Ph.D Dissertation

SEA STATE DETERMINATION USING GNSS-R TECHNIQUES:
CONTRIBUTIONS TO THE PAU INSTRUMENT

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This work, conducted as part of the award “Passive Advanced Unit (PAU): A Hybrid L-band Radiometer, GNSS Reflectometer and IR-Radiometer for Passive Remote Sensing of the Ocean” made under the European Heads of Research Councils and European Science Foundation European Young Investigator (EURYI) Awards scheme in 2004, was supported in part by the Participating Organizations of EURYI, and also by Department of Universities of the Catalan Autonomous Government and the European Social Fund (Formació d'Investigadors FI grant, 2004-2008).
With the upcoming launch of the ESA’s Soil Moisture and Ocean Salinity (SMOS) mission, the retrieval of Sea Surface Salinity (SSS) from space will benefit both the oceanography and climatology communities. However, the impact of the sea roughness on the radiometric measurement has to be accurately modeled and accounted for first, so that to reduce the induced error on the retrieved SSS and yield meaningful values. In recent years the use of reflected Global Navigation Satellite System Signals (GNSS-R) has shown its potential to retrieve geophysical parameters, mainly altimetry and more recently sea state. The approach consisted of comparing the measured waveform (correlation at different delays) with a modeled one. One of the rationales that motivated the submission of the Passive Advanced Unit (PAU) project to the EURYI foundation was to study the direct relationship between the radiometric brightness temperature and some to-be-defined GNSS-R observables by obtaining co-located measurements with an L-band radiometer and a GPS reflectometer, and perform the actual SSS retrieval with the aid of an infrared radiometer. The PAU project has been developed by the Passive Remote Sensing Group of the Remote Sensing Lab, at the Department of Signal Theory and Communications of the Universitat Politènica de Catalunya.

The present PhD dissertation describes the work undertaken between 2004 and 2008 in both theoretical and hardware issues within the field of GNSS-R reflectometry. More specifically, in chapter 1 a brief introduction to SSS retrieval is given along with the description of the PAU project. Chapter 2 is devoted to the basics of GNSS reflectometry, whereas chapter 3 is focused into the simulation of the chosen GNSS-R observable, the whole Delay-Doppler Map (DDM). A new approach to DDM simulation is introduced along with the review of the classical implementation of the Zavorotny-Voronovich expressions for the reflected GNSS signal. After that, the parameterization of the DDMs is studied in chapter 4, where some of the derived parameters are to be linked to the actual sea state. Chapter 5 offers a detailed description of the design, implementation and validation of a real-time FPGA-based DDM reflectometer, whereas chapter 6 describes the ALBATROSS 08 measurement campaign, where the developed PAU-GNSS/R was tested. The conclusions and the future work are listed in chapter 7.
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<td>Two Scale Composite Model</td>
</tr>
<tr>
<td>ADC</td>
<td>Analog to Digital Converter</td>
</tr>
<tr>
<td>ALBATROSS</td>
<td>Advanced L-Band emissivity and Reflectivity Observations of the Sea Surface</td>
</tr>
<tr>
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<td>Commercial Service</td>
</tr>
<tr>
<td>DDM</td>
<td>Delay Doppler Map</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>DMC</td>
<td>Disaster Monitoring Constellation</td>
</tr>
<tr>
<td>DMSS</td>
<td>Directional Mean Squared Slope</td>
</tr>
<tr>
<td>ECMWF</td>
<td>European Center for Medium range Weather Forecasting</td>
</tr>
<tr>
<td>EM</td>
<td>ElectroMagnetic</td>
</tr>
<tr>
<td>ESA</td>
<td>European Space Agency</td>
</tr>
<tr>
<td>ESTAR</td>
<td>Electronically Steered Thinned Array Radiometer</td>
</tr>
<tr>
<td>EURYI</td>
<td>European Young Investigator award</td>
</tr>
<tr>
<td>FDMA</td>
<td>Frequency Division Multiple Access</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>FOV</td>
<td>Field of View</td>
</tr>
<tr>
<td>FPGA</td>
<td>Field-Programmable Gate Array</td>
</tr>
<tr>
<td>GLONASS</td>
<td>GLObalnaya NAVigatsionnaya Sputnikovaya Sistema</td>
</tr>
<tr>
<td>GNSS</td>
<td>Global Navigation Satellite System</td>
</tr>
<tr>
<td>GO</td>
<td>Geometric Optics</td>
</tr>
<tr>
<td>GODAE</td>
<td>Global Ocean Data Assimilation Experiment</td>
</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>HP</td>
<td>High Precision</td>
</tr>
<tr>
<td>IF</td>
<td>Intermediate Frequency</td>
</tr>
<tr>
<td>IR</td>
<td>InfraRed</td>
</tr>
<tr>
<td>IRNSS</td>
<td>Indian Regional Navigational Satellite System</td>
</tr>
<tr>
<td>KM</td>
<td>Kirchhoff Model</td>
</tr>
<tr>
<td>LAURA</td>
<td>L-band AUtomatic RAdiometer</td>
</tr>
<tr>
<td>LEO</td>
<td>Low Earth Orbit</td>
</tr>
<tr>
<td>LFSR</td>
<td>Linear Feedback Shift Register</td>
</tr>
<tr>
<td>MAC</td>
<td>Multiply and ACCumulate</td>
</tr>
<tr>
<td>MEO</td>
<td>Medium Earth Orbit</td>
</tr>
<tr>
<td>MIRAS</td>
<td>Microwave Imaging Radiometer by Aperture Synthesis</td>
</tr>
<tr>
<td>MLS</td>
<td>Maximum Length Sequence</td>
</tr>
<tr>
<td>MSB</td>
<td>Most Significant Bit</td>
</tr>
<tr>
<td>MSE</td>
<td>Mean Squared Error</td>
</tr>
<tr>
<td>MSS</td>
<td>Mean Squared Slope</td>
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<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
</tr>
<tr>
<td>NIR</td>
<td>Noise Injection Radiometer</td>
</tr>
<tr>
<td>OLED</td>
<td>Organic Light Emitting Diode</td>
</tr>
<tr>
<td>PAU</td>
<td>Passive Advance Unit for ocean monitoring</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PLD</td>
<td>Programmable Logic Device</td>
</tr>
<tr>
<td>PRN</td>
<td>Pseudo-Random Noise</td>
</tr>
<tr>
<td>PRS</td>
<td>Public Regulated Service</td>
</tr>
<tr>
<td>PSU</td>
<td>Practical Salinity Unit</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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<td>--------------</td>
<td>------------------------------------</td>
</tr>
<tr>
<td>RHCP</td>
<td>Right Hand Circularly Polarized</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Squared</td>
</tr>
<tr>
<td>ROM</td>
<td>Read Only Memory</td>
</tr>
<tr>
<td>SDRAM</td>
<td>Synchronous Dynamic Random Access Memory</td>
</tr>
<tr>
<td>SMOS</td>
<td>Soil Moisture and Ocean Salinity</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>SoL</td>
<td>Safety of Life</td>
</tr>
<tr>
<td>SP</td>
<td>Standard Precision</td>
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<tr>
<td>SPM</td>
<td>Small Perturbation Method</td>
</tr>
<tr>
<td>SSS</td>
<td>Sea Surface Salinity</td>
</tr>
<tr>
<td>SST</td>
<td>Sea Surface Temperature</td>
</tr>
<tr>
<td>SWH</td>
<td>Significant Wave Height</td>
</tr>
<tr>
<td>TOW</td>
<td>Time of Week</td>
</tr>
<tr>
<td>TPR</td>
<td>Total Power Radiometer</td>
</tr>
<tr>
<td>TSIP</td>
<td>Trimble Standard Interface Protocol</td>
</tr>
<tr>
<td>UART</td>
<td>Universal Asynchronous Receiver-Transmitter</td>
</tr>
<tr>
<td>ULPGC</td>
<td>Universidad de Las Palmas de Gran Canaria</td>
</tr>
<tr>
<td>UPC</td>
<td>Universitat Politècnica de Catalunya</td>
</tr>
<tr>
<td>USB</td>
<td>Universal Serial Bus</td>
</tr>
<tr>
<td>UV</td>
<td>Ultra Violet</td>
</tr>
<tr>
<td>VHDL</td>
<td>Very High Speed Integrated Circuit Hardware Description Language</td>
</tr>
<tr>
<td>VHSIC</td>
<td>Very High Speed Integrated Circuit</td>
</tr>
<tr>
<td>WAF</td>
<td>Woodward Ambiguity Function</td>
</tr>
<tr>
<td>WS</td>
<td>Wind Speed</td>
</tr>
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</table>
1. INTRODUCTION

1.1 IMPORTANCE OF THE SEA SURFACE SALINITY RETRIEVAL

Water plays a key role in all the geological and biological processes that take place in our planet. Cycling endlessly between oceans, atmosphere and land, it triggers and supports life, shapes the Earth and drives the weather and the climate (Fig. 1.1). Recalling that oceans account for more than 96 % of water on Earth, it is important to study the mechanisms that govern the ocean-to-atmosphere interface.

![Fig. 1.1 The water cycle on Earth [1].](image)

The sea surface salinity (SSS) is an oceanographic parameter that depends on the balance between precipitation and fresh water river discharge, ice melting, atmospheric evaporation, and mixing and circulation of the ocean surface water with deep water below. It is usually expressed using the practical salinity unit (PSU) of the practical Salinity Scale of 1978 (PSS-78).
1. INTRODUCTION

In the open ocean the SSS ranges between 32 psu and 38 psu, with an average value of 35 psu. It is maximum in sub-tropical latitudes, where evaporation is more important than precipitation. Conversely, the salinity drops below the average around the Equator, where there is more precipitation, and in polar regions, due to ice melting and snowfall (Fig. 1.2). Salinity and temperature are the two variables that control the density of the ocean water, which increases with increasing salinity and decreasing temperature. Density itself is a very important oceanographic parameter, since ocean currents are generated by horizontal differences in density, and also its vertical profile determines the effect that surface winds, heating, and cooling have on subsurface waters.

Salinity, through density, also determines the depth of convection at high latitudes. During the formation of sea ice, composed mainly of fresh water, dense cold salty water masses remain in the surface. At some point the water column loses its balance and denser water sinks. This vertical circulation is one of the engines of the global oceanic circulation known as the thermohaline circulation (Fig. 1.3). This sort of oceanic conveyor belt is a key component of the Earth’s heat engine, and therefore strongly influences the weather and the climate. Therefore, SSS is directly linked to the climatic cycle.

Moreover, the salt water content strongly influences the ecosystems of fish and other marine animals. Consequently, it is a significant factor to manage and forecast the yearly yield of fisheries. From one side salinity is linked to the sea water pH (between 8 and 8.3), which is an important biological constraint. From the other side, the annual variations of SSS affect the
depth and stability of the surface mixed layer, which generally has a low content of nutrients. High SSS results in high water density, which makes the mixed layer more stable. This results in a reduction of the nutrient flux from deeper waters towards the surface, thus affecting the productivity.

Fig. 1.3 Thermohaline circulation acts as a global conveyor belt that redistributes heat throughout the whole planet [3].

Salinity also determines the behavior of the ocean-air interface where gas and heat exchange take place. The increased precipitation of tropical latitudes can locally create pockets of fresh water where the upper layer is more stable, thus reducing the gas transfer. SSS also influences the vapor pressure of sea water, thus controlling the evaporation rates.

Nowadays Sea Surface Temperature (SST) along with other oceanographic parameters such as Wind Speed (WS) or sea surface topography are monitored on a regular basis from spaceborne sensors. However, SSS retrieval from space has not been possible so far [1]. Therefore, while ocean circulation models already incorporate satellite SST, WS and altimetry, they lack of accurate SSS data. To overcome this limitation usually temperature-salinity correlations are used, based on the density conservation principle over a certain water volume [4]. However, the validity of this principle is seriously questioned at the surface, where heat and gas exchange between sea and air takes place [5]. This results in modeling errors that hinder the modeling of surface currents. The severity of this lack of data is clearly understood considering that SSS has never measured for 42% of the ocean surface, and that has been measured less than four times over the past 125 years for 88% of the ocean surface [6] (Fig. 1.4).
1.2 SSS RETRIEVAL BY MEANS OF RADIOMETRY

Microwave radiometry is acknowledged as the best technique to significantly increase the available SSS measurements using remote sensing techniques. The basics of such measurements are summarized in the following sections.

1.2.1 Thermal Emission

All matter at an absolute temperature above 0 K emits electromagnetic radiation throughout the whole electromagnetic spectrum. This emission is caused by the electron transitions from higher to lower energy bands. Since the transition probability depends upon the density and the kinetic energy of the particles, an increase of the absolute temperature results in an increase of the energy radiated by the object being considered.

\[
B_{\nu} = \frac{2 \cdot h \cdot f^3}{c^2} \left( \frac{1}{e^{\frac{hf}{kT}} - 1} \right) \left[ \frac{W}{m^2 \cdot sr \cdot Hz} \right],
\]

where \( h = 6.63 \cdot 10^{-34} \) J·s is Planck’s constant, \( c \) is the speed of light in m/s, \( k = 1.38 \cdot 10^{-23} \) J/K is Boltzmann’s constant, \( f \) is the radiation frequency in Hz, and \( T \) is the absolute physical
temperature in K. Planck’s law fits all the experimental data and agrees well with the Rayleigh-Jeans and Wien’s laws derived earlier for the lower and higher frequency parts of the spectrum. For instance, for short wavelengths Eqn. 1.1 turns into the Wein’s law, found previously to Planck’s works:

$$B_f \equiv \frac{2 \cdot h \cdot f^3}{c^2} \cdot e^{-\frac{hf}{kT}} \cdot \frac{W}{m^2 \cdot sr \cdot Hz}.$$  \hspace{1cm} (1.2)

On the other hand, at microwave frequencies (roughly from 1 to 30 GHz, with wavelengths between 20 and 1 cm) it is possible to approximate Eqn. 1.1 with a 1\textsuperscript{st} degree Taylor polynomial, since $hf/KT_o \ll 1$, thus obtaining the Rayleigh-Jeans formula [8]:

$$B_f \approx \frac{2 \cdot f^2 \cdot kT}{c^2} = \frac{2kT}{\lambda^2},$$  \hspace{1cm} (1.3)

where $\lambda = c/f$ is the electromagnetic wavelength. In it the spectral brightness temperature and the physical temperature have a lineal dependence. The disagreement between Planck’s law and the Rayleigh-Jeans approximation is not significant for most of the microwave spectrum (error smaller than 0.08 % K for $f = 10$ GHz at 300 K), and therefore the latter will be subsequently used (Fig. 1.6).
Fig. 1.6 Planck’s law represented along with its low-frequency (Rayleigh-Jeans law) and high-frequency (Wien’s law) approximations at 300 K [7].

Assuming an ideal antenna and receiver surrounded by a blackbody at a constant physical temperature $T$, and considering the bandwidth of the receiver narrow enough so that the spectral brightness density can be considered flat throughout the whole frequency range, it can be demonstrated that the power collected by the antenna is given by [7]:

$$P_{bb} = kTB.$$  \hfill (1.4)

An analog expression was proposed by Nyquist for the noise power $P_n$ available at the terminals of a resistance at a physical temperature $T$:

$$P_n = kTB.$$  \hfill (1.5)

Thus, the power delivered by a blackbody and collected by the antenna is the same as the one delivered by a resistance, provided that they are at the same physical temperature (Fig. 1.7).

Fig. 1.7 Equivalence between the power available the terminals of an antenna placed inside of a blackbody at a physical temperature $T$ (left) and the power available at the terminals of a resistor at the same physical temperature (right) [7].
However, most natural objects are not blackbodies, since they do not absorb all the radiation that reaches them but reflect a part of it. Therefore, the absorbed radiation that is re-emitted afterwards is smaller than that of a blackbody: they are called gray bodies. The power emitted by a gray body is also proportional to the physical temperature, and the proportionality constant is called emissivity:

\[ T_b = e \cdot T \]  

which is dependent on the dielectric constant and the roughness of the object, and also on the observation angles (θ, φ) and polarization of the electromagnetic wave. Its value ranges from zero (perfect reflectors such as ideal metals) to one (blackbodies).

The detection of variations in the brightness temperature of objects is the core of passive remote sensing techniques. The radiation emitted from the Earth’s surface can be collected to infer geophysical parameters of the surface under survey. Whereas the so-called active instruments rely on a transmitter to send energy towards the surface and ‘illuminate’ it, passive sensors rely on the natural emission of the Earth. At the upper end of the spectrum used for remote sensing, radiometers and imaging cameras measure the emissivity in the infrared (IR), visible, and ultraviolet (UV) bands. These sensors, such as Landsat or SPOT, rely critically on favorable atmospheric conditions. Clouds, for instance, can interfere or even block the radiation of interest. Conversely, at the low-end of the microwave range the impact of the atmosphere is nearly negligible (Fig. 1.8).

Fig. 1.8 Atmospheric attenuation as a function of frequency. Red line marks the protected passive observation band at 1.4 GHz [9].

1.2.2 \( T_b \) Measured by a Radiometer

The radiation reaching an antenna pointing towards the Earth’s surface from the space is composed of the power emitted by the surface (\( T_b = e \cdot T \)), the power emitted by the atmosphere in the upwards direction (\( T_{up} \)), and the power from outside the Earth which is
reflected on the ocean surface \( (T_{sc}) \). Assuming a scattering free atmosphere, the so-called apparent temperature \( T_{ap} \) can be expressed as (Fig. 1.9):

\[
T_{ap}(\theta, \varphi) = T_{ap} + \frac{1}{L_a} (T_b + T_{sc}), \tag{1.7}
\]

where \( L_a \) is the atmospheric attenuation. From Eqn. 1.7 it can be seen for high \( L_a \) values the desired term \( T_b \) is heavily attenuated, and the apparent temperature converges to the upwelling one. Again, at L-band such attenuation is low, thus enabling Earth’s surface remote sensing.

![Fig. 1.9 Components of the apparent temperature \( T_{ap} \).](image)

The term \( T_{sc} \) is composed by both atmospheric \( (T_a) \) and extraterrestrial components \( (T_{ext}) \). This \( T_{ext} \) contains contributions from the isotropic cosmic background, galactic radiation, and the sun:

\[
T_{ext} = T_{cos} + T_{gal} + T_{sun}. \tag{1.8}
\]

Additionally, the polarization of the radiation changes as it passes through the ionosphere. This effect is known as Faraday rotation. Therefore, in order to use the measured \( T_{ap} \) to perform SSS retrieval, first it is necessary to account for all the terms in Eqn. 1.7 not related with the sea surface itself (Fig. 1.10).
The power detected at the output of an ideal (lossless) antenna is known as antenna temperature ($T_a$). It is obtained by integrating $T_{ap}(\theta, \phi)$:

$$T_a = \frac{1}{\Omega_p} \iint_{4\pi} T_{ap}(\theta, \phi) |F_n(\theta, \phi)|^2 \cdot d\Omega,$$

(1.9)

where $F_n(\theta, \phi)$ is the normalized antenna radiation voltage pattern, and $\Omega_p$ is the antenna pattern solid angle:

$$\Omega_p = \iint_{4\pi} |F_n(\theta, \phi)|^2 \cdot d\Omega,$$

(1.9)

1.2.3 Sea Surface Brightness Temperature

The emissivity of seawater at microwave frequencies can be calculated, for example, using the Klein and Swift model [11] for given Sea Surface Temperature (SST) and SSS values. As stated in [12], the sensitivity of the emissivity to SSS increases as the frequency decreases from 1.5 to 0.4 GHz, for typical oceanic SST values. On the other hand, the sensitivity to errors in SST increases over that range. Thus, a frequency between 0.8 and 1.5 GHz is optimal. The emissivity for vertically polarized radiation is significantly more sensitive to salinity variations than for horizontal polarization (as it will be noted in Fig. 1.13). The protected frequency band at 1.413 GHz (1.400-1.427 GHz) is the lowest available for passive observations, and satisfies the constraints to perform successful SSS retrievals with the highest sensitivity from space [13], with almost negligible atmospheric effects. Recalling Eqn. 1.6, the sea brightness temperature is related to the emissivity as:

$$T_b(\theta, p) = e(\theta, p) \cdot SST.$$

(1.10)
The sea surface can be considered a flat, semi-infinite body. Then, the emissivity can be expressed as the complementary of the Fresnel power reflection coefficient:

$$e(\theta,p) = 1 - R(\theta,p).$$  \hspace{1cm} (1.11)

where $R(\theta,p)$ is the Fresnel power reflection coefficient at $p$ polarization ($h$: horizontal or perpendicular to the plane of incidence, or $v$: vertical or parallel to the plane of incidence), that depends on the incidence angle $\theta$ and on the complex dielectric constant (or dielectric permittivity) of the sea water $\varepsilon_r$:

$$R_h = \frac{\cos \theta - \sqrt{\varepsilon_r - \sin^2 \theta}}{\cos \theta + \sqrt{\varepsilon_r - \sin^2 \theta}}^2,$$

$$R_v = \frac{\varepsilon_r \cos \theta - \sqrt{\varepsilon_r - \sin^2 \theta}}{\varepsilon_r \cos \theta + \sqrt{\varepsilon_r - \sin^2 \theta}}^2.$$  \hspace{1cm} (1.12)

However, as it will be seen further on this dissertation, it is useful to obtain the expressions associated to circular polarizations. According to [14], the polarization matrix can be expressed as:

$$\begin{bmatrix} R_{RR} & R_{LR} \\ R_{RL} & R_{LL} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} R_h + R_v & R_h - R_v \\ R_h - R_v & R_h + R_v \end{bmatrix},$$  \hspace{1cm} (1.13)

where the subindexes $R$ and $L$ stand for the right hand and left hand circular polarization states. To summarize, the permittivity of sea water links the SSS with the measured $T_b$, and also depends on SST [11]. Therefore, the SSS retrieval by means of radiometry is feasible if all the other factors affecting $T_b$ are accounted for.

![Fig. 1.11 Sea brightness temperature dependence on SSS and SST at L-band for nadir observations [15].](image-url)
The dynamic range of the \( T_b \) variations due to SSS changes is pretty small, even though L-band offers the best retrieval conditions (Fig. 1.11). Therefore, SSS retrieval by means of radiometry calls for high sensitivity and accuracy. Fig. 1.12 shows the brightness temperature dependence on the SST for several SSS values, considering a flat surface and nadir incidence angle. It is remarkable that the sensitivity to SSS decreases as the SST also decreases, so that the salinity retrieval in cold water areas becomes even more demanding. The sensitivity of the brightness temperature \( T_b \) at L-band to SSS has been widely studied. For instance, at nadir the sensitivity is 0.5K/psu for a sea surface temperature of 20 °C decreasing down to 0.25K/psu for an SST of 0 °C. On average, this sensitivity varies between 0.2 and 0.8 K/psu depending on the SST, the incidence angle and polarizations [10].

![Graph showing brightness temperature dependence on SSS and SST](image)

Fig. 1.12 Dependence of the brightness temperature with SSS and SST for nadir incidence[16].

Up to now the sea surface has been considered flat whenever deriving the \( T_b \) dependence on SSS and SST. However, changes in the sea surface roughness can result in a variation of the brightness temperature (\( \Delta T_b \)) of several K for very rough sea surface [12], having a major contribution to the overall \( T_b \) than the SSS itself. In fact, the sea roughness is the major contributor to the deviations of the brightness temperature with respect to the flat sea model [17]. Taking these considerations into account the polarization-dependent brightness temperature can be expressed as:

\[
T_{b,p}(\theta) = T_{b,p,\text{flat}\,\text{sea}}(\theta,f,\text{SSS},\text{SST}) + \Delta T_{b,p}(\theta,\bar{\rho})
\]

(1.14)

where \( \bar{\rho} \) is a generic vector of parameters that defines the sea state. The sea state is usually parameterized in terms of the 10-meter height wind speed \( (U_{10}) \), as the most widely available data source, or the significant wave height \( \text{(SWH)} \) [18], or both [19], but none of them have been found fully satisfactory at L-band. One of the main goals of the present PhD dissertation is to introduce an efficient yet simple approach to account for the term \( \Delta T_b \).
1.2.4 Microwave Radiometers

A microwave radiometer is an instrument designed to measure the thermally-induced noise emission. It is typically composed of an antenna to collect the radiation, a front-end for detecting and amplifying the received signal within a given frequency band, and a control unit to process and store the radiometric data [20] and [21]. The total noise is made up of two parts: the signal collected by the antenna and the noise generated by the receiver, which are comparable (Gaussian random variables). Both contributions have the same statistical properties.

