# Mean Field Theory of Fluid Neural Networks

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**Abstract**

Fluid Neural Networks (FNN) are a mathematical framework where the phenomenon of self-synchronization in ant colonies can be explained, predicting the model a critical density, i.e. a density where oscillations appear, observed in real ant colonies. However, up to now all results have been solely numerical. In this paper we put forward a simpler FNN with the same phenomenology as the original one, but an analytical approximation can be performed in such a way that critical densities can be computed, offering a good approximation to the numerical ones.

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# 1 Introduction

Collective behavior in ant colonies is undoubtedly a fascinating subject. Hundreds or thousands of small *simple* insects display coordinated collective patterns of complex behaviors, such as raid patterns, food foraging, brood care, task allocation or nest building, showing how, generally speaking, simple *interacting* individuals can behave as a whole in unexpected ways. Nuisance of reductionists, swarm behavior has attracted the attention of physicists working on critical phase transitions or dynamical systems. It has been shown that behind some phenomena such as trail patterns [1] or self-synchronization of activity [2] there are mechanisms well known to physicists, where noise-induced transitions [3] or marginal stability [4] are but a few examples. Beyond the classical interest in swarm behaviour from biologists [5], Complexity sciences have taken ant colonies as one of the main subjects of study in their quest for laws behind complex phenomena [6].

The mechanisms underlying swarm intelligence, as is also called, are certainly not few in number though we will be interested mainly in mechanisms by which the global performance of the colony goes beyond that of individuals, such as interactions by means of laying pheromones or by physical contact among individuals. These different ways of interaction may generate striking behaviors, such as stigmery (a stigmergic process, following Wilson [7], is a process by which it is the work already accomplished that induces the insects to perform additional labor) or the one we will focus on in this paper: Self-synchronization.

Some experiments with *Leptothorax acervorum* ants by N. Franks [8] and *Leptothorax allardycei* by B. J. Cole [2] revealed the existence of short-term rhythms of activity. This synchronization in activity seems to be specially apparent in nurse workers, where cycles of approximately 20 min. (15 min. of quiescence plus 2-5 min. of activity) have been measured. There exist some mathematical models of this behavior in very different

frameworks: Differential equations [9], probabilistic process algebra [10] or Fluid Neural Networks (FNN) [11], though up to now is far from clear which one is best fitted to the phenomenon under study. The interest in self-synchronization is not solely a biological one, since this phenomenon has been shown to be related to mutual exclusion in brood care [12] and task allocation [13]. Thus, the possible usefulness of self-synchronization in ant colonies makes it also interesting for distributed asynchronous algorithm designers.

In this paper we offer an analytical approximation to FNNs, one of the mathematical models of self-synchronization in ant colonies [14]. After reviewing the original FNN in section 2, in section 3 we introduce and justify a simpler FNN, with the same phenomenology as the original one, where analytical approximations can be made in order to obtain critical densities near to the ones computed numerically. This reveals and explains some curious results already obtained in the original FNN, such as lattice size dependence of critical density. Finally, we discuss in section 4 the approximations we have performed and suggest possible ways of improving analytical results.

# 2 Fluid Neural Networks

FNNs are defined as formal neurons [15] moving on a lattice. Each "neuron-ant" has a continuous state $S_i(t) \in \mathbf{R}$, at each time step $t$. Interaction with nearest individuals, located in the neighborhood $B(i)$ defined by the eight nearest lattice sites, is defined by

$$S_i(t+1) = \Phi\left[\, g\left\{ J_{ii}S_i(t) + \sum_{i\neq j\in B(i)} J_{ij}S_j(t) \; - \Theta_i \right\}\right]$$

where $J_{ii} \neq 0$. For simplicity we use the threshold $\Theta_i = 0$, and we take $\Phi(z) = \tanh(gz)$ where $g$ is a gain parameter. Each automaton can be either *active* or *inactive*, depending on the state $S_i(t)$ and, if active, it moves randomly to one of the eight nearest cells (if no space is available, no movement takes place). In FNN a given automaton will be active if it is above some threshold $\theta_{act}$, $S_i(t) > \theta_{act}$, and inactive otherwise. Once an automaton

becomes inactive, it can return to the active state (with a *spontaneous activity level* $S_a$) with some probability $p_a$. The coupling matrix $\mathbf{J}$ is not fixed. Connections are local and changing over time as a consequence of movement. They are also state-dependent i.e. $J_{ij}$ will be a simple function of the states of the actually interacting pair $(i,j)$ of automata, i.e. $J_{ij} = f(a_i^t, a_j^t)$, where $a_i^t = \Theta\left[S_i(t) - \theta_{act}\right]$. In our case, where two basic states are defined the connection matrix reduces to the following $2 \times 2$ table

$$\mathbf{\Lambda} = \begin{bmatrix} \lambda_{11} & \lambda_{10} \\ \lambda_{01} & \lambda_{00} \end{bmatrix}$$

At a given time step, the interaction $J_{ij}$ between the $i$-th and the $j$-th elements is equal to $\lambda_{a_i^t a_j^t} \in \mathbf{\Lambda}$ by depending on the activity states of the given elements. More precisely, $J_{ij}$ will be equal to: $\lambda_{11}$ when both ants are active, to $\lambda_{10}, \lambda_{01}$ when one is active and the other inactive and to $\lambda_{00}$ if both automata are inactive.

