

Basic Geometric Operations in Ruler-and-Compass Constraint Solvers using Interval Arithmetic

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Abstract

In this paper we report on a successful application of interval arithmetic to extend the basic geometric operations underlying the ruler-and-compass geometric constraint solving technique to deal with problems where geometric constraints take values in the interval domain.

Keywords: Geometric constraint solving, interval arithmetic, under-constrained problems.

1 Introduction

One of the most promising paradigms in computer aided design is the geometric constraint-based design. In this technique, the designer specifies a rough sketch of an object annotated with geometric and dimensional constraints that are not yet satisfied. If the object is consistently defined, a geometric *constraint solver* evaluates the set of constraints and generates an instance of the object.

Depending on the way geometric constraints are represented, constraint solvers are based either on equational or on constructive techniques. In equational solvers, systems of geometric constraints are computed using numerical techniques. Although they are quite general, they have no way to provide geometric reasoning about the object under design. In constructive solvers, however, geometric constraints are expressed by predicates and they are interpreted using the axioms of Euclidean geometry. These solvers generate a construction plan such that, when properly executed, builds an instance of the geometric object. If the sequence of construction steps in the construction plan can be performed using ruler and compass alone, the solver is a *ruler-and-compass* constructive solver. For a thorough review on geometric constraint solving see [4].

In general, current geometric constraint solving approaches operate on constraints with values in the real numbers. But, computing feasible ranges of parameters, or solving under-constrained designs, are examples of problems that need to be solved in the interval domain.

A naive extension of the existing geometric constraint solving techniques yields too wide intervals when based on the standard interval arithmetic, [17]. To cope with this problem, these techniques should be extended in a more convenient way. Since ruler-and-compass construction steps are a small set of simple and well characterized operations, ruler-and-compass geometric constraint solving is specially well suited to be extended to the interval domain.

In this paper we report on a successful application of interval arithmetic to extend the basic geometric operations underlying the ruler-and-compass geometric constraint solving technique. With this extension, ruler-and-compass solvers can deal with problems where geometric constraints take values in the interval domain.

The paper is organized as follows. Section 2 introduces the basics of interval arithmetics and the set of geometric operations available in ruler-and-compass solvers. In Section 3, we characterize ruler-and-compass operations in the interval domain; we define intersection operations and geometric transformations. We conclude in Section 4.

2 Basic Concepts

First, we recall the basic definitions related with interval arithmetics. Then, we characterize the basic geometric operations in ruler-and-compass geometric constraint solving.

2.1 Interval Arithmetics

An interval is a closed bounded set of real numbers represented by an ordered pair $[\underline{x}, \bar{x}]$. The classical interval arithmetic is defined on the set of intervals $\mathbb{IR} = \{[\underline{x}, \bar{x}] \mid \underline{x}, \bar{x} \in \mathbb{R}, \underline{x} \leq \bar{x}\}$, [17]. Unfortunately, this arithmetic is somehow limited to deal with certain kind of problems because additive and multiplicative inverse intervals are not in \mathbb{IR} . The completion of the classical interval arithmetic is the extended interval arithmetic [6, 13]. It also extends real numbers to intervals, but without the requirement of $\underline{x} \leq \bar{x}$. The extended interval arithmetic, defined on the set $\mathbb{IR} = \{[\underline{x}, \bar{x}] \mid \underline{x}, \bar{x} \in \mathbb{R}\}$, has then group structure.

Using the extended interval arithmetic reduces the widening effect of the classical interval computations because the order of the endpoints provides additional information on the functions to be evaluated. An interval \mathbf{x} is said to be proper if $\underline{x} \leq \bar{x}$; otherwise, it is improper. The direction of an interval may represent the monotonicity of a function of one variable. Although this is the principle on which the arithmetic of directed intervals is founded [15], it is shown that the arithmetic of directed intervals is equivalent to the extended interval arithmetic in [3]. In this paper, we will refer to \mathbb{IR} although we shall apply results of both interval structures.

An interval $\mathbf{x} \in \mathbb{IR}$ can be either represented by a generalized interval $[\underline{x}, \bar{x}]$ or by the pair $(pr(\mathbf{x}); mod(\mathbf{x}))$, where $pr(\mathbf{x})$ is the proper part of the interval and $mod(\mathbf{x})$ is its direction, or modality. These two operations are defined as follow

$$pr(\mathbf{x}) := \begin{cases} [\underline{x}, \bar{x}] & \text{if } \mathbf{x} \text{ is proper,} \\ [\bar{x}, \underline{x}] & \text{if } \mathbf{x} \text{ is improper.} \end{cases} \quad mod(\mathbf{x}) := \begin{cases} +1 & \text{if } \mathbf{x} \text{ is proper,} \\ -1 & \text{if } \mathbf{x} \text{ is improper.} \end{cases}$$

2.2 Ruler-and-Compass Geometric Constraint Solvers

There is a general architecture for constructive geometric constraint solving systems that has been proved to be useful when all the constraints defined by the user are valuated, [5, 12]. This architecture splits the solution procedure into two main phases: The analysis phase and the construction phase. In the analysis phase, a constraint graph is analyzed and a sequence of construction steps is produced. In the construction phase, the geometric elements are positioned with respect to each other according to the sequence of construction steps. If all construction steps can be performed using only ruler and compass, we say that the object is ruler-and-compass constructive. A more formal definition of ruler-and-compass constructivity is given below.

