The Width-size Method for General Resolution is Optimal

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The Width-size Method for General Resolution is Optimal

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Abstract

The Width-Size Method for resolution was recently introduced by Ben-Sasson and Wigderson ([BW98]: *Short Proofs are Narrow - Resolution Made Simple* STOC 99). They found a trade-off between two complexity measures for Resolution refutations: the *size* (i.e. the number of clauses) and the *width* (i.e. the size of the largest clause). Using this trade-off they reduced the problem of giving lower bounds on the size to that of giving lower bounds on the width and gave a unified method to obtain all previously known lower bounds on the size of Resolution refutations. Moreover, the use of the width as a complexity measure for Resolution proofs suggested a new very simple algorithm for searching for Resolution proof.

Here we face with the following question (also stated as an open problem in [BW98]): can the size-width trade-off be improved in the case of unrestricted resolution ? We give a negative answer to this question showing that the result of [BW98] is optimal. Our result, also holds for the most commonly used restrictions of Resolution like Regular, Davis-Putnam, Positive, Negative and Linear.

A consequence of our result is that the width-based algorithm proposed by [BW98] for searching for Resolution proofs cannot be used to show the automatizability of Resolution and its restrictions.

1 Introduction

Proof Complexity Theory is concerned with proving non-trivial lower bounds on the length of proofs in sound and complete propositional proof systems. After obtaining lower bounds for somewhat powerful systems like Bounded-Depth Frege, there is a renewed interest in devoloping new techniques for obtaining lower bounds for simple but non-trivial systems like Resolution. This is a consequence of the following facts:

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- several lower bound techniques gave insights for developing new algorithms for searching for proofs in Resolution;
- Proof-search algorithms for restricted versions of Resolution are the main algorithms implemented in Automated Theorem Proving applications.

Recently Ben-Sasson and Wigderson in [BW98] introduced a new complexity measure for Resolution refutations. The *width* of a refutation is defined as the maximal number of literals in any clause of the refutation. The importance of this new measure is twofold. On one side they were able to give a general relationship between the width and the length of a refutation, reducing the problem of giving lower bounds on the length to that of giving lower bounds on the width. This way they obtained an unified method to prove all the previously known lower bounds for Resolution. On the other side they made explicit a new simple proof-search algorithm based on searching for clauses of increasing size.

The width-size relation can be stated as follows:

If F, an unsatisfiable formula over n variables, has a resolution refutation of size S, then it has a resolution refutation of width $O(\sqrt{n \log S})$.

In this paper we are face with the following question (also stated in [BW98] among the open problems). Can the width-size trade-off be improved ? We give a negative answer to this question showing that the result of [BW98] is optimal. Namely we find an unsatisfiable 3-CNF F over $O(n^2)$ variables such that:

- F has a polynomial size resolution refutation; and
- Any resolution refutation of F requires a clause of size $\Omega(n)$.

A tree-like refutation is a refutation that can be arranged as a tree. The size-width trade-off for tree-like Resolution (the system in which we only allow tree-like refutation) is better than the corresponding trade-off for general resolution. Namely,

if F, an unsatisfiable formula over n variables, has a tree-like resolution refutation of size S_T, then it has a tree-like resolution refutation of width $O(\log S_T)$.

[BW98] showed that this last trade-off is optimal. Anyway the difference between the two cases leaves open the question whether for other known restrictions of Resolution (that eventually lie in between tree-like and general resolution) it could be possible to improve the size-width trade-off given for general resolution. Here we also show that it is not the case for the following restrictions of Resolution: Regular, Davis-Putnam, Positive, Negative and Linear Resolution. Indeed we show that in all these cases the size-width result for general resolution is also optimal. Moreover this result is obtained by using the same formula F used to give the result for general resolution.

A propositional proof system S is *automatizable*, if there is an algorithm that for every tautology F finds a proof of F in S in time polynomial in the length of the shortest proof of F in S. [BW98] gave the following simple width-based algorithm for searching proofs.

Set $w = 1$. Starting with the axiom try to derive all clauses of width (=size) w. If the empty clause is derived then stop. Else increase w by 1 and repeat.