This model is able to account for the oscillations observed in the experiments (see Fig.1). The model also allows one to define a critical density of active elements, i.e. a density where oscillations appear, that is approximately the same density observed usually in ant colonies [14]: $\rho_c \approx 0.2$. Furthermore, recent work has shown that noise is a determinant in the mechanism of oscillations, through spontaneous activation, suggesting that oscillations appear at a noise induced transition [16] (see [3] for a description of noise induced transitions). An order parameter for FNNs was also found in [16]: Assuming the transition to be noise-induced we can define an order parameter by using the stationary density of active elements $P(\rho^+)$ (computed by means of histograms). If we define $\rho_m^+$ such that

$$P(\rho_m^+) = \max_{\rho^+ \in [0,1]} P(\rho^+)$$

the order parameter will be defined by:

$$\Gamma(\rho, p_a) = 1 - \rho_m^+$$

As seen in Fig.2 the value of $\Gamma$ is zero after the transition and non-zero before the transition and, as was shown in [16], the critical density $\rho_c$ is the same as the one that was formerly determined in [14] by means of the Shannon-Kolmogorov Entropy.

There has been experimental work measuring some parameters of FNN, such as $J_{ij}$ (interaction between "neuron-ants", assumed to be 1 in theoretical simulations) and $g$ (the gain parameter of the non-linear interaction among individuals, assumed to be 0.1 in the model) [17].

# 3 Simple Fluid Neural Networks

Some features of the original FNN, as defined in sect. 2, can be considerably simplified. We will define the Simple FNN (SFNN) in the following way: We have $N$ individuals $S_i(t) \in \mathbf{R}$ that change their state according to:

$$S_i(t+1) = gS_i(t) + g\sum_{j_i^*} J_{ij_i^*} S_{j_i^*}(t) + S_a\Theta[\theta_{act} - S_i(t)]I_i^t \tag{1}$$

where $I_i^t \in \{0,1\}$ with probability $P(I_i^t = 1) = p_a$ and we have made a first order approximation of tanh: $\tanh(x) \approx x$ removing one of the nonlinearities of the original FNN. The meaning of $J_{ij}$, $S_a$, $p_a$ and $g$ is the same as in the original FNN (section 2). Active states will be defined by $a_i^t = \Theta[S_i(t) - \theta_{act}]$.

What does it mean $j_i^*$?, it is the *neighborhood.* At a given time step $t$, the local field $h_i(t) = \sum_{j_i^*} J_{ij_i^*} S_{j_i^*}(t)$ will be computed for all $i$ *before* the change of state $S_i(t+1)$ is performed. In order to do so, *for each individual* $S_i(t)$, $K$ random connections to some individuals will be established (these individuals will be called *the neighbours*). $K$ is chosen randomly from the distribution

$$P(K = k) = \binom{V}{k}\rho^k(1-\rho)^{V-k} \tag{2}$$

This has the same effect as if we threw, at each time step *and* for each element $S_i(t)$, all the $N$ elements upon a $L \times L$ lattice (then $\rho = \frac{N}{L^2}$ will be the density of elements), in

order to compute the corresponding local field $h_i(t)$. Thus, we get some kind of "annealed" movement. This is similar to the mean field approximation made in spatially distributed epidemic models [18], where movement was dependent on a parameter $m$ such that the limit $m \to \infty$ was in fact the same as throwing randomly all the elements upon the lattice at each time step. In our case we do so to compute each local field $h_i(t)$, so our system is, in this sense, more disordered.

Considering the density of active individuals at time $t$

$$\rho_t^+ = \frac{1}{N}\sum_{i=1}^{N}\Theta[S_i(t) - \theta_{act}] \tag{3}$$

we can see in Fig.3 that $\rho_t^+$ in SFNN has the same temporal behavior as $\rho_t^+$ in FNN: Irregular behavior at low densities and more ordered oscillatory behavior for growing $\rho$. This allows one to apply the FNN order parameter in this case too. We can see $\Gamma(\rho, p_a)$, as defined in Section 2, in Fig.4, computed for a SFNN.

To sum up, we have a simple FNN, where some nonlinearities have been removed and where each individual, at each time step, establishes connections randomly, as if we had some kind of "annealed" movement. This has simplified considerably the model without loss of interesting behavior because both, FNN and SFNN are phenomenologically identical. In the rest of the paper we will explore the relation between the critical density $\rho_c$ and activation probability $p_a$ in SFNN (as we did numerically with FNN in [16]). Throughout the paper, the values of the parameters will be $g = 0.1$, $V = 4$, $S_a = 0.1$, $J_{ij} = 1$ for all $i, j$ and $\theta_{act} = 10^{-16}$.

# 4 Analytic approximation of $\rho_c$

The analysis will be performed for $\rho > \rho_c$, that is, in the region of well developed oscillations. There the behavior of activity spreading is quite well defined: As we see in Fig.5 the role of spontaneous activation is merely that of starting the process of activ-

ity propagation, process that continues by means of interaction among individuals until activity reaches the whole system ($\rho_t^+ = 1$). This would allow us to analyze separately activity propagation and inactivation, assuming in both cases that there is no spontaneous activation.

First of all we will find a condition on $V$ and $g$ to assure the decay of the system to a state where all $N$ elements are inactive. With the above mentioned assumption the evolution for $S_i(t)$ will be

$$S_i(t+1) = gS_i(t) + g\sum_{j_t^*} S_{j_t^*}(t)$$

To see the global evolution of the $N$ individuals we can derive a discrete equation for the state average $< S(t) >= \frac{1}{N}\sum_{i=1}^{N} S_i(t)$ if we approximate the term $\sum_{j_t^*} S_{j_t^*}(t)$ by the mean field version $V\rho < S(t) >$ so that

$$< S(t) >= (g(1+V\rho))^t < S(0) >$$

and we can assure activation decay if $g(1+V\rho) < 1$. If we assume that $\rho$ is as largest as possible ($\rho = 1$), we get the condition

$$g < \frac{1}{1+V}$$

that is satisfied in our case, because $g = 0.1$ and $V = 4$. Of course, if $< S(t) >$ tends to 0, $\rho_t^+$ will tend to 0 too. Let us remark that, though $< S(t) >$ tends to zero exponentially but smoothly, $\rho_t^+$ goes to 0 in very few time steps (as can be seen in Fig.3 when $\rho > 0.2$).

