic distortional buckling moment about y-y axis from CUFSM, reduced thickness

\[ \lambda_d := \sqrt{\frac{M_{\text{ney}}}{M_{\text{crdx}}}} = 0.693 \]

slenderness

(Eq. F4.1-3)

\[ M_{\text{ne.nety}} := S_{\text{fnety}} \cdot F_n = 5.015 \text{ kN} \cdot \text{m} \]
\[ \lambda_{d2} = 0.673 \cdot \left( 1.7 \cdot \left( \frac{M_{ney}}{M_{ne,ney}} \right)^{2.7} \right) = 0.639 \]  
(Eq. F4.2-5)

\[ \lambda_d \leq \lambda_{d2} = 0 \]  
will use subchapter F4.1 (members without holes)

\[ M_{ndy} := \left( 1 - 0.22 \cdot \left( \frac{M_{crdy}}{M_{ney}} \right)^{0.5} \right) \cdot \left( \frac{M_{crdy}}{M_{ney}} \right)^{0.5} \cdot M_{ney} = 4.429 \text{ kN} \cdot \text{m} \]  
(Eq. F4.2-4)

\[ M_{ndy} \]  
nominal flexural strength about the y-y (2-2) axis  
(Eq. F4.2-2)

\[ M_{sy} := \min (M_{ney}, M_{nly}, M_{ndy}) = 4.429 \text{ kN} \cdot \text{m} \]

Verfication:

\[ \frac{P}{P_a} = 0.908 \]

\[ \frac{M_x}{M_{ax}} = 0.047 \]

\[ \frac{M_y}{M_{ay}} = 0.068 \]

\[ \frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} = 1.023 \]

5.2.3 Global, local+global and distortional buckling, using only \( A_{net,\text{min}} \)

In this model only the minimal cross section area will be used.

a. Members in compression (chapter E)

The available axial strength \( P_a \) (factored resistance) shall be the smallest of the available axial strength \( F_n \) for global buckling, nominal axial strength \( P_m \) for local buckling interacting with global buckling, and the nominal axial strength \( P_{nd} \) for distortional buckling.

a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]

\[ A_{net,\text{min}} := 547.741 \text{ mm}^2 \]  
minimum net cross-section area

\[ F_y := 370 \text{ MPa} \]  
yield stress

\[ P_{cre} := 236979 \text{ N} \]  
minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP, for reduced thickness
\[ F_{\text{cre}} := \frac{P_{\text{cre}}}{A_{\text{net.min}}} = 432.648 \text{ MPa} \] least of the applicable elastic global (flexural, torsional and flexural torsional) buckling stress

\[ \lambda_c := \sqrt{\frac{F_y}{F_{\text{cre}}}} = 0.925 \] slenderness (Eq. E2-4)

\[ \lambda_c \leq 1.5 = 1 \quad F_n := (0.658 \lambda_c^2) \cdot F_y = 258.672 \text{ MPa} \] (Eq. E2-2)

\[ F_n \] Compressive stress

\[ P_{\text{ne}} := A_{\text{net.min}} \cdot F_n = 141.685 \text{ kN} \] nominal axial strength (Eq. E2-1)

**a.2 Local Buckling Interacting with Yielding and Global Buckling [E3.2]**

\[ P_{n_l} \] nominal axial strength

\[ P_{\text{ne}} = 141.685 \text{ kN} \]

\[ F_n := \frac{P_{\text{ne}}}{A_{\text{net.min}}} = 258.672 \text{ MPa} \]

\[ P_{\text{ua}} := 45.59 \text{ kip} = (2.028 \cdot 10^5) \text{ N} \] experimental ultimate compressive strength of the stub column test

\[ Q := \frac{P_{\text{ua}}}{A_{\text{net.min}} \cdot F_y} = 1.001 \]

Using Q RMI (see report from T. Pekoz, August 2015)

\[ Q := 1 \]

\[ P_{n_l} := F_n \cdot \left[ 1 - (1 - Q) \cdot \left( \frac{F_n}{F_y} \right)^Q \right] \cdot A_{\text{net.min}} = [141.685] \text{ kN} \]

**a.3 Distortional Buckling [E4.2]**

\[ P_{n_d} \] nominal axial strength for distortional buckling

\[ F_y = 370 \text{ MPa} \]

\[ A_{\text{net.min}} := 547.741 \text{ mm}^2 \]

\[ P_y := A_{\text{net.min}} \cdot F_y = 202.664 \text{ kN} \] (Eq. E4.2-7)

Using \( P_{\text{crd}} \) calculated with CUFSM

\[ \text{Load} := 1000 \text{ N} \]
load.factor := 258.5945

\[ P_{crd} := Load \cdot load.factor = 258.595 \text{ kN} \] critical elastic distortional column buckling load
determined by analysis in CUFSM, for reduced thickness

\[ \lambda_d := \sqrt{\frac{P_y}{P_{crd}}} = 0.885 \] slenderness \hspace{1cm} (Eq. E4.2-3)

\[ \lambda_d > 0.561 = 1 \]
\[ P_{nd} := \left(1 - 0.25 \cdot \left(\frac{P_{crd}}{P_y}\right)^{0.6}\right) \cdot \left(\frac{P_{crd}}{P_y}\right)^{0.6} \cdot P_y = 166.698 \text{ kN} \] \hspace{1cm} (Eq. E4.1-2)

\[ P_a := \min(P_{ne}, P_{nl}, P_{nd}) = 141.685 \text{ kN} \]

b. Members in flexure (chapter F)

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and
global buckling \( M_{ne} \), the available flexural strength due to the interaction of the yielding or global buckling
with local buckling \( M_{nl} \) and the nominal flexural strength for distortional buckling \( M_{nd} \).

b.1. Flexure about x-x (1-1) axis

b.1.1 Yielding and Global Buckling [F2]

\[ M_{crex} := \left(4.42029 \cdot 10^7\right) \text{ N-mm} \] critical elastic bending moment from CUTWP, reduced thickness

\[ I_{xx} := 583565.0981 \text{ mm}^4 \] moment of inertia about the x-x axis

\[ y_g := 37.1196 \text{ mm} \]

\[ S_{fnetx} := \frac{I_{xx}}{y_g} = \left(1.572 \cdot 10^4\right) \text{ mm}^3 \] net section modulus

\[ F_{cre} := \frac{M_{crex}}{S_{fnetx}} = \left(2.812 \cdot 10^3\right) \text{ MPa} \] critical elastic lateral-torsional buckling stress

\[ F_{cre} \geq 2.78 \cdot F_y = 1 \] \hspace{1cm} (Eq. F2.1-1)

\[ F_n := F_y = 370 \text{ MPa} \]

\[ M_{nex} := S_{fnetx} \cdot F_n = 5.817 \text{ kN-m} \] nominal axial strength for yielding
and global buckling about the x-x (1-1) axis \hspace{1cm} (Eq. F2.1-1)
b.1.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