1.2.4.1 Real Aperture Radiometers

These radiometers employ antennas with a size proportional to the radiation wavelength. Thus, they have large antennas, but the electronics is easier to implement than in their synthetic aperture counterparts. The three basic real aperture radiometers are the Total Power Radiometer (TPR), the Dicke Radiometer and the Noise Injection Radiometer (NIR), following an increasing complexity and stability order.
1.2.4.1.1 Total Power Radiometer

This radiometer is in essence a superheterodyne receiver with a given bandwidth $B$ and gain $G$, directly fed by an antenna (Fig. 1.14). It is composed of a pre-detection stage and a power detection stage.

![Block Diagram of a Total Power Radiometer](image)

The predetection section is composed of a radiofrequency amplifier, in charge of filtering the input signal by amplifying the receiver’s frequency band $B$, a mixer to down convert from RF to IF the selected band, and a IF amplifier. On the other hand, the power detection is usually accomplished by means of a quadratic diode and a low-pass filter that performs the power integration.

The radiometric sensitivity is defined as the minimum input temperature variation that the radiometer is able to detect. For a TPR it is given by:

$$\Delta T'_n \equiv \frac{T'_A + T'_{\text{rec}}}{\sqrt{B \cdot \tau}} - \frac{T_{\text{sys}}}{\sqrt{B \cdot \tau}},$$

where $T'_A$ is the antenna temperature at the output of a real aperture antenna ($T'_A = T'_A \cdot \eta_0 + T_{\text{ph}} (1-\eta_0)$, where $\eta_0$ is the antenna ohmic efficiency and $T_{\text{ph}}$ its physical temperature), $T'_{\text{rec}}$ is receiver noise equivalent temperature, $T_{\text{sys}}$ is the noise system temperature, and $\tau$ is the integration time. One major drawback of a TPR is related to the gain fluctuations of the receiver chain. The long term ones (thermal drifts, aging...) are compensated by performing periodic calibrations. However, the short-term drifts (~Hz) limit the radiometric sensitivity, since they appear as actual variations of $T'_A$:
\[ \Delta T' \equiv T_{\text{sys}} \left( \frac{\Delta G}{G} \right), \quad (1.16) \]

where \( \Delta G \) is the RMS gain variation. Therefore, considering that the noise and the gain fluctuations are independent from each other, the resulting radiometric sensitivity is:

\[ \Delta T = \sqrt{\left( \Delta T_n \right)^2 + \left( \Delta T_G \right)^2} = T_{\text{sys}} \sqrt{\frac{1}{B\tau} + \left( \frac{\Delta G}{G} \right)^2}. \quad (1.17) \]

From Eqn. 1.17 it is clear that the TPR sensitivity is bound to be strongly influenced by the gain fluctuations that could occur during the radiometer integration time. To overcome this limitation the Dicke radiometer was envisioned.

1.2.4.1.2 Dicke Radiometer

The Dicke radiometer is an evolution of the TPR to minimize the impact of the gain fluctuations on the radiometric sensitivity (Fig. 1.15).

The main additions with respect to the TPR are:

- An input switch that multiplexes between the antenna and a matched noise reference load,
- A synchronous demodulator at the output of the quadratic detector composed of one switch and two amplifiers with unit gain and opposite sign, and

- A square wave generator operating at the sampling frequency of the system to synchronize the operation of the two switches.

In order to avoid the gain impact on the radiometric sensitivity is necessary for the sampling frequency to be higher than the maximum frequency of the gain fluctuations. Now, the sensitivity can be expressed as:

\[
\Delta T = \sqrt{\frac{2 \left( T_A' + T_{REC}' \right)^2}{B\tau} + \frac{2 \left( T_{REF}' + T_{REC}' \right)^2}{B\tau} + \left( \frac{\Delta G}{G} \right)^2 \left( T_A' - T_{REF}' \right)^2}, \quad (1.18)
\]

where \( T_{REF} \) is the noise temperature of the reference load. In the case that the antenna temperature is equal to that of the reference load, the radiometer is “balanced” and Eqn. 1.18 becomes:

\[
\Delta T = \frac{2 \left( T_A' + T_{REC}' \right)}{\sqrt{B\tau}} = 2 \cdot \Delta T_{IDEAL}, \quad (1.19)
\]

where \( \Delta T_{IDEAL} \) is the radiometric sensitivity of a TPR without gain fluctuations. It is clearly seen that the gain fluctuations have no impact at all, but the sensitivity has degraded by a factor of two, since the antenna observation time has halved. However, if the condition of \( T_A = T_{REF} \) is not fulfilled these fluctuations are not totally compensated.

1.2.4.1.3 Noise Injection Radiometer

It is an evolution of a Dicke radiometer in which a certain noise power \( T_I \) is added to the input so that \( T_A + T_I = T_{REF} \). In doing so, the radiometer output is always zero, so that both the gain fluctuations and the power detector linearity errors are compensated (Fig. 1.16). There are a number of ways to achieve it: by injecting noise pulses of constant amplitude and variable duty cycle, by injecting a variable noise power during the whole half period the switch is in position 1 (which can be achieved by inserting a variable attenuator or by varying the polarization current of the noise diode), etc.
The NIR radiometric sensitivity, considering that the antenna temperature $T_a$ is equal to the ambient temperature $T_0$ (~290 K), can be expressed as:

$$\Delta T = \frac{2(T_{\text{REF}} + T_{\text{REC}})}{\sqrt{B\tau}} = 2 \cdot \Delta T_{\text{IDEAL}}, \quad (1.20)$$

which coincides with the sensitivity of a balanced Dicke radiometer (Eqn. 1.19).

1.2.4.1.4 The AQUARIUS Mission

The goal of the NASA/CONAE Aquarius mission [12] is to provide global mapping of SSS and soil moisture to improve the studies and modeling related to the global water cycle. Instead of a synthetic aperture radiometer, Aquarius will employ a 3-beam (3 NIR) push-broom radiometer. It will also include a scatterometer to measure and correct for the sea surface roughness. This 1.26 GHz active sensor shall measure the power backscattered by the sea surface. This direct approach to correct for the roughness-induced bias in the retrieved SSS has the drawback of being power consuming, and the scatter plots $\sigma^\circ-\Delta T_b$ still seem to exhibit a too large scatter.

1.2.4.2 Synthetic Aperture Radiometers

The spatial resolution of real-aperture radiometers depends on the antenna size. Therefore, they usually have large antennas with narrow beams to scan the field of view. However, the retrieval of some geophysical parameters, such as the soil moisture or the ocean salinity, has demanding requirements on spatial resolution (10-20 Km). This implies antenna sizes on the
order of 20 m of diameter from a LEO satellite, which at present are not possible to implement.

The aperture synthesis approach allows for lighter structures composed of small antennas that effectively ‘synthesize’ a larger one, able to meet the required spatial resolution [22]. The drawback is the increase in hardware, data processing and calibration complexity.

The Electronically Steered Thinned Array Radiometer (ESTAR) was an L-band airborne radiometer that served as the first demonstrator for a spaceborne synthetic aperture radiometer to measure soil moisture [23]. The first spaceborne radiometer mission for soil moisture and ocean salinity retrieval is briefly described next.

1.3 THE SMOS MISSION

Despite the key role SSS plays in the water cycle, so far no global and systematic measurements are available. The Soil Moisture and Ocean Salinity mission will measure these parameters for the first time systematically from the space. As stated in [24], its goal is to provide global and frequent soil moisture and sea surface salinity maps. Both variables are crucial in weather, climate and extreme-event forecasting and they will be provided on spatial and temporal scales compatible with applications in the fields of climatology, meteorology and large scale hydrology [25]. It was selected by the European Space Agency (ESA) as an Earth Explorer Opportunity missions within the wider frame of the ESA Living Planet Program. The Earth Explorer missions are designed to address critical and specific issues that have been raised by the scientific community whilst demonstrating breakthrough technology in observing techniques. Earth Explorer missions are, in turn, split in two categories: the so-called “core” missions and the “opportunity” missions [26]. Unlike the core missions, the opportunity missions are smaller and more focused on a specific issue, and aims at demonstrating the feasibility of emerging technologies. SMOS was selected for feasibility studies in May 1999 by ESA’s Program Board for Earth Observation. Since then, a successful Phase A feasibility study (2000-2001) and a Phase B (2002) for further definition and critical breadboarding have been completed (the Phase B payload design was completed in October 2003). Approval for full implementation was given in November 2003. Phase C/D started in mid-2004. The Critical Design Review of the payload took place in November 2005[27]. As of June 2007 the payload is ready and integrated with the satellite, waiting for the launch. SMOS will carry the first-ever, polar-orbiting, space-borne, 2-D interferometric radiometer. SMOS has a Sun-synchronous polar dawn-dusk circular orbit. The orbit altitude is 763 km, the inclination is 98.4° with 06.00 hrs local solar time at ascending node, and the latitude coverage is at least ±80°. The launch of the SMOS spacecraft is planned on a Rockot launch vehicle from Plesetsk cosmodrome, in Russia, in July 2009. The minimum foreseen lifetime is 3 years, in order to cover at least two seasonal cycles. Temporal resolution is 3 days revisit time at Equator. Spatial resolution spans from 32 km at its best up to 100 km in the field of view (FOV) swath edge (Fig. 1.17).
Considering the spatial resolution constraints, the overall goal for SSS retrieval from SMOS data is to meet the Global Ocean Data Assimilation Experiment (GODAE) optimized requirement for open ocean SSS. The pilot experiment GODAE aimed at demonstrating the feasibility of real-time global ocean modeling and data assimilation systems, both in terms of their implementation and their utility [28]. Following recommendations of the Ocean Observing System Development Panel, the proposed GODAE accuracy requirement for satellite SSS is specified as 0.1 psu for a ten-day and 2°x2° resolution for global ocean circulation studies.

SMOS payload is MIRAS (Microwave Imaging Radiometer by Aperture Synthesis), an L-band, two-dimensional, synthetic aperture radiometer with multi-angular and dual/fully-polarimetric imaging capabilities, able to measure the brightness temperature of the Earth within a wide field of view and without any mechanical antenna sweeping. It has a Y-shaped deployable structure, consisting of 3 coplanar arms, 120° apart each other. The total arm length is about 4.5 m with an angular resolution of approximately 2°. The range of incidence angles is variable (spanning from 0° to almost 65°) within the FOV and depends on the distance between the pixel and the sub-satellite path. To achieve an even greater angular excursion and fully exploit its viewing capability the array will be tilted 32° with respect to nadir.

Fig. 1.17 SMOS observation geometry. The field of view has a hexagon-like shape of 1000 km approximately [1].
As seen in Table 1.1, the roughness estimation is performed either using an scatterometer (AQUARIUS) or by means of auxiliary data (SMOS). Another feasible approach is to use the reflection of opportunity signals over the sea surface to perform the roughness estimation and correction, as recently proposed in [29] or [30]. Particularly, the signals of the Global Navigation Satellite Systems such as GPS or GLONASS could be received and processed after being reflected over the sea to infer the roughness state. This GNSS-R approach has been also recently proposed for the future follow-on SMOS mission SMOSops (time schedule still to be defined). The payload in charge of performing such processing is known as a reflectometer. The present PhD dissertation aims to design, implement, and test a GNSS-R demonstrator to validate the GNSS-R approach to SSS roughness-induced bias correction.

**Table 1.1 Comparison between the AQUARIUS and the SMOS missions [31]**

<table>
<thead>
<tr>
<th></th>
<th>AQUARIUS</th>
<th>SMOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planned lifetime</td>
<td>2 years</td>
<td>3 years</td>
</tr>
<tr>
<td>Orbit</td>
<td>Sun-synchronous, ascending node at 0600 local time; 8 day revisit</td>
<td>Sun-synchronous at 755 km; ascending node at 0600 local time; 3 day revisit</td>
</tr>
<tr>
<td>Instruments</td>
<td>L-band radiometer; L-band scatterometer</td>
<td>L-band 2D interferometer</td>
</tr>
<tr>
<td>Radiometer Frequency</td>
<td>1.4 GHz</td>
<td>1.4 GHz</td>
</tr>
<tr>
<td>Radiometer Polarization</td>
<td>H-H and V-V</td>
<td>H-H and V-V (full-polarization mode optional)</td>
</tr>
<tr>
<td>Antenna Type</td>
<td>parabolic</td>
<td>Thinned array, Two-dimensional aperture synthesis</td>
</tr>
<tr>
<td>Incidence angle</td>
<td>23.3°, 33.7°, 41.7°</td>
<td>15 - 50°</td>
</tr>
<tr>
<td>Swath Width (km)</td>
<td>250</td>
<td>620 to 1050</td>
</tr>
<tr>
<td>Spatial Resolution (km)</td>
<td>70-90 km</td>
<td>35-50 km</td>
</tr>
<tr>
<td>Single observation error (psu)</td>
<td>0.43 at mid-latitudes</td>
<td>1.2 in warm seas</td>
</tr>
<tr>
<td>Roughness Estimation</td>
<td>Radar</td>
<td>Effective U_{10} and ECMWF aux. data</td>
</tr>
</tbody>
</table>
1.4 PAU CONCEPT

As discussed, the collocated measurements of sea brightness temperature and reflected GNSS signal can result in a significant improvement of the retrieved SSS. The Passive Advance Unit (PAU) project aims at demonstrating this sensor synergy. It was proposed in 2003 to the European Science Foundation (ESF) within the frame of the European Young Investigator (EURYI) Awards program, and was funded in 2004 [32]. Its scientific goals are to perform ocean monitoring by passive remote sensing, improve the knowledge of the relationship of the GNSS-R signal with the sea state, and improve the knowledge on the relationship between L-band brightness temperature and sea state. To accomplish these goals the PAU sensor consists of three instruments that operate in a synergetic way:

1) PAU-RAD: an L-band radiometer to measure the brightness temperature of the sea surface,

2) PAU-GNSS/R: a reflectometer to measure the sea state using reflected global positioning system (GPS) opportunity signals, and

3) PAU-IR: an off-the-shelf 8–14-μm thermal infrared (IR) radiometer to measure the sea surface temperature.

One technological goal of the PAU concept was to prove the feasibility of using the same hardware front-end for the radiometer and the reflectometer. To do so a new radiometer topology was envisioned (Fig. 1.18): The antenna output is connected to a Wilkinson power splitter, thus dividing the signal in two in-phase signals \( S_{A1} \) and \( S_{A2} = S_{A1} \). Additionally, the 100 \( \Omega \) resistance of the Wilkinson splitter adds two noise signals that are 180° apart \( S_{n1} \) and \( S_{n2} = -S_{n1} \). Thus, the incoming signal is not chopped, as required to track the GNSS signal. The PAU concept is being implemented in both a real-aperture (PAU-16, [33]) and synthetic aperture (PAU-SA, [34]) prototypes.
The design constraints for the radiometer beam were to have a 20° -3 dB beamwidth with main beam efficiency (MBE) better that 95 %. This beam should be steered up to ±20° from the boresight direction (45° from nadir). These constraints are fulfilled using a 4 x 4 array of elements spaced 0.63 wavelengths with a “triangular” weighting function [1 2 2 1].

As depicted in Fig. 1.19, the output signals of the receivers are digitized and enter both the radiometer (PAU-RAD) and reflectometer (PAU-GNSS/R) digital processing sections. The
1. INTRODUCTION

effective sampling frequency is of 5.745 MHz. The processed GNSS-R and radiometric observables are eventually sent to a terminal computer. The actual implementation of PAU-16 is seen in Fig. 1.20.

1.4.2 Synthetic Aperture PAU (PAU-SA)

The PAU-SA demonstrator was conceived to test several improvements over the current MIRAS (SMOS payload), namely digital I/Q down-conversion, digital filtering and power estimation using digital techniques, and improved calibration strategies. It is composed by a Y-shaped array of 8 antennas per arm plus one in the center, and an additional dummy antenna at the end of each arm. Additionally, the GNSS/R section employs the 4 center antennas plus 3 additional ones to create a steerable beam that point to the GPS specular reflection point (Fig. 1.21).

Fig. 1.20 Actual implementation of the PAU-16 demonstrator. In (a), the thermal control section is seen, along with the FPGA’s and ADC’s housing, and the receivers’ location. (b) View of the antenna array and the ground plane.
The front-ends used are simplified versions of those used in PAU-16, since they do not use the Wilkinson power splitter (Fig. 1.22). This happens because the GNSS-R section here uses dedicated receivers, and thus there is no problem in chopping the incoming signal. It also reduces the hardware complexity and the routing of the signals to the FPGA’s.

PAU-SA and PAU-16 are complex and heavy instruments, designed for accurate measurements, already in their latter implementation stages. To readily test several of the underlying concepts the PAU-One Receiver (PAU-OR) was implemented, which is a single-receiver version of PAU-16 (further explained in chapter 5). Additionally, the PAU-One Receiver Airborne (PAU-ORA) was conceived to fit into a remote control plane.

1.5 CONCLUSIONS

The retrieval of SSS from space will benefit several different areas of the oceanography and climatology studies. L-band radiometry offers a good potential to perform this retrieval,
provided that the roughness impact on the radiometric observable is accounted for. The use of GNSS-R signals is a potentially efficient approach to perform this roughness correction. To demonstrate its feasibility, the PAU project was envisioned. It considers the synergetic operation of a L-band radiometer and GPS reflectometer sharing the same front-end, along with and IR radiometer. One of the main contributions of the present thesis is the design, implementation and validation of the PAU-GNSS/R section.
2. USE OF GNSS-R FOR SEA ROUGHNESS DETERMINATION

Global Navigation Satellite Systems (GNSS) are satellite constellations that cover the entire Earth with navigation signals to provide time and position information to users located on or near the Earth’s surface. Such systems are used nowadays in a wide range of everyday situations, such as fleet management, search and rescue, wildlife tracking, vehicle guidance or leisure interactive maps, among many others. So far the American GPS is the only fully operational GNSS. The Russian GLONASS system is partially deployed, whereas the European GALILEO is scheduled to be operational in 2013. There are other planned GNSS systems, such as the Chinese COMPASS or the Indian IRNSS that are to be operative in the future. Altogether, more than 75 GNSS satellites will be available when all the currently planned systems are deployed. The retrieval of geophysical parameters using these GNSS signals as a source of opportunity (GNSS-R) can in principle be used to measure ocean surface roughness, altimetry, soil moisture, or ice properties. It is true that there are other systems that already perform those measurements, such as active scatterometers (roughness), radar altimeters, or microwave radiometers. For the particular application of sea state determination, the GNSS-R approach boasts a low mass and power constraints, since there is no transmitter and a small antenna can be used. Also, the bistatic scattering geometry ensures a strong signal return in the specular direction, in opposition to the weak return for monostatic off-nadir configurations. It is also noteworthy that GNSS-R is directly sensitive to ocean slopes at L-band, and inherently less sensitive to power calibration.

2.1 GPS

The American Global Positioning System (GPS) was designed to provide 3D positioning anytime anywhere on Earth. To fulfill that goal at least four satellites have to be observed simultaneously at a given place and moment. In order to ensure the service even when one satellite fails it is necessary to consider a minimum of five visible satellites. These considerations result in a constellation of at least 24 satellites distributed in six orbital planes spaced 60° through the Equator with an inclination of 55°. The at least four satellites in the same orbital plane are not equally spaced, but distributed so that the effects of a single satellite failure are minimized. Their orbital period is of 12 sidereal hours, which implies that the ground track repeats daily with a time shift of four minutes. The near circular orbits (eccentricity smaller than 0.02) have a medium height of 20163 Km above the Earth’s surface, and result in a mean satellite speed of 3.87 km/s approximately. The actual satellite visibility depends on the latitude, but there is always a minimum of 5 satellites in view, and for more than 80 % of the time this minimum number is of 7 [35].
2. USE OF GNSS-R FOR SEA ROUGHNESS DETERMINATION

2.1.1 PRN Codes

The GPS signal structure was designed to allow multiple transmitters using the same frequency band and to have a certain tolerance to multipath and jamming (a serious issue for military applications). It was also conceived to have a low power spectral density to avoid mutual interference with other microwave systems, and to allow estimating the ionospheric delay for accurate range determination. These features are achieved by means of spread spectrum techniques. In short, this implies to spread the bandwidth of the navigation signal (bi-phase modulation with a symbol rate of 50 Hz) by mixing it with a pseudo-random rectangular pulse train that has a much higher frequency than the data. The higher the spreading frequency the higher the power spectral density decrease is for a given total radiated power. The spreading sequences are known as pseudo-random noise (PRN) since they have autocorrelation and cross-correlation properties similar to those of Gaussian noise, but with the advantage that they can be accurately generated and regenerated, since they are in essence deterministic. Each GPS satellite has its own PRN code that not only allows discriminating between transmitters, but also grants the required jamming and multipath resilience and provides range estimations to determine the user position by triangulation.

To understand the structure and properties of the PRN codes it is useful to consider first the case of a pure random sequence of pulses of width $T_c$ (Fig. 2.1 and Eqn. 2.1):

$$P(t) = \sum_{n=-\infty}^{\infty} x_n \cdot \Pi\left(\frac{t-nT_c}{T_c}\right),$$  \hspace{1cm} (2.1)

where $x_n$ takes the values ±1 with equal probability.

![Fig. 2.1 Random sequence of pulses [36].](image)

Each individual pulse that composes the sequence is known as *chip* in opposition to *bit*, since it does not carry any information. The autocorrelation of $P(t)$ is (Fig. 2.2 and Eqn. 2.2):
\[ R_p(\tau) = \begin{cases} 1 - \frac{|\tau|}{T_c}, & |\tau| \leq T_c \\ 0, & |\tau| > T_c \end{cases}, \]  \hspace{1cm} (2.2)

where \( \tau \) is the time lag. The actual codes to use cannot be strictly random, since it is necessary to regenerate the spreading sequence used by the transmitter at the receiver to decode the navigation signal and retrieve the range observable. Therefore, as already mentioned, the PRN codes will be deterministic and periodic sequences but with autocorrelation properties similar to those of a pure random sequence.

\[ \text{Fig. 2.2 Autocorrelation of a random sequence of pulses.} \]

The Coarse Acquisition (C/A) codes are used for the open-access civil service. They have a period of 1 ms to allow quick signal acquisition, with a length of 1023 chips. This implies a chip rate of 1.023 MHz and a bandwidth 2.46 MHz (Fig. 2.3).

\[ \text{Fig. 2.3 Spectrum of the C/A PRN codes [37].} \]

This C/A codes are obtained as the product of two maximal length sequences (MLS) G1 and G2, so that the cross-correlation properties of the single MLS are improved. Both G1 and G2 are generated by linear feedback shift registers (LFSR) of 10 stages driven by a 1.023 MHz clock.
The actual satellite ID is determined by the relative delay between G1 and G2. This delay is determined by the position of the two connectors of the cells that compose the G2 LFSR. There are only 37 delay combinations: 32 of them are reserved for the satellites, and 5 are used for other applications such as ground transmission. Fig. 2.4 summarizes this code generation:

![Diagram](image)

Fig. 2.4 Generation of the C/A code as the product of two MLS sequences [37].