On a given formula F such an algorithm works in time $T(n) = n^{O(w)}$ where w is the minimal width of any refutation of F. Moreover by the previous width-size relations it is easy to see that $T(n)$ is quasi-polynomial ($S_T^{O(\log n)}$) in the minimal size tree-like refutation S_T of F and is sub-exponential ($\exp(\sqrt{(n \log S)})$) in the minimal size general resolution refutation S of F.

As a consequence of our result we therefore obtain that the algorithm of [BW98] is not an efficient algorithm to automatize resolution refutations or many of its restrictions.

2 Preliminaries

Resolution is a refutation proof system for formulas in CNF based on the following inference rule:

$$\frac{C \vee x \quad D \vee \bar{x}}{C \vee D}$$

A Resolution refutation for an inital set Σ of clauses is a derivation of the empty clause from Σ using the above inference rule. Several restrictions of the resolution proof system have appeared in the literature. Here we consider the following five:

1. the *regular* resolution system in which the proofs are restricted in such a way that any variable can be eliminated at most once in any path from an initial clause to the empty clause.

2. the *Davis-Putnam* resolution system in which the proofs are restricted in such a way that there exists an ordering of the variables such that if a variable x is eliminated before a variable y on any path from an initial clause to the empty clause, then x is before y in the ordering. This restriction is also sometimes called *ordered* resolution in the Automated Theorem Proving community.

3. the *negative* resolution system, or N-resolution for short, where in each application of the resolution rule one of the premises must not contain any positive literals.

4. the *positive* resolution system, or P-resolution for short, where in each application of the resolution rule one of the premises must not contain any negative literals.

5. The *linear* resolution system in which the empty clause is *linearly resolvable* from the initial clauses Σ *with respect to a clause* $C \in \Sigma$: i.e. there is a sequence of clauses $(C_0, C_1, \ldots, C_n)$ such that $C_0 = C$, C_n is the empty clause and for all i, $1 \leq i \leq n$ in the resolution step $\frac{C_{i-1} \quad B_{i-1}}{C_i}$, the clause B_{i-1} is either a initial clause (i.e. a clause in Σ) or such that $B_{i-1} < C_j$ for some $j < i$.

Let $R \vdash F$ (resp. $R \vdash_{tl} F$) denote that R is a general (resp. tree-like) resolution refutation of F. The *size* $|R|$ of a refutation R in any of the above systems is defined as the number of clauses used in R. The size complexity $S(\vdash F)$ (respectively $S_T(\vdash F)$) of deriving a CNF formula F in general resolution (respectively in tree-like resolution) is defined as $\min_{R \vdash F} |R|$ (respectively $\min_{R \vdash_{tl} F} |R|$).

The *width* $w(F)$ of a CNF formula is defined to be the size (i.e. the number of literals) of the largest clauses in F The *width* $w(R)$ of a refutation R is defined as the size of the greatest clause appearing in R. The width $w(\vdash F)$ (resp. $w(\vdash_{tl} F)$) of deriving a formula F in general (resp. tree-like) resolution is defined as $\min_{R \vdash F} w(R)$ (resp. $\min_{R \vdash_{tl} F} w(R)$. The size-width relations obtained by [BW98] are given in the following Theorem

Theorem 2.1 *Let F be any unsatisfiable formula over n variables. Then*

- $S_T(\vdash F) \geq 2^{(w(\vdash tlF) - w(F))}$;
- $S(\vdash F) \geq \exp(\Omega(\frac{(w(\vdash F) - w(F))^2}{n}))$.

Our target question " Is the size-width result for general resolution optimal ?" can be therefore formalized as follows: can one find an unsatisfiable k-CNF formula F over n variables such that $w(F) = k$, $S(\vdash F) \leq O(n^{O(1)})$ and $w(\vdash F) \geq \Omega(\sqrt{n})$?

We consider the CNF formula GT_n expressing the negation of the property that in any directed graph closed under transitivity and with no cycles of size two there is a source node. We obtain the following clauses:

$$\begin{array}{lll} (1) & x_{i,j} \wedge x_{j,k} \rightarrow x_{i,k} & i,j,k \in [n], i \neq j \neq k \\ (2) & x_{i,j} \rightarrow \bar{x}_{j,i} & i,j \in [n], i \neq j \\ (3) & x_{1,j} \vee \ldots x_{j-1,j} \vee x_{j+1,j} \ldots x_{n,j} & j \in [n] \end{array}$$

where the clauses in (1) encode the transitivity closure property, those in (2) the property that there are no cycles of size two and those in (3) say that each node receives at least an edge from any other node (i.e. there is no source node). This formula was firstly formulated by Krishnamurthy in [Kr85] where he conjectured its hardness for resolution. Subsequently Stalmark in [St 96] refuted the conjucture giving polynomial size resolution proofs for the formula GT_n.