As the Q cannot be determined I will use subchapter F3.2.2 of AISI S-100

\[ M_{nex} = 5.817 \, kN \cdot m \]

\[ M_{crdx} := 14869.8255 \cdot 1000 \, N \cdot mm = 14.87 \, kN \cdot m \]

\( \lambda_i := \sqrt{\frac{M_{nex}}{M_{crdx}}} = 0.625 \) (Eq. F3.2.1-3)

\( \lambda_i < 0.776 = 1 \) (Eq. F3.2.1-1)

\[ M_{nlx} := M_{nex} = 5.817 \, kN \cdot m \]

b.1.3 Distortional Buckling [F4.2]

\[ F_y = 370 \, MPa \]

\[ M_{yx} := S_{netx} \cdot F_y = 5.817 \, kN \cdot m \] (Eq. F4.1-4)

\[ M_{crdx} := 10338.2261 \cdot 1000 \, N \cdot mm = 10.338 \, kN \cdot m \]

\[ \lambda_d := \sqrt{\frac{M_{yx}}{M_{crdx}}} = 0.75 \] slenderness (Eq. F4.1-3)

\( \lambda_d > 0.673 = 1 \)

\[ M_{ndx} := \left( 1 - 0.22 \cdot \left( \frac{M_{crdx}}{M_{yx}} \right)^{0.5} \cdot \left( \frac{M_{crdx}}{M_{yx}} \right)^{0.5} \cdot M_{yx} \right) = 5.48 \, kN \cdot m \]

\[ M_{ndx} \]

nominal flexural strength about the x-x (1-1) axis (Eq. F4.1-2)

\[ M_{ax} := \min (M_{nex}, M_{nlx}, M_{ndx}) = 5.48 \, kN \cdot m \]

b.2. Flexure about y-y (2-2) axis

b.2.1 Yielding and Global Buckling [F2]

\[ M_{crey} := (1.35583 \cdot 10^7) \, N \cdot mm \] critical elastic bending moment from CUTWP, reduced thickness

\[ I_{yy} := 483242.9716 \, mm^4 \] moment of inertia about the y-y axis
\[ x_g := 35.5852 \text{ mm} \]
\[ S_{nely} := \frac{l_{yy}}{x_g} = \left(1.358 \cdot 10^4\right) \text{ mm}^3 \]  
net section modulus

\[ F_{cre} := \frac{M_{crey}}{S_{nely}} = 998.41 \text{ MPa} \]
critical elastic lateral-torsional buckling stress

\[ F_{cre} \geq 2.78 \cdot F_y = 0 \]  
(Eq. F2.1-1)

\[ 2.78 \cdot F_y > F_{cre} \geq 0.56 \cdot F_y = 1 \]

\[ F_n := \frac{10}{9} \cdot F_y \cdot \left(1 - \frac{10 \cdot F_y}{36 \cdot F_{cre}}\right) = 368.791 \text{ MPa} \]

\[ M_{nely} := S_{nely} \cdot F_n = 5.008 \text{ kN \cdot m} \]  
nominal axial strength for yielding and global buckling about the y-y (2-2) axis  
(Eq. F2.1-1)

\[ b.2.2 \text{ Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]} \]

\[ M_{nely} = 5.008 \text{ kN \cdot m} \]

\[ M_{cly} := 6000.4396 \cdot 1000 \text{ N \cdot mm} = 6 \text{ kN \cdot m} \]  
critical elastic local buckling moment from CUFSM, reduced thickness

\[ \lambda_1 := \sqrt{\frac{M_{nely}}{M_{cly}}} = 0.914 \]
\[ \lambda_1 > 0.776 = 1 \]

\[ M_{nely} := \left(1 - 0.15 \cdot \left(\frac{M_{cly}}{M_{nely}}\right)^{0.4}\right) \cdot \left(\frac{M_{cly}}{M_{nely}}\right)^{0.4} \cdot M_{nely} = 4.516 \text{ kN \cdot m} \]  
(Eq. F3.21-1)

\[ M_{ynyety} := S_{nely} \cdot F_y = 5.025 \text{ kN \cdot m} \]  
yield moment of net cross-section  
(Eq. F3.2.2-2)

\[ M_{nly} \leq M_{ynyety} = 1 \]

\[ b.2.3 \text{ Distortional Buckling [F4.2]} \]

\[ F_y = 370 \text{ MPa} \]  
yield stress

\[ M_{yy} := S_{nely} \cdot F_y = 5.025 \text{ kN \cdot m} \]  
yield moment  
(Eq. F4.1-4)

\[ M_{cly} := 7399.1076 \cdot 1000 \text{ N \cdot mm} = 7.399 \text{ kN \cdot m} \]  
critical elastic distortional buckling moment about y-y axis from CUFSM, reduced thickness
\[ \lambda_d := \sqrt{\frac{M_{yx}}{M_{crd}}} = 0.75 \quad \text{slenderness} \quad \text{(Eq. F4.1-3)} \]

\[ \lambda_d > 0.673 = 1 \]

\[ M_{ndy} := \left( 1 - 0.22 \cdot \left( \frac{M_{crdy}}{M_{yy}} \right)^{0.5} \right) \cdot \left( \frac{M_{crdy}}{M_{yy}} \right)^{0.5} \cdot M_{yy} = 4.47 \text{kN} \cdot \text{m} \quad \text{(Eq. F4.2-4)} \]

\[ M_{ndy} \quad \text{nominal flexural strength about the y-y (2-2) axis} \quad \text{(Eq. F4.2-2)} \]

\[ M_{ay} := \min (M_{ney}, M_{nly}, M_{ndy}) = 4.47 \text{kN} \cdot \text{m} \]

**Verification:**

\[ \frac{P}{P_a} = 0.925 \]

\[ \frac{M_x}{M_{ax}} = 0.045 \]

\[ \frac{M_y}{M_{ay}} = 0.067 \]

\[ \frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} = 1.037 \]

