Computed-Torque-Plus-Compensation-Plus-Chattering Controller of Robot Manipulators

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1. Introduction

Robot control is a modern technology that requires of innovation in control theory. The robot system is a complex and nonlinear system involving mechanics, electronics, and computer science. With technological innovation in electronics, more complex controllers can be designed and implemented in robotic systems to conceive a computer controlled robot manipulator. In this sense, a robot system can be viewed as a mechanical arm that operates under computer control in order to have a reprogrammable - and thus multifunctional - manipulator designed to move material, parts, or performing tracking motion for a great variety of tasks. However, there still exists an important challenge: to cope with friction that can degrade the performance of our robot system.

Friction is a natural phenomenon that affects almost all mechanical systems. This phenomenon has been extensively studied for many years, as it is hard to model and, in some situations, hard to predict because of several factors that vary over time (wasting, humidity, and temperature). For these reasons, friction is usually ignored at the controller design stage. Although there are many controllers based on friction models such as (Orlov et al., 2003), (Aguilar et al., 2003), and (Guerra & Acho, 2007), the real implementation of these controllers requires on-line final tuning. In other words, those controllers that were designed by neglecting the friction perturbations have to be robust against them. From the robot control point of view, there have been many controllers based on frictionless robot modeling: PD and P"D" control with gravity compensation, computed-torque plus control, etc. (Kelly et al., 2005). From the engineering point of view, it is of interest to redesign some of these controllers to make them robust against friction perturbations. Friction mitigation is an important topic in the high-precision control of mechanisms (Weiping & Xu, 1994). It is well known that chattering controllers can deal with model uncertainties like friction, (Orlov et al., 2003). Chattering is a fast commuting term that is added to a given controller.

The computed-torque-plus-compensation controller of robot manipulators, that was originally called *computed-torque control with compensation*, has been well documented, *e.g.* (Kelly et al., 2005). According to (Kelly et al., 2005), for the academic robotics community, the global stability of the closed-loop system with this controller is still an open problem. Here, a chattering term is added to the previous controller to improve the global asymptotic stability. We call it the *computed-torque-plus-compensation-plus-chattering* controller of robot manipulators. Moreover, according to numerical experiments applied to the tracking control of a robot manipulator with two degrees of freedom, this new controller represents an important and robust improvement over the original one, especially when the system is operated under Coulomb friction effects. Lyapunov theory is employed in proving the global uniform asymptotic stability of the closed-loop system.

This work is structured as follows. Section 2 introduces the computed-torque plus compensation controller of robot manipulators. The dynamic notation for an n-degree-of-freedom (n-DOF) robot manipulator is also presented. In Section 3, the chattering version of the computed-torque plus compensation controller is defined. Global uniform asymptotic stability is achieved by invoking Lyapunov theory. Section 4 studies the performance and robustness of the proposed controller and compares it with the performance of the original controller through numerical experiments on a 2-DOF vertical robot manipulator with Coulomb friction. This kind of robot is one that is affected by gravity. Finally, Section 5 states the conclusions.

2. Computed-torque plus compensation control of robot manipulators

Consider the following general equation describing the dynamic of an n-degrees-of-freedom (n-DOF) rigid robot manipulator in joint space:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau, \tag{1}$$

where $q \epsilon R^n$ is the vector of generalized coordinates, $\tau \epsilon R^n$ is the vector of external torques, $M(q) \epsilon R^{nxn}$ is the positive-definite inertia matrix, $C(q,\dot{q})\dot{q} \epsilon R^n$ is the vector of Coriolis and centrifugal torques, and $G(q) \epsilon R^n$ is the vector of gravitational torques. The equation for the computed-torque control plus compensation is given by (Kelly et al., 2005):

$$\tau = M(q)\left[\ddot{q}_d + K_v\dot{\tilde{q}} + K_p\tilde{q}\right] + C(q,\dot{q})\dot{q} + G(q) - C(q,\dot{q})\upsilon, \tag{2}$$