The resulting C/A codes have high autocorrelation peaks to clearly identify an acquired satellite and low cross-correlation peaks so that the satellites do not interfere between them (Fig. 2.5). In order to discriminate a weak signal surrounded by strong ones it is necessary for the autocorrelation peak of the weak signal to be higher than the cross-correlation peaks of the stronger signals. In the ideal case of using random sequences the codes would be orthogonal and the cross-correlations zero. The used PRN codes are almost orthogonal, and the cross-correlation values are as low as -65/1023 (12.5 % of the time), -1/1023 (75 % of the time), or 63/1023 (12.5 % of the time).
There is also a precise code (P) used for the restricted military signal. It has a chipping rate 10 times faster than the C/A code (10.23 MHz) that results in a ten-fold increase of the pseudorange observable accuracy. The code period is of 1 week, so that the direct acquisition of the code (i.e., the estimation of the code offset) is pretty cumbersome. Therefore, to acquire this P code special data fields of the navigation frames (Z-count and Time of Week TOW) are used. To increase even more the code robustness it is possible to switch the system operation to use an encrypted version of the P code, noted as P(Y) [37].

2.1.2 Transmitted GPS Signals

The C/A and P codes are modulated in-phase & quadrature on the L1 carrier:

\[
S_1(t) = \sqrt{2P_{\text{C/A}}} D(t) CA(t) \cos(\omega_1 t + \phi_1) + \sqrt{2P_P} D(t) P(t) \sin(\omega_1 t + \phi_1),
\]  

where \(S_1(t)\) is the signal transmitted by a given GPS satellite, \(P_{\text{C/A}}\) is the transmitted power for the civil signal at L1, and \(P_P\) is the transmitted power for the restricted signal at L1. On L2 for a long time only the P code was broadcast (Eqn. 2.4):
It is noteworthy that no navigation signal is usually broadcast on L2. As of November 2008 there are already 6 satellites (IIR-M block) broadcasting a civil signal at L2 (LC2) in quadrature with the P(Y) code. The LC2 sequence has the same chip rate of the C/A signal, but it is composed of two PRN codes of different length. From one side the moderate length code (CM) is 10230 chips long and repeats every 20 ms. It is modulated with navigation data. From the other side, the long code (CL) has 767250 chips, repeats every 1.5 s and has no data modulation. Both CM and CL codes are generated using the same 27-state LFSR, which is restarted every one CM or CL period. The initial state of the LFSR determines the ID of the satellite the generated code belongs to. Each code is generated at 511.5 MHz and then multiplexed on a chip-by-chip basis to obtain the composite signal at a rate of 1.023 chips/s (Fig. 2.6). A detailed description can be found in [38].

![Fig. 2.6 Multiplexation of the CM and CL codes to obtain the LC2 signal [36].](image)

Navigation information such as ephemeris, almanacs, or corrections and constellation health are conveyed by the 50 Hz bi-phase code $D(t)$. All the bit/chip transitions in the C/A, P and D codes are synchronous, since they are all driven by the same clock. These various signals are broadcast within the L-band (Fig. 2.7), thus suffering low atmospheric attenuation and not being disturbed by rain. The carrier frequencies are multiples of 10.23 MHz:

$$
\begin{align*}
L1 &= 1575.42 \text{ MHz} = 154 \cdot 10.23 \text{ MHz}, \\
L2 &= 1227.60 \text{ MHz} = 120 \cdot 10.23 \text{ MHz}, \\
L5 &= 1176.45 \text{ MHz} = 115 \cdot 10.23 \text{ MHz},
\end{align*}
$$

The frequency spacing between L1 and L2 allows estimating the ionospheric delay:

$$
\Delta t_i = \frac{f_2^2}{f_1^2 - f_2^2} \delta(\Delta t),
$$

where $\Delta t_i$ is the time delay at the frequency L1 due to the ionosphere, $f_1$ and $f_2$ are the L1 and L2 frequencies and $\delta(\Delta t)$ is the measured time difference between frequencies $f_1$ and $f_2$. 52
Additionally, atomic clocks onboard the satellites ensure high frequency stability and precise time synchronization.

The minimum received power for the L1 P signal is of -163 dBW for a 0 dBic RHCP antenna. At L1 the C/A signal is transmitted with additional +3 dBW over the P signal. At L2 the P code is sent +3 dBW below the L1 P signal. Within the GPS satellite antenna field of view (FOV), the different signal attenuation due to different propagation losses and atmospheric absorption is compensated with the antenna pattern itself (Fig. 2.8). More specifically, the edge of the Earth is 14° off the antenna boresight, and therefore the pattern maximum is located at this angle. The power transmitter over this angular value is used by other satellites as an additional navigation aid.

![Fig. 2.7 GPS Signal Generation](https://example.com/fig27.png)

The transmitted signal is right-hand circularly polarized, and so it is immune to the atmospheric Faraday rotation. The power signal-to-noise ratio (SNR) for the direct signal ranges between 39 dBHz and 52 dBHz, depending on the geometry, the actual transmitted power, and the instrumental and propagation losses.
2. USE OF GNSS-R FOR SEA ROUGHNESS DETERMINATION

2.1.3 L5 Signal

The new L5 signal designed for Safety of Life (SoL) applications is expected to be available from 2009 onwards. It will have the bandwidth of the restricted P signal and has been designed to coexist with the Aeronautical Navigation Services existing in the assigned band. It will be more resistant to interference and exhibit a better weak signal performance. It is composed of two in-phase quadrature multiplexed signals: a navigation data channel and a data free channel (more robust carrier phase tracking). A complete signal description can be obtained from [39].

2.2 GLONASS

The Russian GLObalnaya NAvigatsionnaya Sputnikovaya Sistema (GLONASS) plans a full constellation of 24 satellites deployed in 3 orbital planes separated by 120°. The orbits are nearly circular, with a height of 19100 km and an orbital period of 11 h 15 m approximately. The satellites broadcast a standard precision (SP) signal and a restricted high precision (HP) signal at both L1 and L2 bands. All the satellites transmit the same spreading sequence for SP, and employ a frequency division multiple access (FDMA) approach that differs from the CDMA approach that GPS or GALILEO use:

\[
\begin{align*}
  f_{11,n} &= 1602 + n \cdot 0.5625 \text{ MHz}, n \in [-7, 7] \\
  f_{12,n} &= 1246 + n \cdot 0.4375 \text{ MHz}, n \in [-7, 7]
\end{align*}
\]
where \( n \) is the frequency channel number. Signals are right-hand circularly polarized. The 24 satellites employ only 15 frequency channels; since antipodal satellite pairs use identical frequencies, so that an Earth-located receiver will never receive both satellites simultaneously. The HP signal has 10 times more bandwidth than the SP one, and both are multiplexed in phase quadrature. As of November 2008, the constellation consists of 17 elements. It is planned that the full operative constellation will be deployed in 2011.

2.3 GALILEO

The European GALILEO GNSS was conceived as an alternative to both GLONASS and GPS, since Russia or the USA could deny the access to their respective navigation systems in case of war or political disagreement. The first GALILEO satellite was launched in December 2005, with the aim of having fully deployed the 30 element constellation by 2013. The satellites are distributed in 3 orbital planes with an inclination on 56°. The Open Service (OS) will perform similarly to the GPS C/A service, whereas the encrypted Commercial Service (CS) will be available under subscription. Two additional Public Regulated Service (PRS) and Safety of Life Service (SoL) modes will be available. The frequency allocation for Galileo, Glonass, and GPS is shown in Fig. 2.9.

![Fig. 2.9](image)

**Fig. 2.9** Radio Navigation Satellite Service band distribution after the World Radio Conference, Istanbul, 8 May-2 June 2000, which discussed the allocation of the GALILEO signal spectrum. E and C bands (blue) are assigned to GALILEO, L bands (green) are for GPS, and G bands (red) are reserved for the GLONASS signals [14].

2.4 COMPASS

The planned Chinese Compass (or Beidou II) is an independent system similar to GPS or Galileo. It uses a CDMA scheme to discriminate between satellites and will offer two levels of
service: open and restricted (for military purposes). The whole constellation consists of 5 geostationary satellites and 30 medium Earth orbit (MEO) satellites with a nearly circular orbit and a height of 21.150 km with an inclination of 55.5°. The signal structure has not been publicly disseminated, but it seems that again phase-quadrature multiplex is used to broadcast at the same frequencies the public and the restricted codes.

2.5 SCATTERING PROCESS OF THE GNSS SIGNAL OVER THE OCEAN SURFACE

The GNSS-R approach allows retrieving geophysical parameters related to the observed surface. This is possible because the scattering process that watermarks the incoming GNSS signal depends on the surface characteristics. Therefore, it is necessary to understand the underlying scattering mechanism in order to perform the geophysical retrieval.

2.5.1 Electromagnetic Models for Bistatic Scattering

To acquire the GNSS signal it is necessary to correlate it with a locally generated replica of the PRN code and a phasor to compensate the Doppler frequency shift due to the relative movement of the transmitter and the receiver. The correlation value is proportional to the power scattered at different delay and Doppler bins, and it is known as Delay Doppler Map [40]. For a specular surface the DDM will be very similar to that obtained when acquiring direct signal, except for a scale factor due to the Fresnel reflection coefficient of the surface. However, the rougher the surface the more spread the DDM. As explained in [40], the region of the scattering surface from where reflections that reach the receiver originate is known as the glistening zone. Its extension depends on the roughness, and for a perfectly flat sea it will be reduced to a single point (the specular reflection point). It can be quantitative defined as the locus of the surface points whose associated scattering coefficient $\sigma^0$ is higher than a certain threshold ($\sigma^0_{\text{max}}/e$, for instance). This glistening zone comprises several different delay and Doppler bins, so that the resulting signal that reaches the receiver is made up of a variety of components with different delay and Doppler shifts. The correlation process selects a given surface patch and extracts the power reflected on that particular surface region. The sea state parameterizes the $\sigma^0$, which in turn determines the extension and shape of the glistening zone. Thus, the measured DDM conveys information regarding the sea state.
Fig. 2.10 Sketch of the GNSS-R bistatic geometry, showing the isorange (set of annuli) and isoDoppler lines [41].

For a completely calm sea, the signal scattering becomes a signal reflection where the Snell reflection law applies; i.e., the incidence and reflection angles are equal, and the specular point where the reflection takes place defines the shortest path between transmitter, surface and receiver. The correlation at the specular delay will have contributions from all those surface points within the glistening zone whose associated delay does not differ more than 1 chip from the specular delay. This happens because the PRN correlation function can be approximated by a triangle function of width ±1 chip (Eqn. 2.2). Therefore, the contributions from points that differ more than 1 chip in delay are filtered out. This defines an ellipse around the specular point. If the glistening zone is larger than this first iso-range ellipse, some scattered power will be measured when correlating with PRN replicas with higher delay shifts. These regions compose a set of elliptical rings or annuli, named isorange lines. Moreover, the signal reflected at a given surface point \((x, y)\) has suffered a Doppler shift \(f_d(x, y)\) in its carrier frequency. The integration time of the correlation integral acts as a filter that rejects the signal components with a Doppler frequency \(f_d(x,y)\) that verifies that \(|f_d - f_d(x,y)| > 1/T_i\). This filter has a sinc-shape, and defines a set of isoDoppler lines on the sea surface. The isolines are depicted in Fig. 2.10. Since they are defined as the locus of the points with equal delay and Doppler, they strongly depend on the scenario geometry, mainly defined by the elevation of the transmitter and the height of the receiver (Fig. 2.11).

Summing up, at a given instant received signal is composed by several contributions from different sea surface cells. The correlation value of a given delay-Doppler bin is proportional to the signal scattered over the surface cell associated to those delay-Doppler coordinates \((\tau, f_d)\). The shape of the DDM depends on both the scenario geometry and the sea roughness. For a given \((\tau, f_d)\) coordinates, the DDM amplitude depends on the contributions of the surface points that verify [40]:

- They are located within the field of view of the transmitter and receiver antennas.
- They are located within the glistening zone; i.e., \(\sigma^0(x,y)\neq 0\).
2. USE OF GNSS-R FOR SEA ROUGHNESS DETERMINATION

- The difference between their associated delay $\tau(x,y)$ and the DDM bin delay $\tau$ is smaller than 1 chip.
- The difference between their associated Doppler shift and $f_d$ is smaller than the integration bandwidth $1/T_i$.

![Fig. 2.11 Influence of the scenario geometry on the distribution of the isolines: incidence angles of 10° (left) and 40° (right). The resolution is 1 chip (delay) and 500 Hz (Doppler) [41].](image)

The glistening zone not only is determined by the sea state, but also by the geometry. Thus, for a given sea roughness condition, the higher over the surface the receiver is the more isorange zones that will lay within the glistening zone, and so the higher the sensitivity to sea roughness of the system will be, but also the more delay-Doppler bins that will have to be sampled. The bistatic scattering of the incident GNSS signal is composed of a specular and a diffuse term. The former is characterized by a high directivity, whereas the latter decomposes the incident signal into an ensemble of low amplitude signals over a wide range of directions. Closed-form models for the electromagnetic scattered field from random surfaces are asymptotic solutions of the Maxwell equations. The two limits usually considered are refereed to as the Kirchhoff method (KM) [42] and the Small Perturbation Method (SPM) [43]. In addition to these, the Two Scale Composite model (2SCM) combines both KM and SPM. A brief description of these different approaches to the scattering problem follows:

- KM: Applies a linear relationship between the incident and the scattered field through the Fresnel coefficients. The integral equations of the electromagnetic (EM) field on the surface simplify to the integration of the incident field over the surface. This accounts only for the induced currents on the surface, allowing solving the integral equation through the Green’s theorem considering only the tangential fields. This approximation is valid provided that both the correlation length of the sea surface and its average radius of curvature are significantly larger than the wavelength of the incident field. This method properly models the quasi-specular scattering, but it is not sensitive to the field polarization. The geometric optics (GO) or stationary phase approximation applies when the surface roughness (standard deviation of the surface height) is large compared to the wavelength. The surface is decomposed in a set of
elementary facets that reflect the signal specularly and independently from the others. The resulting field is the incoherent sum from all the properly-oriented mirrors.

- **SPM**: Derived under the small slopes and short waves assumptions. It models properly the polarizations effects, but it is unable to predict the quasi-specular return, because disregards the longer scale features. It solves the partial differential boundary equation by expanding the field in a series of perturbations of the slopes of the sea surface. It is suitable for Bragg scattering and polarimetric modeling.

- **2SCM**: Linear combination of the large scale contribution modeled with the KM and the small scale effect on the scattered field applying the SPM solution averaged over the statistics of the tilt of the large scaled surface characterization (Fig. 2.12).

Fig. 2.12 The two-scale model decomposes the sea surface in a large-scale contribution (low wavenumber representing the gravitational waves) and a small-scale (larger wavenumber associated to the capillary waves) contribution.

As stated in [40], the 2SCM performs better than the other models, which are not able to represent properly all the sea states individually. However, the computational burden of the 2SCM is much higher than that of the KM-GO, whereas the difference is not significant for the cross-polar component (the predominant in the GNSS-R scenarios) for incidence angles over 40°. In addition to that, the small scale roughness contribution of the 2SCM seems to be significant only for large incidence angles. Additionally, the selection of a cut-off wavelength to split the two regimes (usually, $K_{GPS}/3$) is somewhat arbitrary, and thus results in an additional tuning parameter. Therefore, the KM-GO is the approach usually employed in GNSS-R studies such as [40], [14], or [41], even though it is acknowledged that it is a coarse approximation to the actual scattering process [44]. Other methods such as the Small Slope Approximation (SSA) have been recently developed at L-band, but have not yet been applied to GNSS-R.

2.5.2 CORRELATION POWER AND AMPLITUDE OF THE SCATTERED GNSS SIGNAL

The Geometric Optics approximation of the Kirchhoff model assumes that the local radius of curvature is much larger than the radiation wavelength (Fig. 2.13):
Fig. 2.13 Large (left) and small (right) radius of curvature compared to the radiation wavelength, from [42].

The field at the GPS receiver can be thought of as being created by a distributed radiating surface [42]:

$$u(\bar{r},t) = \frac{1}{4\pi} \int \left[ D(\bar{r},t) \mathcal{R} \left( \frac{\partial}{\partial N} u(\bar{r}) \frac{\mathcal{R}R}{R} \right) \right] d^2\bar{r}, \quad (2.8)$$

where $u(\bar{r})$ is the field at the surface point $\bar{r}$, $D(\bar{r},t)$ is the antenna gain projected onto the scattering surface, $\mathcal{R}$ is the Fresnel reflection coefficient, $R$ is the distance between the scattering point and the receiver, and $\frac{\partial}{\partial N}$ stands for the derivative with respect to the local normal of the surface. The incident field on the scattering surface is:

$$U(\bar{r}) = \text{PRN} \left( t - \frac{R_0 + R}{c} \right) \frac{e^{i(\mathcal{R}R_0 - \omega t)}}{R_0}, \quad (2.9)$$

where $\text{PRN}(t)$ is the transmitted pseudorandom code, and $R_0$ is the distance between the GPS satellite and the scatter, and where the physics convention of $e^{i\omega t} \cdot e^{-i\omega t}$ is used as it has been usual in this field ([45], [36]). Substituting Eqn. 2.9 in Eqn. 2.8, the field at the receiver results in ([45]):

$$u(\bar{r},t) = e^{-i\omega t} \int D(\bar{r}) \text{PRN} \left( t - \frac{R + R_0}{c} \right) g(\bar{r},t) d^2\bar{r}, \quad (2.10)$$

where $g(\bar{r},t)$ is a complex random variable that accounts for the surface geometry:

$$g(\bar{r},t) = \frac{q^2 \mathcal{R}}{4\pi^2j} \frac{e^{i(\mathcal{R}R_0)}}{R_0R}, \quad (2.11)$$
being $\mathbf{q}$ the scattering vector, defined as $\mathbf{q} \equiv K(\mathbf{\hat{n}} - \mathbf{\hat{m}})$, where $\mathbf{\hat{n}}$ and $\mathbf{\hat{m}}$ are the unit vectors of the incident and scattered waves respectively. The receiver performs the correlation of the incoming signal with a local PRN replica and a phasor spinning at the expected Doppler frequency $f_d$. Thus, the obtained correlation amplitude is a function of the code delay $\tau$, the compensation Doppler frequency $f_d$, and the time $t$:

$$Y(t, \tau, f_d) = \int_0^T u(t + t') \cdot PRN(t + t' + \tau) \cdot e^{i\phi(t + t')} \cdot dt',$$  

(2.12)

where $T_i$ is the coherent integration time, during which the surface is considered frozen. The random phase due to the surface scattering limits the maximum coherent integration time. The navigation bit poses another upper bound for this integration time (20 ms). Substituting Eqn. 2.10 into Eqn. 2.12 and operating [45]:

$$Y(t, \tau, f_d) = e^{-i\phi} e^{-i\tau\pi\hat{f}_d} T_i \int \int D(\hat{r}) \cdot \chi(\Delta \tau(\hat{r}), \Delta f_d(\hat{r})) \cdot g(\hat{r}, t + \tau) d^2 \hat{r},$$  

(2.13)

where $\chi$ is known in radar terminology as the Woodward ambiguity function, and it can be approximated as:

$$\chi(\Delta \tau, \Delta f_d) = \Lambda(\Delta \tau(\hat{r})): S(\Delta f(\hat{r})), \quad (2.14)$$

where $\Lambda(\Delta \tau)$ can be approximated with Eqn. 2.2, and $S(\Delta f_d)$ is a complex sinc function:

$$S(\Delta f) = \frac{\sin(\pi: \Delta f: T_i)}{\pi: \Delta f: T_i} e^{-i\pi: \Delta f: T_i}, \quad (2.15)$$

The amplitude waveform is a stochastic process because of the surface randomness. To study it, it is necessary to obtain statistical descriptors. The autocorrelation of the amplitude waveform is:

$$R_s(\tilde{t}, \tau, f_d) = E\{Y(t + \tilde{t}, \tau, f_d)Y^*(t, \tau, f_d)\}, \quad (2.16)$$

where $\tilde{t}$ is the time lag. A closed expression is given in [46]:

$$R_s(\tilde{t}, \tau, f_d) = T_i \cdot e^{-i\tau\pi(\hat{f}_d - \hat{f}_0)} \int \int \left[3\pi \cdot D'(\hat{\rho}) \cdot \Lambda'((\Delta \tau) \cdot S(\Delta f_d)) \cdot q^\prime(\hat{\rho}) \cdot P_j \left( -\frac{\mathbf{q}}{q_j} \right) \cdot e^{-i\pi\omega_0 l} \cdot d^2 \hat{\rho}. \quad (2.17)$$
In Eqn. 2.17 the probability density function of the surface slopes $P_{\nu}$ appears. The terms inside the slope PDF account for the required orientation of the facet so that a specular reflection takes place. The last exponential term accounts for the signal decorrelation: the longer the time lag the more important the phase jump will be. The power correlation distribution as a function of the PRN is retrieved as the autocorrelation for a zero time lag:

$$
\left\langle |Y(\tau, f_d)|^2 \right\rangle = T_s^2 \int \int \frac{3 |\rho^2 \cdot D^2(\rho) \cdot \Lambda^2(\Delta \tau) \cdot \mathcal{L}(\Delta f_d)|^2 \cdot q_s(\rho) \cdot \rho \cdot \rho \cdot \rho \cdot \rho}{4 R_s^2(\rho) \cdot R_s^2(\rho)} \cdot P_{\nu} \left( -\frac{q_s}{R_s} \right) \cdot d\rho \cdot d\rho \cdot \rho \cdot \rho \cdot \rho \cdot \rho .
$$

Equation 2.18 is usually referred to as the Zavorotny-Voronovich model [45], and it is widely used to model the DDM. The ambiguity function within the power Delay Doppler Map expression selects the region of the glistening zone that contributes to the power at a given $\tau$ and $f_d$ bin, and masks out the other surface points:

$$
\Lambda^2(\Delta \tau) = 0, \quad |\Delta \tau| > T_{chip},
$$

$$
|\mathcal{L}(\Delta f_d)|^2 = 0, \quad |\Delta f| > \frac{1}{T_i}.
$$

In fact, $\tau$ and $f_d$ can be thought of as a new coordinate system to map the sea surface. The locus of the points with constant delay is referred to as isorange, whereas the constant Doppler areas constitute the isoDoppler lines.

### 2.5.3 NOISE IMPACT AND SNR

The thermal noise power on an electronic system can be expressed as:

$$
P_n = k T_0 B, \quad (2.20)
$$

where $P_n$ is the noise power, $k$ is the Boltzmann constant, $T_0$ is the system temperature, and $B$ is the noise bandwidth of the system, determined by the pre-correlation bandwidth and the integration time. The signal that the receiver processes is affected by this additive Gaussian noise:

$$
s(t) = u(t) + n(t), \quad (2.21)
$$

being $u(t)$ the GNSS signal and $n(t)$ the noise term. Thus, applying Eqn. 2.21 into Eqn. 2.12, the noisy DDM expression is:

$$
Y_s(t, \tau, f_d) = Y(t, \tau, f_d) + \int_0^{t} n(t + t') \cdot PRN(t + t' + \tau) \cdot e^{j\omega(t + t')} \cdot dt'. \quad (2.22)
$$
The product of the PRN with the noise term has the same properties of noise itself. Following [40], the noise DDM is:

\[
\left\langle \left| Y_\tau (t,R) \right|^2 \right\rangle = \left\langle \left| \int_0^T n(t+\delta t) \cdot PRN(t+\delta t) \cdot e^{i\phi(t+\delta t)} \cdot dt \right|^2 \right\rangle = T_0 \cdot k \cdot T_0. \tag{2.23}
\]

Combining Eqns. 2.18 and 2.23, the signal-to-noise ratio (SNR) for the correlation power is:

\[
SNR_N = \frac{\left\langle \left| Y_\tau (t,R) \right|^2 \right\rangle}{\left\langle \left| Y_\tau (t,R) \right|^2 \right\rangle} = \frac{T_0}{k \cdot T_0} \int \int [R^2 \cdot D^2 (\delta \rho) \cdot A^2 (\delta \tau)] \cdot \left| S(\Delta f_d) \right|^2 \cdot q^4 (\rho) \cdot q_{\rho} (\rho) \cdot \left( \frac{-a_+}{a_\rho} \right) \cdot d^2 \delta \rho. \tag{2.24}
\]

When obtaining DDM from real data, the amplitude peak to noise floor ratio is usually used rather than the above defined SNR. The relationship between the two signal quality ratios is [40]:

\[
SNR_p \propto \left( Y_\tau (t,R) \right)_{\text{max}}^2 \cdot T_0. \tag{2.25}
\]

Apparently, increasing the correlation interval (the so-called coherent integration time) improves the DDM quality. However, the received power decreases, since the isoDoppler stripe decreases in width proportionally to 1/T_i, and thus there is no SNR improvement. On the other hand, increasing T_i decreases both the standard deviation and the mean of the DDM. The incoherent averaging consists of summing up several power correlation DDMs. As a result the dispersion of the measure decreases, but there is no reduction of the noise floor.