5.2.4 Global, local+global buckling

This model **won't take into account the effect of distortional buckling.**

**a. Members in compression (chapter E)**

The available axial strength \( P_a \) (factored resistance) shall be the smallest of the available axial strength \( F_n \) for global buckling, nominal axial strength \( P_{nl} \) for local buckling interacting with global buckling

*a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]*

\[ A_g := 694 \text{mm}^2 \quad \text{gross cross-section area from Consteel} \]

\[ F_y := 370 \text{MPa} \quad \text{yield stress} \]

\[ P_{cre} := 236979 \text{N} \quad \text{minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP, for reduced thickness} \]

\[ F_{cre} := \frac{P_{cre}}{A_g} = 341.468 \text{MPa} \quad \text{least of the applicable elastic global(flexural, torsional and flexural torsional) buckling stress} \]
\( \lambda_c := \sqrt{\frac{F_y}{F_{cre}}} = 1.041 \)  
slenderness  
(Eq. E2-4)

\[ \lambda_c \leq 1.5 = 1 \]

\[ F_n := \left(0.658 \lambda_c^2\right) \cdot F_y = 235.093 \text{ MPa} \]  
(Eq. E2-2)

\( F_n \)  
Compressive stress

\[ P_{ne} := A_g \cdot F_n = 163.154 \text{ kN} \]  
nominal axial strength  
(Eq. E2-1)

a.2 Local Buckling Interacting with Yielding and Global Buckling [E3.2]

\[ P_{nl} \]  
nominal axial strength

\[ P_{ne} = 163.154 \text{ kN} \]

\[ A_{net.min} := 547.741 \text{ mm}^2 \]  
minimum net cross-section area

\[ F_n := \frac{P_{ne}}{A_g} = 235.093 \text{ MPa} \]

\[ P_{ua} := 45.59 \text{ kip} = \left(2.028 \cdot 10^5\right) \text{ N} \]  
experimental ultimate compressive strength of the stub column test

\[ Q := \frac{P_{ua}}{A_{net.min} \cdot F_y} = 1.001 \]

Using Q RMI (see report from T. Pekoz, August 2015)

\[ Q := 1 \]

\[ P_{nl} := F_n \cdot \left[1 - (1 - Q) \cdot \left(\frac{F_n}{F_y}\right)^Q\right] \cdot A_g = [163.154] \text{ kN} \]

\[ P_a := \min(P_{ne}, P_{nl}) = 163.154 \text{ kN} \]

b. Members in flexure (chapter F)

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and global buckling \( M_{ne} \), the available flexural strength due to the interaction of the yielding or global buckling with local buckling \( M_{nl} \).

b.1. Flexure about x-x (1-1) axis

b.1.1 Yielding and Global Buckling [F2]

\[ M_{crex} := 4.42029 \cdot 10^7 \text{ N} \cdot \text{mm} \]  
critical elastic bending moment from CUTWP, reduced thickness
\( S_{fx} := 18107 \text{ mm}^3 \) 

\( F_{cre} := \frac{M_{crex}}{S_{fx}} = (2.441 \times 10^3) \text{ MPa} \) 

Critical elastic lateral-torsional buckling stress

\( F_{cre} \geq 2.78 \cdot F_y = 1 \) 

(Eq. F2.1-1)

\( F_n := F_y = 370 \text{ MPa} \)

Nominal axial strength for yielding and global buckling about the x-x (1-1) axis

b.1.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

As the Q cannot be determined I will use subchapter F3.2.2 of AISI S-100

\( M_{nex} = 6.7 \text{ kN} \cdot \text{m} \)

\( M_{crlx} := 14869.8255 \cdot 1000 \text{ N} \cdot \text{mm} = 14.87 \text{ kN} \cdot \text{m} \) 

Critical elastic local buckling moment from CUFSM, reduced thickness (the same as for the global buckling)

\[ \lambda_j := \sqrt{\frac{M_{nex}}{M_{crlx}}} = 0.671 \]  

(Eq. F3.2.1-3)

\[ \lambda_j < 0.776 = 1 \]  

(Eq. F3.2.1-1)

\( M_{nlx} := M_{nex} = 6.7 \text{ kN} \cdot \text{m} \)

\( I_{xx} := 583565.0981 \text{ mm}^4 \) 

Moment of inertia about the x-x axis

\( y_g := 37.1196 \text{ mm} \)

\( S_{fnetx} := \frac{I_{xx}}{y_g} = (1.572 \times 10^4) \text{ mm}^3 \) 

Net section modulus

\( M_{ynetx} := S_{fnetx} \cdot F_y = 5.817 \text{ kN} \cdot \text{m} \) 

Yield moment of net cross-section 

(Eq. F3.2.2-2)

\[ M_{nlx} \leq M_{ynetx} = 0 \] 

so 

\[ M_{nlx} := M_{ynetx} = 5.817 \text{ kN} \cdot \text{m} \]

\[ M_{ax} := \min(M_{nex}, M_{nlx}) = 5.817 \text{ kN} \cdot \text{m} \]

b.2. Flexure about y-y (2-2) axis

b.2.1 Yielding and Global Buckling [F2]

\( M_{crey} := 1.35583 \times 10^7 \text{ N} \cdot \text{mm} \) 

Critical elastic bending moment from CUTWP, reduced thickness
Sfy := 13429 mm³  

\[ F_{cre;} = \frac{M_{crey}}{S_{fy}} = (1.01 \cdot 10^3) \text{ MPa} \]

gross cross section modulus

\[ F_{cre} \geq 2.78 \cdot F_y = 0 \]

(Eq. F2.1-1)

\[ 2.78 \cdot F_y > F_{cre} \geq 0.56 \cdot F_y = 1 \]

\[ F_n := \frac{10}{9} \cdot F_y \left( 1 - \frac{10 \cdot F_y}{36 \cdot F_{cre}} \right) = 369.261 \text{ MPa} \]

\[ M_{ney} := S_{fy} \cdot F_n = 4.959 \text{ kN} \cdot \text{m} \]

nominal axial strength for yielding and
global buckling about the y-y (2-2) axis

(Eq. F2.1-1)

b.2.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

\[ M_{ney} = 4.959 \text{ kN} \cdot \text{m} \]

\[ M_{crey} := 6000.4396 \cdot 1000 \text{ N} \cdot \text{mm} = 6 \text{ kN} \cdot \text{m} \]

critical elastic local buckling moment from
CUFSM, reduced thickness

\[ \lambda := \sqrt{\frac{M_{ney}}{M_{crey}}} = 0.909 \]

\[ \lambda > 0.776 \geq 1 \]

\[ M_{nly} := \left( 1 - 0.15 \cdot \left( \frac{M_{crey}}{M_{ney}} \right)^{0.4} \right) \cdot \left( \frac{M_{crey}}{M_{ney}} \right)^{0.4} \cdot M_{ney} = 4.485 \text{ kN} \cdot \text{m} \]