Now let us study the propagation of activation through the system. In [16] we put forward the hypothesis that only two factors were important in order to understand FNN oscillations: the average time $\tau'(\rho, p_a)$ one individual is active between two inactive states and activity propagation $Y_{\lfloor Np_a \rfloor}(\rho, p_a)$, that is, the average number of time steps necessaries to reach the state of $\rho_t^+ = 1$ from an initial state where $\lfloor Np_a \rfloor$ individuals are active, i.e. the mean (integer) number of individuals that would activate spontaneously

with probability $p_a$ in a system with all $N$ elements inactive. These are precisely the factors we will analyze in order to compute analitically $\rho_c$. Intuitively, if $Y_{\lfloor Np_a \rfloor}(\rho, p_a)$ is less than $\tau'(\rho, p_a)$ the state of maximum activation will be reached *before* individuals start the process of inactivation, then we will observe oscillations. So then, $\rho_c$ will be such that

$$Y_{\lfloor Np_a \rfloor}(\rho_c, p_a) = \tau'(\rho_c, p_a) \tag{4}$$

Activity spreading can be treated as a branching process if when considering activity by interaction we take into account only the state $a_i^t$ of each individual. In this way, we will say that an inactive individual is activated by its neighbours if there is at least one of them active (it is obvious that this is not the *exact* mechanism by which individuals activate each other, since an individual with all neighbours active, each one with a very small $S_i(t)$, might not be activated). The probability of having at least an active individual as a neighbour is easy to compute, because of the "annealed movement" we have introduced. If we have $i$ active individuals, the above mentioned probability is

$$\gamma_i = 1 - \left(1 - \frac{i}{L^2}\right)^V \tag{5}$$

As we are only considering activity spreading in the oscillations phase, there will be no activity decay, allowing us to compute the probability of $j$ active individuals having $i$ individuals active in the previous time step

$$P_{ij} = P(A_{t+1} = j \mid A_t = i) = \begin{cases} \binom{N-i}{j-i} \gamma_i^{j-i} (1-\gamma_i)^{N-j} & \text{if } i \leq j \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where $A_t$ is the number of active individuals at time $t$. This defines a branching process that will finish when $A = N$. We will treat this process as a Markov chain [19] with stochastic matrix

$$\overline{P} = \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1N} \\ 0 & P_{22} & \cdots & P_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & P_{NN} \end{pmatrix} \tag{7}$$

with which we can compute the mean number of steps before being absorbed by the unique clossed class of our system, the one element set $\{N\}$. In order to perform the calculations, the $\overline{P}$ matrix has to be rearranged to get the canonical form

$$\overline{P^*} = \begin{pmatrix} P_1 & 0 \\ R & Q \end{pmatrix} = \left(\begin{array}{c|ccc} P_{NN} & 0 & \cdots & 0 \\ \hline P_{1N} & P_{11} & \cdots & P_{1(N-1)} \\ P_{2N} & 0 & \cdots & P_{2(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ P_{(N-1)N} & 0 & \cdots & P_{(N-1)(N-1)} \end{array}\right) \tag{8}$$

so that the *fundamental matrix* $M = [I - Q]^{-1}$ of the Markov chain can be found. The matrix $M$ plays a central role in transient analysis of Markov chains [19]. $M$ gives immediately the quantity we want to compute. It is easy to verify that the $ij-th$ element of $Q^k$, $q_{ij}^{(k)}$ is the probability of a transition from the state $A = i$ to the state $A = j$ in exactly $k$ steps. The average number of times that starting in state $A = i$ the process reaches state $A = j$ before it leaves transient states and enters the clossed class is

$$q_{ij}^{(0)} + q_{ij}^{(1)} + q_{ij}^{(2)} + \ldots + q_{ij}^{(k)} + \ldots$$

that is precisely $M_{ij}$, since the identity

$$M = [I - Q]^{-1} = I + Q + Q^2 + \ldots + Q^k + \ldots$$

follows from the fact that $Q$ has all the eigenvalues strictly inside the unit circle (the eigenvalues of $Q$ are $\lambda_j = P_{jj}$ for $1 \leq j \leq N-1$ and $\lambda_j < 1$ since $P$ is a stochastic matrix). If $\mathbf{1}$ is a column vector whose components are all equal to 1, the mean number of steps before reaching the state of all individuals active, taking as a departure point a

state $A = i$, is the $i - th$ component of the vector $M\mathbf{1}$. So, if $Y_j = (M\mathbf{1})_j$, solving the linear system

$$M^{-1}\mathbf{Y} = (I - Q)\mathbf{Y} = \mathbf{1}$$

we can compute any $Y_j$ with the recurrence

$$Y_j = (1 - P_{jj})^{-1}\left(1 + \sum_{i=j+1}^{N-1} P_{ji}Y_i\right) \tag{9}$$

Now that we have $Y_{\lfloor Np_a \rfloor}(\rho, p_a)$ we need to compute $\tau'(\rho, p_a)$. This is a rather difficult calculation and we have approximated $\tau'(\rho, p_a)$ by the inactivation time $\tau_{ind}$ of a solitary individual

$$S_i(t+1) = g^t S_i(0)$$

with $S_i(0) = S_a$, the spontaneous activity level. Making $g^{\tau_{ind}} S_a = \theta_{act}$ we get

$$\tau_{ind} = \frac{\log(\theta_{act}/S_a)}{log(g)}$$

which is $\tau_{ind} = 15$ for the set of parameters given at the end of section 3. Of course this is just a first approximation, since if we compute numerically the mean number of steps $< \tau >= \tau'(\rho, p_a)$ in the SFNN (see Fig.6) we see that it is density dependent.