Definition 2.1 ([7]) *A point $P \in \mathbb{R}^2$ is constructive if there exists a finite sequence of points in the plane, $P_0, P_1, \dots, P_n = P$, with the following property. Let*

$$\mathcal{P}_j = \{P_0, P_1, \dots, P_j\} \quad \text{for } 1 \leq j \leq n$$

For each j , with $2 \leq j \leq n$, P_j is either

- (i) the intersection of two distinct straight lines, each joining two points of \mathcal{P}_{j-1} , or*
- (ii) a point of intersection of a straight line joining two points of \mathcal{P}_{j-1} and a circle with center a point of \mathcal{P}_{j-1} and radius the distance between two points of \mathcal{P}_{j-1} , or*
- (iii) a point of intersection of two non concentric circles, each with center a point of \mathcal{P}_{j-1} and the radius the distance between two points of \mathcal{P}_{j-1} .*

Ruler-and-compass construction plans are then characterized as follows. Let a cluster S_i be a set of points whose position relative to each other has already been determined. If S_1, S_2 and S_3 are three clusters that pairwise share a point (see Figure 1a),

$$S_1 \cap S_2 = P_1, \quad S_1 \cap S_3 = P_2 \quad \text{and} \quad S_2 \cap S_3 = P_j$$

and $\mathcal{P}_j = \{P_1, \dots, P_j\}$ is the union set of the points belonging to S_1, S_2 and S_3 , for each construction step the following operations are performed:

1. Point P_j is positioned with respect to cluster S_1 by intersecting either two lines, or a line and a circle, or two circles.
2. Rigid-body transformations are applied to points in S_2 and S_3 to align the three clusters accordingly.
3. S_1, S_2 and S_3 are merged into a single cluster $S = \{S_1 \cup S_2 \cup S_3\}$.

In the ruler-and-compass geometric constraint solving paradigm, the basic geometric operations are then: Intersecting two straight lines, intersecting a straight line and a circle, intersecting two non-concentric circles, and rigid geometric transformations.

3 Ruler-and-Compass Operations in the Interval Domain

In this section, we extend the ruler-and-compass basic geometric operations to the interval domain. However, we need to provide first a suitable representation for the primitive geometric objects.

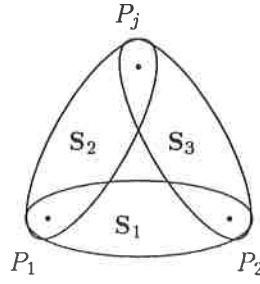


Figure 1: Ruler-and-compass general construction step.

3.1 Primitive Geometric objects

The following definitions are illustrated in Figure 2.

Definition 3.1 An interval point $\mathbf{P} = (\mathbf{p}_1, \mathbf{p}_2)$ is defined by the interval vector $\mathbf{P} = [\underline{P}, \overline{P}]$, where $\underline{P} = [p_1, p_2]$ and $\overline{P} = [\bar{p}_1, \bar{p}_2]$.

Definition 3.2 A pencil is a tuple (\mathbf{P}, \mathbf{v}) where $\mathbf{P} = [P, P]$ is a degenerated interval point, called pivot, and \mathbf{v} is either an angle α or a distance r , which we will refer to as a dimensional parameter.

Definition 3.3 A pencil of lines $\mathbf{L} = (\mathbf{P}, \alpha)$ is defined as the set of rays with origin on point \mathbf{P} that makes an angle α with the positive x -axis, belonging to the oriented interval $([\underline{\alpha}, \overline{\alpha}]; s) \in \mathbb{IR}$.

Parameter s determines the orientation of the pencil of lines: $s = +1$ if the pencil is oriented counterclockwise, otherwise $s = -1$. In some applications, we may want pencils to be defined counterclockwise. This can always be achieved using the transformation $(\mathbf{P}, ([\lambda, \mu]; s)) = (\mathbf{P}, ([\mu, \lambda]; -s))$ with $\lambda, \mu \in [0, 2\pi]$. However, improper interval angles, $[\lambda, \mu]$ with $\lambda > \mu$, may occur because of the $[0, 2\pi]$ representation. An improper interval angle can be translated into a proper interval using a wider representation than $[0, 2\pi]$. Then, $[\lambda, \mu]$ can be represented by $[\lambda, \mu + 2\pi]$.

Definition 3.4 A pencil of circles $\mathbf{C} = (\mathbf{P}, r)$ is defined as the set of circles centered on point \mathbf{P} and radii r belonging to the interval $[\underline{r}, \overline{r}]$.