(Eq. F3.21-1)

\[ I_{yy} := 483242.9716 \text{ mm}^4 \]

moment of inertia about the y-y axis

\[ x_g := 35.5852 \text{ mm} \]

\[ S_{fnety} := \frac{I_{yy}}{x_g} = (1.358 \cdot 10^4) \text{ mm}^3 \]

net section modulus

\[ M_{ynety} := S_{fnety} \cdot F_y = 5.025 \text{ kN} \cdot \text{m} \]

yield moment of net cross-section

(Eq. F3.2.2-2)

\[ M_{nly} \leq M_{ynety} = 1 \]

\[ M_{ay} := \min (M_{ney}, M_{nly}) = 4.485 \text{ kN} \cdot \text{m} \]

Verification:

\[ \frac{P}{P_a} = 0.803 \]

\[ \frac{M_x}{M_{ax}} = 0.042 \]

Design of pallet rack upright frames subject to compression
\[ \frac{M_y}{M_{ay}} = 0.067 \]
\[ \frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} = 0.912 \]

### 5.2.5 Global, local+global, distortional buckling, using the section without perforations

This a model for the **section without perforations**.

- **\( P_{nt} := 131 \text{ kN} \)** axial force from first-order analysis, with the structure restrained against lateral translation
- **\( M_{nt,x} := 0.1 \text{ kN} \cdot \text{m} \)** moment from first-order elastic analysis, with the structure restrained against lateral translation along the x-x axis (y-y for EU)
- **\( M_{nt,y} := 0.3 \text{ kN} \cdot \text{m} \)** required flexural strength along the y-y axis (z-z for EU)

#### a. Required axial strength

- **\( P \)** required second-order axial strength. It is permitted to be taken as \( P_{nt} + P_{it} \)
- **\( P_{it} := 0 \)** axial force from first-order analysis, with the structure restrained against lateral translation
- **\( P := P_{nt} + P_{it} = 131 \text{ kN} \)\) (Eq. C1.2.1.1-2)

- **\( \alpha := 1 \)**

- **\( P_{e1} := 954732 \text{ N} \)** elastic critical buckling strength of the member in the plane of bending, calculated based on the assumption of no lateral translation at member ends
b. Required flexural strength about the x-x axis

\[ M_1 := -0.04 \text{ kN} \cdot \text{m} \]

\[ M_2 := 0.011 \text{ kN} \cdot \text{m} \]

\[ \frac{M_1}{M_2} = -3.636 \]

\[ c_m \]

\[ c_m := 0.6 - 0.4 \cdot \left( \frac{M_1}{M_2} \right) = 2.055 \quad \text{(Eq. C1.2.1.1-4)} \]

\[ B_1 := \frac{c_m}{1 - \alpha \cdot \left( \frac{P}{P_{01}} \right)} = 2.381 \quad \text{(Eq. C1.2.1.1-3)} \]

\[ B_1 \geq 1 = 1 \]

\[ B_1 = 2.381 \]

\[ M_h := 0 \cdot \text{kN} \cdot \text{m} \]

\[ M_h \] moment from first-order analysis due to lateral translation of the structure only

\[ B_2 := 0 \]

multiplier to account for P-\(\Delta\) effects, determined for each story of the structure and each direction of lateral translation of the story

\[ M_x := B_1 \cdot M_{nt,x} + B_2 \cdot M_n = 0.238 \text{ kN} \cdot \text{m} \quad \text{(Eq. C1.2.1.1-1)} \]

\[ M_x \]

required flexural strength about the x-x axis

c. Required flexural strength about the y-y axis

\[ M_1 := 0.17 \text{ kN} \cdot \text{m} \]

\[ M_2 := 0.33 \text{ kN} \cdot \text{m} \]

\[ \frac{M_1}{M_2} = 0.515 \]

\[ c_m \]

\[ c_m := 0.6 - 0.4 \cdot \left( \frac{M_1}{M_2} \right) = 0.394 \quad \text{(Eq. C1.2.1.1-4)} \]
\[ B_1 := \frac{c_m}{1 - \alpha \cdot \frac{P}{P_{e1}}} = 0.457 \quad \text{(Eq. C1.2.1.1-3)} \]

\[ B_1 \geq 1 = 0 \]

\[ B_1 := 1 \]

\[ M_n := 0 \cdot kN \cdot m \quad \text{moment from first-order analysis due to lateral translation of the structure only} \]

\[ B_2 := 0 \quad \text{multiplier to account for P-\Delta effects, determined for each story of the structure and each direction of lateral translation of the story} \]

\[ M_y := B_1 \cdot M_{nl,y} + B_2 \cdot M_n = 0.3 \ kN \cdot m \quad \text{(Eq. C1.2.1.1-1)} \]

\[ M_y \quad \text{required flexural strength about the y-y axis} \]

\[ P_a \quad \text{available axial strength determined in accordance with Chapter E} \]

\[ M_{ax}, M_{ay} \quad \text{available flexural strengths determined as required in Section F} \]

**a. Members in compression (chapter E)**

The available axial strength \( P_a \) (factored resistance) shall be the smallest of the available axial strength \( F_n \) for global buckling, nominal axial strength \( P_{nl} \) for local buckling interacting with global buckling, and the nominal axial strength \( P_{nd} \) for distortional buckling.