Once we have $Y_{\lfloor Np_a \rfloor}(\rho, p_a)$ and $\tau'(\rho, p_a) \approx \tau_{ind}$ we can find a density $\rho_c$ such that $Y_{\lfloor Np_a \rfloor} \approx \tau_{ind}$. This analytically computed $\rho_c^{an}$ is compared with numerically determined $\rho_c^{nu}$ in Fig.7. $\rho_c^{an}$ and $\rho_c^{nu}$ have both a linear dependence on $\log(p_a)$ but our analytical approximation gives a critical density above the one computed numerically with the order parameter $\Gamma(\rho, p_a)$.

# 5 Discussion

In this paper we have explored the emergence of oscillatory behaviour in a mean-field model of FNNs. FNN have been successfully used as models of collective behavior, from oscillations [11,14] to the problem of universal computation [20]. The introduction of

mobility by simple units enables us to go beyond the classical models based on excitable systems [21] though also makes difficult to reach analytic results. Here, following a mean field approach to the original FNN with sigmoidal response, we have derived a simple analytic result that provides a good understanding for the origin of oscillatory behavior and the existence of noise-induced transitions.

Some simplifications on the FNN model have allowed us to define the SFNN, with identical phenomenology but with the possibility of performing analytical work. Working on the $\rho > \rho_c$ phase we have assumed no spontaneous activation and we have analyzed separately the inactivation process and the activation process, that is, the two parts that compose one oscillation in the $\rho_i^+$ curve. We have derived a relation between $g$ and $V$ in order to assure inactivation and we have checked an hypothesis put forward in [16] in order to approximate $\rho_c$. This hypothesis and approximations of $Y_{\lfloor Np_a \rfloor}(\rho, p_a)$ and $\tau'(\rho, p_a) \approx \tau_{ind}$ provide a $\rho_c^{an}$ value that reproduces the linear dependence on $\log(p_a)$ but overestimates $\rho_c^{nu}$ (obtained from $\Gamma(\rho, p_a)$). The approximations we have done are obviously not exact, but the coarser one $\tau'(\rho, p_a) \approx \tau_{ind}$ is the responsible for the inaccuracy of $\rho_c^{an}$. Work in progress is focusing on a better analytical approximation of $\tau'(\rho, p_a)$.

The suggested mechanism of activity propagation (eqns (5), (6) and (7)) provides an explanation of a phenomenon observed in both FNN and SFNN [16]: Critical density dependence on lattice linear size $L$. As we can see in eqns. (5) and (6) activity propagation depends on $N$ and $L$ *separately*, in such a way that $\tau'(\rho, p_a) > Y_{\lfloor Np_a \rfloor}(\rho, p_a)$ for a certain $\rho$ (we have oscillations) and, for the same $\rho$ but greater $L$, $\tau'(\rho, p_a) < Y_{\lfloor Np_a \rfloor}(\rho, p_a)$ loosing oscillatory behaviour.

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# 6 References


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# Figures

[1] Temporal behavior of $\rho_t^+$, with parameters $L = 50$, $S_a = 0.1$, $g = 0.1$, $\theta_{act} = 10^{-16}$, $p_a = 0.01$ and (A) $\rho = 0.10$; (B) $\rho = 0.15$; (C) $\rho = 0.20$; (D) $\rho = 0.25$; (E) $\rho = 0.30$; (F) $\rho = 0.35$;

[2] Order Parameter $\Gamma(\rho, p_a)$ with parameters $L = 50$, $S_a = 0.1$, $g = 0.1$, $\theta_{act} = 10^{-16}$ and (A) $p_a = 0.01$, (B) $p_a = 0.001$.

[3] Temporal behavior of $\rho_t^+$ in SFNN, with parameters $L = 100$, $S_a = 0.1$, $g = 0.1$, $\theta_{act} = 10^{-16}$, $p_a = 0.01$ and $V = 4$.

[4] Order Parameter $\Gamma(\rho, p_a)$ in SFNN with parameters $L = 100$, $S_a = 0.1$, $g = 0.1$, $\theta_{act} = 10^{-16}$, $p_a = 0.01$ and $V = 4$. Each point has been computed with $10^4$ time steps after $2 \times 10^3$ transitories.

[5] Temporal behaviour of the number of new active individuals due to interactions and spontaneous activations. *Values in the spontaneous activation curve are multiplied by* 10. Parameters $L = 100$, $S_a = 0.1$, $g = 0.1$, $\theta_{act} = 10^{-16}$, $p_a = 0.01$, $V = 4$ and $\rho = 0.25$.

[6] The mean time an individual is active between two inactivations. We can see that $< \tau >$ is density dependent with a clear change in the shape of the curve around $\rho_c$ ($\rho_c \approx 0.212$ in this case). Parameters $L = 100$, $S_a = 0.1$, $g = 0.1$, $\theta_{act} = 10^{-16}$, $p_a = 0.01$ and $V = 4$. $< \tau >$ is computed from $10^3$ time steps after $10^3$ transitories.

[7] $\rho_c^{nu}$ (empty dots) and $\rho_c^{an}$ (filled dots) as a function of $\log(p_a)$ Parameters $L = 100$, $S_a = 0.1$, $g = 0.1$, $\theta_{act} = 10^{-16}$, and $V = 4$. $\rho_c^{nu}$ is computed from $\Gamma(\rho, p_a)$ with $10^4$ time steps after $2 \times 10^3$ transitories, averaged over 10 samples.