where K_p and K_v are symmetric positive-definite design matrices, $\tilde{q}(t) = q_d(t) - q(t)$ denotes the position error vector, and thus $\dot{\tilde{q}}(t) = \dot{q}_d(t) - \dot{q}(t)$ is the velocity error vector. $q_d(t)$ is the given reference trajectory vector which is assumed to be smooth and bounded in its first and second time derivatives. Finally, $\upsilon \epsilon R^n$ is obtained by filtering $\tilde{q}$ and $\dot{\tilde{q}}$ (Kelly et al., 2005):

$$\upsilon = -\frac{bp}{p+\lambda}\dot{\tilde{q}} - \frac{b}{p+\lambda}\left[K_v\dot{\tilde{q}} + K_p\tilde{q}\right], \tag{3}$$

where $p = \frac{d}{dt}$ is the differential operator, and λ and b are scalar positive constants given by the designer. For simplicity, we can set $b = 1$ as in (Kelly et al., 2005). The above equation can be expressed as follows:

$$\dot{\upsilon} + \lambda\upsilon = -[\ddot{\tilde{q}} + K_v\dot{\tilde{q}} + K_p\tilde{q}]. \tag{4}$$

The controller in equations (2) and (4) applied to the robot system in equation (1) satisfies the next motion control objective (Kelly et al., 2005), that is,

$$\lim_{t\to\infty} \tilde{q}\,(t) = 0. \tag{5}$$

3. Computed-torque-plus-compensation-plus-chattering control of robot manipulators

We now introduce the chattering version of the computed-torque-plus-compensation controller:

$$\tau = M(q)\left[\ddot{q}_d + K_v\dot{\tilde{q}} + K_p\tilde{q}\right] + C(q,\dot{q})\dot{q} + G(q) - C(q,\dot{q})v - K_\alpha sgn(v), \tag{6}$$

where v is obtained from equation (4), $K_\alpha = diag\{k_a\}$, with $k_a > 0$; $sgn(v)^T = [\; sgn(v_1) \;\; sgn(v_2) \;\; \cdots \;\; sgn(v_n)\;]$, and $v^T = [v_1 \;\; \cdots \;\; v_n]$. The function $sgn(\cdot)$ is the signum function, which is 1 if its argument is positive, -1 if it is negative, and 0 if it is zero. The closed-loop system in equations (1), (4) and (6) yields

$$M(q)\left[\ddot{\tilde{q}} + K_v\dot{\tilde{q}} + K_p\tilde{q}\right] - C(q,\dot{q})v - K_\alpha sgn(v) = 0, \tag{7}$$

which, after invoking equation (4), produces

$$M(q)[\dot{v} + \lambda v] + C(q,\dot{q})v + K_\alpha sgn(v) = 0. \tag{8}$$

Consider now the following nonnegative Lyapunov function, which is also used in (Kelly et al., 2005),

$$V(t,v) = \tfrac{1}{2}v^T M(q)v = \tfrac{1}{2}v^T M(q_d - \tilde{q})v\ . \tag{9}$$

Its time derivative is

$$\dot{V}(t,v) = \tfrac{1}{2}v^T \dot{M}(q)v + v^T M(q)\dot{v}\ . \tag{10}$$

Solving equation (8) for $M(q)\dot{v}$ and substituting it in equation (10), we arrive at

$$\dot{V}(t,v) = v^T\left[\frac{1}{2}\dot{M}(q) - C(q,\dot{q})\right]v - \lambda v^T M(q)v - v^T K_\alpha sgn(v),$$

where the term $v^T[\frac{1}{2}\dot{M}(q) - C(q,\dot{q})]v$ can be canceled thanks to the fact that $\frac{1}{2}\dot{M}(q) - C(q,\dot{q})$ is a skew-symmetric matrix. Thus,

$$\dot{V}(t,v) = -\lambda v^T M(q)v - v^T K_\alpha sgn(v).$$

On one hand, there exists a real positive number α such that

$$\|M(q)\|_2 \le \alpha\|I\|_2,$$

and using the Cauchy-Schwartz inequality, it follows that

$$V(t,v) = \tfrac{1}{2}v^T M(q)v \le \tfrac{\alpha}{2}\|v\|_2^2.$$

Using that $\|v\|_2 \le \|v\|_1$, we obtain

$$V(t,v) \le \frac{\alpha}{2}\|v\|_2^2 \le \frac{\alpha}{2}\|v\|_1^2,$$

or equivalently,

$$k\sqrt{V(t,v)} \le \|v\|_1, \quad \text{where } k = \sqrt{\frac{2}{\alpha}}\,. \tag{11}$$