**a.1 Yielding and Global (Flexural, Flexural-Torsional and Torsional) Buckling [E2]**

\[ A_g := 694 \ mm^2 \]

\[ F_y := 370 \ MPa \quad \text{yield stress} \]

\[ P_{cre} := 300891 \ N \quad \text{minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP, for gross cross section} \]

\[ F_{cre} := \frac{P_{cre}}{A_g} = 433.561 \ MPa \quad \text{least of the applicable elastic global(flexural, torsional and flexural torsional) buckling stress} \]

\[ \lambda_c := \sqrt{\frac{F_y}{F_{cre}}} = 0.924 \quad \text{slenderness} \quad \text{(Eq. E2-4)} \]

\[ \lambda_c \leq 1.5 = 1 \quad F_n := (0.658 \lambda_c^2) \cdot F_y = 258.867 \ MPa \quad \text{(Eq. E2-2)} \]

\( F_n \quad \text{Compressive stress} \)
For the model without holes, instead of applying the Q RMI method (which is suitable for models with holes only), I will apply the Direct Strength Method from AISI S-100. The value of \( P_{nl} \) will be calculated using a critical elastic local buckling load, \( P_{cr,l} \), calculated in CUFSM.

\[
P_{nl} = \frac{P_{ne}}{P_{cr,l}} = 0.512 \quad \text{(Eq. E3.2.1-3)}
\]

\[
\lambda_{l} = 0.776 = 1
\]

\[
P_{nl} = P_{ne} = 179.653 \text{ kN} \quad \text{(Eq. E3.2.1-1)}
\]

**a.3 Distortional Buckling [E4.1]**

\[
P_{nd} = \text{nominal axial strength for distortional buckling}
\]

\[
F_{y} = 370 \text{ MPa}
\]

\[
P_{y} = A_{g} \cdot F_{y} = 256.78 \text{ kN} \quad \text{(Eq. E4.1-7)}
\]

Using \( P_{cr,d} \) calculated with CUFSM

\[
Load := 1000 \text{ N}
\]

\[
load.factor := 404.3171
\]

\[
P_{cr,d} = Load \cdot load.factor = 404.317 \text{ kN} \quad \text{critical elastic distortional column buckling load determined by analysis in CUFSM, for gross cross section}
\]

\[
\lambda_{d} = \sqrt{\frac{P_{y}}{P_{cr,d}}} = 0.797 \quad \text{slenderness} \quad \text{(Eq. E4.1-3)}
\]

\[
\lambda_{d} > 0.561 = 1
\]
\[ P_{nd} = \left(1 - 0.25 \cdot \left(\frac{P_{ord}}{P_y}\right)^{0.6}\right) \cdot \left(\frac{P_{ord}}{P_y}\right)^{0.6} \cdot P_y = 226.49 \text{ kN} \quad \text{(Eq. E4.1-2)} \]

\[ P_a := \min \left(P_{ne}, P_{nl}, P_{nd}\right) = 179.653 \text{ kN} \quad \text{from global buckling} \]

b. Members in flexure (chapter F)

The available flexural strength shall be the smallest of the nominal flexural strength for yielding and global buckling \(M_{ne}\), the available flexural strength due to the interaction of the yielding or global buckling with local buckling \(M_{nl}\) and the nominal flexural strength for distortional buckling \(M_{nd}\).

b.1. Flexure about x-x (1-1) axis

b.1.1 Yielding and Global Buckling [F2]

\[ M_{crex} := 5.00633 \cdot 10^7 \text{ N} \cdot \text{mm} \quad \text{critical elastic bending moment from CUTWP, gross section} \]

\[ S_{fx} := 18107 \text{ mm}^3 \]

\[ F_{cre} := \frac{M_{crex}}{S_{fx}} = \left(2.765 \cdot 10^3\right) \text{ MPa} \quad \text{critical elastic lateral-torsional buckling stress} \]

\[ F_{cre} \geq 2.78 \cdot F_y = 1 \]

\[ F_n := F_y = 370 \text{ MPa} \]

\[ M_{nex} := S_{fx} \cdot F_n = 6.7 \text{ kN} \cdot \text{m} \quad \text{nominal axial strength for yielding and global buckling about the x-x (1-1) axis} \quad \text{(Eq. F2.1-1)} \]

b.1.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.2]

As the Q cannot be determined I will use subchapter F3.2.2 of AISI S-100

\[ M_{nex} = 6.7 \text{ kN} \cdot \text{m} \]

\[ M_{crfx} := 23121.2747 \cdot 1000 \text{ N} \cdot \text{mm} = 23.121 \text{ kN} \cdot \text{m} \quad \text{critical elastic local buckling moment from CUFSM, gross cross section} \]

\[ \lambda_i := \sqrt{\frac{M_{nex}}{M_{crfx}}} = 0.538 \]

\[ \lambda_i < 0.776 = 1 \]

\[ M_{nlx} := M_{nex} = 6.7 \text{ kN} \cdot \text{m} \]
b.1.3 Distortional Buckling [F4.1]

\[ S_{tx} := 11959 \text{ mm}^3 \]

\[ F_y = 370 \text{ MPa} \]

\[ M_{yx} := S_{tx} \cdot F_y = 4.425 \text{ kN} \cdot \text{m} \]

\[ M_{crdx} := 15307.6696 \cdot 1000 \text{ N} \cdot \text{mm} = 15.308 \text{ kN} \cdot \text{m} \]

\[ \lambda_d := \sqrt{\frac{M_{yx}}{M_{crdx}}} = 0.538 \]

\[ \lambda_d < 0.673 = 1 \]

\[ M_{ndx} := M_{yx} = 4.425 \text{ m} \cdot \text{kN} \]

\[ M_{ndx} \quad \text{nominal flexural strength about the x-x (1-1) axis} \]

\[ M_{ax} := \min\left( M_{nex}, M_{nlx}, M_{ndx} \right) = 4.425 \text{ kN} \cdot \text{m} \]

b.2. Flexure about y-y (2-2) axis

b.2.1 Yielding and Global Buckling [F2]

\[ M_{crey} := 1.53646 \cdot 10^7 \text{ N} \cdot \text{mm} \]

\[ S_{ty} := 13429 \text{ mm}^3 \]

\[ F_{cre} := \frac{M_{crey}}{S_{ty}} = \left( 1.144 \cdot 10^3 \right) \text{ MPa} \]

\[ F_{cre} \geq 2.78 \cdot F_y = 1 \]

\[ F_n := F_y = 370 \text{ MPa} \]

\[ M_{ney} := S_{ty} \cdot F_n = 4.969 \text{ kN} \cdot \text{m} \]

\[ M_{ney} \quad \text{nominal axial strength for yielding and global buckling about the y-y (2-2) axis} \]
b.2.2 Local Buckling Interacting with Yielding and Global Buckling [F3.2.1]