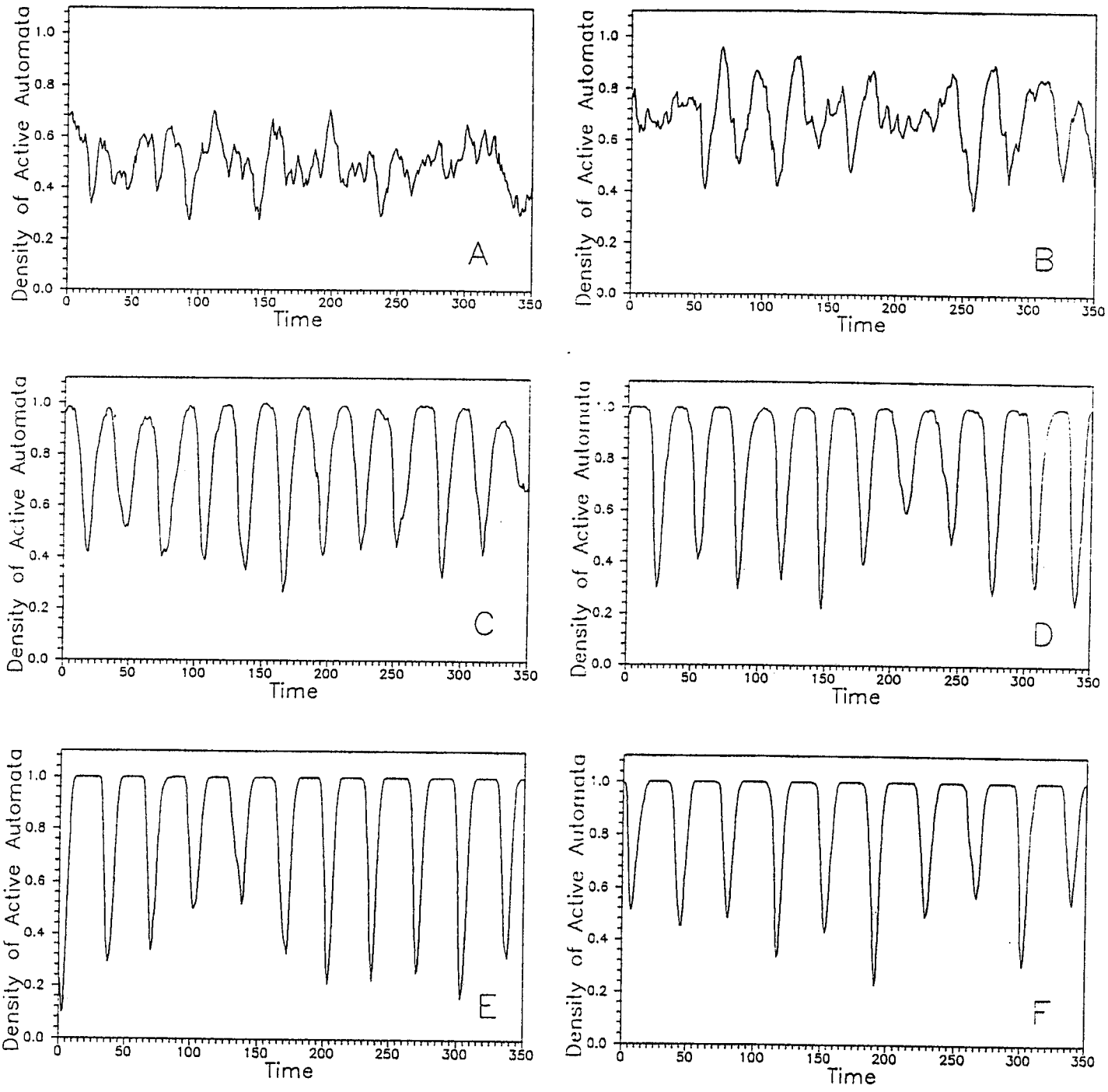
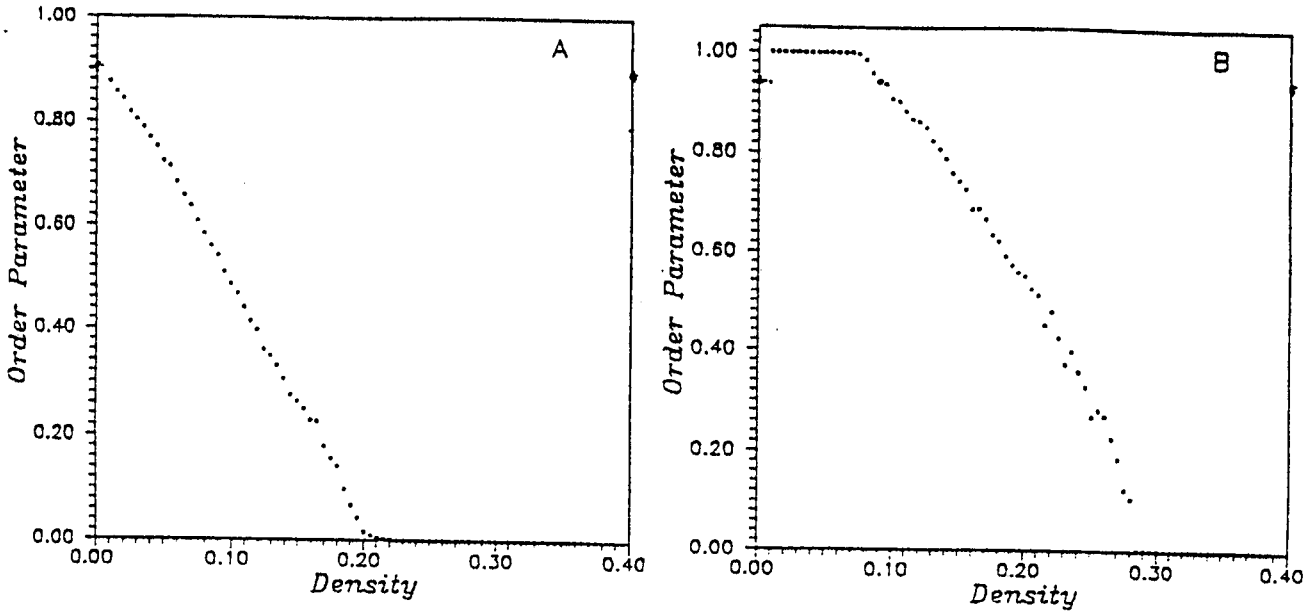