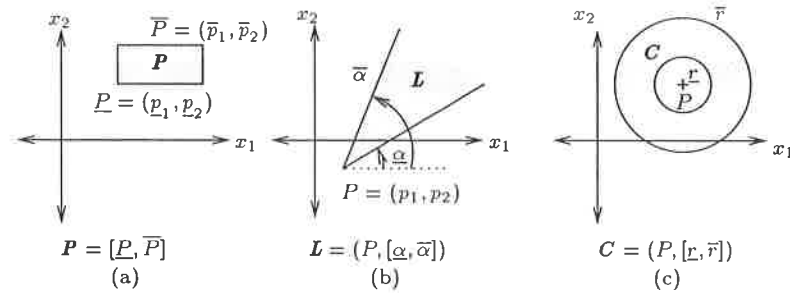


Figure 2: (a) Interval point. (b) Pencil of Lines. (c) Pencil of circles.

3.2 Intersection Operations

Intersecting two straight lines, or a line and a circle, or two circles in \mathbb{R}^2 are operations with a well-defined semantics. When straight lines and circles are pencils of lines and pencils of circles, intersection operations are properly extended to the interval domain by combining the natural interval extension with *narrowing operators*.

Assume that $U_1 = (P_1, v_1)$ and $U_2 = (P_2, v_2)$ are two pencils and σ is an intersection operation defined on them. We want to compute the set of points belonging to their intersection, $U_1 \cap U_2$, that is, the range of $\sigma(U_1, U_2)$ for all $U_1 \in U_1$ and $U_2 \in U_2$, denoted by $\sigma(U_1, U_2)$. The main difficulty when defining $\sigma(U_1, U_2)$ is that σ may have no solution for some values of the parameters, and therefore, it is not always possible to find a reasonable interval extension [18]. To cope with this problem, the feasible range of values for the parameters is computed using narrowing constraint operators, and intersection functions are then evaluated in the narrowed domain.

In this section, we give the basic concepts related with narrowing operators in order to define the narrowing operators associated with intersection functions. Then, we extend intersection functions to the interval domain.

3.2.1 Narrowing Operators

A narrowing operator is a mapping such that, given an interval and a function defined on it, computes a subset of the initial interval where the function holds. The general definition of narrowing operator can be stated as follows, [2].

Definition 3.5 *Let σ be an n -ary relation defined in \mathbb{IR}^n . The function $N_\sigma : \mathbb{IR}^n \rightarrow \mathbb{IR}^n$ is a constraint narrowing operator for the relation σ iff for every $U, V \in \mathbb{IR}^n$, the four following properties hold:*

1. *Contractance.* The narrowed intervals are smaller than or equal to the initial intervals: $N_\sigma(U) \subseteq U$.
2. *Correctness.* Every valid solution in the theoretical real numbers lies in the narrowed intervals: $U \cap \sigma \subseteq N_\sigma(U)$.
3. *Monotonicity.* The narrowing preserves the inclusion: $U \subseteq V \rightarrow N_\sigma(U) \subseteq N_\sigma(V)$.
4. *Idempotence.* The narrowed intervals have to be computed only once: $N_\sigma(N_\sigma(U)) = N_\sigma(U)$.

In particular, when σ is either the intersection of two lines (*intLL*), or the intersection of a line and a circle (*intLC*), or the intersection of two circles (*intCC*), we define N_σ as follows.

Definition 3.6 *The constraint narrowing operator associated to the intersection function $\sigma(U_1, U_2)$, $N_\sigma(U_1, U_2)$, is defined as follows:*

$$N_\sigma((P_1, v_1), (P_2, v_2)) = ((P_1, v_1 \cap v'_1), (P_2, v_2 \cap v'_2)) = (U'_1, U'_2)$$

where the intervals v'_1 and v'_2 are the feasible ranges of values for the dimensional parameters of the pencils.