\[ M_{vell} = 4.969 \, kN \cdot m \]

\[ M_{cly} := 40572.583 \cdot 1000 \, N \cdot mm = 40.573 \, kN \cdot m \]

\[ \lambda_l := \sqrt{\frac{M_{vell}}{M_{cly}}} = 0.35 \]

\[ \lambda_l \leq 0.776 = 1 \quad \text{(Eq. F3.1-1)} \]

\[ M_{nly} := M_{vell} = 4.969 \, kN \cdot m \]

b.2.3 Distortional Buckling [F4.1]

\[ F_y = 370 \, MPa \quad \text{yield stress} \]

\[ M_{yy} := S_y \cdot F_y = 4.969 \, kN \cdot m \quad \text{yield moment} \quad \text{(Eq. F4.1-4)} \]

\[ M_{crdy} := 10525.145 \cdot 1000 \, N \cdot mm = 10.525 \, kN \cdot m \]

\[ \lambda_d := \sqrt{\frac{M_{yy}}{M_{crdy}}} = 0.538 \quad \text{slenderness} \quad \text{(Eq. F4.1-3)} \]

\[ \lambda_d \leq 0.673 = 1 \quad \text{(Eq. F4.1-1)} \]

\[ M_{ndy} := M_{yy} = 4.969 \, m \cdot kN \]

\[ M_{ndy} \quad \text{nominal flexural strength about the y-y (2-2) axis} \]

\[ M_{ay} := \min (M_{vell}, M_{nly}, M_{ndy}) = 4.969 \, kN \cdot m \]

\[ \text{Verification:} \]

\[ \frac{P}{P_a} = 0.729 \]

\[ \frac{M_x}{M_{ax}} = 0.054 \quad \frac{P}{P_a} + \frac{M_x}{M_{ax}} + \frac{M_y}{M_{ay}} = 0.843 \]

\[ \frac{M_y}{M_{ay}} = 0.06 \]
6. US - Simplified Effective Length Method

6.1 US - SELM Column 0.07 inch

\[ P = 18.78 \text{kip} = (8.354 \cdot 10^4) \text{N} \quad \text{ultimate load from the experimental test} \]

Calculating the reduced thickness for global and distortional buckling (acc. to T. PEKOZ "Design of Rack Columns Considering Distortional Buckling")

\[ t := 1.72 \text{ mm} \quad \text{is the thickness of the steel sheet} \]

\[ L := 2 \text{ in} = 50.8 \text{ mm} \]
\[ L_{np.w} := 0.914 \text{ in} = 23.216 \text{ mm} \quad \text{length of the unperforated part of the web as shown in the figure} \]
\[ L_{np.f} := 1.469 \text{ in} = 37.313 \text{ mm} \quad \text{length of the unperforated part of the flange as shown in the figure} \]

\[ k_d := 0.8 \]
\[ k_g := 0.6 \]

\[ t_{g,w} \quad \text{reduced thickness of the strip containing web perforations} \]
\[ t_{g,f} \quad \text{reduced thickness of the strip containing flange perforations} \]
\[ t_{d,w} \quad \text{reduced thickness of the strip containing web perforations} \]
\[ t_{d,f} \quad \text{reduced thickness of the strip containing flange perforations} \]

GLOBAL BUCKLING \[ \text{will be introduced in CUTWP} \]

\[ t_{g,w} := k_g \cdot t \cdot \left( \frac{L_{np,w}}{L} \right) = 0.472 \text{ mm} \]

\[ t_{g,f} := k_g \cdot t \cdot \left( \frac{L_{np,f}}{L} \right) = 0.758 \text{ mm} \]
DISTORTIONAL BUCKLING will be introduced in CUFSM

\[ t_{d.w} = k_d \cdot t \left( \frac{L_{np.w}}{L} \right)^{\frac{1}{3}} = 1.06 \text{ mm} \]

\[ t_{d.f} = k_d \cdot t \left( \frac{L_{np.f}}{L} \right)^{\frac{1}{3}} = 1.242 \text{ mm} \]

All the buckling lengths are taken from a report from August 2015 (T. Pekoz to RMI)
- Lx RMI determination of the flexural buckling parameter with taking the effective length as 0.7 times the flexurally unsupported length of the column (60°). For the frame tested Lx RMI = .7 * 60
  - buckling length along y-y (1-1) \( KL := 0.7 \cdot 1524 \text{ mm} = (1.067 \cdot 10^3) \text{ mm} \)

- Lx RMI determination of the flexural buckling parameter with taking the effective length as 1 times the flexurally unsupported length of the column.
  - buckling length along z-z (2-2) axis \( KL := 1 \cdot 1092.2 \text{ mm} = (1.092 \cdot 10^3) \text{ mm} \)

- Lt RMI determination of the torsional buckling parameter with taking the effective length as 0.8 times the torsionally unsupported length of the column (between the braces, 43°). For the frame tested Lt RMI = .8 * 43"
  - tors. buckling length along z-z (3-3) axis \( KL := 0.8 \cdot 1092.2 \text{ mm} = 873.76 \text{ mm} \)

a. GLOBAL BUCKLING

\[ A_{net.min} := 0.551 \text{ in}^2 = 355.483 \text{ mm}^2 \] minimum net cross-section area

\[ A_g := A_{net.min} = 355.483 \text{ mm}^2 \] net cross-section area from the experiment

\[ F_y := 430 \text{ MPa} \] yield strength

\[ P_y := A_g \cdot F_y = (1.529 \cdot 10^8) \text{ N} \] (Eq. 1.2.1-4)

\[ P_{cre} := 148419 \text{ N} \] minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP

\[ \lambda_c := \sqrt{\frac{P_y}{P_{cre}}} = 1.015 \] (Eq. 1.2.1-3)

\[ \lambda_c \leq 1.5 \] (Eq. 1.2.1-1)

\[ P_{ne} := 0.658 \lambda_c^2 \cdot P_y = 99.329 \text{ kN} \]
b. DISTORTIONAL BUCKLING

\[ \text{Load} := 1000 \text{ N} \]

\[ \text{load.factor} := 99.447 \]

\[ P_{\text{crd}} := \text{Load} \cdot \text{load.factor} = 99.447 \text{ kN} \]  
\text{critical elastic distortional column buckling load determined by analysis in CUFSM}

\[ \lambda_d := \sqrt{\frac{P_y}{P_{\text{crd}}}} = 1.24 \]