On the other hand,

$$\dot{V}(t,v) = -\lambda v^T M(q)v - v^T K_\alpha sgn(v) \le -k_a v^T sgn(v) \le -k_a\|v\|_1, \tag{12}$$

and substituting (11) in equation (12) we arrive at,

$$\dot{V}(t,v) \le -k_a\|v\|_1 \le -k_a k\sqrt{V(t,v)}; \quad \Rightarrow \quad \dot{V}(t,v) + k_a k\sqrt{V(t,v)} \le 0,$$

thus, there exists a settling time, t_s, such that $\lim_{t\to t_s} v(t) = 0$ and $v(t) = 0$ for all $t \ge t_s$. For details, see Theorem 4.2 on finite-time stability in (Bhat & Bernstein, 2000). From equation (4) and using that $v(t) = 0$ (and $\dot{v}(t) = 0$) for all $t \ge t_s$, we have $\ddot{\tilde{q}} + K_v\dot{\tilde{q}} + K_p\tilde{q} = 0$, which is a linear time-invariant and asymptotically stable system. In summary, we have obtained the following main result stated in Theorem 1.

Theorem 1.- *The controller in equations (6) and (3) (or (4)) global-uniformly-asymptotically stabilizes the robot system described in equation (1) at the equilibrium point* $(\tilde{q}, \dot{\tilde{q}}, \upsilon) \equiv 0$.
Remark 1.- *Although the closed-loop system contains discontinuity terms in the right-hand side, its solution is continuous and locally Lipschitz everywhere except at the origin. Hence, every set of initial conditions in* $R^n \backslash \{0\}$ *has a unique solution in forward time on a sufficiently small time interval. The chattering appears at the origin. This justifies the use of Lyapunov theory for this special case of non-smooth dynamical systems.*

4. Numerical experiments

The performance of the controller specified in Theorem 1 is compared with that of the computed-torque plus compensation controller in equations (2) and (4). Consider a 2-DOF robot manipulator moving in a vertical plane (see Figure 1). The characterization of this manipulator is taken from (Berghuis & Nijmeijer, 1993),

$$M(q) = \begin{bmatrix} 8.77 + 1.02\cos(q_2) & 0.76 + 0.51\cos(q_2) \\ 0.76 + 0.51\cos(q_2) & 0.62 \end{bmatrix},$$

$$C(q, \dot{q}) = \begin{bmatrix} -0.51\sin(q_2)\dot{q}_2 & -0.51\sin(q_2)(\dot{q}_1 + \dot{q}_2) \\ 0.51\sin(q_2)\dot{q}_1 & 0 \end{bmatrix},$$

$$G(q) = g \begin{bmatrix} 7.6\sin(q_1) + 0.63\sin(q_1 + q_2) \\ 0.63\sin(q_1 + q_2) \end{bmatrix},$$

where g is the gravity acceleration. Moreover, let us assume that the robot system is subject to a Coulomb friction perturbation, that is, the robot with added friction is given by (Orlov et al., 2003)

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + F(\dot{q}) = \tau,$$

where $F(\dot{q}) = K_f sgn(\dot{q})$ is the friction force vector (which can be seen as the un-modeled dynamics). We use $K_f = diag\{9.9\}$, and, to complete the numerical experimental platform, we set $K_p = diag\{100\}$, $K_v = diag\{50\}$, and $\lambda = 10$, for the original controller, and $K_\alpha = diag\{10\}$ for the proposed controller. We set the reference trajectory vector, $q_d(t) = [q_{d1}(t) \quad q_{d2}(t)]^T = [\pi + 0.5\sin(2\pi t) \quad 0.5\pi + 0.5\sin(2\pi t)]^T$. The simulation results are shown in Figures 2 and 3. From these two figures, it is clear that the proposed chattering controller represents an important performance improvement.