| σ | U_1 | U_2 | v'_1, v'_2 |
|----------|-------|-------|---|
| $intLL$ | L_1 | L_2 | $\alpha'_1 = \begin{cases} [0, 2\pi] & \text{if } \pi \in [\underline{\alpha}_2, \bar{\alpha}_2] \\ [\min(0, \underline{\alpha}_2), \max(0, \bar{\alpha}_2)] & \text{if } \pi \notin [\underline{\alpha}_2, \bar{\alpha}_2] \end{cases}$ $\alpha'_2 = \begin{cases} [0, 2\pi] & \text{if } 0 \in [\underline{\alpha}_1, \bar{\alpha}_1] \\ [\min(\underline{\alpha}_1, \pi), \max(\bar{\alpha}_1, \pi)] & \text{if } 0 \notin [\underline{\alpha}_1, \bar{\alpha}_1] \end{cases}$ |
| $intLC$ | L_1 | C_2 | $\alpha'_1 = \begin{cases} [0, 2\pi] & \text{if } d \leq \bar{r}_2 \\ [2\pi - \arcsin(\bar{r}_2/d), \arcsin(\bar{r}_2/d)] & \text{if } d > \bar{r}_2 \end{cases}$ $r'_2 = \begin{cases} [0, \infty] & \text{if } 0 \in [\underline{\alpha}_1, \bar{\alpha}_1] \\ [d \sin \lambda, \infty] & \text{if } 0 \notin [\underline{\alpha}_1, \bar{\alpha}_1] \end{cases}$ <p>where $\lambda = \min(\underline{\alpha}_1 , \bar{\alpha}_1 , \frac{\pi}{2})$ with $\underline{\alpha}_1, \bar{\alpha}_1 \in [-\pi, \pi]$.</p> |
| $intCC$ | C_1 | C_2 | $r'_1 = \begin{cases} [\underline{r}_2 - d, \bar{r}_2 + d] & \text{if } d < \underline{r}_2 \\ [0, \bar{r}_2 + d] & \text{if } \underline{r}_2 < d < \bar{r}_2 \\ [d - \bar{r}_2, \bar{r}_2 + d] & \text{if } d > \bar{r}_2 \end{cases}$ $r'_2 = \begin{cases} [\underline{r}_1 - d, \bar{r}_1 + d] & \text{if } d < \underline{r}_1 \\ [0, \bar{r}_1 + d] & \text{if } \underline{r}_1 < d < \bar{r}_1 \\ [d - \bar{r}_1, \bar{r}_1 + d] & \text{if } d > \bar{r}_1 \end{cases}$ |

Table 1: Feasible range of values for the dimensional parameters involved in the intersection functions.

To characterize the intersection $U_1 \cap U_2$, first we transform the pencils in such a way that P_1 is the origin of coordinates and the half-line defined by $\overline{P_1 P_2}$ is the positive x -axis. If d is the distance between the two pivots, then v'_1 and v'_2 are computed as shown in Table 1. The way we define those parameters guarantees that the interval function $N_\sigma(U_1, U_2)$ is a narrowing operator. For a proof, see [10].

Figure 3 illustrates how we compute α'_1 and α'_2 when intersecting two pencils of straight lines L_1 and L_2 . If pivot P_1 is in pencil L_2 (Figure 3a), then any arbitrary line through P_1 intersects L_2 . However, if pivot P_1 is not in pencil L_2 (Figure 3b), the pencil of lines intersecting L_2 is (P_1, α'_1) . Analogously, if pivot P_2 is in pencil L_1 (Figure 3c), then any arbitrary straight line through P_2 intersects L_1 . Otherwise, the pencil of lines intersecting L_1 is (P_2, α'_2) (Figure 3d).

We characterize the intersection of a pencil of lines and a pencil of circles, L_1 and C_2 as follows. See Figure 4. If P_1 is an interior point of circle $\overline{C_2}$, then any arbitrary straight line through P_1 intersects C_2 . Otherwise, the pencil of lines intersecting C_2 is (P_1, α'_1) (Figure 4a). If pivot P_2 is in pencil L_1 , then any arbitrary circle centered in P_2 intersects L_1 . However, if pivot P_2 is not an interior point of L_1 , the pencil of circles intersecting L_1 is (P_2, r'_2) (see Figures 4b and 4c).

Finally, the feasible range of values for the dimensional parameter r'_1 associated with the intersection of two pencils of circles is illustrated in Figure 5. If pivot P_1 is inside the pencil of circles, that is, P_1 is in $\underline{C_2}$, then any circle centered in P_1 with radius $r'_1 \in [\underline{r}_2 - d, \bar{r}_2 + d]$ intersects C_2 (Figure 5a). If pivot P_1 belongs to pencil C_2 , then any arbitrary circle centered in P_1 and radius $r'_1 \in [0, \bar{r}_2 + d]$ intersects C_2 (Figure 5b). If pivot P_1 is outside the pencil of circles, that is, P_1 is outside $\overline{C_2}$, then any circle centered in P_1 with radius $r'_1 \in [d - \underline{r}_2, \bar{r}_2 + d]$ intersects C_2 (Figure 5c). Analogously, we obtain the feasible range of values for the dimensional parameter r'_2 .