\[ \lambda_d > 0.561 = 1 \]

\[ P_{\text{nd}} := \left(1 - 0.25 \cdot \left(\frac{P_{\text{crd}}}{P_y}\right)^{0.6}\right) \cdot \left(\frac{P_{\text{crd}}}{P_y}\right)^{0.6} \cdot P_y = 95.292 \text{ kN} \]

c. DISTORTIONAL BUCKLING + GLOBAL BUCKLING

\[ P_y := P_{\text{ne}} = 99.329 \text{ kN} \]

\[ \text{Load} := 1000 \text{ N} \]

\[ \text{load.factor} := 99.447 \]

\[ P_{\text{crd}} := \text{Load} \cdot \text{load.factor} = 99.447 \text{ kN} \]  
\text{critical elastic distortional column buckling load determined by analysis in CUFSM}

\[ \lambda_d := \sqrt{\frac{P_y}{P_{\text{crd}}}} = 0.999 \]

\[ \lambda_d > 0.561 = 1 \]

\[ P_{\text{nd}} := \left(1 - 0.25 \cdot \left(\frac{P_{\text{crd}}}{P_{\text{ne}}}\right)^{0.6}\right) \cdot \left(\frac{P_{\text{crd}}}{P_{\text{ne}}}\right)^{0.6} \cdot P_{\text{ne}} = 74.532 \text{ kN} \]

d. LOCAL BUCKLING + GLOBAL BUCKLING

\[ F_n := \frac{P_{\text{ne}}}{A_{\text{net.min}}} = 279.42 \text{ MPa} \]

\[ P_{\text{ua}} := 23.75 \text{ kip} = (1.056 \cdot 10^5) \text{ N} \]  
\text{experimental ultimate compressive strength of the stub column}

\[ Q := \frac{P_{\text{ua}}}{A_{\text{net.min}} \cdot F_y} = 0.691 \]

6.1.1 Q RMI

Using Q RMI (see report from T. Pekoz, August 2015)
\[ P_{nl} := F_n \cdot \left[ 1 - (1 - Q) \cdot \left( \frac{F_n}{F_y} \right)^Q \right] \cdot A_{net, min} = [76.554] \text{ kN} \]

Verification for global, local+global buckling, using Q RMI

\[ P_{ne} = 99.329 \text{ kN} \]
\[ P_{nl} := 76.554 \text{ kN} \]
\[ P_a := \min \left( P_{ne}, P_{nl} \right) = 76.554 \text{ kN} \]
\[ \frac{P}{P_a} = 1.091 \]

Verification for global, distortional, local+global buckling, using Q RMI

\[ P_{ne} = 99.329 \text{ kN} \]
\[ P_{nd} := 95.292 \text{ kN} \]
\[ P_{nl} := 76.554 \text{ kN} \]
\[ P_a := \min \left( P_{ne}, P_{nd}, P_{nl} \right) = 76.554 \text{ kN} \]
\[ \frac{P}{P_a} = 1.091 \]

Verification for global, distortional+global, local+global buckling, using Q RMI

\[ P_{ne} = 99.329 \text{ kN} \]
\[ P_{nd} := 74.532 \text{ kN} \]
\[ P_{nl} := 76.554 \text{ kN} \]
\[ P_a := \min \left( P_{ne}, P_{nd}, P_{nl} \right) = 74.532 \text{ kN} \]
\[ \frac{P}{P_a} = 1.121 \]

6.1.2 Q Sarawit

Using Q Sarawit (see report from T. Pekoz, August 2015)

\[ P_{nl} := F_n \cdot \left[ 1 - (1 - Q) \cdot \left( \frac{F_n}{F_y} \right)^Q \right] \cdot A_{net, min} = 87.636 \text{ kN} \]
Verification for global, local+global buckling, using Q Sarawit

\[ P_{ne} = 99.329 \text{ kN} \]
\[ P_{nl} = 87.636 \text{ kN} \]
\[ P_a := \min (P_{ne}, P_{nl}) = 87.636 \text{ kN} \]
\[ \frac{P}{P_a} = 0.953 \]

Verification for global, distortional, local+global buckling, using Q Sarawit

\[ P_{ne} = 99.329 \text{ kN} \]
\[ P_{nd} = 95.292 \text{ kN} \]
\[ P_{nl} = 87.636 \text{ kN} \]
\[ P_a := \min (P_{ne}, P_{nd}, P_{nl}) = 87.636 \text{ kN} \]
\[ \frac{P}{P_a} = 0.953 \]

Verification for global, distortional+global, local+global buckling, using Q Sarawit

\[ P_{ne} = 99.329 \text{ kN} \]
\[ P_{nd} = 74.532 \text{ kN} \]
\[ P_{nl} = 87.636 \text{ kN} \]
\[ P_a := \min (P_{ne}, P_{nd}, P_{nl}) = 74.532 \text{ kN} \]
\[ \frac{P}{P_a} = 1.121 \]

6.2 US - SELM Column 0.105 inch

\[ P := 28.64 \text{kip} = (1.274 \times 10^5) \text{ N} \]

Calculating the reduced thickness for global and distortional buckling (acc. to T. PEKOZ "Design of Rack Columns Considering Distortional Buckling")
$t := 2.69 \text{ mm}$ is the thickness of the steel sheet.

$L := 2 \text{ in} = 50.8 \text{ mm}$

$L_{np,w} := 0.914 \text{ in} = 23.216 \text{ mm}$ length of the unperforated part of the web as shown in the figure.

$L_{np,f} := 1.469 \text{ in} = 37.313 \text{ mm}$ length of the unperforated part of the flange as shown in the figure.