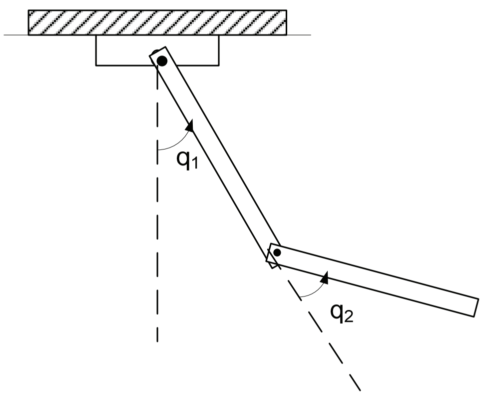

Fig. 1. 2-DOF vertical robot manipulator.

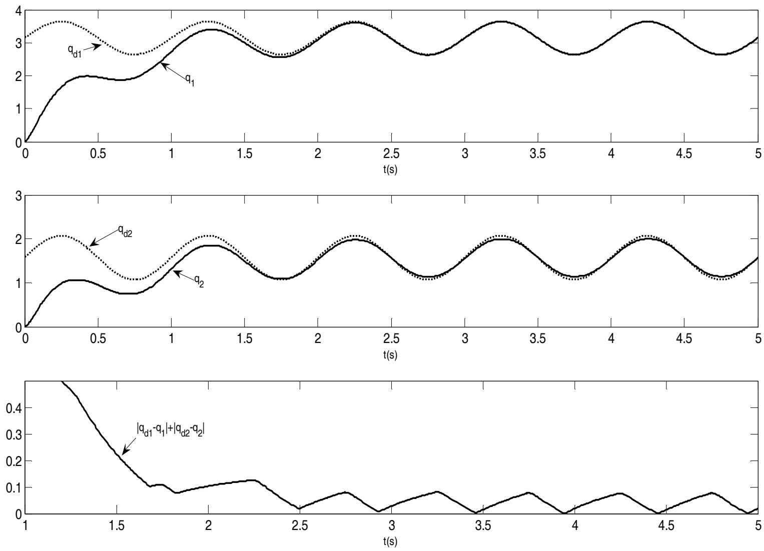

Fig. 2. Simulation results on the computed-torque plus compensation controller.

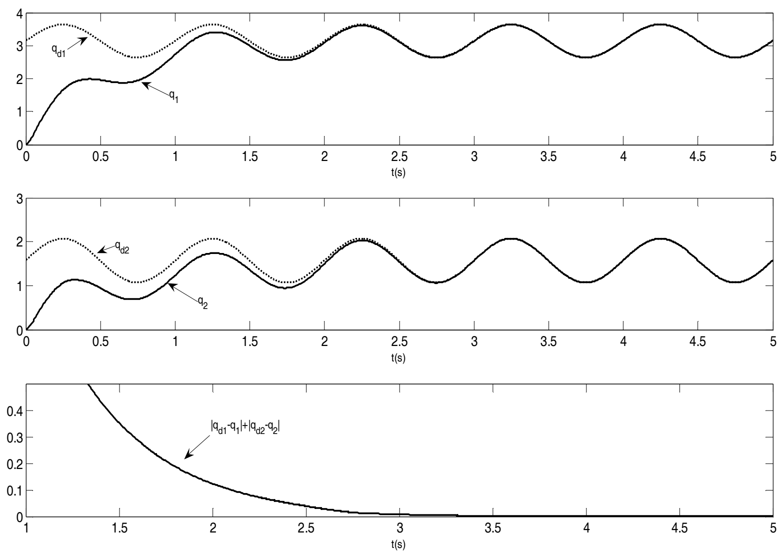

Fig. 3. Simulation results on the computed-torque-plus-compensation-plus chattering controller.

Figure 2 shows the system trajectories and their comparison with respect to the desired ones. The graph of $|q_{d1} - q_1| + |q_{d2} - q_2|$ versus time captures the 1-norm error position. Here, an oscillating error is obtained because of friction. In some applications, this tracking error can be unacceptable. For instance, repeatability (the measure of how close a manipulator can return to a previously taught point) is perturbed, as well as accuracy (the measure of how close the manipulator can approach a given point within its workspace). However, using our controller (Figure 3), the oscillatory error behavior is precluded, thus improving the repeatability and accuracy performance. Moreover, the tracking error shown in Figure 3 can be inside of the controller resolution (the smallest increment that the controller can sense). When this happens, our controller rejects completely the effects of friction on the robot system. Figures 4 and 5 show the control signals for both cases. We can appreciate that both control signals are alike. Only small chattering appears in our case. This chattering has small amplitude ant it is not persistent, like the chattering that appears, for instance, in (Orlov et al., 2003).