Figure 1


A
B
Order Parameter
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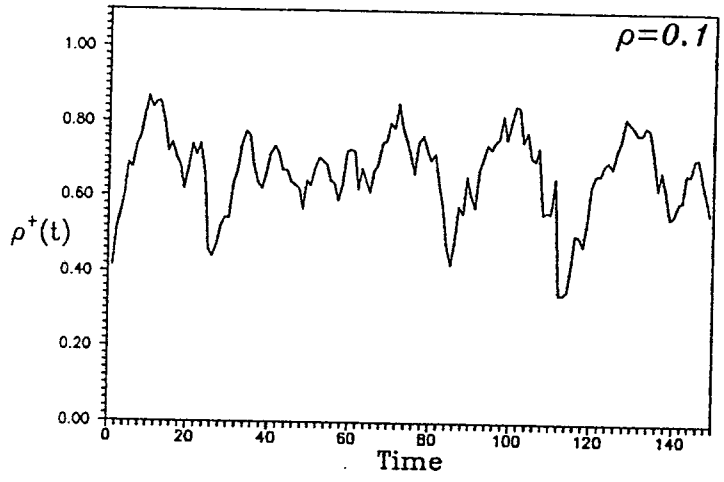
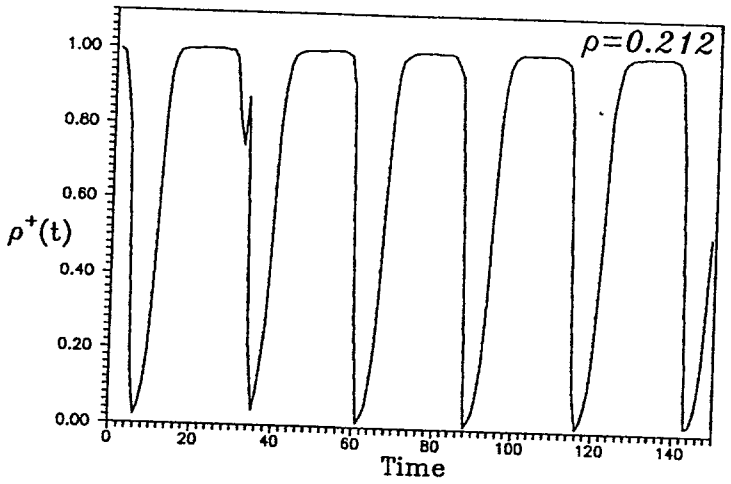
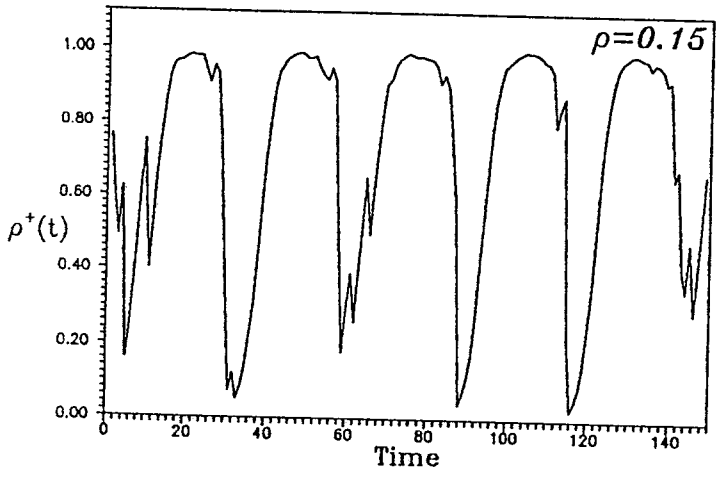
0.20
0.00
0.10
0.20
0.30
0.40