3 Polynomial Size Refutations of GT_n

[St 96] gave a polynomial size unrestricted resolution proof for the GT_n unsatisfiable formula. We slightly modify his proof in order to show that actually it also works for the following restrictions of resolution: regular, positive, davis-putnam and linear resolution.

Theorem 3.1 *There are polynomial size proofs for GT_n in the following proof systems: (i) general resolution, (ii) Davis-Putnam resolution, (iii) Regular resolution, (iv) Positive Resolution, (v) Linear Resolution.*

Proof. We start by giving the general resolution refutation, then we discuss why this proof falls in any of the restricted versions of resolutions We adopt the following abbreviations.

Let:

$$\begin{array}{ll} A(i,j,k) := x_{i,j} \wedge x_{j,k} \to x_{i,k} & i \neq j \neq k \neq i \in [n] \\ B(i,j) := x_{i,j} \to \bar{x}_{j,i} & i \neq j \in [n] \\ C_m(j) := \bigvee_{1 \leq i \leq m \ i \neq j} x_{i,j} & j \in [n], m \in [n] \\ C_m := \bigwedge_{j=1\ldots n} C_m(j) & m \in [m] \\ D^j_{k-1}(i) := C_{k-1}(j) \vee \bar{x}_{i,k} & k \in [n]/\{1\}, i \in [k-1], j \in [n] \\ E^j_{k-1}(i) := C_{k-1}(j) \vee \bigvee_{\ell=i,\ldots,n}(x_{\ell,k}) & i \in [n]/\{1\}, j \in [n], k \in [n] \end{array}$$

The proof proceeds by steps downward from n to 2. At the k-th step, for each $j = 1, \ldots, n$, we prove $C_{k-1}(j)$ using the inital clauses $A(1,k,j)$, $B(k,j)$ and the clauses $C_k(j)$ and $C_k(k)$ obtained at the previous step. At the end we have proved C_2 from which a contradiction is obtained in 2 steps using $B(1,2)$. Now we give a description of how to perform the k-th step obtaining in parallel the clauses $C_{k-1}(1), C_{k-1}(2), \ldots, C_{k-1}(n)$. For a generic value $j \in [n]$ we obtain $C_{k-1}(j)$ by the following steps:

(a): Perform in parallel the following resolutions steps, each one resolving the variable $x_{k,j}$:

$$(1) \quad \frac{C_k(j) \quad A(1,k,j)}{D^j_{k-1}(1)}$$

$$(2) \quad \frac{C_k(j) \quad A(2,k,j)}{D^j_{k-1}(2)}$$

$$\vdots$$

$$(j-1) \quad \frac{C_k(j) \quad A(j-1,k,j)}{D^j_{k-1}(j-1)}$$

$$(j) \quad \frac{C_k(j) \quad B(j,k)}{D^j_{k-1}(j)}$$

$$(j+1) \quad \frac{C_k(j) \quad A(j+1,k,j)}{D^j_{k-1}(j+1)}$$

$$\vdots$$

$$(n) \quad \frac{C_k(j) \quad A(n,k,j)}{D^j_{k-1}(n)}$$

(b): $C_{k-1}(j)$ is obtained by the following tree-like refutation in which we are resolving along the variables $x_{1,k}, x_{2,k}, \ldots, x_{k-1,k}$:

$$(1) \quad \frac{C_k(k) \quad D^j_{k-1}(1)}{E^j_{k-1}(1)}$$

$$(2) \quad \frac{E^j_{k-1}(1) \quad D^j_{k-1}(2)}{E^j_{k-1}(2)}$$

$$\vdots$$

$$(n) \quad \frac{E^j_{k-1}(n) \quad D^j_{k-1}(k-1)}{C_{k-1}(j)}$$

It is easy to see that such a refutation is a Positive resolution, indeed at each resolution step one of the involved clauses is always made by positive literals.