2.6 SEA SURFACE CHARACTERIZATION

The sea surface can be thought of as a random process in both space and time. On a Cartesian reference frame, the height at a given time \( t \) and at a surface point \( \vec{r} = (x,y) \) is the random variable \( \eta(\vec{r},t) \). The autocorrelation of the process is:

\[
\rho(\vec{r},t) = E \{ \eta(\vec{r},t) \cdot \eta(\vec{r}+\vec{r}_0,t+t_0) \}, \tag{2.26}
\]

where \( \vec{r}_0 \) and \( t_0 \) are arbitrary position and time variables, and assuming that the surface is homogeneous in space and stationary in time. The Fourier transform of Eqn. 2.26 is the 3D power spectrum of the sea surface:

\[
\phi(\vec{k},\omega) = \frac{1}{(2\pi)^3} \int \int \int \rho(\vec{r},t) e^{i\vec{k} \cdot \vec{r} \cdot \omega} \cdot d^2 \vec{r} \cdot dt. \tag{2.27}
\]
Unfortunately, this full spectrum cannot be measured, since there is a technological limitation in monitoring both the space and time variation of the sea surface. Therefore, two particular cases of Eqn. 2.27 are used to work with the surface model:

- **Wavenumber Spectrum**: snapshot of the ocean surface at a given time instant:

\[
\psi(\vec{K}) = \int_{-\infty}^{\infty} \phi(\vec{K}, \omega) \cdot d\omega .
\] (2.28)

- **Frequency Spectrum**: time evolution of a given surface point

\[
\psi_f(\omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \phi(\vec{K}, \omega) \cdot d\vec{K} .
\] (2.29)

In order to relate both spectra it is necessary to establish the dispersion relationship \( \omega = f(K) \). For infinitesimally small waves in deep water the relationship is:

\[
\omega^2 = gK ,
\]

\[
K = |K| ,
\] (2.30)

where g is the gravitational acceleration constant \( (g = 9.81 \text{ m/s}) \). Expressing the frequency spectrum (Eqn. 2.29) in polar coordinates:

\[
\psi_f(\omega) = \int_{0}^{2\pi} \int_{0}^{\infty} \phi(K, \theta, \omega) \cdot dK \cdot d\theta .
\] (2.31)

Using the above given equation it is possible to apply the dispersion equation, that determines the wavenumber of the existing waves at a given time frequency \( \omega \):

\[
\psi_f(\omega) = \int_{0}^{2\pi} \int_{0}^{\infty} \phi(\frac{\omega^2}{g}, \theta, \omega) \cdot \delta(\omega - \omega) \cdot \frac{d\theta}{\frac{\omega^2}{g}} - \frac{2\omega}{g} \int_{0}^{2\pi} \phi(\frac{\omega^2}{g}, \theta) \cdot d\theta .
\] (2.32)

From the other side, the omnidirectional wavenumber spectrum is defined as:

\[
S(K) = \int_{0}^{2\pi} \psi(K, \theta) d\theta .
\] (2.33)

So the direct relationship between frequency and wavenumber spectra is:

\[
\psi_f(\omega) = \frac{2\omega^3}{g^2} S\left(\frac{\omega^2}{g}\right) .
\] (2.34)
Oceanographic buoys can directly measure \( \psi, \omega \), and thus provide an indirect estimate of \( S(K) \). An important surface descriptor is the directional slope defined as:

\[
Z_x(x,y,t) = \lim_{\Delta x \to 0} \frac{\eta(x+\Delta x, y, t) - \eta(x, y, t)}{\Delta x},
\]

\[
Z_y(x,y,t) = \lim_{\Delta y \to 0} \frac{\eta(x, y+\Delta y, t) - \eta(x, y, t)}{\Delta y},
\]

(2.35)

where \( Z_{x,y}(x,y,t) \) is the slope of the surface point \((x,y)\) at the instant \(t\) along the \(x\) or \(y\) axis. However, since the surface is random such a definition has a limited use. The parameters that are actually useful are the slope variances or mean squared slopes (MSS):\]

\[
\text{MSS}_x \equiv \sigma^2_{x,x} = \mathbb{E} \left\{ Z_x^2 \right\} = \lim_{\Delta x \to 0} \mathbb{E} \left\{ \frac{[\eta(x+\Delta x, y, t) - \eta(x, y, t)]^2}{\Delta x^2} \right\},
\]

\[
\text{MSS}_y \equiv \sigma^2_{y,y} = \mathbb{E} \left\{ Z_y^2 \right\} = \lim_{\Delta y \to 0} \mathbb{E} \left\{ \frac{[\eta(x+\Delta y, y, t) - \eta(x, y, t)]^2}{\Delta y^2} \right\}.
\]

(2.36)

The wavenumber spectrum can be used to obtain the total MSS [7]:

\[
\text{MSS} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (K_x^2 + K_y^2) \cdot \psi(K_x, K_y) \cdot dK_x \cdot dK_y = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} K^2 \cdot \psi(K, \theta) \cdot K \cdot dK \cdot d\theta = \int_{-\infty}^{\infty} K^2 \cdot S(K) \cdot dK.
\]

(2.37)

The directional MSS (DMSS) is usually decomposed in its upwind and crosswind components:

\[
\text{MSS}_u = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} K_x^2 \cdot \psi(K_x, K_y) \cdot dK_x \cdot dK_y = \int_{0}^{\infty} \int_{-\infty}^{\infty} K^2 \cdot \cos^2 \theta \cdot \psi(K, \theta) \cdot K \cdot dK \cdot d\theta,
\]

\[
\text{MSS}_c = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} K_y^2 \cdot \psi(K_x, K_y) \cdot dK_x \cdot dK_y = \int_{0}^{\infty} \int_{-\infty}^{\infty} K^2 \cdot \sin^2 \theta \cdot \psi(K, \theta) \cdot K \cdot dK \cdot d\theta,
\]

(2.38)

where the upwind direction is aligned with the \(x\) axis and the crosswind direction with the \(y\) axis. The DMSS’s and the total MSS are closely related:

\[
\text{MSS} = \text{MSS}_u + \text{MSS}_c \equiv \sigma_u^2 + \sigma_c^2.
\]

(2.39)

The PDF of the surface’s slopes can be approximated as the product of a two-dimensional Gaussian distribution times a two-dimensional Gram-Charlier distribution:

\[
P(\bar{Z}) = P_{\text{Gauss}}(\bar{Z}; \sigma_u, \sigma_c) \left[ 1 + \sum_{i,j=1}^{\infty} c_{ij} H_i(\bar{z}) H_j(\bar{z}) \right],
\]

(2.40)

where \( \bar{Z} = (Z_u, Z_c) \) is the vector of the slopes, \( \sigma_u \) and \( \sigma_c \) are the standard deviations of the upwind & cross-wind slopes, and \( H_i \) and \( H_j \) are Hermite polynomials with \( c_{ij} \) coefficients to be determined. In [47] optical data was used to establish with a semi-empirical approach the value of these \( c_{ij} \) coefficients. However, whenever trying to use those results for microwave L-
band data it is necessary to recall that at optical wavelengths the curvature radius of the surface is much larger than the radiation wavelength. This does not hold for GPS: at the L1 band the wavelength is ~19 cm, whereas the typical surface curvature radii range between 2 cm and 3 cm. This renders false the GO assumption. The two approaches used to overcome this limitation are:

- Define a cut wavelength to set an upper bound to the integral of the spectrum

\[ \text{MSS}^* = \int_{0}^{K^*} \int_{-\pi}^{\pi} K^2 \cdot \psi(K, \phi) \cdot K \cdot dK \cdot d\phi, \]  

(2.41)

where the typical value used for the cut wavelength is \( K^* = K_{\text{gps}}/3 \) [45].

- Apply a correction factor to reduce the overall MSS

\[ \text{MSS}^* = \alpha \cdot \text{MSS}, \]  

(2.42)

At L1 a value of \( \alpha = 0.45 \) is reported in [48] to yield a good fit with actual data.

It is important to emphasize that the PDF of the slopes defines the scattering coefficient of the surface. In turn, this coefficient will influence the received GNSS signal. Thus, the GNSS-R receiver ‘senses’ the slopes of the surface. These slopes characterize the sea state, which as already noted depends on several factors such as the WS, the swell or the currents. In relation to this, the Cox and Munk data does not account for the impact of swell on the overall surface roughness.

2.7 CONCLUSIONS

The reflection of GNSS signals can be used to infer geophysical parameters related to the scattering surface. The Kirchhoff Model in its Geometrics Optics approximation (KM-GO) is a first approach to the actual scattering problem. Nevertheless, it is commonly used in GNSS-R because of its simplicity and acceptable performance. By applying it the Zavorotny-Voronovich model for the GNSS-R basic observable, the DDM, is obtained. This Delay-Doppler map is a distribution of the power scattered by each of the surface cells that contribute to the total scattered power. These cells are determined by the intersection of isorange and isoDoppler lines over the scattering surface. The retrieval of the sea surface roughness is possible the scattering coefficient determines the extent of the glistening zone, and thus also the shape of the DDM. The sea is modeled as a random surface; being the mean squared slope (MSS) the most commonly used roughness descriptor.
3. DDM SIMULATION

In order to study the dependence of the GNSS-R observable on both the scattering geometry and the sea state it is useful to be able to simulate 'synthetic' DDMs. Moreover, these simulations can be compared to the actual output of the GNSS-R receiver to validate its implementation. The Zavorotny-Voronovich DDM expressions introduced in chapter 2 are the core of the implemented simulator, as has been the case in other studies [40] [14] [41]. As they state, the Kirchhoff approximation in the Geometric Optics limit (KM-GO) is just a rough approach to the actual bistatic scattering problem. Nevertheless, it yields good enough results to perform analysis near the specular reflection direction.

More specifically, the present chapter will consider the time evolution of the instantaneous amplitude DDM, the implementation of the Zavorotny-Voronovich model itself (expectation of the power DDM), and will eventually introduce an efficient approach to alternatively obtain the power DDM using the analytical expression of the isorange and isoDoppler lines.

3.1 SIMULATION SCENARIO

3.1.1 Reference System

The recreation of a GNSS-R scenario requires placing the transmitter, the receiver and the scattering surface within a reference frame. A usual choice is to define a local Cartesian coordinate system centered at the specular point, with the Transmitter-Specular Point-Receiver plane parallel to the Y-Z plane. The receiver height $h$ and the transmitter elevation $\gamma$ define univocally the scenario (Fig. 3.1). The positions of the receiver, transmitter, and an arbitrary surface point are:

$$\vec{R}_t = \left(0, \frac{h_0}{\tan \gamma}, h_0\right),$$  \hspace{1cm} (3.1)

$$\vec{R}_r = \left(0, -\frac{h}{\tan \gamma}, h\right),$$  \hspace{1cm} (3.2)

and
3. DDM SIMULATION

\[
\vec{r} = (x, y, z),
\]

where \(h_0\) and \(h\) are the transmitter and receiver heights over the tangent plane at the specular point. Additionally, the mean Earth radius \(R_e \approx 6.371\) km and the GPS orbit height \(R_{GPS} \approx 20.189\) km are used.

Depending on the receiver height, the spherical Earth cap can be approximated by the tangent plane at the specular point. The incurred error increases with the receiver height, and it is not uniform throughout the scattering surface. Therefore, for the tangent approximation the mean surface height is \(z_{\text{mean}} = 0\), whereas for the spherical cap it can be obtained from the equation of a sphere of radius \(R_e\) centered at \((0, 0, -R_e)\):

\[
z_{\text{mean}} = \sqrt{R_e^2 - x^2 - y^2} - R_e.
\]

Fig. 3.1 Flat (dashed line) and Spherical Earth (solid line) scenario geometry. The scenario is determined with the elevation \(\gamma\) and the receiver \((R_e)\) height \(h\) over the tangent plane at the specular point. \(R_e\) and \(R_{GPS}\) are the Earth and GPS orbit radii, \(h_0\) is the transmitter \((T_x)\) height over the flat surface, and \(\vec{n}\) and \(\vec{n}\) are unit vectors in the incident and scattering directions.
Either way, at each surface point both the incident and scattered wave vectors can be defined as:

$$\hat{n}_i = \left( \hat{r} - \hat{R}_i \right) / \left| \left( \hat{r} - \hat{R}_i \right) \right| ,$$

(3.5)

and

$$\hat{n}_s = \left( \hat{R}_s - \hat{r} \right) / \left| \left( \hat{R}_s - \hat{r} \right) \right| .$$

(3.6)

The scattering vector introduced in chapter 2 is:

$$\vec{q} = K \cdot (\hat{n}_s - \hat{n}_i) ,$$

(3.7)

and plays an important role in the computation of the scattering coefficient, since it points in the ideal orientation of the normal of an individual facet so that it would reflect the signal specularly.

### 3.1.2 Surface Generation

The sea surface roughness is accounted for by the scattering coefficient $\sigma_0$. This coefficient provides the average return from a given surface point. However, it would also be interesting to simulate the actual surface dynamics and the associated $Y(t, \tau, f_d)$ (Eqn. 2.12). The surface height is obtained as [49]:

$$\eta(\hat{r}, t) = \text{IFFT} \left\{ \eta_0(\vec{K}) e^{j \alpha(k) t} + \eta_0^*(\vec{K}) e^{-j \alpha(k) t} \right\} ,$$

(3.8)

where $\alpha(k) = \sqrt{\sigma_k}$ is the dispersion relation for the deep water approximation, and $\eta_0(\vec{K})$ is the bidimensional Fourier transform of the surface height $\eta(\hat{r})$ for $t = 0$:

$$\eta_0(\vec{K}) = \frac{\xi_r + j \xi_i}{\sqrt{2}} \sqrt{\psi(K_x, K_y)} ,$$

(3.9)

being $\psi(k_x, k_y)$ the wavenumber spectrum, and $\xi_r$ and $\xi_i$ two independent draws of a normally distributed one dimensional random variable.

### 3.1.3 Delay and Doppler Computation

The absolute delay associated to a surface point is:
3. DDM SIMULATION

\[ \tau_{xy,\text{obs}} = \frac{|\vec{r} - \vec{R}| + |\vec{R} - \vec{r}|}{c}, \] \hspace{1cm} (3.10)

whereas the delay associated to the specular point is:

\[ \tau_{xy,\text{spec}} = \frac{|\vec{R}| + |\vec{R}|}{c}. \] \hspace{1cm} (3.11)

In GNSS-R it is usual to refer the used delay to the specular point, thus:

\[ \tau_{xy} = \tau_{xy,\text{obs}} - \tau_{xy,\text{spec}}. \] \hspace{1cm} (3.12)

By definition, the Doppler shift depends on the relative speed \( \vec{V} \) between transmitter and receiver, the unitary vector defining the transmitter-receiver direction \( \hat{\vec{r}} \), and the radiation wavelength \( \lambda \):

\[ f_d = \frac{\vec{V} \cdot \hat{\vec{r}}}{\lambda}. \] \hspace{1cm} (3.13)

For the GNSS-R geometry, and disregarding the Doppler induced by the surface motion [45], the Doppler shift of at a given surface point can be retrieved as:

\[ f_{d,xy} = \frac{\vec{V}_t \cdot \hat{n}_t - \vec{V}_r \cdot \hat{n}_s}{\lambda}, \] \hspace{1cm} (3.14)

where \( \vec{V}_t \) and \( \vec{V}_r \) are the transmitter and receiver velocities, respectively.

3.1.4 Antenna Pattern

Not only the scenario geometry or the scattering surface influence the GNSS-R observable, but also the receiver itself has a strong impact. Its antenna pattern determines the extension of the illuminated area. For instance, a too thin beam whose projection over the surface lies within the first isochip area will have a limited sensitivity to the sea state. Conversely, a beam wider than the glistening zone will have a low gain that will reduce the signal SNR. In the simulations performed a generic antenna pattern with revolution symmetry has been used. It has been parameterized by the -3 dB angular bandwidth \( \Delta\theta_{-3\text{dB}} \):
\[ D(\theta)_{\text{dB}} = G_0 - 12\left(\frac{\theta}{\Delta \theta - 3dB}\right)^2, \quad (3.15) \]

where \( \theta \) is the angle with respect to the antenna broadside.

### 3.2 GENERATION OF AMPLITUDE AND POWER DDMs

The amplitude correlation of the reflected GNSS-R signal was introduced in chapter 2:

\[
Y(t, \tau, f_d) = e^{-j \phi} e^{-j 2\pi (f_d - f_0) \tau} \sqrt{\int \int D(\tilde{r}) \cdot \Lambda(\tau - \tau_p) \cdot S(f_d - f_{d,xy}) \cdot \left(\frac{q^2 \Re e^{iK(R_0 + R)}}{qz 4\pi j R_0 R} \right)} d\tilde{r}. \quad (3.16)
\]

Each surface point has its own associated phase \( K \cdot (R_0 + R) \). This results in a speckle-like behavior for the computed time series of DDMs.

---

Fig. 3.2 (Top) Time Series for a ground-based receiver (as described in chapter 6, double click to play the video). (Bottom, left to right) Incoherent and coherent means.
The implemented approach has been used to simulate both the spaceborne PAU/SEOSAT scenario described in chapter 5 and the ALBATROSS 08 ground-based campaign introduced in chapter 6. The resulting time series are shown in Fig. 3.2 and Fig. 3.3 along with their resulting coherent and incoherent means. The power correlation introduced in Eqn. 2.18 is reproduced here for convenience:

\[
\left\langle |Y(\tau, f_d)|^2 \right\rangle = T^2 \int \int \left| \mathbf{X}^2 \right| \cdot D^2(\rho) \cdot A^2(\Delta \tau) \cdot S(\Delta f) \cdot q^4(\rho) \cdot p_r \left( \frac{\bar{q}_r}{q_r} \right) \cdot d^2 \bar{f} . \tag{3.17}
\]

The actual computation of the power DDM calls for evaluating all the terms inside the integral for each \((x,y)\) coordinate, and then add them to obtain the power correlation at the considered delay-Doppler bin.

The meshing of the surface has to incorporate the entire glistening zone. This implies that the higher the receiver is the larger the surface to be simulated, and thus the more computational
resources needed. For a LEO receiver, the glistening zone extends over hundreds of kilometers (≈240 km of diameter for a 10 m height wind speed $U_{10} = 4$ m/s and ≈ 400 km for $U_{10} = 10$ m/s at nadir incidence [50]. As stated in chapter 2, the extension of the glistening zone is directly related to the surface scattering coefficient $\sigma^o$. If the antenna pattern is smaller than the glistening zone, a certain part of the scattered power will not be detected by the receiver. This truncation will be more severe as the sea roughness increases, or as the off-nadir geometry becomes more accentuated (Fig. 3.4).

![Fig. 3.4 Effect of the antenna beamwidth on the truncation of the received scattered power. Higher roughness conditions (here qualitative described as 3 m/s vs. 10 m/s wind speed) or increasing off-nadir geometries result in a truncation of the scattering coefficient, i.e., of the glistening zone.](image)

3.3 A NOVEL EFFICIENT APPROACH TO THE GENERATION OF POWER DDM

A first step to accelerate the simulation consists of expressing the DDM as a two-dimensional convolution [50], [51] that can be efficiently computed by means of 2D FFTs:

$$\left\langle \mathcal{Y}(\tau, f_d) \right\rangle = \mathcal{X}^2(\tau, f_d) * \Sigma(\tau, f_d) .$$  \hspace{1cm} (3.18)

In radar, Eqn. 3.18 is analog to the so-called Radar Mapping equation [52], whereas $\mathcal{X}$ is the already introduced ambiguity function (Eqn. 2.14). This ambiguity function is independent of the pixel position and can be computed in a straightforward manner and only once for all the
pixels. \( \chi \) can be understood as the impulse response to the scattered signal from a single delay-Doppler cell. On the other hand, \( \Sigma \) is given by:

\[
\Sigma(\tau, f_d) = \tau^2 \left( \int_0^1 \frac{D^2(\hat{\rho})}{4\pi R^2(\hat{\rho})} \delta(\tau - \tau(\hat{\rho})) \delta(f_d - f_d(\hat{\rho})) d^2 \rho \right),
\]

and accounts for the surface geometry, the antenna patterns and other terms from the bistatic radar equation, assigning a ‘weighting factor’ to each delay-Doppler cell of the scene. The advantage of Eqn. 3.19 over Eqn. 3.17 is that instead of evaluating \( \Lambda^2 \) and \( |S|^2 \) at every \((\tau, f_d)\) coordinate they are just computed for all the \((\tau, f_d)\) values at a time (to obtain \( \chi^2 \)) and then convolved with \( \Sigma \). Nevertheless, the integral in Eqn. 3.19 is still a simulation bottleneck that may prevent computing DDMs at all. Next, to author’s knowledge, a new and efficient approach to overcome this limitation is described.

### 3.3.1 Analytical approach to DDM Simulation

Instead of performing the integration of Eqn. 3.19 over the \( xy \) surface, it is possible to apply a change of variables using the geometrical expressions that link \( \tau_f \) to \( x-y \):

\[
\begin{align*}
\tau_{xy} &= \tau(x, y), \\
f_{d,xy} &= f_d(x, y).
\end{align*}
\]

The surface’s differential \( d^2 \rho \) in Eqn. 3.19 becomes:

\[
d^2 \rho = |J(\tau_{xy}, f_{d,xy})| \cdot df_{d,xy} \cdot d\tau_{xy},
\]

where \( df_{d,xy} \) and \( d\tau_{xy} \) are the differentials of the new integration variables, and \( |J| \) stands for the absolute value of the Jacobian of the change of variables defined in Eqn. 3.20. Substituting Eqn. 3.21 into Eqn. 3.19 yields:

\[
\Sigma(\tau, f_d) = \tau^2 \left( \int_0^1 \frac{D^2(\hat{\rho})}{4\pi R^2(\hat{\rho})} \delta(\tau - \tau(\hat{\rho})) \delta(f_d - f_d(\hat{\rho})) |J(\tau_{xy}, f_{d,xy})| df_{d,xy} \cdot d\tau_{xy} \right).
\]
where $G'$ is the integration domain that results of applying the change of variables defined in Eqn. 3.20 to the domain $G$. Applying the properties of the Dirac delta, Eqn. 3.22 becomes:

$$
\Sigma(\tau, f_d) = T^2 \frac{D^2(\tilde{\rho}(\tau, f_d)) \sigma^0(\tilde{\rho}(\tau, f_d))}{4\pi R_0^2(\tilde{\rho}(\tau, f_d)) R^2(\tilde{\rho}(\tau, f_d))} |f(\tau, f_d)| .
$$

(3.23)

Finally, substituting Eqn. 3.23 in Eqn. 3.18:

$$
\left\langle |Y(\tau, f_d)|^2 \right\rangle = \lambda^2(\tau, f_d)^\ast \left\{ T^2 \frac{D^2(\tilde{\rho}(\tau, f_d)) \sigma^0(\tilde{\rho}(\tau, f_d))}{4\pi R_0^2(\tilde{\rho}(\tau, f_d)) R^2(\tilde{\rho}(\tau, f_d))} |f(\tau, f_d)| \right\} .
$$

(3.24)

A similar result can be also derived applying the defined change of variables directly to Eqn. 3.17. Thus, the DDM can be fully determined with a 2D integral (convolution) over the $(\tau, f_d)$ domain, which is much smaller than the $(x, y)$ domain (scattering surface) used in Eqn. 3.19 to obtain $\Sigma$. Therefore, it is only necessary to evaluate Eqn. 3.24 at the desired Delay-Doppler coordinates, regardless of the size of the physical surface to simulate. This allows performing simulations corresponding to spaceborne scenarios that otherwise would require very high performance computers.