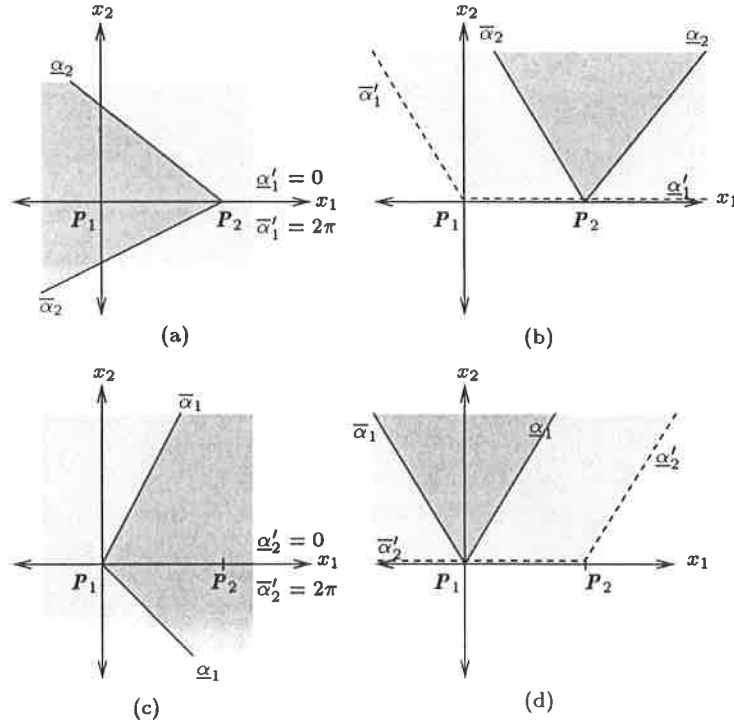


Figure 3: Computing parameters α'_1 and α'_2 depending on the relative position of the pivots P_1 and P_2 with respect to the pencils L_2 and L_1 , respectively.

3.2.2 Evaluating Intersection Functions

Since the interval function $N_\sigma(U_1, U_2)$ is a narrowing operator, the extended geometric intersection functions, $intLL$, $intLC$ and $intCC$, can be solved in the narrowed intervals. We represent the intersection of two pencils, U_1 and U_2 , by an interval point P_j which is evaluated as follows

$$P_j = U_1 \cap U_2 = \sigma(N_\sigma(U_1, U_2)) = \sigma(U'_1, U'_2)$$

Note that the range of $\sigma(U'_1, U'_2)$ can be computed from some extreme points of the pencils. First, we intersect the upper and the lower bounds of the pencils, that is, $\sigma(\underline{U}'_1, \underline{U}'_2)$, $\sigma(\underline{U}'_1, \overline{U}'_2)$, $\sigma(\overline{U}'_1, \underline{U}'_2)$ and $\sigma(\overline{U}'_1, \overline{U}'_2)$. Second, we characterize extreme points in configurations involving pencils of circles. Then, we compute the range by evaluating, component-wise, the upper and lower bounds for this set of points. In Figure 6, we give an example for each type of intersection function. It is worth to note that P_j is sharp, but it is not monotonic because local extrema occur at interior points of the interval.

3.3 Geometric Transformations

In ruler-and-compass construction plans, geometric transformations position subsets of points with respect to each other. For instance, in the example of Figure 1, once point P_j has been positioned with respect to P_1 and P_2 , points belonging to clusters S_2 and S_3 are aligned with respect to S_1 before they are merged into a new single cluster.

Geometric transformations are usually composed of translations and rotations about the

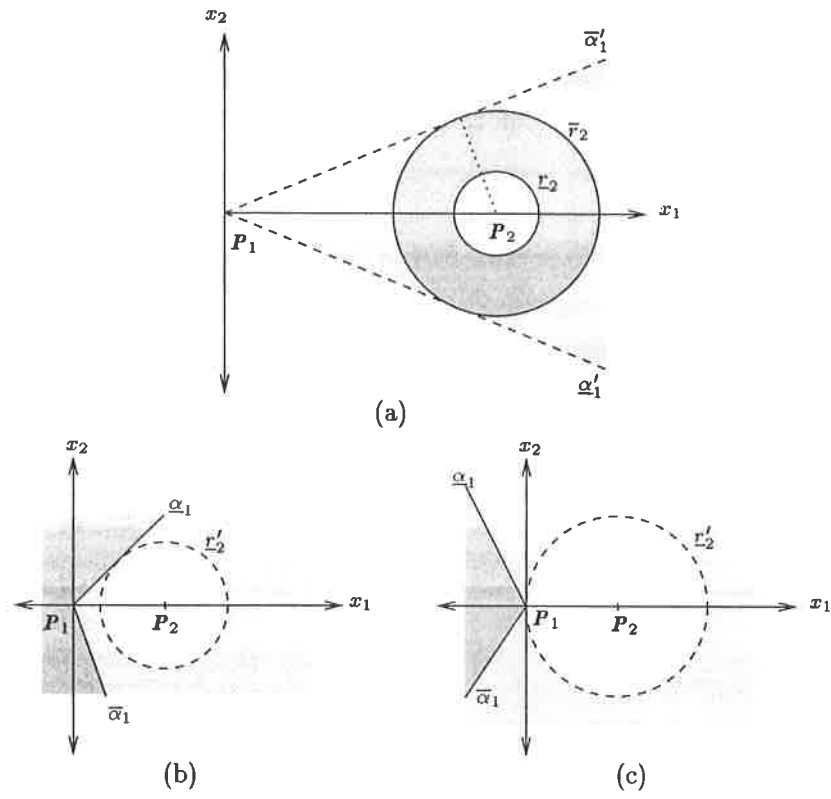


Figure 4: Feasible range of values for the dimensional parameters α'_1 and r'_2 when intersecting a pencil of lines and a pencil of circles.