It is also easy to see that the following orders of elimination of the variables is respected:

$x_{n,1}, x_{n,2}, \ldots, x_{n,n-1}$

$x_{1,n}, x_{2,n}, \ldots, x_{n-1,n}$

$x_{n-1,1}, x_{n-1,2}, \ldots, x_{n-1,n}$

$x_{1,n-1}, x_{2,n-1}, \ldots, x_{n-2,n-1}$

$\vdots$

$x_{2,1}$
$x_{1,2}$
Therefore the refutation is a Davis-Putnam resolution as well as a Regular resolution.
To see that the refutation Linear observe that the following sequence of clauses define the order of the linear elimination:

$C_n(n)$,
$\quad C_n(1), B(n,1), A(2,n,1), A(3,n,1), \ldots, A(n-1,n,1)$
$\qquad D_n^1(1), \ldots, D_n^1(n), E_n^1(1), \ldots, E_n^1(n)$,
$\quad C_n(2), A(1,n,2), B(n,2), A(3,n,2) \ldots, A(n-1,n,2)$,
$\qquad D_n^2(1), \ldots, D_n^2(n), E_n^2(1), \ldots, E_n^2(n)$,
$\quad \vdots$
$\quad C_n(n-1), A(1,n,n-1), A(2,n,n-1), \ldots, A(n-2,n,n-1), B(n,n-1)$
$\qquad D_n^{n-1}(1), \ldots, D_n^{n-1}(n-1), E_n^{n-1}(1), \ldots, E_n^{n-1}(n-1)$,
$C_{n-1}(n-1)$,
$\quad C_{n-1}(1), B(n-1,1), A(2,n-1,1), A(3,n-1,1), \ldots, A(n-2,n-1,1)$,
$\qquad D_{n-1}^1(1), \ldots, D_{n-1}^1(n-1), E_{n-1}^1(1), \ldots, E_{n-1}^1(n-1)$,
$\quad C_{n-1}(2), A(1,n-1,2), B(n-1,2), A(3,n-1,2) \ldots, A(n-2,n-1,2)$,
$\qquad D_{n-1}^2(1), \ldots, D_{n-1}^2(n-1), E_{n-1}^2(1), \ldots, E_{n-1}^2(n-1)$
$\quad \vdots$
$\quad C_{n-1}(n-2), A(1,n-1,n-2), A(2,n-1,n-2), \ldots, A(n-3,n-1,n-2), B(n-1,n-2)$
$\qquad D_{n-1}^{n-2}(1), \ldots, D_{n-1}^{n-2}(n-1), E_{n-1}^{n-2}(1), \ldots, E_{n-1}^{n-2}(n-1)$,
$C_{n-2}(n-2)$,
$\vdots$
$\vdots$
$\{\}$

□

It is easy to show, by considering an appropriate definition of critical assignments for the GT_n formula and using the "large clause" method of Beame and Pitassi [BP96], that any resolution proof of GT_n must contain a "large" clause different from the initial ones.

Theorem 3.2 *Any resolution proof of GT_n contains a clause different from the inital ones of size $\Omega(n)$*

The previous Theorem does not imply that the width-size method of [BW98] is optimal since the initial clauses in GT_n have size $O(n)$. In the next section we consider a modification of the formula GT_n that will make us to overcoming the problem of large inital clauses.

4 Tightness Results for the Width-Size Method

We modify the clauses of GT_n in such a way to make the lenght of initial clauses bounded by a constant but preserving the lower bound of $\Omega(n)$ on the size of the clauses needed in any resolution proof of these clauses.

We introduce for each $j \in [n]$ n new variables $y_{j,0}, \dots y_{j,j-1}, y_{j,j+1}, \dots y_{j,n}$ and modify the clauses in (3) by the following clauses (the modification is similar to the reduction used from SAT to 3-SAT and is also reported in [BW98].)

$$(3') \quad \bar{y}_{j,0} \wedge \bigwedge_{i=1\dots n, i\neq j}(y_{i-1,j} \vee x_{i,j} \vee \bar{y}_{i,j}) \wedge y_{n,j}$$

Define the 3-CNF formula MTG_n as the conjunction of the clauses in (1), (2) and (3′).