![Fig. 3.5 Delay-Doppler mapping](image)

Fig. 3.5 Delay-Doppler mapping: Each delay-Doppler bin is associated to two delay-Doppler cells that may have different area over the mapped surface.

In this approach, the Jacobian accounts for the area associated to a given Delay-Doppler cell on the x-y space (Fig. 3.5). As stated, the geometrical relationships that link a given surface coordinate and its associated delay and Doppler values at the defined scenario are the key to compute the Jacobians. Therefore, it is necessary to derive analytical expressions for them. Another remark is that the change of variables is not univocal, since a single delay-Doppler
coordinate corresponds to two points in the physical space (Fig. 3.5). Thus, the modules of the Jacobians associated to each of the two solutions must be properly combined, since each of them is associated to a different section of the observed surface. Thus, Eqn. 3.23 results in:

\[
\Sigma(\tau,f_d) = \frac{T^2}{4\pi} \left( \frac{D^o(\rho_1(\tau,f_d)) \cdot \sigma^o(\rho_1(\tau,f_d))}{R^o(\rho_1(\tau,f_d))} \left| j_1(\tau,f_d) \right| + \frac{D^o(\rho_2(\tau,f_d)) \cdot \sigma^o(\rho_2(\tau,f_d))}{R^o(\rho_2(\tau,f_d))} \left| j_2(\tau,f_d) \right| \right),
\]

(3.25)

where \( \rho_1, \rho_2 \) are each of the two space points associated to one delay-Doppler bin. The Jacobians have the following expression:

\[
|J_i(\tau_{xy}, f_{dy})| = \det \begin{vmatrix} \frac{\partial x_i}{\partial \tau_{xy}} & \frac{\partial x_i}{\partial f_{dy}} \\ \frac{\partial y_i}{\partial \tau_{xy}} & \frac{\partial y_i}{\partial f_{dy}} \end{vmatrix},
\]

(3.26)

where “det” is the determinant, and \( i = 1, 2 \) stands for the 1\(^{\text{st}}\) or 2\(^{\text{nd}}\) solution of the \( x-y \) coordinates. Its practical computation calls for substituting the derivatives by their associated central finite difference approximations:

\[
\frac{\partial x}{\partial \tau_{xy}} \approx \frac{x\left(\tau_{xy} + \Delta \tau/2, f_{dy,xy}\right) - x\left(\tau_{xy} - \Delta \tau/2, f_{dy,xy}\right)}{\Delta \tau},
\]

\[
\frac{\partial x}{\partial f_{dy}} \approx \frac{x\left(\tau_{xy}, f_{dy,xy} + \Delta f_d/2\right) - x\left(\tau_{xy}, f_{dy,xy} - \Delta f_d/2\right)}{\Delta f_d},
\]

(3.27)

where \( \Delta \tau, \Delta f_d \) are the delay and Doppler resolutions respectively of the DDM being computed. The derivatives of \( y \) with respect to \( \tau \) and \( f_d \) are obtained in an analog way. The computation of these Jacobians is critical around the borders of the DDM domain, and so the central differences approximation is preferred.
3.3.2 Validation of the Theoretical Approach Using a “Toy Model” for the Isolines

In order to verify the approach introduced in the previous section it is useful to use a “toy model” for the isorange and isoDoppler lines:

\[
\delta = y, \\
\tau^2 = (x - \tau)^2 + y^2,
\]

(3.28)

where \(\delta\) and \(\tau\) are analogous to the actual delay (\(\tau\)) and Doppler (\(f_d\)) parameters. The obtained isolines are depicted in Fig. 3.6:

![Isoline \(\tau\) (truncated over 10 / below -10)](image)

Then, it is possible to compute the inverse transform \((\tau, \delta) \rightarrow (x, y)\):

\[
\begin{align*}
x &= \tau \pm \sqrt{\tau^2 - \delta^2}, \\
y &= \delta.
\end{align*}
\]

(3.29)

From Eqn. 3.29 it is clear that there are two \((x, y)\) pairs associated to a single \((\tau, \delta)\) coordinate. Both solutions have complementary validity regions, as clearly seen in Fig. 3.7: the dark-blue triangle-shaped regions are associated to a zero error between the retrieved \((x_1, y_1)\) or \((x_2, y_2)\) solution and the actual \((x, y)\) coordinate. The union of the error-free areas of the first (left) and second (right) solutions give the full \((x, y)\) domain.
Fig. 3.7 Absolute error between the actual and the two retrieved x coordinates after performing a return transformation \((x, y) \rightarrow (\tau, \delta) \rightarrow (x_1, y_1)\) and \((x_2, y_2)\).

Now, for obtaining the Jacobians of the change of coordinates it is necessary to compute the partial derivatives. Applying Eqn. 3.26 into Eqn. 3.29 yields:

\[
J(\tau, \delta) = \begin{bmatrix}
\frac{\partial x}{\partial \tau} & \frac{\partial x}{\partial \delta} \\
\frac{\partial y}{\partial \tau} & \frac{\partial y}{\partial \delta}
\end{bmatrix} = \begin{bmatrix}
\frac{\tau}{\sqrt{\tau^2 - \delta^2}} & \pm \frac{\delta}{\sqrt{\tau^2 - \delta^2}} \\
0 & 1
\end{bmatrix},
\]

(3.30)

and the absolute value of the determinant is:

\[
|J(\tau, \delta)| = 1 \pm \frac{\tau}{\sqrt{\tau^2 - \delta^2}}.
\]

(3.31)

The resulting Jacobians are depicted in Fig. 3.8:

Fig. 3.8 The two solutions of the Jacobian for the change of variables between the transformed \((\tau, \delta)\) and the space \((x, y)\) domains.

Let’s now consider the Jacobian of the transform in the opposite direction, \((x, y) \rightarrow (\tau, \delta)\):
\[ J(x,y) = \begin{bmatrix} \frac{\partial \tau}{\partial x} & \frac{\partial \tau}{\partial y} \\ \frac{\partial \delta}{\partial x} & \frac{\partial \delta}{\partial y} \end{bmatrix} . \] \tag{3.32} 

Substituting Eqn. 3.28 into 3.32 one obtains:

\[ J(x,y) = \begin{bmatrix} \frac{1}{2} - \frac{1}{2} \left( \frac{y}{x} \right)^2 & \frac{y}{x} \\ 0 & 1 \end{bmatrix} . \] \tag{3.33} 

Now, taking the absolute value of the determinant yields:

\[ |J(x,y)| = \frac{1}{2} \left( \frac{y}{x} \right)^2 . \] \tag{3.34} 

Using Eqn. 3.34 it is possible to determine where exactly the Jacobian \( J(x,y) \) is equal to zero or tends to infinity:

\[ |J(x,y)| = 0; \quad \frac{1}{2} - \frac{1}{2} \left( \frac{y}{x} \right)^2 = 0; \quad 1 - \left( \frac{y}{x} \right)^2 = 0; \quad 1 = \left( \frac{y}{x} \right)^2 ; \quad y = \pm x \ . \] \tag{3.35} 

\[ |J(x,y)| = \infty; \quad \frac{1}{2} - \frac{1}{2} \left( \frac{y}{x} \right)^2 = \infty; \quad \begin{cases} y = \infty \\ x = 0 \end{cases} . \] \tag{3.36} 

Fig. 3.9 Meshing of the \((x, y)\) domain using the defined \((\tau, \delta)\) isolines. The blue dashed line is the locus of the cells with maximal area; i.e., with a Jacobian value that tends to zero.
In Fig. 3.9 a set of these \((\tau, \delta)\) isolines is plotted, along with the region where the Jacobian tends to zero (blue dashed line, Eqn. 3.35). In this region the area of the cells has its highest value. Therefore, the area weighting to apply would be at its minimum value, since it corresponds to the inverse of \(J(x,y)\). This holds when the Jacobian tends to infinity, which is associated to having very small cells \((x = 0\) or \(y \to \infty\)). The inverse of \(J(x,y)\) is plotted in Fig. 3.10. It is very similar to the Jacobians \(J_1(\tau, \delta)\) and \(J_2(\tau, \delta)\), each of them used within their respective validity regions.

![Fig. 3.10 Inverse of the Jacobian associated to the transform from the direct to the transformed domains. In this case a given \((x, y)\) coordinate results in just a \((\tau, \delta)\) pair.](image)

The difference between \(J_1(\tau, \delta)\) and \(J_2(\tau, \delta)\) with respect to \(J(x, y)\) follows (Fig. 3.11):

![Fig. 3.11 Difference between \((J(x,y))^{-1}\) and \(J_1(\tau, \delta)\) and \(J_2(\tau, \delta)\). The dark blue areas have a zero difference and the light blue a difference of two. The saturated points are located over the discontinuities.](image)
Again, the union of the triangle-shaped error-free regions yield the whole \((x,y)\) domain without overlapping.

The practical implementation of a DDM simulator applying this new approach will require sampling the \((\tau, \delta)\) space with a regular grid. Therefore, there will be \((\tau, \delta)\) coordinates that do not correspond to a real point in the ‘physical’ \((x, y)\) space. This will be detected because the \((x, y)\) obtained will not be real but complex. This shall allow ‘flagging out’ and discarding such points, as further explained in the following section. Also, instead of infinitesimals finite differences will be used. Thus, the expression of the Jacobians should account for this finite resolution. Recalling here Eqn. 3.29 for convenience:

\[
 x_1 = \tau + \sqrt{\tau^2 - \delta^2}.
\]

\[
 x_2 = \tau - \sqrt{\tau^2 - \delta^2}.
\]

\[
 y = \delta.
\]

(3.37)

An approximation to the partial derivatives can be expressed as:

\[
 \frac{\partial x_i}{\partial \tau} \approx \frac{\Delta x_i}{\Delta \tau} = \frac{x_i(\tau+\Delta \tau)-x_i(\tau)}{\Delta \tau} = 1 + \frac{\sqrt{(\tau+\Delta \tau)^2 - \delta^2} - \sqrt{\tau^2 - \delta^2}}{\Delta \tau}.
\]

(3.38)

From Eqn. 3.38 it is possible to obtain the actual analytical expression of the partial derivative. By making \(\Delta \tau \to 0\) and applying L’Hôpital theorem one obtains:

\[
 \lim_{\Delta \tau \to 0} \left(1 + \frac{\sqrt{(\tau+\Delta \tau)^2 - \delta^2} - \sqrt{\tau^2 - \delta^2}}{\Delta \tau}\right) = 1 + \lim_{\Delta \tau \to 0} \frac{\partial}{\partial \Delta \tau} \left(\frac{\sqrt{(\tau+\Delta \tau)^2 - \delta^2} - \sqrt{\tau^2 - \delta^2}}{\Delta \tau}\right) = 1 + \frac{\tau}{\sqrt{\tau^2 - \delta^2}}.
\]

(3.39)

which coincides with the first solution of Eqn. 3.31. Returning to Eqn. 3.38, the finite difference approximation of the first solution of the Jacobian is:

\[
 \det J_1(\tau, \delta) = \left| \frac{\partial x_i}{\partial \tau} \right| = \left| \frac{\Delta x_i}{\Delta \tau} \right| = 1 + \frac{\sqrt{(\tau+\Delta \tau)^2 - \delta^2} - \sqrt{\tau^2 - \delta^2}}{\Delta \tau}.
\]

(3.40)

A straightforward conclusion is that the Jacobian approximation depends on the meshing resolution of the transformed domain. Several decreasing values for \(\Delta \tau\) have been used to obtain Fig. 3.12:
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Fig. 3.12 Incremental approximation to the first solution of the Jacobian using several resolution values. (Right column) Absolute error with respect to the actual analytical solution of the Jacobian.

It is clear now that the finite difference Jacobian indeed converges to the true analytical Jacobian. Next the actual expression of the isorange and isoDoppler lines will be considered to develop an operative method for the DDM simulation.
3.3.3 Flat Earth Scenario

For low-to-mid altitude airborne scenarios, the Earth’s surface can be considered flat around the specular point without incurring in a significant error. This approximation simplifies the expressions of the change of variables between the surface and the delay-Doppler domains, resulting in faster processing times. The position of an arbitrary surface point now is:

\[
\tilde{r} = (x, y, 0) .
\]  

(3.41)

Additionally, the absolute delay associated to a surface point is:

\[
\tau_{xy,abs} = |\tilde{r} - \tilde{R}_i| + |\tilde{R}_y - \tilde{r}| .
\]  

(3.42)

Then, substituting Eqns. 3.1, 3.2 and 3.41, into Eqn. 3.42 results in:

\[
\tau_{xy,abs} = \sqrt{x^2 + \left(y - \frac{h_0}{\tan \gamma}\right)^2 + h_0^2} + \sqrt{x^2 + \left(y + \frac{h}{\tan \gamma}\right)^2 + h^2} .
\]  

(3.43)

The first square root can be further simplified considering that \(h_0 \gg x, y\) (assumption of very far away transmitter), and keeping the terms up to first order only:

\[
\tau_{xy,abs} = \sqrt{x^2 + \left(y + \frac{h}{\tan \gamma}\right)^2 + \frac{h_0}{\sin \gamma} - y \cos \gamma} .
\]  

(3.44)

It is usual to express the code delay with respect to the specular point delay (surface point with \(x = y = 0\)); then:

\[
\tau_x = \tau_{xy,abs}(x, y, h, h_0, \gamma) - \tau_{xy,abs}(0, 0, h, h_0, \gamma) = \sqrt{x^2 + \left(y + \frac{h}{\tan \gamma}\right)^2 + \frac{h_0}{\sin \gamma}} - \frac{h}{\sin \gamma} - y \cos \gamma .
\]  

(3.45)

The Doppler shift of each point is derived in an analog way (very far away transmitter approximation):
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\[
V_x \cdot x + V_y \cdot \left( y + \frac{h}{\tan \gamma} \right) - V_z \cdot h \sqrt{x^2 + \left( y + \frac{h}{\tan \gamma} \right)^2 + h^2}, \quad (3.46)
\]

where \( \vec{V}_t \) and \( \vec{V}_r \) are the transmitter and receiver velocities, respectively. To compute the Jacobians it is necessary to express the \( x-y \) coordinates as a function of \( \tau_{xy}, f_{d,xy} \). After some lengthy, but straightforward manipulations, the expressions \( f_1, f_2, g_1, g_2 \) for the change of variables between delay-Doppler and space domains are obtained (please refer to appendix A for the detailed expressions):

\[
\begin{align*}
    x_1 &= f_1(\tau_{xy}, f_{d,xy}, h, \gamma, \vec{V}_t, \vec{V}_r), \\
    y_1 &= g_1(\tau_{xy}, f_{d,xy}, h, \gamma, \vec{V}_t, \vec{V}_r), \\
    x_2 &= f_2(\tau_{xy}, f_{d,xy}, h, \gamma, \vec{V}_t, \vec{V}_r), \\
    y_2 &= g_2(\tau_{xy}, f_{d,xy}, h, \gamma, \vec{V}_t, \vec{V}_r),
\end{align*}
\]

which allows to compute the Jacobians associated to each solution using Eqn. 3.27. The DDM simulation using the Jacobian approach starts by setting a Cartesian mesh over an area of the delay-Doppler space with \( \Delta \tau \) and \( \Delta f_d \) resolution steps. Then, the Jacobian value at each \( (\tau_{xy}, f_{d,xy}) \) coordinate for each solution \( (x_1, y_1) \) and \( (x_2, y_2) \) (Eqn. 3.26) is obtained, along with the associated \( D, \sigma_0 \) and propagation losses values. Eventually, applying Eqn. 3.24 the DDM is obtained. The use of a Cartesian grid to map the delay-Doppler space implies that ‘forbidden’ delay-Doppler coordinates (delay-Doppler pairs that do not match to any existing delay-Doppler bin over the surface) may also be evaluated. When that happens, the obtained complex \( x-y \) coordinates must be discarded. The border between ‘forbidden’ and ‘allowed’ delay-Doppler coordinates follows the line of DDM maxima. The criterion of discarding all the complex points of the Jacobian (‘allowed’ delay-Doppler coordinates should produce pure real Jacobian values) fails around the mentioned border. Then, it is necessary to define a mask to set to zero the incorrect values: first the pure-real points are selected, and then the adjacent pixels to the selected ones are also considered (Fig. 3.13a). These pixels actually provide the highest contribution to the DDM, and by any means cannot be discarded. They can be thought as points corresponding to a pair of isorange and isoDoppler lines that do not intersect themselves, but the area around them associated to the delay and Doppler resolution widths does (Fig. 3.13b). The non-masked and masked jacobians appear in Fig. 3.13c and Fig. 3.13d.
To test this approach, the implemented simulator was able to generate DDMs by both the new Jacobian approach and by the classical approach reviewed on section 3.2 (Fig. 3.14). Simulation parameters are $\gamma = 60^\circ$, $h = 680$ km, $\Delta \tau = 0.1$ chip, $\Delta f_d = 20$ Hz, and 10 m height wind speed $U_{10} = 5$ m/s. This last parameter is used to derive the directional mean squared slope values (DMSS) that parameterize the sea surface slope’s PDF. The DMSSs have been computed using [47], scaled by 0.45 as in[48]. A desktop computer with a 3.4 GHz Pentium 4 processor and 2 GB of RAM memory is used. The difference is computed after normalizing both DDMs by their peak value. As seen in Fig. 3.15, in the worst case this difference is smaller than 1% the peak value. The computing times are ~25.000 s for the classical approach and ~5 s for the Jacobian one when meshing the xy space with a cartesian 1500x1500 point mesh. The mesh density of the scattering surface grid has been progressively increased until convergence of the computed DDM using the classical approach. This DDM is used as a reference to assessed the accuracy of the proposed method.

The flat Earth approximation simplifies the Jacobian expressions, but implies a mapping error that increases with the receiver height. For a satellite-borne receiver, the actual isolines differ
significantly from the ones obtained using the flat Earth approximations (Fig. 3.16). This difference is clearly seen in the obtained DDM for the actual spherical Earth and the flat approximation (Fig. 3.17). Therefore, a spherical Earth model would greatly improve the DDM modeling for spaceborne scenarios.

Fig. 3.14 Obtained DDM with (a) the xy integration and (b) the Jacobian approaches for the flat Earth.

Fig. 3.15 DDM difference between the xy and the Jacobian approaches after peak normalization (flat Earth).
3.3.4 Spherical Earth Scenario

The reference system defined in section 3.1.1 remains the same. However, the $z$ values of the surface are no longer considered zero, but assigned a height according to:

$$z = \sqrt{R_e^2 - x^2 - y^2} - R_e,$$

where $R_e$ is the Earth’s radius. Thus, both the plane and sphere surfaces are tangential at the specular point. Within this new reference frame the new expressions of the delay and Doppler are easily derived considering a non-zero $z$:

$$\tau_{xy} = \sqrt{x^2 + \left(y + \frac{h}{\tan \gamma}\right)^2 + \left(h - z\right)^2 - \frac{h}{\sin \gamma} - y \cos \gamma - z \sin \gamma},$$

and
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\[
f_{d,xy} = \vec{V}_i \cdot \hat{n}_i - \vec{V}_r \cdot \hat{n}_r = -V_y \cdot \cos \gamma - V_z \cdot \sin \gamma + \frac{V_x \cdot x + V_y \cdot \left( y + \frac{h}{\tan \gamma} \right) + V_z \cdot \left( z - h \right)}{\sqrt{x^2 + \left( y + \frac{h}{\tan \gamma} \right)^2 + (h - z)^2}}. \tag{3.50}
\]

Fig. 3.17 Difference of the DDM obtained with the Jacobian approach for flat Earth and the actual DDM (xy integration for the Spherical Earth Scenario). DDMs are normalized to 1 at the peak value.

For the sake of simplicity, without loss of generalization, in the derivation of Eqn. 3.50 the incident vector has been assumed to be constant and equal to \( \hat{n}_i = (0, -\cos \gamma, -\sin \gamma) \) for all surface points, as in Eqn. 3.46. This assumption may seem less accurate, since now \( z \neq 0 \), but as it will be seen later on, the difference with the DDM obtained using Eqn. 3.17 is acceptably small. The next step is the derivation from Eqns. 3.48-3.50 of a similar expression to Eqn. 3.47. However, no closed-form solutions of the following form have been found:

\[
\begin{align*}
x_i &= f_i \left( \tau_{xy}, f_{d,xy}, h, \gamma, \vec{V}_i, \vec{V}_r \right), \\
y_i &= g_i \left( \tau_{xy}, f_{d,xy}, h, \gamma, \vec{V}_i, \vec{V}_r \right), \\
z_i &= k_i \left( \tau_{xy}, f_{d,xy}, h, \gamma, \vec{V}_i, \vec{V}_r \right), \tag{3.51}
\end{align*}
\]
where $i = 1, 2$ denotes the first or the second solutions and $f_i, g_i, k_i$ denote the change of variables between the delay-Doppler and the space domains. A possible bypass to this limitation calls for considering $z$ as another parameter (such as $h$ or $y$), so that it is possible to find explicit (though very long) expressions for:

$$
x_i = f_i \left( \tau_{xy}, f_{d,xy}, h, \gamma, \vec{v}_t, \vec{v}_r, z_i \right),
$$
$$
y_i = g_i \left( \tau_{xy}, f_{d,xy}, h, \gamma, \vec{v}_t, \vec{v}_r, z_i \right). \tag{3.52}
$$

The problem is that the $(x, y, z)$ values of the surface points are not known a priori, starting from a delay-Doppler mesh. The use of the following iterative procedure overcomes the problem:

1. A first-guess value of $z_1 = z_2 = 0$ (flat Earth) is chosen.
2. An approximation to the actual $(x, y)$ values is obtained using Eqn. 3.52.
3. Eqn. 3.48 is then applied to retrieve the corresponding new values of $z_1$ and $z_2$.
4. Steps 2 and 3 are repeated until convergence to obtain an accurate estimate of $(x, y)$ as a function of $\tau_{xy}, f_{d,xy}$.
5. These estimates are then used to compute the finite difference approximation to the Jacobians (Eqns. 3.26 and 3.27), and to compute the DDM as for the flat Earth case.

![Fig. 3.18 (Top) MSE of the retrieved $z$ value at each iteration, and (bottom) difference between the actual $z$ and the retrieved $z_i$ over the simulated $(x, y)$ surface.](image)

An important issue is to determine the number of iterations needed so that the retrieved $(x, y, z)$ approximations have a minimum error. To do so, every iteration each of the two retrieved values of $z$ is compared to the actual value of $z$ of the simulated scenario, and their mean squared error (MSE) is obtained. For $h = 680$ km and $\gamma = 60^\circ$ (Low Earth Orbit LEO satellite), after 7 iterations the error is below 1 m (Fig. 3.18). The DDMs obtained applying the classical
and the Jacobian approaches for a spherical Earth surface are compared in Fig. 3.19. The difference is smaller than 7% (peak error) for the worst case, and usually well below 4%. These values are significantly smaller than the measurement variances of a real system [54], and can be used to perform systematic simulations in different configurations and sea states.