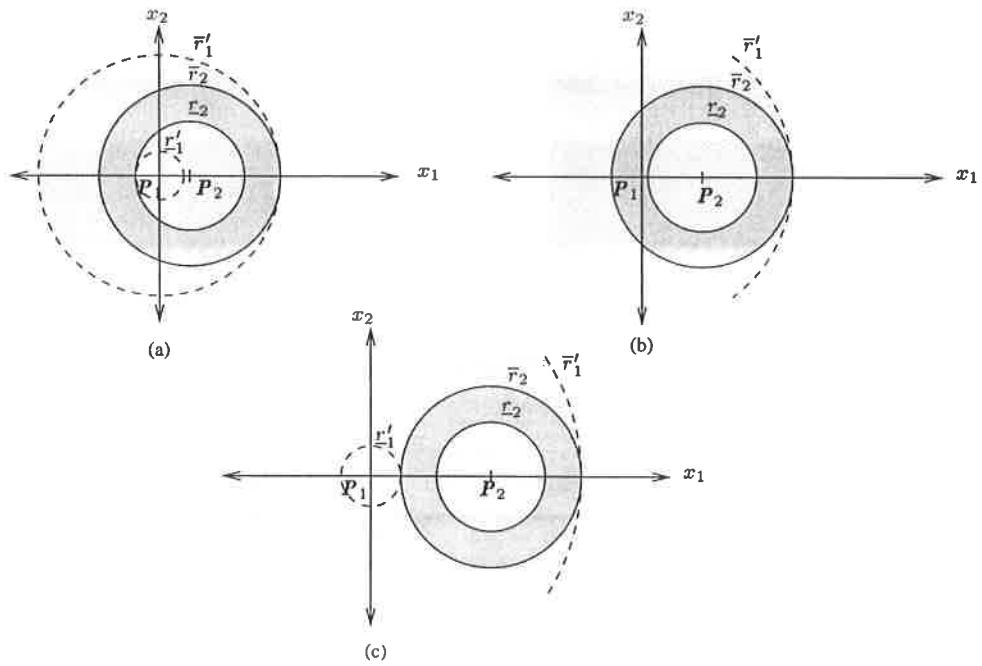


Figure 5: Feasible range of values for r'_1 when intersecting two pencils of circles: (P_1, r'_1) and (P_2, r_2) .

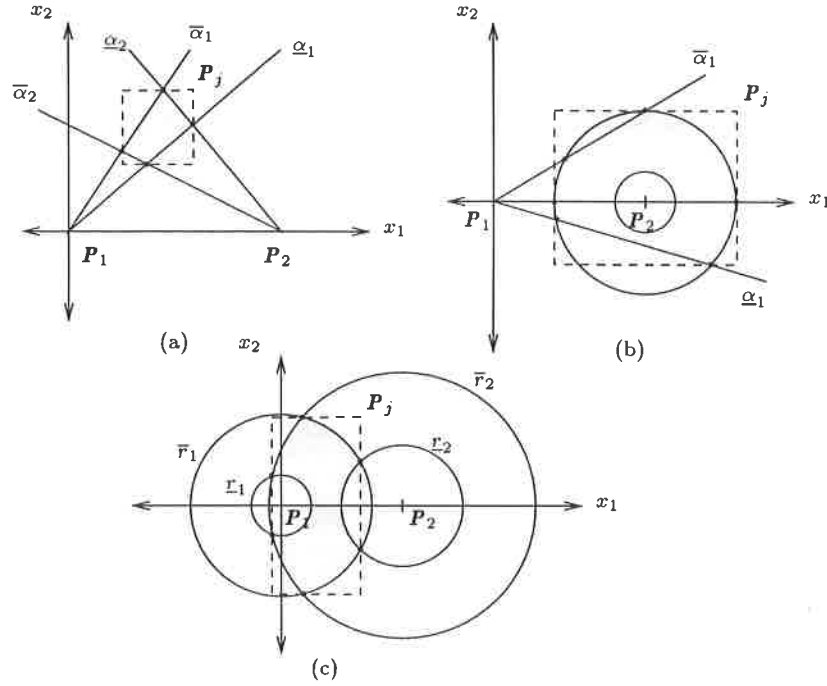


Figure 6: (a) Intersecting two pencils of lines, (b) a pencil of lines and a pencil of circles, (c) two pencils of circles.

origin. When we extend them to the interval domain, we find that the natural interval extension overestimates the range and, therefore, the resulting intervals are not sharp. In the next sections, we define the geometric transformations *translation* and *rotation* in the interval domain, and we discuss in which conditions sharpness is achieved.

3.3.1 Translations

A translation is a rigid, straight-line movement of an object from one position to another. A translation of point $P = (p_1, p_2)$ to a new position $Q = (q_1, q_2)$, $translation(P, Q)$, is represented by the translation vector $V = P - Q$.