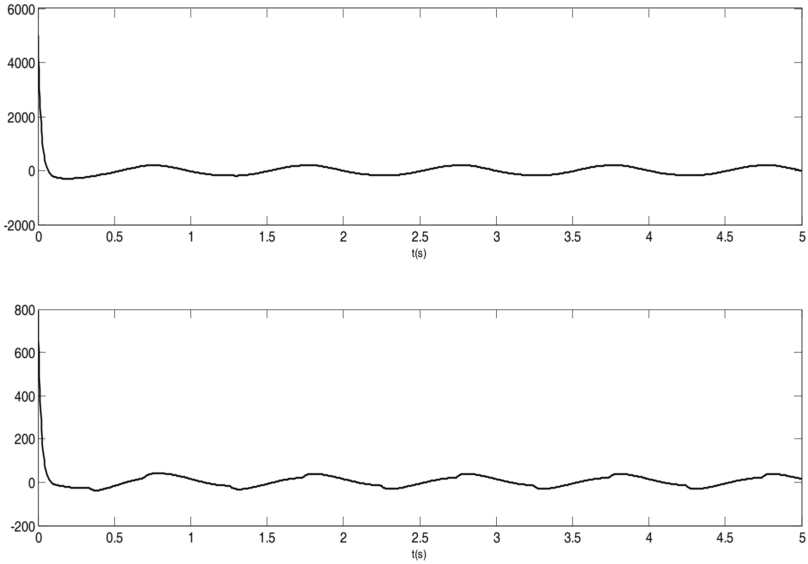

Fig. 4. Simulation results on the computed-torque plus compensation controller: the applied torque (N-m) to the first link (top) and the applied torque (N-m) to the second link (bottom).

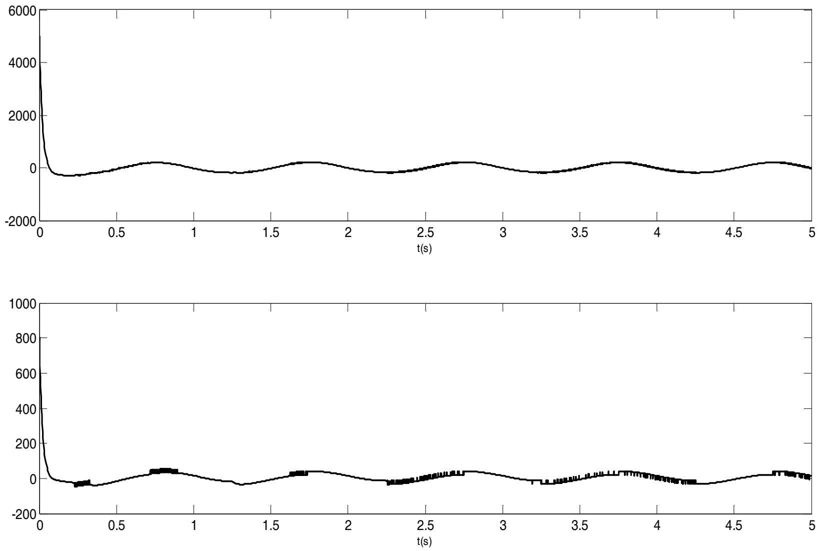

Fig. 5. Simulation results on the computed-torque-plus-compensation-plus chattering controller: the applied torque (N-m) to the first link (top) and the applied torque (N-m) to the second link (bottom).

Let us test the controllers performance by means of a more general case of perturbation. Consider that the robot system is subject to external perturbation; that is, consider the system:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) = \tau + d(t),$$

where $d(t)\epsilon R^2$ is a bounded external perturbation. This perturbation can be introduced into the robot system, for instance, when working on a ship since wave motion induces vertical force perturbation. Let us set $d^T(t) = [\sin(t) \quad \sin(2t)]$. Simulation results are shown in Figures 6, 7, 8 and 9. When the proposed controller is used, the tracking error between the system trajectory and the reference trajectory is clearly improved for the second joint. Thus, when the external perturbation is present, our controller outperforms the original one.