![Image](image_url)

Fig. 3.19 (Top) Obtained DDM with the Jacobian approach for Spherical Earth. (Bottom) difference with the actual DDM (xy integration for the Spherical Earth Scenario).

The result is significantly better than the obtained comparing the flat Earth Jacobian method with the precise, but extremely resource consuming method of integrating over the xy domain (Eqn. 3.17) using an spherical Earth surface (Fig. 3.17), where the error even exceeded 80%. Using the same computer described in section III, the computing time of this iterative jacobian
approach is of 600 s (7 iterations), still significantly lower than the 25000 s needed for the 1500 x 1500 points DDM.

3.4 OVERVIEW OF THE IMPLEMENTED SIMULATOR

The DDM simulation methods described above have been implemented into a simulator. The interface is shown in Fig. 3.20. The field of the upper section describes the geometry of the scenario, whereas the middle fields set the DDM parameters. Both the elevation and the wind speed can be entered as a vector so that a sweep for all the desired parameter is performed.

![Simulator interface.](image)

Fig. 3.20 Simulator interface. It allows the generation of either amplitude or power DDMs. Those last ones can also be generated with the novel Jacobian approach (DDM TF).

A detailed description of the input parameters is given in Table 3.1 and Table 3.2:
3. DDM SIMULATION

### Table 3.1 Input parameters that define the scenario in the implemented DDM simulator

<table>
<thead>
<tr>
<th>SCENARIO PARAMETERS</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_c$</td>
<td>Carrier frequency of the GNSS signal</td>
</tr>
<tr>
<td>$U_{10,\text{dir}}$</td>
<td>Direction of the 10 m height wind speed</td>
</tr>
<tr>
<td>$U_{10}$</td>
<td>Module of the 10 m height wind speed</td>
</tr>
<tr>
<td>Elevation</td>
<td>Elevation of the transmitting GNSS satellite</td>
</tr>
<tr>
<td>Receiver height</td>
<td>Receiver height over the tangent plane at the specular point</td>
</tr>
<tr>
<td>$G_0$</td>
<td>Antenna gain of the receiver</td>
</tr>
<tr>
<td>Beamwidth</td>
<td>Antenna beamwidth of the receiver</td>
</tr>
<tr>
<td>$V_t$</td>
<td>Transmitter speed (3-component vector)</td>
</tr>
<tr>
<td>$V_r$</td>
<td>Receiver speed (3-component vector)</td>
</tr>
<tr>
<td>$x_{\text{min}}, y_{\text{min}}$</td>
<td>Minimum value of the x/y coordinate of the scenario</td>
</tr>
<tr>
<td>$x_{\text{max}}, y_{\text{max}}$</td>
<td>Maximum value of the x/y coordinate of the scenario</td>
</tr>
<tr>
<td>$N_x, N_y$</td>
<td>Number of points used to sample the x/y dimension</td>
</tr>
</tbody>
</table>

### Table 3.2 Input parameters that define the simulated DDM

<table>
<thead>
<tr>
<th>DDM PARAMETERS</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\text{coh}}$</td>
<td>Coherent integration time</td>
</tr>
<tr>
<td>$d\tau, df_d$</td>
<td>Resolution in the delay/Doppler domain. The impact of the resolution is clearly appreciated in Fig. 3.21.</td>
</tr>
<tr>
<td>Threshold</td>
<td>Amplitude threshold used to compute the area/volume of the DDM. Further described in chapter 4</td>
</tr>
<tr>
<td>$N_{\tau}$</td>
<td>Number of delay bins</td>
</tr>
<tr>
<td>$N_{f_d}$</td>
<td>Number of Doppler bins</td>
</tr>
<tr>
<td># looks</td>
<td>Number of looks used when generating time series of amplitude DDMs</td>
</tr>
</tbody>
</table>
Once the parameters are set it is possible to start the DDM generation (‘start’ button in Fig. 3.19). By default both the Zavorotny-Voronovich (Z-V) and the Jacobian approaches are computed, even though it is possible to disable either of the two solutions (‘Disable’ box in Fig. 3.19). The simulation flow for the Z-V DDM can be summarized as follows:

a) Computation of the surface coordinates.

b) Obtention of the incidence and scattering vectors at each surface coordinate.

c) Computation of the delay and Doppler values over the surface applying Eqns. 3.11, 3.12, and 3.14 (Fig. 3.22).

d) Obtention of the scattering coefficient $\sigma^0$ and the antenna pattern $D$ values at each surface point by applying Eqns. 2.17 and 3.15.
e) Double loop for all the delay and Doppler bins of the desired DDM to evaluate Eqn. 3.17. In Fig. 3.23 the intersection of the delay annulus, the Doppler hyperbola, the antenna pattern, and the scattering coefficient is shown on the left. The already computed delay-Doppler bins appear on the right.

![Fig. 3.23](left) intersection of the delay annulus, the Doppler hyperbola, the scattering coefficient, and the antenna pattern. The power contribution to a given delay-Doppler bin is proportional to the sum of the pixels that intersect. (Right) DDM generation in progress.

On the other side, the implementation of the Jacobian approach for the flat Earth approximation implies the following steps:

a) Lay a Cartesian grid over the delay-Doppler space defined by the associated input parameters.

b) Obtain the WAF sampled with the input delay and Doppler resolution parameters applying Eqn. 2.14.

c) Obtain the space coordinates \((x_1, y_1)\) and \((x_2, y_2)\) associated to each \((\tau, f_d)\) (Eqn. 3.27, explicitly given in appendix A, Eqns. A.10-A.13). Then, retrieve the scattering coefficient and the antenna pattern values \(J_1(x_1, y_1), J_2(x_2, y_2), \sigma_1(x_1, y_1),\) and \(\sigma_2(x_2, y_2)\).

d) Retrieve the two jacobian solutions \(J_1\) and \(J_2\) using Eqn. 3.27.

e) Evaluate the \(\Sigma\) function throughout the \((\tau, f_d)\) space (Eqn. 3.25) and obtain the DDM convolving the WAF and the \(\Sigma\) functions (Eqn. 3.24).
For the spherical Earth scenario the steps c) and d) are repeated iteratively, as already described in section 3.3.4. The simulator also allows generating time series of DDMs by implementing Eqn. 3.16, using the ‘Time Series’ option shown in Fig. 3.19. Additionally, an interface to compare the DDM generated using the several approaches can be opened by selecting the ‘Compare’ option (Fig. 3.24).

Fig. 3.24 DDM comparison interface. The two small figures (A and B) are the operands to compare. Several unary operations can be performed on each of them (Unary operand dialog on the bottom left). The difference (or sum) between A and B is shown of the right. More specifically, the A operand has been computed assuming a flat Earth, whereas the B operand is associated to an spherical Earth scenario. The difference has been computed after the normalization of both DDM by their peak value. The extreme values (dark blue / red) take almost ±1 values. The main contribution to the difference comes from the change in the isolines due to the spherical versus flat surface models.

3.5 CONCLUSIONS

There are different approaches to the simulation of DDM. The Zavorotny-Voronovich expressions can be straightforwardly implemented to obtained either amplitude or power correlations.

Spaceborne scenario simulations increase the consumption of resources (time and memory) as the surface gets larger, until becoming not affordable at all. The use of explicit expressions for the delay and Doppler values allows very fast computation of the DDMs using the Jacobians of this change of variables between space and delay-Doppler domains. This significantly reduces the number of points to be evaluated: from all the surface points (hundreds of thousands or millions) to just the number of bins of the desired DDM.
Despite the approximations made on the isolines expressions, the obtained DDM differs less than 1% with respect to the reference one (obtained without any approximation) for the flat Earth scenario. For the spaceborne case the flat Earth approximation no longer holds, and an iterative approach to compute the Jacobians is introduced. In this case, the retrieved DDM differs less than 7% (peak difference) with respect to the reference one, which is well below the noise error in a real system [54]. This significant speed improvement allows one to undertake simulations of satellite reflections that were too slow or not feasible at all.

The overall Jacobian approach described in section 3.3 is understood as a general reference frame. Closed-form expressions for other scenarios apart of those described in the present paper can be derived in an straightforward manner and used in a similar way, for instance a low height static receiver. The described simulation approaches have been implemented into a simulator to readily obtain DDMs, compare the different approaches, and allow validating the DDM measured in field campaigns.
4. DDM PARAMETERIZATION

Throughout this work the DDM has been chosen as the optimum GNSS-R observable to be related to the sea state, since it contains all the information, as opposed to the Delay Map (also referred to as ‘waveform’). In former chapters it has been shown that each delay-Doppler bin is associated to the power scattered at different surface patches. The use of a single parameter or a set of parameters to characterize the whole DDM, and thus the sea state, requires assuming that all the considered bins lie within a single sea-state area. Such an assumption depends on the actual sea state and receiver geometry (mainly the height and the antenna pattern), but it is also a common assumption in radar scatterometers or altimeters, wind radiometers, etc. The parameterization of the DDM can accomplish a two-fold task:

- To reduce the data throughput when substituting a whole DDM with just one (or a small number) of scalars.
- To allow the study of the correlation of a chosen sea state descriptor with the DDM associated parameter. More specifically, the aim is to relate the DDM with the brightness temperature change induced by the sea state \( \Delta T_{sb}(\theta, \bar{p}) \).

Two different semi-empirical approaches to the DDM simulation are described next.

4.1 SEMIEMPIRICAL MODEL FITTING

The aim of the parameterization introduced here is to substitute the DDM itself by a set of parameters that allows regenerating the DDM at a further point in the processing chain (Fig. 4.1). To do so it is necessary to define an empirical model for the DDM. A first approximation is to use a normalized two-dimensional Gaussian function defined on the delay and Doppler domains (4.1) to model the peak normalized WAF:

\[
\text{DDM}_{\text{gauss}}(\tau, f_d) = e^{-\frac{1}{2(1-\rho^2)} \left( \frac{(\tau-m_\tau)^2}{\sigma_\tau^2} - 2\rho \frac{(\tau-m_\tau)(f_d-m_{fd})}{\sigma_\tau \sigma_{fd}} + \frac{(f_d-m_{fd})^2}{\sigma_{fd}^2} \right)},
\]

(4.1)

where \( \tau \) and \( f_d \) set the delay and Doppler coordinates, \( m_\tau \) and \( m_{fd} \) define the peak position, \( \sigma_\tau \) and \( \sigma_{fd} \) define the width of the model, and \( \rho \) governs the correlation between domains (Fig. 4.2a).
4. DDM PARAMETERIZATION

Fig. 4.1 Rationale of the parameter estimation for data throughput reduction from the receiver to the processing point.

Since the line-of-maximums of the reflected DDMs obtained from a spaceborne received has a characteristic ‘parabolic’ shape (Fig. 3.14), the following empirical quadratic transformation is introduced:

$$\tau' = \alpha \cdot (f_d - m_f)^2 + \tau \cdot \rho.$$  \hspace{1cm} (4.2)

Thus, the resulting DDM model becomes (Eqn. 4.3):

$$DDM(\tau, f_d) = e^{-\left\{ -\frac{1}{2(1-\rho^2)} \left[ \frac{1}{\sigma_f^2} \left( f_d - m_f \right)^2 + \tau - m_f \right] -\frac{1}{\sigma_f^2} \left( f_d - m_f \right)^2 + \tau - m_f \right\} \left[ \frac{m_d}{\sigma_d} \right]} \cdot} \hspace{1cm} (4.3)

The transformed Gaussian function is shown in Fig. 4.2b. Thus, the proposed fitting function has six adjusting parameters:

- The peak coordinates \((m_c, m_f)\), readily obtained from the GNSS-R observable.
- Width factors in the delay and Doppler domains \(\sigma_c\) and \(\sigma_f\).
- Correlation factor between domains \(\rho\).
- Curvature factor \(\alpha\) that determines the parabolic parameter.

Fig. 4.2 (a) Gaussian model of the DDM. (b) Transformed model of the DDM. The characteristic “boomerang-like” shape can be clearly seen.
Montecarlo simulations for a spaceborne GNSS-R receiver (further described in section 4.2) were performed to generate DDMs and derive the associated descriptive parameters for a variety of scenario geometries (Fig. 4.3). The outcome is that the proposed parameters perform well in order to describe a whole DDM, even though it is clear that the agreement is better around the DDM peak. Therefore, the parametric model allows using only six parameters to store a single DDM instead of 400 x 80 points (as in Fig. 4.3).

Fig. 4.3 (Left) Simulated DDM and (right) Parametric model DDM for (top) $U_{10} = 6$ m/s and $\theta_{inc} = 0^\circ$, (center) $U_{10} = 8$ m/s and $\theta_{inc} = 10^\circ$, and (bottom) $U_{10} = 10$ m/s and $\theta_{inc} = 40^\circ$. 
Another important issue is to relate these DDM-derived parameters to the sea state (in this study, and since no other data is available, described only by the surface wind speed). The dependence of the parameters $\alpha$, $\rho$, $\sigma_x^2$, and $\sigma_y^2$ on the roughness parameter $U_{10}$ for several incidence angles is given in Figs. 4.4 and 4.5 for two different satellite geometries.

Unfortunately, no clear relationship is appreciated, even though it is true that for low incidence angles the $\alpha$ parameter exhibits a linear correlation with the wind speed parameter $U_{10}$. Therefore, other DDM-derived parameters which have a clearer relationship with the sea roughness are needed.
Fig. 4.5 Dependence of the parameters $\alpha$, $\rho$, $\sigma^2_x$, and $\sigma^2_y$ on the wind speed for several incidence angles, using SV 7.

4.2 AREA AND VOLUME OF THE NORMALIZED DDM

It has already being noted that a sea roughness increase results in an enlargement of the glistening zone. This means that more delay-Doppler bins far off the specular point contribute to the total scattered signal, thus resulting in a wider DDM but with a lower peak. The two proposed parameters try to estimate this widening:

- The volume under the normalized power DDM (peak equal to one), which increases with increasing roughness, since the region from which the signals are scattered enlarges:

$$\text{Volume} = \int_{\tau_{\text{min}}}^{\tau_{\text{max}}} \int_{f_{d,\text{min}}}^{f_{d,\text{max}}} \overline{\text{DDM}(\tau, f_d)} \cdot d\tau \cdot df_d ,$$

(4.4)

where $\tau_{\text{min}}$, $\tau_{\text{max}}$, $f_{d,\text{min}}$, $f_{d,\text{max}}$ are the minimum and maximum values for the code offset and Doppler shift respectively, and $\overline{\text{DDM}(\tau, f_d)}$ is the normalized DDM. However, since
the numerical computation of the volume requires an infinite domain of integration both in delay and Doppler (in practice, a very large one), it is proposed to use a threshold below the maximum to truncate the section of the DDM used to compute the volume:

\[ Volume = \int_{\text{DDM}(\tau, f_d) - \text{threshold}} \int \text{DDM}(\tau, f_d) \cdot d\tau \cdot df_d, \]  

(4.5)

- The area of a section of the normalized DDM at a given threshold below the maximum:

\[ Area = \int_{\text{DDM}(\tau, f_d) - \text{threshold}} d\tau \cdot df_d, \]  

(4.6)

This threshold must be carefully selected in order to provide maximum sensitivity to the geophysical parameters. The convention used in this work is to define the threshold a percentage of the maximum. For instance, a threshold of 10% from a normalized DDM would mean to consider only the points with amplitude over 0.1. The normalization is performed using the peak power of the DDM so that its maximum is equal to one.

4.3 DDM PARAMETER DEPENDENCE ON THE SEA BRIGHTNESS TEMPERATURE

To study the behavior of the area/volume of the normalized DDM, a number of simulations have been performed with the PAU/SeoSAT simulator [51]. The SeoSAT satellite’s orbit was not yet determined at the time of performing the study, but a polar circular orbit (inclination = 98.9°) with a height of 681 km (v = 7.5 km/s) has been assumed. The orbital elements of the GPS satellites have been obtained from data given by the Center for Space, Standard and Innovation [55] and the GPS satellites and SeoSAT relative geometry is computed in 20 s steps. PAU’s antenna consists of a 7 patch hexagonal array with a half-power beamwidth of ~25°, providing a footprint of ~260 km, that best matches the glistening zone at medium wind speeds. Figures 4.6a and 4.6b show the computed DDMs (steps \( \Delta \tau = 1 \) chip, \( \Delta f_d = 500 \) Hz) for wind speeds equal to 3 and 10 m/s in the particular case that the GPS satellite is exactly located in the zenith of the GPSR receiver. A logarithmic scale has been used to better show how the tails of the function extend towards higher delays when the sea surface roughness increases. Note that the envelope (border between the bins with and without power contribution) is the same in both cases, but in the second one (Fig. 4.6b) the tails extend over a longer area.
Fig. 4.6 Simulated DDMs (log-scale) when GPS scattered signal is at nadir of the GNSS-R receiver (h = 681 km) for wind speeds equal to (a) 3 m/s and (b) 10 m/s. A reduction of 4.5 dB(AU) for the peak power is observed (AU arbitrary unit).

Figures 4.7a and 4.7b show the DDMs computed for the same wind speeds as seen in Figs. 4.6a and 4.6b, respectively. In this case the GPS scattered signal is not at nadir position, but at 12° off-nadir. As it can be appreciated, the symmetry is now broken and one tail is larger than the other. The shape of the envelope in Figs. 4.7a and 4.7b is the same, but it is different from Figs. 4.6a and 4.6b, since it is only determined by geometrical factors.

Fig. 4.7 Simulated DDMs (log-scale) when GPS scattered signal is at the half-power beamwidth of the GNSS-R receiver (h = 681 km) for wind speeds equal to (a) 3 m/s and (b) 10 m/s.

The threshold influence is shown in Figs. 4.8a and 4.8b for wind speeds from 0 to 14 m/s. The shape of the area follows also the same trend as the volume, and it is similar to that of the mean squared slope (MSS) versus wind speed [30], which suggests that both observables are also related to the same geophysical parameter. As expected, both the volume and the area decrease as the threshold increases, and the lower the threshold, the larger the sensitivity (largest output change for same input change) with respect to the wind speed. Similarly, the
\( \Delta T_{p, \theta} (\theta=0^\circ, U_{10}) \) (increase of the brightness temperature for the p polarization at nadir position as a function of the surface wind speed \( U_{10} \)) associated to the same roughness conditions was computed as described in [56]. It is then possible to link the DDM with the \( \Delta T_{p, \theta} (\theta=0^\circ, U_{10}) \), which can be used to compensate for this term in the sea surface salinity retrieval algorithms. At present, this dependence is only empirically known with respect to the wind speed and/or the significant wave height ([18] or [19]) or through numerical models that rely on the wind speed dependence [57] (and eventually others such as the wave age [58] and other surface processes [59]).

Even though \( \Delta T_{p, \theta} (\theta, \nabla_{VDDM}) \) and \( \Delta T_{p, \theta} (\theta, \nabla_{A_{DDM}}) \) will only be known when the DDMs and the brightness temperatures are measured simultaneously, an estimate can now be obtained by using the wind speed as intermediate variable \( \Delta T_{p, \theta} (\theta, \nabla_{VDDM} (U_{10})) \) and \( \Delta T_{p, \theta} (\theta, \nabla_{A_{DDM}} (U_{10})) \). This relationship is shown in Figs. 4.9a and 4.9b, which have been obtained by replacing in Figs. 4.8a and 4.8b the wind speed dependence of the L-band brightness temperature at nadir derived using the Small Perturbation Method/Small Slope Approximation (SPM/SSA) [58], [60] (Fig. 4.10). This approach is one of the three algorithms implemented in the SMOS salinity retrieval processor [61].
Fig. 4.9 Relationship between $\Delta T_{p,p} (\theta = 0^\circ, \bar{\rho})$ and the (a) area and (b) volume under the normalized DDM.

Fig. 4.10 $\Delta T_{p,p} (\theta = 0^\circ, U_{10})$ as a function of wind speed computed with the SPM/SSA method and Elfouhaily’s sea surface spectrum.

4.3.1 Practical Considerations

The results presented in Figs. 4.9a and 4.9b assume a perfectly measured DDM in steps $\Delta \tau = 1$ chip, $\Delta f_d = 500$ Hz computed over a very large delay-Doppler region so that the contributions of the tails outside the integration region are negligible. In this Section four aspects that appear in the practical implementation of the DDM generator are considered:

1) The finite size in both the delay and Doppler variables: $[\tau_{\min, \max}] \times [f_{d,\min, f_{d,\max}}]$. 

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2) The DDM resolution: step width $\Delta \tau$ and $\Delta f_d$,

3) The quantization of the DDM, or number of levels in which it is coded, and

4) The effect of the noise.

Since the sea state dependence is very similar both for the DDM area and volume (Figs. 4.8a and 4.8b), in the remaining only results for the DDM volume are presented.

### 4.3.2 DDM Finite Size

From a practical point of view, it is convenient to reduce the size of the computed DDM, either using a Field-Programmable Gate Array (FPGA) [62] or in software after the signals are acquired. The idea is to find the minimum size around the DDM peak that provides an estimated area or volume close enough to the ideal case (Figs. 4.8a and 4.8b). Results are shown in Figs. 4.11a and 4.11b for the estimated volume under the DDM. As expected, due to the widening of the DDM tails with increasing wind speed, the saturation occurs at higher delays, while the saturation Doppler frequency slightly increases. Consequently, the minimum DDM size so that truncation errors are negligible is 205 chips and ± 11 KHz (≤ 2% error) for $U_{10}=10$ m/s and 220 chips and ± 11 KHz (≤ 2% error) for $U_{10}=14$ m/s. This represents an area which is much larger than the antenna footprint, which is actually limiting the field of view from which reflections can be seen (~260 km). This footprint is compatible with the size of the final salinity products to be derived, from 100 to 300 km of spatial resolution every 10 to 30 days ([63] and [12]).

![Fig. 4.11 Volume under the normalized DDM as a function of the normalized DDM size (chips x KHz around DDM maximum) for wind speeds equal to (a) 3 m/s and (b) 10 m/s.](image)
4.3.3 DDM Resolution

The DDM computation in real-time is also limited by the number of points used to sample the DDM in each variable \( N_t = (\tau_{\text{max}} - \tau_{\text{min}})/\Delta \tau + 1 \) and \( N_d = (\Delta f_{d\text{max}} - \Delta f_{d\text{min}})/\Delta f_d + 1 \). Therefore, the accuracy of the estimated area or volume of the normalized DDM is also affected by the DDM resolution.

Even though the DDM is a wide and slowly varying function, undersampling it results in a bias in the volume or area estimates, which translates into a bias in the estimated brightness temperature correction associated due to the sea state \( (\Delta T_{\text{b,p}}(\theta, \bar{\rho})) \). To estimate this impact, the DDMs have been computed for several wind speed conditions decimating both in the delay and Doppler variables \( (\Delta \tau = N_{\text{decim. factor}, \tau}, [\text{chips}], \Delta f_d = M_{\text{decim. factor}, f_d}, [\text{Hz}]) \), and results have been compared to the obtained one using \( \Delta \tau = 1 \) chip, \( \Delta f_d = 500 \) Hz. Simulation results for the bias in \( \Delta T_{\text{b,p}}(\theta, \bar{\rho}) \) as a function of the decimation factors \( N_{\text{decim. factor}, \tau} \) and \( M_{\text{decim. factor}, f_d} \) are shown in Figs. 4.12a and 4.12b for wind speeds 3 and 10 m/s. In order to have a bias in the brightness temperature correction due to sea state smaller than, for example 0.05 K (~0.1 psu SSS error), the maximum decimating factors can be \( N_{\text{decim. factor}, \tau} = 1 \) (1 chip), and \( M_{\text{decim. factor}, f_d} = 2 \) (1 KHz).

![Fig. 4.12a](image1.png)

![Fig. 4.12b](image2.png)

Fig. 4.12 \( \Delta T_{\text{b,p}}(\theta = 0^\circ, \bar{\rho}) \) error as a function of the decimation factor in delay and Doppler frequency for wind speeds equal to (a) 3 m/s and (b) 10 m/s.