In ruler-and-compass construction plans, there are points which are simultaneously represented with respect to different frameworks. When two clusters containing one of such points have to be merged, they define the translation vector needed to align the clusters. If one of the two points involved is a degenerated interval, then the translation vector is sharp, [17]. However, if P and Q are not degenerated, the interval translation vector $V = P - Q$ is not sharp because the interval arithmetic operations consider these two points to be independent. If the interval points P and Q are monotonic in \mathbb{IR} , and the translation vector is computed as $V = P - dual(Q)$, then the natural interval extension of the translation matrix in homogeneous coordinates

$$translation(P, Q) = \begin{pmatrix} 1 & 0 & v_1 \\ 0 & 1 & v_2 \\ 0 & 0 & 1 \end{pmatrix}$$

is sharp. To prove this statement is routine, just see that applying $translation(P, Q)$ to

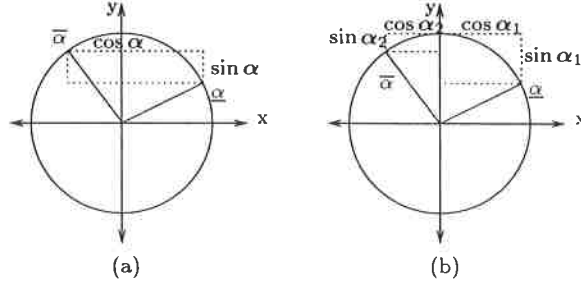


Figure 7: Since $\sin \alpha$ is not monotonic in $[\underline{\alpha}, \overline{\alpha}]$, α is subdivided into the set of intervals α_1 and α_2 . Then, the rotation matrix is piecewise monotonic. in $\alpha = \alpha_1 \cup \alpha_2$.

the interval point Q renders the interval point P .

3.3.2 Rotations

A rotation is a transformation of object points along circular paths specified by the rotation angle α . An interval rotation is the set of rotations about the origin at an angle $\alpha \in [\underline{\alpha}, \overline{\alpha}]$. When we define the natural interval extension of the rotation function in the interval domain, sharpness is lost because of the dependency problem [9]. The problem arises from the well known fact that if a given variable enters in the evaluation of an interval function more than once, then the variable contributes to the lack of sharpness. This always happens because in the rotation function occur both $\sin \alpha$ and $\cos \alpha$, and more than once.

Coefficient dependence can be solved by characterizing the shape of the solution set [1], or by using monotonicity conditions of interval functions in the extended interval arithmetic, [14, 16, 18]. Clearly, for a given rotation angle $\alpha = [\underline{\alpha}, \overline{\alpha}]$ in one quadrant of the unit circle, if $\sin \alpha$ is increasing, $\cos \alpha$ is decreasing, and vice versa. Then, we define a sharp version of the rotation matrix in \mathbb{IR} as follows

$$rotation(\alpha) = \begin{pmatrix} \cos \alpha & \sin \alpha & 0 \\ -\text{dual}(\sin \alpha) & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} [\cos \underline{\alpha}, \cos \overline{\alpha}] & [\sin \underline{\alpha}, \sin \overline{\alpha}] & 0 \\ [-\sin \underline{\alpha}, -\sin \overline{\alpha}] & [\cos \underline{\alpha}, \cos \overline{\alpha}] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

If we allow α to take values in the unit circle, the interval rotation function we have defined is not monotonic because neither $\sin \alpha$ nor $\cos \alpha$ are monotonic functions. However, since the behavior of both functions is known and local extrema can easily be determined, monotonicity conditions can be used to subdivide the initial interval into parts, so that the rotation matrix is monotonic in each subinterval. See an example in Figure 7.

3.4 Sharpness and Monotonicity of Ruler-and-Compass Operations

In Section 3.2, we have shown how to compute tight intersection operations with interval parameters by evaluating them at the endpoints of the pencils. In Section 3.3, we have defined a sharp interval rotation matrix which transforms interval points for the upper and lower bounds of the angle of rotation. However, when applying a sequence of ruler-and-compass construction steps, sharpness is lost because interval points are not monotone. For example, when a rotation is applied to the intersection of two pencils, new exact

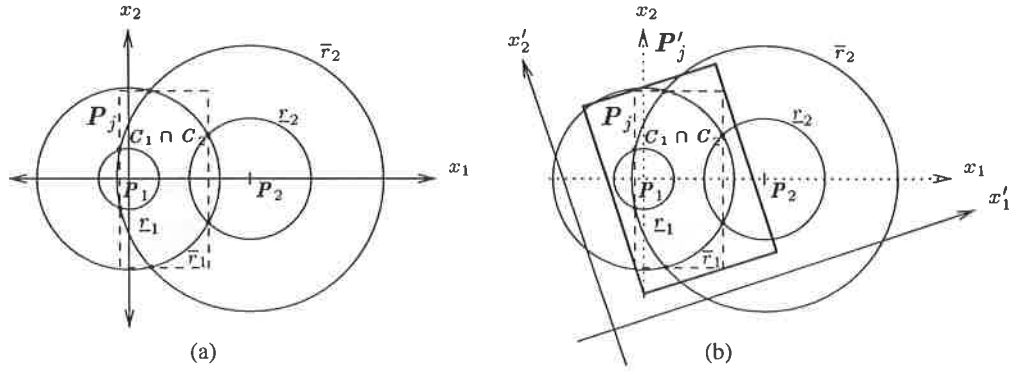


Figure 8: Applying a geometric transformation to the intersection of two pencils of circles.