Theorem 4.1 *There are polynomial size refutations for the formula MTG_n in any of the following systems: (i) general resolution, (ii) positive resolution, (iii) davis-putnam resolution, (iv) regular resolution, (v) linear resolution.*

Proof. The proof proceeds in the following way. From the clauses in (3′) obtain the clauses in (3) eliminating once at time the y variables. Then we apply the polynomial size proof for GT_n to these new clauses. Observe that the first part of the proof is in fact a tree-like proof of size quadratic in n and, since the y variables are different for different $j \in [n]$, the regularity of the proof is preserved. It is also easy to see that the new first part of the proof is a Davis-Putnam resolution since the following order of elimination of the y variables is respected:
$y_{0,1}, \dots, y_{n,1},$
$y_{0,2}, \dots, y_{n,2},$
$\vdots$
$y_{0,n}, \dots, y_{n,n},$
Moreover if for each $j \in [n]$ we start by elminating the $y_{j,n}$ variable it is easy to see that the new first part is also a positive resolution. Finally, to prove that this proof is a Linear resolution proof, consider for $j = 1, \dots, n$ the following definition:

$$G_j(i) := \begin{cases} y_{n,j} & \text{if } i = n \\ (x_{n,j} \vee x_{n-1,j} \vee \dots \vee x_{i,j} \vee y_{i-1,j}) & \text{for any } i = 1, \dots, n-1 \\ C_n(j) & \text{if } i = 0 \end{cases}$$

Then the order of the clauses in the linear resolution of MTG_n is obtained by the order of the linear resolution for GT_n by putting for each $j = 1, \dots, n$ the sequence of clauses $G_j(n), \dots G_j(1)$ just before the clause $C_n(j)$ (that actually corresponds to $G_j(0)$). □

4.1 Optimality for Unrestricted Resolution

Let A_j be the conjunction of the clauses $x_{i,j} \to \bar{x}_{j,i}$ for all $i \in [n], i \neq j$. Let B_j the clauses in (3′). Consider the formula C_j defined as the conjunction of $A_j \wedge B_j$. First of all an observation about assignments for these clauses: if we want an assignment α such that $\alpha(C_j) = 1$ for some $j \in [n]$, then α must verify the following conditions:

- at least one of the variables $x_{i,j}$ has value 1 under α;
- the $y_{j,i}$ variable will be assigned in such a way that there is one $k \in [n]/\{j\}$ such that $x_{j,k} = 1$ and $y_{j,i} = 0$ for all $i < k$ and $y_{j,i} = 1$ for all $i \geq k$;

- if $x_{i,j}$ has value 1 then $x_{j,i}$ must have value 0.

On the other hand, when α falsifies some C_j we are not sure that all the $x_{i,j}$ variables have assigned a 0 value. It could be the case that $\alpha(C_j) = 0$ because of some particular way of α to assign values to the $y_{j,i}$ variable (e.g. $\alpha(y_{0,j}) = 1$).

Theorem 4.2 *Any resolution proof of MTG_n must have a clause of size $\Omega(n)$*

Previous Theorem joint with Theorem 4.1 implies the main result of the paper.

Theorem 4.3 *The width-size metod is optimal. That is, there is a $k-CNF$ F on $O(n^2)$ variables verifying the following two properties:*

- *F has polynomial size resolution refutations;*
- *Any resolution refutation of F contains a clause having at least $\Omega(n)$ variables.*

Proof of Theorem 4.2
Recall the definition of the formula C_j. For each $I \subseteq [n]$ let C_I defined as $\bigwedge_{i \in I} C_i$ We define a measure μ giving measure of the complexity of any clause in resolution proof of MTG_n. $\mu(C)$ is the size of the minimal $I \subseteq [n]$ such that $C_I \models C$ on all assignments. $\mu(C_i) \leq 1$, $\mu(\{\}) = n$, and μ is obviously subadditive, therefore in any resolution proof of MTG_n there is a clause such that $\frac{n}{3} \leq \mu(C) \leq \frac{2n}{3}$. Fix such a clause. We show that this clause will contain $\Omega(\frac{n}{6})$ literals. Assume for the sake of contradiction that $|C| < \frac{n}{6}$. We will prove a contradiction. First of all notice that since $\mu(C) \geq \frac{n}{3}$ the following claim holds:

Claim 4.1 *There exists at least an $l \in I$ such that no variable from C_l belongs to C.*