4.3.4 DDM Quantization

The quantization of the DDM values into a finite number of levels results in round-off errors that also produce a bias in the area or volume estimate. And then, it translates into a bias in the brightness temperature correction to be applied due to sea state. This bias decreases as the number of levels increases. The impact of the number of quantization levels in the DDM
volume has been studied for wind speeds from 3 to 14 m/s. Figure 4.13a shows the bias in the DDM volume estimate as a function of the quantization levels from 2 (1 bit) to 1024 (10 bits). Figure 4.13b shows the resulting bias in the brightness temperature correction. As it can be appreciated, for a bias smaller than 0.05 K, the number of levels must be larger than 256, and above 512 the bias is nearly independent on the sea state. This means that the GNSS receiver to be implemented in chapter 5 has to compute each power DDM bin with at least 9 bits.

![Image](image_url)

**Fig. 4.13** Error in (a) estimated volume under the normalized DDM and (b) associated $\Delta T_{_{sp}}(\theta = 0^\circ, \beta)$ error due to quantization.

### 4.3.5 Noise Impact

In the previous sections the effect of additive noise has not been taken into account. However, the performance of the brightness temperature correction ultimately depends on the amount of noise present in the DDMs. Taking into account that: 1) the correlation time of the sea surface’s backscatter at L-band is on the order of a few milliseconds, and 2) the possibility of having a sign change associated to the navigation bit, a 1 ms coherent integration time has been assumed, followed by incoherent integration. In [40], the average noise level from a receiver in a LEO is given by:

$$N = T_s^2 \left( \frac{K_s \cdot P + K_p T_{\text{sys}} B}{\sqrt{N_{\text{incoherent}}}} \right), \quad (4.7)$$

which includes two terms: a speckle one, that depends on the scattered power $P$ and $K_p = 1/\sqrt{h}$ which is related to the satellite height, and a Gaussian thermal noise which depends
on the Boltzmann constant \(k_B\), the system temperature \(T_{sys}\), and the system bandwidth \(B\).

According to [40], the noise of the DDM module has a Rayleigh-Rice distribution, since the noise in the real and imaginary parts are zero-mean Gaussian noises. This means that a noise increment not only increases the DDM variance, but its mean as well. This is a critical point, since the brightness temperature correction \(\Delta T_{bp}(\theta, \hat{\theta})\) is computed through the DDM volume, which also increases with the noise level, resulting in a bias in \(\Delta T_{bp}(\theta, \hat{\theta})\). In order to compensate for this factor, the noise level has to be accurately estimated. To do this, the mean value of the DDM points corresponding to “forbidden” delay-Doppler coordinates (coordinates that cannot exist for a given geometry, dark regions in the left of Figs. 4.6 and 4.7) is computed, and then subtracted from the noisy DDM.

As shown in Fig. 4.14a, the volume estimate improves as the incoherent integration time increases. Figure 4.14b shows the values of estimated brightness temperature corrections for a coherent integration time of 1 ms, and 1 ms, 10 ms and 100 ms incoherent integration time. Figure 4.14c shows the associated error when comparing Fig. 4.14b with the noiseless ideal case. As it can be appreciated, the impact of noise increases with wind speed (sea state), and to achieve an error in the brightness temperature correction smaller than 0.05 K, the integration time must be 100 ms (1 ms coherent integration, followed by 100 ms incoherent integration), for wind speeds below 11 m/s, (at 14 m/s this error is 0.13 K for the same incoherent integration time).

Fig. 4.14 a) Estimated normalized DDM mean volume as a function of wind speed after subtraction of the estimated noise floor, b) Estimated \(\Delta T_{bp}(\theta = 0^\circ, \hat{\theta})\), and c) associated error as a function of the wind speed. DDM computed for 1 ms coherent integration time and variable incoherent integration time (1, 10, 100 ms).
4.4 Conclusions

Two semi empirical approaches to the DDM parameterization have been proposed. The first one allows to regenerate a reflected DDM coded with six scalar values, but it is not useful to derive a relationship with respect to the sea state. The second approach integrates the normalized DDM to obtain the area and/or volume under it. These descriptors have been related to the wind speed used as single descriptor of the sea state. This relationship has been used to link the brightness temperature correction due to the sea state, parameterized also in terms of the wind speed only, with the area or volume under the normalized DDM. This must be considered an intermediate step, since the ultimate goal is to directly relate the area or volume under the DDM to the brightness temperature correction, when measurements are available in the future. It has been shown that both the area and the volume as a function of the wind speed exhibit the saturation trend for higher wind speeds. Moreover, the requirements of the DDMs to be computed in order to accurately correct for the sea state impact on the L-band brightness temperature. These requirements are summarized as: DDM size 220 chips x ±11 KHz, DDM resolution = 1 chip x 1 KHz, DDM quantization of at least 9 bits, and $T_i = 1$ ms coherent integration followed by 100 ms incoherent integration assuming a GNSS-R receiver at 680 km height and a 25° beamwidth antenna. These results are in line with the limited experimental data available from the UK-DMC GPS-R experiment [41]. As seen in Fig. 4.15, an incoherent integration time of 100 ms brings the DDM above the noise floor.

Fig. 4.15 UK-DMC GPS-R data (courtesy of SSTL) processed with UPC-developed software for different incoherent integration times: a) 10 ms, b) 40 ms, c) 80 ms, d) 120 ms, e) 200 ms, and f) 800 ms. It is appreciated that for an incoherent integration time of some 100 ms the shape of the DDM becomes clearly defined.
5. GNSS-R RECEIVER DESIGN, IMPLEMENTATION, AND VALIDATION

5.1 RECEIVER DESIGN GOALS

The usefulness of the DDM as the GNSS-R observable to the related to the sea state has been described in former chapters. Therefore, the next logical step in order to study the DDM relationship with respect to the sea roughness is to acquire and process GNSS-R data. To do so it was necessary to implement from zero a GNSS-R receiver.

5.1.1 Existing GNSS-R Receivers

To author’s knowledge, as of March 2004, most of the existing GNSS-R receiver architectures just measured the correlation peak or the cut in the delay axis of the DDMs once the Doppler frequency had been compensated, which is called a ‘waveform’. In the following the implementation of a real-time full-DDM GNSS-R receiver will be introduced. As already mentioned, the use of the whole DDM shall provide a better sea roughness estimate, and will allow to further develop other GNSS-R processing approaches.

5.1.2 GNSS-R within the PAU Project

As described in section 1.5, the PAU system operates completely with digitized signals. The sampling frequency determines how many samples constitute the minimum ‘data strip’ to be used. Because the length of the C/A codes is 1 ms, and the sampling frequency for the whole of the PAU system [64] is 5.745 MHz, each data strip has 5745 samples, sampled at 1 bit. This severe quantification of the input signal has a little effect on the GPS signal, just a small loss in signal-to-noise ratio (SNR), while it allows for a much simpler design.

---

1 For simplicity, the sign bit is retained after sampling at 8 bits, calibration and digital beamforming.
2 According to [37], the SNR decreases 1.96 dB and 0.55 dB after 1-bit and 2-bit quantization, respectively.
5.1.3 Specifications

One of the goals of the present Ph.D. Thesis was to implement a real-time GNSS reflectometer with the ability to measure full DDMs, able to be turned into an airborne and spaceborne demonstrator with few additional minor changes. A basic coherent integration time of 1 ms was selected, along with a configurable delay and Doppler resolution (better than 1 chip and 1 KHz respectively) and the ability to track up to 4 satellites simultaneously.

5.2 PRACTICAL ISSUES FOR DDM COMPUTATION

In order to compute the DDMs it is necessary to select the approach. As further discussed on the present section, both a time or frequency domain (FFT) approach to the correlation of the incoming scattered GNSS-R signal with the local replicas are feasible.

5.2.1 Implementable DDM Expression

For the sake of simplicity, let us recall here the expression of the amplitude DDM given in Eqn. 2.12:

\[ DDM(t, \tau, f_d) = \int_0^\infty u(t + t') \cdot CA(t + t' + \tau) \cdot e^{j2\pi(f_d t') \cdot dt'}, \]

where \( u(t) \) is the received signal, \( CA(t) \) the locally generated replica of the C/A PRN code, and \( \omega_d \) the Doppler angular frequency (\( \omega_d = 2\pi f_d \)). Decomposing \( u(t) \) in its in-phase and quadrature (I/Q) components, and expressing the complex exponential in terms of sinus and cosines, it is clear that each delay-Doppler coordinate of a DDM can be obtained from the combination and accumulation of four partial products:

\[
\begin{align*}
DDM &= \int_{CA} (CA \cdot \cos + CA \cdot \sin + j(CA \cdot \cos - CA \cdot \sin)) \cdot dt, \quad f_d > 0Hz, \\
DDM &= \int_{CA} (CA \cdot \cos - CA \cdot \sin + j(CA \cdot \cos + CA \cdot \sin)) \cdot dt, \quad f_d < 0Hz,
\end{align*}
\]

As it will be shown further on, the used approach generates the C/A codes corresponding to all the existing delay coordinates, the demodulating tones associated to all the Doppler bins, and then combines them with the I/Q inputs to form the four partial products (Eqn. 4.9) needed to compute the complex DDM.
5.2.2 Quantization of the Demodulating Tones

It has been stated that the I&Q signal components are 1-bit quantized. However, when trying to quantify in a similar fashion the internally-generated demodulating tones, the shape of the obtained DDM is significantly affected, as seen in Fig. 5.1 here.

The exponential term that appears in Eqn. 4.8 can be decomposed using Euler’s formula:

\[ e^{j\omega t} = \cos \omega t + j \cdot \sin \omega t \quad . \tag{4.10} \]

Then, it is possible to quantize both the sine and cosine with \( N \) bits (\( 2^N \) levels):

\[ Q_N \left[ e^{j\omega t} \right] = Q_N \left[ \cos \omega t \right] + j \cdot Q_N \left[ \sin \omega t \right] \quad , \tag{4.11} \]

where the operator \( Q_N[\cdot] \) denotes the quantization with \( 2^N \) levels. Thus, substituting the exponential term in Eqn. 4.8 by Eqn. 4.11 it is possible to study the impact of the number of levels \( N \) on the generated DDM. On Fig. 5.2 the Doppler strips for a fully compensated delay offset using several quantization levels are depicted, along with the associated absolute error with respect to the non-quantized WAF. The relative error along the whole strip as a function of the number of bits is given in Fig. 5.3. According to it, using 8 bits the error is below 0.2 %, and thus for an 8-bit quantization no there is no significant difference when comparing to the non-quantized demodulation.
5. GNSS-R RECEIVER DESIGN, IMPLEMENTATION, AND VALIDATION

5.2.3 Phase Continuity of the Demodulating Tones

Let’s suppose a direct GPS L1 C/A code signal demodulated to baseband:

\[ s(t) = CA(t - \tau)e^{j\omega_0 t} \]  \hspace{1cm} (4.12)
The correlation with the locally generated codes and tones with their respective delay and Doppler shifts yields the Delay-Doppler map. It reaches the peak value when the code is perfectly aligned and the residual Doppler frequency is fully compensated. The time series of this maximum is to be studied. Let’s first express the DDMs as a time series of elements obtained every 1 ms:

\[
DDM(t' + nT_i) \equiv DDM[n] = \int_{t_i} S(t' + nT_i)CA(t' - \tau)e^{-j\omega_d t'} dt',
\]  

(4.13)

where \(t'\) varies between 0 and \(T_i\) (1 ms), \(n\) is the DDM time ordinal (integer number), and \(\omega_{d2}\) is the locally generated compensation frequency. Substituting 5.5 into 5.6 and operating yields:

\[
DDM[n] = Y[n] = e^{j\omega_{d1} nT_i} \cdot e^{j\frac{\Delta\omega T_i}{2}} \cdot T_i \cdot \sin\left(\frac{\Delta\omega T_i}{2}\right)\]

(4.14)

for a perfectly aligned CA code, where \(\Delta\omega = |\omega_{d1} - \omega_{d2}|\). It is noteworthy than, in the situation described here, every 1 ms the phase of the locally generated tone starts again from zero. This yields a phase that increases linearly as a function of the time index \(n\) (Fig. 5.4):

\[
\angle Y[n] = n \cdot \omega_{d1} \cdot T + \frac{\Delta\omega T_i}{2}
\]

(4.15)

Therefore, according to Eqns. 4.14 and 4.15, the peak amplitude shall remain constant with \(n\), whereas the phase increases linearly. Such a linear increase can be compensated, if necessary for coherent integration purposes, either at the time of generating the DDM or during the post processing. There are two possible scenarios to perform this compensation:

- If \(\Delta\omega = 0\):

  \[
  \angle Y[n] = n \cdot \omega_{d1} \cdot T
  \]

  (4.16)
Then the phase at the peak can be fully compensated by applying a phasor $e^{-j\omega_{1}nT_{s}}$.

- If $\Delta \omega \neq 0$, the compensation will be applied using the local phasor $e^{-j\omega_{2}nT_{s}}$. Then, the resulting compensated phase will be (Fig. 5.5):

$$\angle Y[n]_{\text{compensated}} = n \cdot \Delta \omega_{2} \cdot T_{s} + \frac{\Delta \omega_{2} T_{s}}{2}.$$  \hspace{1cm} (4.17)

Thus, since the residual Doppler is not fully compensated the phase of the maximum will still increase linearly, the faster the higher the frequency difference is.

![Fig. 5.5 DDM peak qualitative phase behavior as a function of the time ordinal n and the residual Doppler frequency $\Delta \omega$. Note that when the latter is zero, the phase is fully compensated between successive DDMs.](image)

5.2.4 FFT Approach to Perform the Correlation

The correlation represented by Eqn. 4.8 can be efficiently implemented by means of Fast Fourier Transforms (FFT), as it will be seen in section 5.4.2.3. However, such an approach provides the correlation value for all the possible time delays of the correlated signal. Since the goal is to obtain the correlation values around the maximum, for a real-time approach the FFT method results in a waste of time and resources, and it is no longer considered.

5.2.5 MAC Approach to Perform Correlation

The multiply and accumulate approach (MAC) is slow for delay determination (i.e., processing the whole range of delay shifts), but performs better than the FFT one when trying to obtain the correlation value of a limited set of delay bins. This can be illustrated considering the approximated number of operations required to compute a single delay strip of the DDM using either the FFT or the MAC approaches. For the MAC approach, a single delay bin implies performing $N$ complex products and $N-1$ complex sums, being $N$ the number of samples in 1 ms (i.e., the period of the GPS CA code). $N$ is further related to the sampling frequency of the system:
Then, the total number of operations needed to obtain a delay strip with $N_t$ bins is:

$$
\#OP_{MAC} = N_t \cdot (2N-1) = N_t \cdot (2 \cdot f_s \cdot 10^{-3} - 1)
$$

(4.19)

On the other hand, according to [65], the required number of operations to compute an FFT/IFFT, supposing that $N$ is a power of two, is of $(N/2) \cdot \log_2 N$ complex products and $N \cdot \log_2 N$ complex sums. Considering that two FFTs, one IFFT, and $N$ complex products are needed to obtain the whole delay strip (refer to section 5.4.2.3), the total number of operations now is:

$$
\#OP_{FFT} = \frac{9}{2} \cdot N \cdot \log_2 N + N = \frac{9}{2} \cdot f_s \cdot 10^{-3} \cdot \log_2 \left( f_s \cdot 10^{-3} \right) + f_s \cdot 10^{-3}
$$

(4.20)

Equating Eqn. 4.19 by Eqn. 4.20, and considering that $N >> 1$, it is possible to determine the maximum number of delay bins for which the MAC approach is more efficient than the FFT one (i.e., the required number of operations becomes equal):

$$
N_{t,\text{max}} = \frac{9}{4} \cdot \log_2 \left( f_s \cdot 10^{-3} \right) + \frac{1}{2}
$$

(4.21)

Particularizing Eqn. 4.21 for $f_s = 5.745$ MHz, $N_{t,\text{max}} \approx 29$ bins. Here it would be necessary to recall that the actual performance of the FFT computation not only depends on the used algorithm, but also on the actual value of $N$ and on other system-related constraints (memory, bus width, etc.), that degrade the performance given in Eqn. 4.20. In the worst case it could be considered that the FFT performs similarly to the full DFT. Then, recalling that a single DFT/IDFT point implies $N$ complex products and $N-1$ complex sums, the number of operations to obtain the delay strip now is:

$$
\#OP_{DFT} = 3 \cdot N \cdot (2N-1) + N = 6N^2 + N = 6 \cdot \left( f_s \cdot 10^{-3} \right)^2 + f_s \cdot 10^{-3}
$$

(4.22)

Combining Eqns. 4.19 and 4.22 yields:

$$
N_{t,\text{max,DFT}} = 3 \cdot f_s \cdot 10^{-3} + \frac{1}{2}
$$

(4.23)

That results in $N_{t,\text{max,DFT}} \approx 17236$ bins for $f_s = 5.745$ MHz. Eventually, considering the former discussion, the MAC approach was selected to be implemented within the GNSS-R receiver.
5.2.6 Feasible Architectures

A number of feasible architectures were considered for implementing Eqn. 4.9 using the MAC approach, each of them located on a different coordinate of the speed-complexity plane (Fig. 5.6). Since the period of a CA code is of 1 ms, and the sampling frequency of 5.745 MHz (refer to section 1.4.1), the basic data strip is composed of 5745 samples. A comparison of the performance of these three approaches for the particular situation of 4 satellites and 256 DDM bins (16 delay bins x 16 Doppler bins) follows:

1. **Serial**: For each delay and Doppler bin the 1 ms long sampled sequences are multiplied sample by sample, one sample at a time, and accumulated until reaching the last sample. It is slow (double loop) but easy to implement, and requires few hardware resources. The minimum system frequency required is (considering an acquisition of 4 satellites and a DDM with 256 bins):

   \[ f_{\text{min}} = 5745 \cdot N_{\text{satellites}} \cdot N_{\text{samples}} \left[ \text{kHz} \right] = \frac{N_{\text{satellites}} = 4}{N_{\text{samples}} = 256} = 5.9 \left[ \text{GHz} \right] \quad . \quad (4.24) \]

   Such value is far from implementable with the nowadays existing digital logic.

2. **Parallel**: The 1 ms long sequences of the I&Q signal components, along with all the possible code offset and Doppler phasor sequences are generated. Then a matrix of multipliers and accumulators obtains all the DDM bins. It is a fast, but very hardware consuming solution (thousands of registers), with a low duty cycle (once in 1 ms). The resulting lower bound for the system frequency now is:

   \[ f_{\text{min}} = N_{\text{satellites}} \left[ \text{kHz} \right] = \frac{N_{\text{satellites}} = 4}{N_{\text{samples}} = 256} = 4 \left[ \text{kHz} \right] \quad . \quad (4.25) \]

3. **Semi-parallel**: Mix of the two former methods. All the codes and phasors associated to the DDM bins are processed simultaneously, multiplying sample by sample, one at a time, and accumulating for each bin until reaching the last samples. It embodies a trade-off between speed and complexity. The minimum system frequency is:

   \[ f_{\text{min}} = 5745 \cdot N_{\text{satellites}} \left[ \text{kHz} \right] = \frac{N_{\text{satellites}} = 4}{N_{\text{samples}} = 256} = 23 \left[ \text{MHz} \right] \quad . \quad (4.26) \]
The semi-parallel approach was the one chosen and implemented, since it balances speed and hardware complexity. A sampling frequency of 23 MHz can be matched without problem by the existing FPGA as of January 2005, able to work up to 100-200 MHz depending on the particular design of the implemented digital system.

5.3 HARDWARE PLATFORM AND DEVELOPMENT ENVIRONMENT

The reflectometer system has been implemented using the AVNet VIRTEX-4 LX 60 development board that includes the FPGA core, SDRAM RAM (32 MB), Flash ROM (8 MB), an UART serial interface, an USB 2.0 serial interface, an Ethernet connection, an OLED display and several other connectors (Fig. 5.7).
The system clock has a frequency of 100 MHz, thus exceeding the minimum required system frequency (Eqn. 4.26). This additional processing capability can be used to either increase the number of bins of the DDM or to allow to track more satellites simultaneously.

5.4 PAU-GNSS/R SYSTEM LAYOUT

The PAU-GNSS/R is a part of the PAU concept described in section 1.4. In the PAU-16 instrument (section 1.4.1) it shares the RF & IF front-end and the A/D converters (ADC) with the PAU-RAD L-band radiometer, in charge of the beamforming. Oppositely, in the PAU-SA instrument (section 1.4.2) the PAU-GNSS/R has its own dedicated receivers and ADCs.

5.4.1 System Overview

The main dataflow path of the system is that of the reflected GPS signal (Fig. 5.8). The PAU-RAD [33] section of the instrument sends the digitized and quantized I/Q raw data corresponding to the particular beams composed (1 or 4) to a buffer located in the reflectometer FPGA. This buffer plays a key role in the system, since it allocates the two I/Q 5745-sample strips (5.745 MHz · 1 ms) for the up to 4 selected beams, to allow them to be processed at a higher clock rate using hardware reuse techniques. The buffer is based in two RAM-like registers that change their respective inputs and output connections. One is being written while the other is being read to generate the DDM. When the input register is full, it is swapped with the other one. From here the data is sent in blocks of 1 ms (5745 samples at the sampling frequency of the system corresponding to the period of the C/A code) to the DDM generator. Then, once the new DDM is ready, it is sent out of the system to a terminal computer, where it will be processed and stored. The transfer of up to 4 complex-valued DDMs every 1 ms is performed using the USB 2.0 protocol. The resulting throughput is 2 (real and imaginary parts) times 32 bits per DDM point times 4 beams times 16 x 16 points = 8 Kbytes every 1 ms, that is 7.8 MB/s. In fact, in addition to the DDM data, the time tag of the raw data and the delay and Doppler center coordinates used for its computation are transmitted as well, since not all of the 32 bytes are used to code the DDM point values. Whenever the first of the 4 DDMs has been successfully received by the host computer, an interrupt is issued in the reflectometer system by the USB FPGA controller and the parameters for the generation of the next DDM are transferred to the DDM generator, which starts the computation using the data of the corresponding beam. After 5745 clock cycles (1 ms of data) the DDM generator activates the ‘data_ready’ signal. The microprocessor program asks the USB controller to read the DDM values from the output interface of the DDM generator, and to send these values to the host.
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PC, closing the cycle of processing and data transfer. The integration time can be freely configured, since every 1 ms a new DDM is obtained. It is up to the receiving program to accumulate as many of them as configured, either coherently (in amplitude and phase) or incoherently (in absolute values).

The direct signal from the satellite is also processed to obtain an accurate estimate of the signal delay. To do so an upwards-looking GPS antenna receives the signal and feeds it into an A/D converter. A second buffer stores these data until they are sent to a ‘delay offset’ block that estimates the delay offset. A third dataflow path is that of the data packets generated by a commercial GPS receiver. They enter the FPGA using an RS-232 interface using the GPS-standard TSIP protocol, and provide the system with parameters of interest such as the elevation, azimuth, power level, and Doppler of the visible satellites, simplifying to a large extent the FPGA design. Inside the FPGA a microprocessor, a data bus and RAM elements are synthetized. Several peripherals are attached to the bus, as it will be further described.

5.4.2 VHDL Peripherals

VHDL stands for VHSIC Hardware Description Language, where VHSIC is the acronym of Very High Speed Integrated Circuit. It is a digital hardware description language widely used for programming Programmable Logic Devices (PLD), Field Programmable Gate Arrays (FPGA), and Application-Specific Integrated Circuits (ASIC). It is also known as the IEEE standard ANSI/IEEE 1076-1993. One of its main features is its platform independence, since it describes the
desired circuit behavior. Then, each specific device will implement such behavior with the particular components it has, translating the VHDL file with its own dedicated synthesizer. Another feature contributing to its worldwide use are its public availability, and the endless code reuse capabilities. In addition to the behavioral description, a digital circuit can also be described using other approaches:

- **DATAFLOW:** Description of concurrent signal assignments.
- **STRUCTURAL:** The described circuit is composed of several instances of basic components. A higher hierarchy design is achieved by connecting the ports of these instances with internal signals or with ports of a higher hierarchy circuit.
- **MIXED:** Combination of any of the behavioral, dataflow, and structural approaches.