coordinate points are computed for the upper and lower bounds of the interval point, but the intersection of the two pencils is not sharp anymore. Figure 8 illustrates this situation. The interval point in Figure 8a accurately represents the intersection of two pencils of circles. When we apply the geometric transformation of Figure 8b, the transformed interval point shown in bold encloses the intersection of the two pencils but, clearly, it is not tight.

In [11] we report on an application of the concepts presented here to a ruler-and-compass geometric constraint solver. It is shown that, using geometric operations in the extended interval set \mathbb{IR} , monotonicity and therefore sharpness, can be achieved in a class of problems where only one parameter is a non-degenerated interval.

4 Conclusions

Constraint solving systems currently evaluate systems of geometric constraints when the assignments of values to the constraint parameters are in the real domain. However, advanced applications in computer aided design sometimes require the values of the constraint parameters to be intervals. For instance, to determine feasible ranges of dimensional parameters, or to solve under-constrained geometric problems.

In this paper, we have successfully extended to the interval domain the set of basic geometric operations involved in constructive solvers based on the ruler-and-compass paradigm.

Future work includes characterizing intersection points in a suitable way in order to solve geometric constraint problems with more than one interval parameter.

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References

- [1] G. Alefeld, V. Kreinovich, and G. Mayer. The shape of the symmetric, persymmetric, and skew-symmetric solution Set. *SIAM J. Matrix Anal. Appl.*, 18(3):693–705, July 1997.
- [2] F. Benhamou and L. Granvilliers. Automatic generation of numerical redundancies for non-linear constraint solving. *Reliable Computing*, 3(3):335–344, 1997.
- [3] N.S. Dimitrova, S.M. Markov, and E.D. Popova. Extended interval arithmetics: new results and applications. In L. Atanassova and J. Herzberger, editors, *Computer Arithmetic and Enclosure Methods*, pages 225–234. IMACS, Elsevier Science Publishers B.V. (North-Holland), 1992.
- [4] I. Fudos. *Constraint Solving for Computer Aided Design*. PhD thesis, Purdue University, Department of Computer Sciences, 1995.
- [5] I. Fudos and C.M. Hoffmann. A graph-constructive approach to solving systems of geometric constraints. *ACM Transactions on Graphics*, 16(2):179–216, April 1997.
- [6] E. Gardeñes and A. Trepát. Fundamentals of SIGLA, an interval computing system over the completed set of intervals. *Computing*, 24:161–179, 1980.
- [7] D.J.H. Garling. *A course in Galois theory*. Cambridge University Press, 1986.
- [8] SIGLA/X Group. Información numérica e intervalos modales. Technical Report IMA 96-08-RR, Universitat de Girona, 1996. (Written in Spanish).
- [9] E.R. Hansen. Sharpness in interval computations. *Reliable Computing*, 3:17–29, 1997.
- [10] R. Joan-Arinyo and N. Mata. Basic geometric operations in constructive constraint solvers using interval arithmetic. Technical Report LSI-98-X-R, Department LiSI, Universitat Politècnica de Catalunya, 1998.
- [11] R. Joan-Arinyo and N. Mata. Interval analysis applied to constraint feasibility in geometric constraint solving. Technical Report LSI-98-3-R, Department LiSI, Universitat Politècnica de Catalunya, 1998.
- [12] R. Joan-Arinyo and A. Soto. A ruler-and-compass geometric constraint solver. In *Product Modeling for Computer Integrated Design and Manufacture*, pages 384 – 393. Chapman and Hall, London, May, 18-23 1997.
- [13] E. Kaucher. Interval analysis in the extended interval space \mathbb{IR} . *Computing*, Suppl.(2):33–49, 1980.
- [14] L. Kupriyanova. Inner estimation of the united solution set of interval linear algebraic system. *Reliable Computing*, 1(1):15–32, 1995.
- [15] S. M. Markov. On the presentation of ranges of monotone functions using interval arithmetic. *Interval Computations*, 4:19–31, 1992.
- [16] S. M. Markov. Some interpolation problems involving interval data. *Interval Computations*, 3:164–182, 1993.

- [17] R.E. Moore. *Methods and Applications of Interval Analysis*. SIAM Studies in Applied Mathematics. SIAM, Philadelphia, PA, 1979.
- [18] V. M. Nesterov. How to use monotonicity-type information to get better estimates of the range of real-valued functions. *Interval Computations*, 4:3–12, 1993.

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