Figure 2



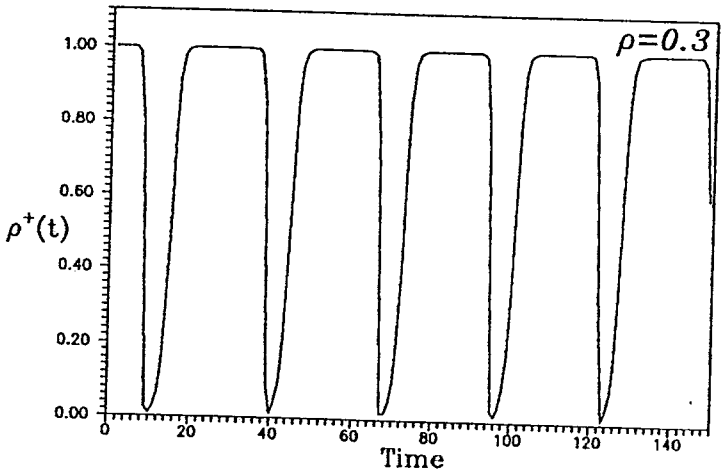




Figure 3

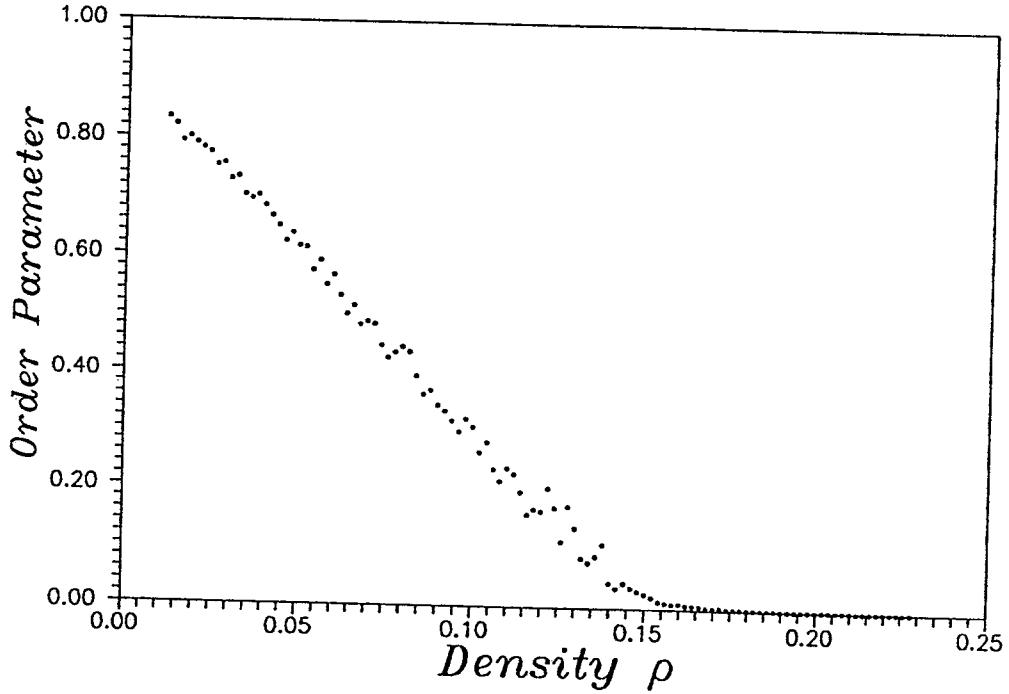
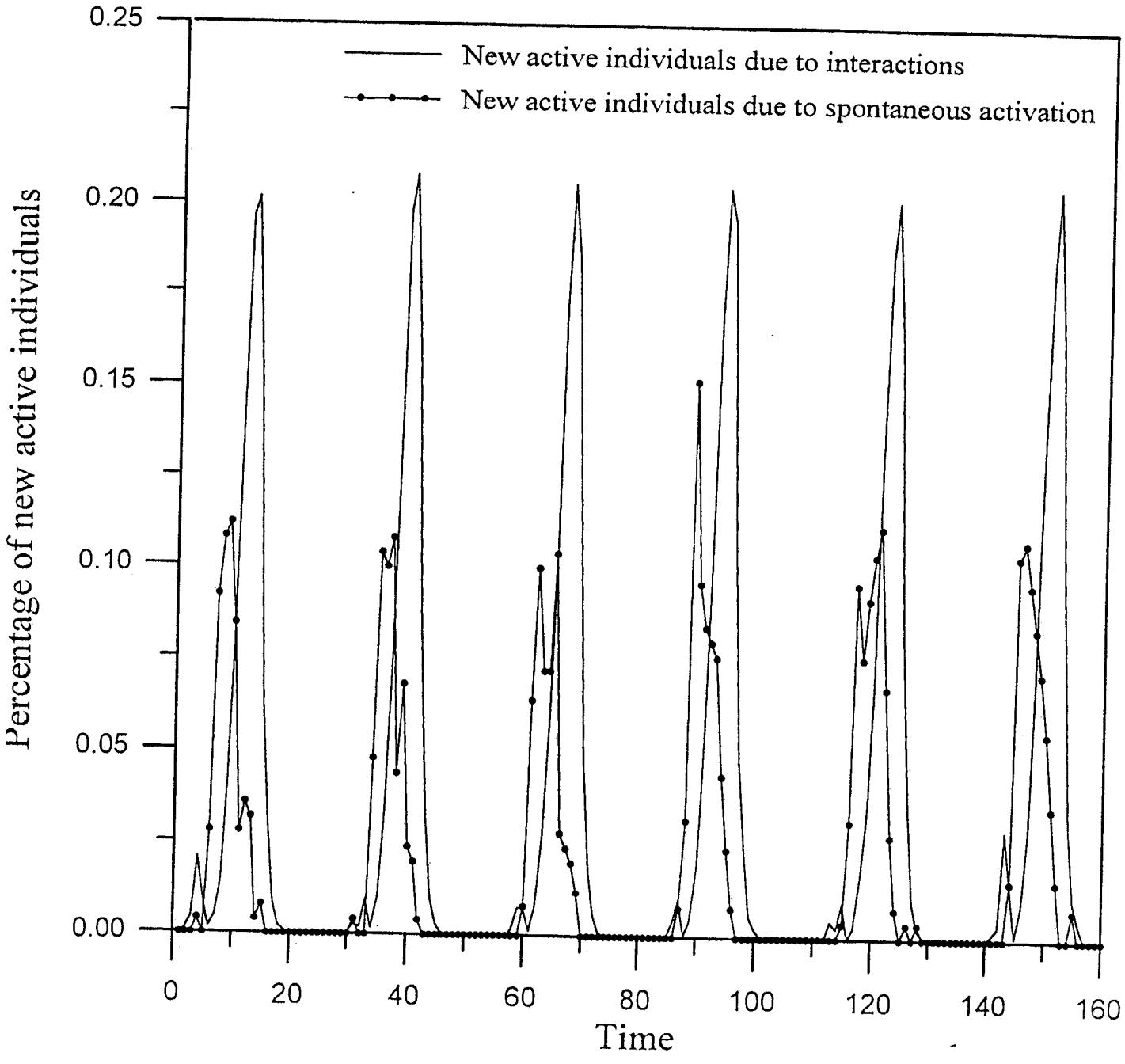