Proof of the Claim
Observe that each C_i contains exactly a negated variable appearing in another C_j (in fact for each $i \neq j \in [n]$ C_i and C_j share the clause $(\bar{x}_{i,j} \vee \bar{x}_{j,i})$). Since $|C| < \frac{n}{6}$, this means that in the worst case C captures variables from at most $\frac{n}{3}$ different C_i. But since $|I| = \mu(C) \geq \frac{n}{3}$, then there is at least an $l \in I$ such that no variable from C_l appears in C.□

Now, given that there is an $l \in I$ such that any variable from C_l does not appear in C, consider any assignment α such that $\alpha(C_l) = 0, \alpha(C) = 0$ and for all $j \in I/\{l\}$ $\alpha(C_j) = 1$. This assignment must exist by the minimality of I. By the previous observation on the assignment α not satisfying a clause C_i we deduce that one of the two following cases occurs for the x variables in C_l:

- either all the variables $x_{i,l}$ are setted to 0 by α, or
- there is some $x_{i,l}$ setted to 1 and $\alpha(C_l) = 0$ because of the way α assignes values to the $y_{i,l}$ variables.

In the latter case we immediately obtain a contradiction noting that since no variable from C_l appears in C we can change the value of the $y_{i,l}$ variables according to the previous observation in such a way that $\alpha(C_l) = 1$. This change does not affect neither the value of C nor the value of the others C_j for $j \in I/\{l\}$. Therefore we have found an assignment satisfying all the C_i for $i \in I$ but falsifying C. And this is not possible.

In the former case we proceed as follows. First of all we extend α to an assignment α' satisfying all the C_i for $i \in [n]/\{l\}$. Defined $J = [n]/I$ we have that $|J| > \frac{n}{3}$ (since $|I| \leq \frac{2n}{3}$). Therefore by the same argument used in Claim 4.1 for the set I we deduce that there is at least a $j \in J$ such that no variable from C_j appears in C.

We build an assignment β from α' such that $\beta(C_i) = 1$ for all $i \in I$ and $\beta(C) = 0$, and this is a contradiction. β is built in the following way: change all $x_{i,j}$ such that $\alpha(x_{i,j}) = 1$ to 0. Change all the symmetric values $x_{j,i}$ such that $\alpha(x_{j,i}) = 0$ to 1. This first change does not affect the value of C since no variable from C_j appears in C. Observe that after this change some variable $x_{i,l}$ will have the value 1. Therefore it remains to change (according to the previous observation) the value of the variable $y_{l,i}$ in such a way to satisfy C_l (i.e. such that $\beta(C_l) = 1$). This change will not affect the value of C since we know that no variable from C_l appears in C. So that β leads to a contradiction.

4.2 Consequences in Restricted versions of Resolution

Our result has several consequences. First of all the width-size method of [BW98] for tree-like resolution joint with Theorem 4.2 give a lower bound of the order $2^{\Omega(n)}$ for tree-like resolution proofs for MTG_n, therefore, by Theorem 4.1 providing to another exponential separation between unrestricted resolution and tree-like resolution as in the case of [BEGJ98].

Theorem 4.4 *Any tree like resolution proof of MTG_n must have size $\Omega(2^n)$.*

Other consequences regard with restrictions of resolution. As we have seen in Section 2, the width-size trade off is more powerful in the tree-like case than in unrestricted one. This fact let us think that (possibly) restricting some way the resolution system it is possible to give better trade-off results than the unrestricted case. We show that this is not the case for the regular, positive, negative, Davis-Putnam and Linear resolution. As we have observed in Section 4 MTG_n has also polynomial size refutation in all the considered restrictions. By Theorem 4.2 any resolution refutation of MTG_n (in particular in any of the considered restrictions) must have a clause of size $\Omega(n)$. This immediately implies that the width-size trade-off for general resolution cannot be improved for the case of regular and positive, davis-putnam and linear resolution. For the case of negative resolution, we consider the unsatisfiable formula $\overline{MTG_n}$ in which the $x_{i,j}$ variables are replaced by $\bar{z}_{i,j}$ whose intended meaning is opposite to that of the x variables. It is easy to see that the positive resolution proof for MTG_n is in fact an negative resolution proof for $\overline{MTG_n}$ and that the lower bound technique can also be applied. Therefore also in the case of negative resolution we cannot improve the width-size trade-off obtained for unrestricted resolution by [BW98].

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