5. Conclusion

A modified version of the computed-torque plus compensation controller was designed by adding a chattering term. Because of this chattering term, the new robot controller outperforms the original one, especially when the robot is subject to Coulomb friction perturbations. Moreover, this new controller facilitates the proof of global stability of the closed-loop system, and also improves the repeatability and accuracy of the robot control system. From the control design point of view, our chattering controller has the following *sliding mode control interpretation*. It is well known that sliding motion occurs when the trajec-

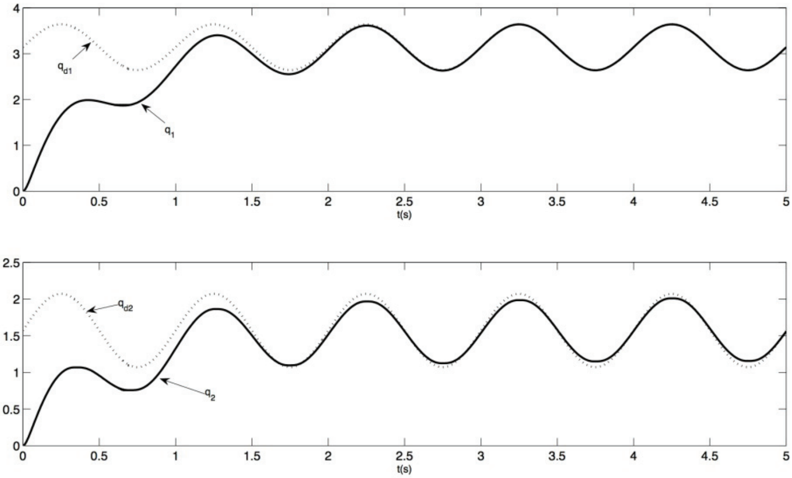

Fig. 6. Simulation results on the computed-torque plus compensation controller.

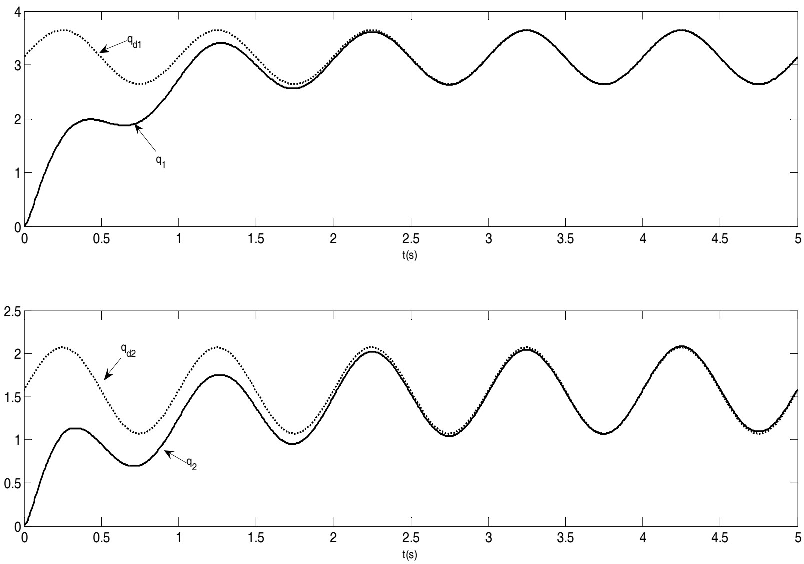

Fig. 7. Simulation results on the computed-torque-plus-compensation-plus chattering controller.

tory of the system is driven (in finite time) towards a sliding surface, where the system has a reduced order behavior, and forced to remain on it where some stability property is satisfied. See, for instance, (Edwards & Spurgeon, 1998), (Perruquetti & Barbot, 2002), and (Spong & Vidyasagar, 1989). Our chattering controller drives the system trajectory, in finite time, to the condition where the non-linear robot system has a linear-time-invariant asymptotically stable behavior given by $\ddot{\tilde{q}} + K_v\dot{\tilde{q}} + K_p\tilde{q} = 0$. This is an important contribution of our chattering control, which is impossible to be fulfilled with the original computed-torque plus compensation controller. So, our controller is in fact a sliding mode controller but designed in an implicit form.

6. References

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