1.00
0.80
0.60
0.40
0.20
0.00
Order Parameter
0.00
0.05
0.10
0.15
0.20
0.25
Density ρ

Figure 4

Figure 5


0.25
0.20
0.15
0.10
0.05
0.00
Percentage of new active individuals
New active individuals due to interactions
New active individuals due to spontaneous activation
0
20
40
60
80
100
120
140
160
Time

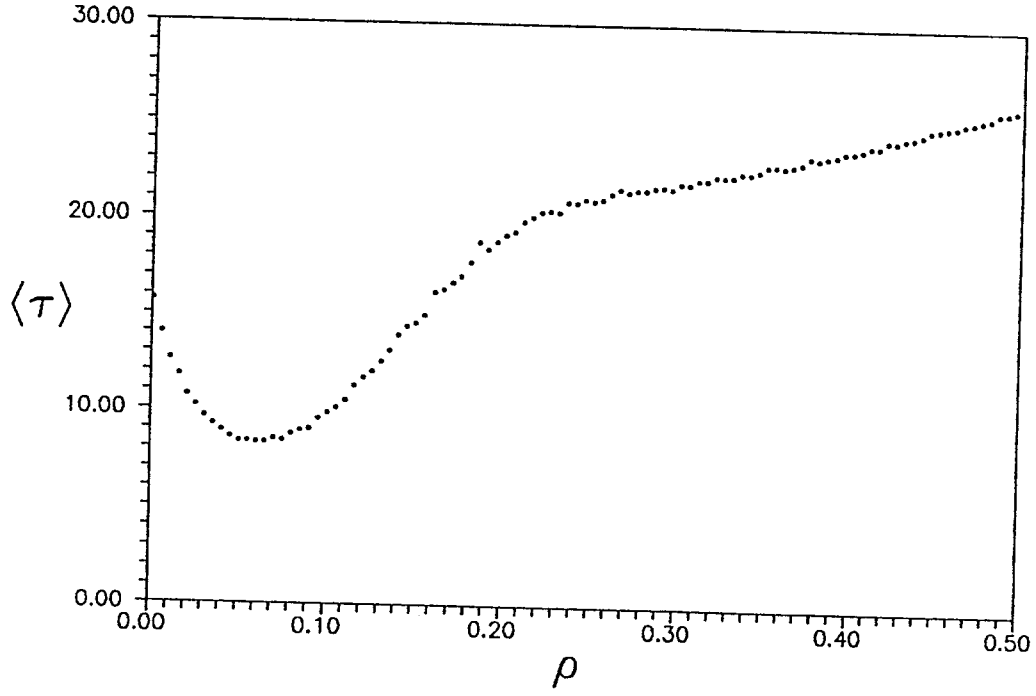



figure 6

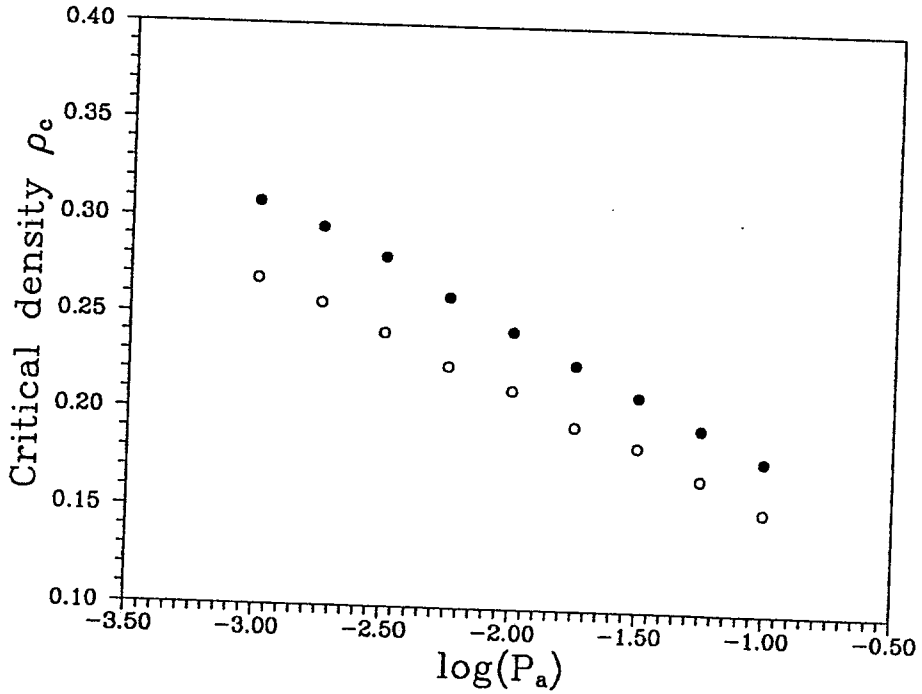

0.40
0.35
0.30
0.25
0.20
0.15
0.10
Critical density $\rho_c$
−3.50
−3.00
−2.50
−2.00
−1.50
−1.00
−0.50
log($P_a$)

figure 7

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