$k_d := 0.8$

$k_g := 0.6$

$t_{g,w}$ reduced thickness of the strip containing web perforations

$t_{g,f}$ reduced thickness of the strip containing flange perforations

$t_{d,w}$ reduced thickness of the strip containing web perforations

$t_{d,f}$ reduced thickness of the strip containing flange perforations

GLOBAL BUCKLING will be introduced in CUTWP

$$t_{g,w} := k_g \cdot t \cdot \left( \frac{L_{np,w}}{L} \right) = 0.738 \text{ mm}$$

$$t_{g,f} := k_g \cdot t \cdot \left( \frac{L_{np,f}}{L} \right) = 1.185 \text{ mm}$$

DISTORTIONAL BUCKLING will be introduced in CUFSM

$$t_{d,w} := k_d \cdot t \cdot \left( \frac{L_{np,w}}{L} \right)^3 = 1.658 \text{ mm}$$

$$t_{d,f} := k_d \cdot t \cdot \left( \frac{L_{np,f}}{L} \right)^3 = 1.942 \text{ mm}$$
All the buckling lengths are taken from a report from August 2015 (T. Pekoz to RMI)

- **buckling length along z-z (2-2) axis**
  
  $KL := 1 \cdot 1092.2 \text{ mm} = (1.092 \cdot 10^3) \text{ mm}$

- **Lx RMI determination of the flexural buckling parameter with taking the effective length as 0.7 times the flexurally unsupported length of the column (60°). For the frame tested Lx RMI = .7 * 60**

- **buckling length along y-y (1-1) axis**
  
  $KL := 0.7 \cdot 1524 \text{ mm} = (1.067 \cdot 10^3) \text{ mm}$

- **Lt RMI determination of the torsional buckling parameter with taking the effective length as 0.8 times the torsionally unsupported length of the column (between the braces, 43°). For the frame tested Lt RMI = .8 * 43**

- **tors. buckling length along z-z (3-3) axis**
  
  $KL := 0.8 \cdot 1092.2 \text{ mm} = 873.76 \text{ mm}$

**a. GLOBAL BUCKLING**

- **$A_{\text{net.min}} := 0.849 \text{ in}^2 = 547.741 \text{ mm}^2$** minimum net cross-section area

- **$A_g := A_{\text{net.min}} = 547.741 \text{ mm}^2$** net cross-section area from the experiment

- **$F_y := 370 \text{ MPa}$** yield strength

- **$P_y := A_g \cdot F_y = (2.027 \cdot 10^5) \text{ N}$ (Eq. 1.2.1-4)**

- **$P_{\text{cre}} := 236979 \text{ N}$** minimum of the critical elastic column buckling load in flexural, torsional or flexural-torsional buckling determined by analysis in CUTWP

- **$\lambda_c := \sqrt{\frac{P}{P_{\text{cre}}}} = 0.925$ (Eq. 1.2.1-3)**

- **$\lambda_c \leq 1.5$ (Eq. 1.2.1-1)**

- **$P_{\text{ne}} := 0.658 \cdot P_y = 141.685 \text{ kN}$**

**b. DISTORTIONAL BUCKLING + GLOBAL BUCKLING**

- **$P_y := 202.664 \text{ kN}$**

- **$Load := 1000 \text{ N}$**

- **$load.factor := 258.5945$**

- **$P_{\text{ord}} := Load \cdot load.factor = 258.595 \text{ kN}$** critical elastic distortional column buckling load determined by analysis in CUFSM
\[ \lambda_d := \sqrt{\frac{P_y}{P_{crd}}} = 0.885 \]

\[ \lambda_d > 0.561 = 1 \]

\[ P_{nd} := \left(1 - 0.25 \left(\frac{P_{crd}}{P_y}\right)^{0.6}\right) \left(\frac{P_{crd}}{P_y}\right)^{0.6} \cdot P_y = 166.698 \text{ kN} \]

c. DISTORTIONAL BUCKLING + GLOBAL BUCKLING

\[ P_y := P_{ne} = 141.685 \text{ kN} \]

\[ \text{Load} := 1000 \text{ N} \]

\[ \text{load.factor} := 258.5945 \]

\[ P_{crd} := \text{Load} \cdot \text{load.factor} = 258.595 \text{ kN} \text{ critical elastic distortional column buckling load determined by analysis in CUFSM} \]

\[ \lambda_d := \sqrt{\frac{P_y}{P_{crd}}} = 0.74 \]

\[ \lambda_d > 0.561 = 1 \]

\[ P_{nd} := \left(1 - 0.25 \left(\frac{P_{crd}}{P_{ne}}\right)^{0.6}\right) \left(\frac{P_{crd}}{P_{ne}}\right)^{0.6} \cdot P_{ne} = 130.368 \text{ kN} \]

d. LOCAL BUCKLING + GLOBAL BUCKLING

\[ F_n := \frac{P_{ne}}{A_{net.min}} = 258.672 \text{ MPa} \]

\[ P_{ua} := 45.59 \text{ kip} = (2.028 \cdot 10^5) \text{ N} \text{ experimental ultimate compressive strength of the stub column} \]

\[ Q := \frac{P_{ua}}{A_{net.min} \cdot F_y} = 1.001 \]

\[ Q := 1 \]

d.1 Using Q RMI (see report from T. Pekoz, August 2015)

\[ P_{nl} := F_n \left[1 - (1 - Q) \left(\frac{F_n}{F_y}\right)^Q\right] \cdot A_{net.min} = [141.685] \text{ kN} \]
d.2 Using Q Sarawit (see report from T. Pekoz, August 2015)

\[
Q := 0.999999999
\]

\[
P_{nl} := F_n \left(1 - (1 - Q) \left(\frac{F_n}{F_y}\right)^{1 - Q}\right) \cdot A_{net,min} = 141.685 \text{ kN}
\]

Due to the fact that \( Q = 1 \), the results obtained by using Q RMI and Q Sarawit are the same.

**Verification for global, local+global buckling**

\[
P_{ne} = 141.685 \text{ kN}
\]

\[
P_{nl} := 141.685 \text{ kN}
\]

\[
P_a := \min(P_{ne}, P_{nl}) = 141.685 \text{ kN}
\]

\[
\frac{P}{P_a} = 0.899
\]

**Verification for global, distortional, local+global buckling**

\[
P_{ne} = 141.685 \text{ kN}
\]

\[
P_{nd} := 166.698 \text{ kN}
\]

\[
P_{nl} := 141.685 \text{ kN}
\]

\[
P_a := \min(P_{ne}, P_{nd}, P_{nl}) = 141.685 \text{ kN}
\]

\[
\frac{P}{P_a} = 0.899
\]

**Verification for global, distortional+global, local+global buckling**

\[
P_{ne} = 141.685 \text{ kN}
\]

\[
P_{nd} := 130.368 \text{ kN}
\]

\[
P_{nl} := 141.685 \text{ kN}
\]

\[
P_a := \min(P_{ne}, P_{nd}, P_{nl}) = 130.368 \text{ kN}
\]

\[
\frac{P}{P_a} = 0.977
\]