The description mode selection depends on many factors (performance on a specific platform, portability, code writing time...) and it is not univocal. However, for complex designs most commonly the mixed approach is used. In Table 5.1 four feasible implementations of a two-input multiplexer are given. A and B are the input ports, S is the selection signal, and X is the output port.

### Table 5.1 Four VHDL implementations of the same two-input multiplexer

<p>| | |</p>
<table>
<thead>
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</table>
| #1 |  \[
X <= A \quad \text{when} \quad S = '1' \quad \text{else} \quad B;\]
| #2 |  \[
\text{with} \quad S \quad \text{select} \quad X <= A \quad \text{when} \quad '1' \quad \text{else} \quad B;\]
| #3 |  \[
\text{process}(A,B,S) \\
\text{begin} \\
\quad \text{case} \quad S \quad \text{is} \\
\quad \quad \text{when} \quad '1' \quad \Rightarrow \quad X <= A; \\
\quad \quad \text{when} \quad \text{others} \quad \Rightarrow \quad X <= B; \\
\quad \text{end case}; \\
\text{end process;}
\]
| #4 |  \[
\text{process}(A,B,S) \\
\text{begin} \\
\quad \text{if} \quad S = '1' \quad \text{then} \\
\quad \quad X <= A; \\
\quad \text{else} \\
\quad \quad X <= B; \\
\quad \text{end if}; \\
\text{end process;}
\]
Other alternative languages to VHDL are ABEL, AHDL, or Verilog. There are several available references on the VHDL topic. Some updates ones include [66], [67], or [68]. Next the implemented VHDL cores to implement the PAU-GNSS/R instrument are described.

5.4.2.1 DDM Generator

The core of the system is a full-custom designed DDM generator that interacts with other in-house and off-the-shelf VHDL cores. As shown in Fig. 5.9, it is a synchronous block that receives the I/Q samples in a serial fashion, as well as the Doppler values and C/A code samples for all the DDM points. The outcome are the four partial products (Eqn. 4.9) that are conveniently added or subtracted (depending on the sign of the Doppler shift) and accumulated during 5745 clock cycles (1 ms of data) to obtain the real and imaginary parts of each DDM coordinate.

![Fig. 5.9 Delay Doppler Map (DDM) generator architecture. The 'Amps to 2C' block performs the two's complement encoding, whereas the 'AC' block accumulates the inputs.](image)

In Fig. 5.9 it is seen that the I, Q and C/A code inputs are delayed by means of shift registers so that they are synchronized with the demodulating tones generated in the block signs_and_amplitudes (Fig. 5.10a).

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The values fed to this component determine the Doppler frequencies that are generated (they are the multiplying factors of the base frequency of the block), and thus the sign (1 bit) and amplitude (7 bits) of each frequency are obtained. Then, the block `signs_combinations` generates, for each DDM point, the sign of the four partial products that compose it (CA·I·SIN, CA·I·COS, CA·Q·SIN and CA·Q·COS). The core of this component is the oscillator block (Fig. 5.10b), that every clock cycle computes one sine and one cosine sample, at a frequency set by the input value \( \alpha \). To do so every clock cycle the value of the variable `full_phase` is increased in \( \alpha \) units (Fig. 5.10b). The 9 most significant bits (trunc_phase) out of 18 are then fed into the oscillator_sinc block to generate the sin and cos outputs. This allows one to have a smaller frequency step without having to use a large number of bits to quantify the phase that is converted into sine/cosine amplitude. The oscillator_sinc component is located at the bottom of the hierarchy level (Fig. 5.9 and Fig. 5.10). To efficiently generate the sine and cosine values, it has been taken into account that the output signal shall be quantized with a finite number of levels. The first Most Significant Bit (MSB) tells whether the sine value is positive or negative, whereas the second MSB has information on the slope sign (Fig. 5.11). The remaining 7 bits indicate the position within the first wave quarter. Instead of dividing the amplitude in equal-length intervals, the phase domain has been divided in such a way, so that the 7-bit phase of the wave quarter have a linear correspondence with the 7 bits that codify the amplitude. Therefore, if no further corrections were made a triangular wave would be obtained. A correction for each level (from 0 to 127) that minimizes the error when comparing it with an ideal quantized sine has been computed, and it is then added to it. At the same time, the cosine is obtained by negating the 7 LSB’s of the truncated phase, thus obtaining the complementary level:

\[
Level_{\text{cos}} = 127 - Level_{\text{sin}},
\]

(4.27)

The cosine sign (the MSB of cos) is computed as the exclusive OR of the sine’s two MSB:

\[
MSB_{\text{cos}} = MSB_{\text{sin}} \oplus (MSB-1)_{\text{sin}},
\]

(4.28)
The sine and cosine waveforms are generated with 8-bit, since the preliminary simulations indicated that using fewer bits resulted in a severe DDM deformation (section 5.2.2).

The simultaneous generation of several different-delay C/A codes is achieved with the block CA_generator (Figs. 5.9 and 5.12). As explained in chapter 2, these codes are the product of two Maximum Length Sequences (MLS) $G_1$ and $G_2$ obtained using their respective linear feedback shift registers (LFSR). The satellite unique C/A code is determined by the delay of $G_2$ with respect to the non-delayed $G_2$. Therefore, to obtain a C/A code delayed n chips, it is necessary to generate a $G_2$ sequence delayed n chips and also a $G_2$ sequence delayed n chips plus the offset that determines the satellite ID. To obtain these delayed sequences the 10-bit M1 and M2 masks determine the values of the G1 and G2 registers that are used to obtain the delayed outputs. These masks are pre-calculated using state-transition matrices, and are stored for all the possible 1022 chip-shift values [69]:

$$M_i(n) = \left[ \begin{array}{c} \vec{p}_i \\ ID \\ 0 \end{array} \right] \cdot M_0^n,$$

where $M_i(n)$ is the mask corresponding to the $i$th sequence ($i = 1, 2$) with a delay of $n$ chips, $\vec{p}_i$ is a row vector containing the coefficients of the polynomial representing the $G_i$ LFSR (0010000001 for $G_1$ and 0110010111 for $G_2$), ID is the square 9-element identity matrix, $\vec{0}$ is a 9-element zero’s column vector and $M_0$ is the mask for a delay offset of 0 chips (0000000001). Thus, the $M_1$ and $M_2$ masks contain not only the C/A code delay offset value, but also the satellite ID to be generated. On the other hand, to allow for a code resolution smaller than one chip the block resampler was conceived (Fig. 5.12). Since the system’s sampling frequency is 5.745 MHz and the period of the C/A code is 1 ms for 1023 chips, the ratio “samples per chip” is 5745/1023, which is equivalent to the irreducible fractional form of 1915/341. Therefore, a modulo-1915 counter increases in 341 units every clock cycle. When the counter “rolls-up” a pulse to drive the $G_1$ and $G_2$ LFSRs is issued (pulse_CA signal in Fig. 5.12). The initial value of
this counter is related to the non-integer desired code delay. For instance, a non-integer chip value of 0.4 chips would correspond to an initial value of 0.4 x 1915 = 766 units.

Fig. 5.12 Sketch of the C/A generator block. The Resampler component drives the Linear Feedback Shift Registers (LFSR) inside the block G1 and G2, out of where the C/A code is obtained. The values of M1, M2 and nint_value determine the code offset and the satellite ID.

5.4.2.2 Buffer

Another capital core of the designed embedded system is the buffer for the reflected signal. Both the direct signal buffer and the reflected signal buffer share the same design (Fig. 5.8). It receives the multiplexed GPS data from PAU-RAD and demultiplexes it so that a DDM for a single beam can be processed at a time. This peripheral is composed of two storage units that alternatively switch their input and output ports. Whenever one is receiving and storing the I/Q data the other is being unloaded (one of the four beams at a time) to compute a new DDM. The clock frequency to receive the data bits is four times the sampling frequency of the data to allow the simultaneous storage of the data of four different beams. On the other hand, the unloading of the raw data and the generation of the DDM are performed at the clock frequency of the reflectometer (100 MHz), which is significantly higher than that of the incoming data (~23 MHz). Thus again, hardware reuse is possible and up to four satellites can be processed in 1 ms. The switching between the storage units takes place whenever the one being written reaches its capacity limit (i.e., 5745 samples for each of the four beams), and an interrupt is issued by the buffer. This interrupt has top priority, since no data loss is allowed, and it triggers the generation of the DDMs associated to the data of the 1ms under consideration. To achieve this, it is necessary to translate the delay and Doppler values of the signals received from different satellites to a set of parameters (the masks M1 and M2 of the LFSR’s that determine the satellite ID and the C/A code offset, and the α values that determine
the step of the frequency synthesizers that compensate the Doppler shifts), and send them to the DDM generator before the computation of a DDM.

5.4.2.3 Delay Estimator

In order to have the DDM maximum in the center of the window of delay and Doppler values it is necessary to have a good estimate of the delay and Doppler frequency of the received signal. At low and moderate altitudes the Doppler shift value for both direct and reflected signals is roughly the same, and the difference lies only in the magnitude of the Doppler spread for the reflected signal. Taking this into account, and also considering that the temporal derivative of these Doppler shifts (~ 1 Hz/s [37]) is much smaller than the 1 s update rate of the parameters provided by the GPS receiver, its value can be used straightforwardly as the Doppler center value. Unfortunately the situation is quite different when it comes to the delay value: it is necessary to estimate the delay difference between the direct and reflected signals from the transmitter-receiver geometry. For example, in the simplest case of a low altitude receiver at constant height \( h \) the excess delay is:

\[
\Delta \tau = \frac{2h}{c} \cos \theta , \tag{4.30}
\]

being \( \theta \) the zenith angle and \( c \) the light speed. This value must be added to the direct signal delay to obtain the reflected signal delay. However, the temporal derivative of the delay cannot be neglected over the 1 s update interval. Therefore, should the delay value obtained through the serial interface be used, the maximum of the DDM will move along the delay domain at a speed depending on the satellite’s position until eventually it gets out of view. This drift is very inconvenient, since integration is needed to improve the low SNR of the reflected signal. If the peak moves, the integration will only further degrade the waveform. Therefore it is of capital importance to have readily available an estimate for the delay updated the more frequently the better. To do so a whole new block is required to perform the circular correlation by means of Fast Fourier Transform operations to find the maximum of the correlation of the direct signal with clean CA code replicas for four different satellites simultaneously. The equation applied is [37]:

\[
R_{x,ca} = \text{IFFT}(\text{FFT}(x) \cdot (\text{FFT}(ca))^*) , \tag{4.31}
\]

where \( x \) is the baseband complex signal \( I + jQ \). Before computing such a correlation it is necessary to compensate the respective Doppler offsets by using the data provided by the uplooking GPS receiver and to generate for each of the four satellites a local replica with zero delay offset of their C/A codes. This block has been implemented with standard Fast Fourier Transform (FFT) and complex multiplier cores, ensuring its performance and preventing an
excessive use of the FPGA resources (Fig. 5.13). Since the number of samples (5745 samples for 1 ms of sampled data) is not a power of 2, as needed to perform an FFT, it is necessary to fill the data values until reaching $8192 = 2^{13}$ samples. This results in two correlation maxima instead of only one. What is more, the amplitude of these maxima is position dependent. Fortunately these peaks are always spaced by the same amount of samples ($8192 - 5745 = 2447$ samples, as shown in Fig. 5.14a). Therefore, if the resulting value lies above a certain threshold, it would be necessary to subtract from it those 2447 samples to obtain the actual offset value. If the value lays beneath the threshold, then it is already the value being searched (Fig. 5.14b). Additional logic performs the translation of this information into the desired 0 to 5744 value for the delay offset.

![Fig. 5.13 Architecture of the delay offset block.](image)

![Fig. 5.14 Correlation of the 5745-sample C/A code using standard FFT blocks with $2^{13} = 8192$ input samples. (a) Two correlation maxima instead of a single one. (b) Relation of the actual code offset and the retrieved one.](image)
5.4.2.4 Beam Selector

As described in chapter 1, the full PAU instrument plans a 4x4 element antenna array to operate the radiometer with beamforming. However, the PAU-GNSS/R section uses only the four central elements. Using a uniform illumination with a linear phase it is possible to point the beam towards the specular reflection direction, using the elevation and azimuth data obtained from the GNSS commercial receiver. Since PAU-RAD is in charge of the beamforming, PAU-GNSS/R has to determine the specular point angular location, and select which one of the synthesizable beams best matches that position. Then the beam selection is transmitted through dedicated data lines to PAU-RAD using the dedicated peripheral ‘beam_selector’. The number of available beams was set to 25 (five angular steps in each angular dimension), according to the coverage simulations depicted in Fig. 5.15. Therefore, the elevation-azimuth space is divided according to a grid and a beam number is assigned to each region (Fig. 5.16.).

Fig. 5.15 Antenna coverage for different number of synthesised beams. On the upper row, the coloured area has directivity better than that of a single antenna. On the lower row, the depicted area has directivity better than the maximum minus -3 dB. The chosen number was of 25 (5 in each angular position).
5. GNSS-R RECEIVER DESIGN, IMPLEMENTATION, AND VALIDATION

5.1.1. Soft Processor

Within the FPGA, along with the described peripherals, a microprocessor is synthesized. This ‘soft’ processor is a Xilinx Microblaze core [70], which executes the control program of the PAU GNSS-R instrument.

5.4.2.5 Control Program Description

The coordinated operation of all the peripherals that compose the reflectometer is ensured by the program running on the MicroBlaze soft processor. This code is written in ANSI C, and controls the GPS incoming data through the UART, the selection of the satellites, the generation of the DDM parameters and the signaling of the beginning of a new DDM computation. This program also transfers the DDM generation parameters to the DDM generator (the masks $M_1$ and $M_2$ of the LFSR’s that determine the satellite ID and the C/A code offset, and the $\alpha$ values that determine the step of the frequency synthesizers that compensate the Doppler shifts). To do so these inputs are mapped to a RAM-like register at the DDM generator side. Then the read/write operations are performed by simply addressing to the DDM generator’s address space. Another task of the code is the decoding of the data packets received from the commercial GPS receiver through the RS-232 link. These packets contain information on the available satellites and their respective delay, Doppler, power, elevation and azimuth values. From these values both the parameters to be fed to the DDM generator and the most suitable array beam for a given satellite are determined.

The program defines several constants such as the DDM resolution (step in both the delay and Doppler domains), the sampling frequency of the incoming data and a set of conversion factors (i.e., to convert from chips to samples). At the same time, the system variables and structures

\[
\begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8 & 9 \\
10 & 11 & 12 & 13 & 14 \\
15 & 16 & 17 & 18 & 19 \\
20 & 21 & 22 & 23 & 24 \\
\end{array}
\]

Fig. 5.16 ID assignation to be available antenna array beams
are also defined. They include the packet structures used to interact with the commercial GPS receiver, the masks of the LFSRs that generate the CA code with a certain delay, or a vector of the selected satellites to work with. The system functions can be grouped in three sets:

- The first one includes those that send and receive data from the GPS receiver,
- A second set selects which of the available satellites are to be used, taking into account their power and position, and finally
- Another set of critical functions computes the DDM parameters and transfers them to the DDM generator.

The system has two sources of interrupts (Fig. 5.17). The buffer for the reflected signal issues an interrupt every 1 ms (i.e., whenever new data is available). This Interrupt Service Routine (ISR) plays an important role in the overall operation of the reflectometer, since the buffer interrupt is the synchronism reference of the system. It comprises the following tasks:

- First, the UART interrupts are disabled,
- Then, for each of the four simultaneous beams/satellites, the buffer is notified which beam has to dump next,
- The corresponding DDM parameters are set into the DDM generator,
- The generation of the DDM is triggered,
- The system waits until the DDM_ready signal goes high,
• Then the DDM data sending through the USB controller is enabled,
• The ISR waits for the USB controller to assert that the sending process is finished, and finally
• Before returning to the main function the UART interrupts are again enabled.

The RS-232 serial port controller issues an interrupt every time new data arrives from the GPS receiver:

• First of all, its associated ISR disables the UART interrupts,
• Then it reads from the UART buffer all the available data bytes and parses them to decode the TSIP report packets,
• Also, depending on the value of a counter, new Elevation & Azimuth packets are requested, and finally
• Before returning to the main program the interrupts for the UART are enabled.

The main program initializes the system and waits for interrupts:

• First, the UART interrupts are enabled in the RS-232 controller,
• Then a configuration command TSIP packet is sent to the GPS receiver, so that it outputs the raw data packet every 1 second,
• The satellite table is initialized and its associated DDM parameters are retrieved,
• The interrupts in the microprocessor and in the interrupt controller are enabled,
• The code waits for interrupts and, after an UART ISR has been executed, gets the new DDM parameters for the four simultaneous beams.

5.4.3  GNSS Off-The-Shelf Receiver

A Trimble ACE III GPS receiver was chosen to provide a priori data on the satellites present on the scenario, along with their associated Doppler shift, elevation, and azimuth values. Such information enters the FPGA embedded system through an RS-232 link, and is decoded within the control program. In order to upgrade the PAU-GNSS/R receiver to a spaceborne demonstrator it would be necessary to replace this Trimble ACE III receiver, since it is not qualified for space. In Table 5.2 a selection of commercial off-the-shelf (COTS) dual frequency receivers is given. In [71] it is stated that NovAtel’s OEM4-G2L performs well within the Low Earth Orbit (LEO) scenario, and has the lowest power and weight constraints. Therefore, it could replace the Trimble ACE III when performing the mentioned upgrade. Alternative the design of a space-qualified VHDL GPS receiver has already started. This should allow having a seamless interaction with the VHDL DDM generator section.
Table 5.2 Dual Frequency GPS receivers for space applications, as in [71]

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Receiver</th>
<th>Channels</th>
<th>Antenna</th>
<th>Power [W]</th>
<th>Weight [kg]</th>
<th>TID [krad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAAB (S)</td>
<td>GRAS/GPSOS</td>
<td>12 CA, P1/2</td>
<td>3</td>
<td>30</td>
<td>30</td>
<td>-</td>
</tr>
<tr>
<td>Laben (I)</td>
<td>Lagrange</td>
<td>16x3 CA, P1/2</td>
<td>1</td>
<td>30</td>
<td>5.2</td>
<td>20</td>
</tr>
<tr>
<td>General Dynamics (US)</td>
<td>Monarch</td>
<td>6-24 CA, P1/2</td>
<td>1-4</td>
<td>25</td>
<td>4</td>
<td>100</td>
</tr>
<tr>
<td>JPL / BRE (US)</td>
<td>Blackjack / IGOR</td>
<td>16x3 CA, P1/2</td>
<td>4</td>
<td>10</td>
<td>3.2 / 4.6</td>
<td>20</td>
</tr>
<tr>
<td>Alcatel (F)</td>
<td>TopStar 3000G2</td>
<td>6x2 CA, L2C</td>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Austrian Aerospace (A)</td>
<td>Inn. GNSS Navigation Receiver</td>
<td>Up to 36 CA, P1/2</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>&gt; 20</td>
</tr>
<tr>
<td>BRE (US)</td>
<td>Pyxis Nautica</td>
<td>16-64 CA, P1/2, L2C, L5</td>
<td>1-4</td>
<td>20</td>
<td>2.5</td>
<td></td>
</tr>
<tr>
<td>NovAtel (CA)</td>
<td>OEM4-G2L</td>
<td>12x2 CA, P2</td>
<td>1</td>
<td>1.5</td>
<td>50</td>
<td>6</td>
</tr>
<tr>
<td>Septentrio (B)</td>
<td>PolaRx2</td>
<td>16x3 CA, P1/2</td>
<td>1 (3)</td>
<td>5</td>
<td>120</td>
<td>9</td>
</tr>
</tbody>
</table>

5.4.4 Data Output

As it has already been pointed out, the computed DDM needs to be swiftly sent out of the FPGA system to be processed. This data link is divided in three areas. The first one is a peripheral synthesized in the FPGA that acts as an external master for the USB controller (CY7C68013 chip). Then there is the USB controller IC itself, composed of a microcontroller and an I/O buffer. The third section is the program running at the terminal computer. The FPGA master controller sends the computed DDM values to the I/O buffer, using additional control signals. It acts as a data bridge between the DDM generator and the USB controller, since it sweeps the output address range of the DDM generator, reading and sending data points away. The data interface is shown in Fig. 5.18. It has been written in Visual C++. After the program starts, it verifies that the USB controller is indeed connected to the computer through an USB cable and then uploads the firmware to it, so that it becomes configured. Additional control buttons allow configuring the integration times and the data files to store. So far two
interfaces have been implemented, each of them associated to one operational mode. In the first one four satellites are tracked simultaneously, and up to 16 x 16 point DDMs can be displayed in real-time. The second mode operates with just one satellite, but combines the hardware resources so that its size is four times larger: 32 x 32.

Fig. 5.18 Implemented interface for receiving, displaying and storing the DDMs. On the left the 4-satellite operational mode is seen, whereas on the right the 1-satellite operational mode is shown. These videos correspond to actual measurements on the Garraf Cliffs, on June 2007 (click to play them).

5.4.5 Implementation Environment

The code to be downloaded into the FPGA has been generated using the Xilinx Platform Studio (XPS 8.1). It is a working environment that allows interconnecting the different user defined and off-the-shelf soft peripherals, set the address space, configure several interrupt sources, and program the soft microprocessor. When the resulting bit stream is downloaded into the FPGA evaluation board it becomes the PAU-GNSS/R instrument. In Fig. 5.19 a snapshot of the main screen of XPS is seen. The main functionality areas of the program are summarized in Table 5.3.
Table 5.3 Functionality areas of XPS highlighted in Fig. 5.19

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Message window</td>
</tr>
<tr>
<td>2</td>
<td>Project information area. The displayed information depends on the active flap (4, 5, or 6)</td>
</tr>
<tr>
<td>3</td>
<td>View and edition of the system hardware components. The view can either display the bus connections, the port connections, or the address space of the peripherals. In Fig. 5.19 the Microblaze processor, the RS232 UART for connecting with the external GPS receiver, the DDM generator, the USB controller, and the input buffer are seen among other cores</td>
</tr>
<tr>
<td>4</td>
<td>The project flap gives access to the project files and the configuration options</td>
</tr>
<tr>
<td>5</td>
<td>The application flap controls the programs to be executed on the soft processor</td>
</tr>
<tr>
<td>6</td>
<td>The peripheral flap lists all the available peripherals (both user defined and off-the-shelf), and allows to add new ones</td>
</tr>
<tr>
<td>7</td>
<td>The connection panel displays the connectivity of the hardware components. Buses are depicted with vertical lines, and peripheral connections to buses with horizontal lines</td>
</tr>
</tbody>
</table>

5.5 SYSTEM VALIDATION

Once the PAU-GNSS/R system was implemented it was necessary to assess the system performance. To do so several consecutive tests were performed.
5.5.1 Comparison of the FFT and MAC Approaches

Simulated direct GPS L1 C/A baseband I/Q data were processed with both using the FFT approach programmed in a MATLAB script and the implemented PAU-GNSS/R. As shown in Fig. 5.20, there is virtually no difference in the obtained results, despite the truncation due to the limited number of bits used within the FPGA (21 bits for both the real and imaginary parts of each amplitude DDM). After that real GPS data was acquired and processed, obtaining similar results (Fig. 5.21), except for a scale factor.

5.5.2 PAU-OR Compact Demonstrator

PAU-OR (one receiver) is a small-scale version of the PAU instrument: a compact instrument to be placed at a fixed point to observe sea and terrain surfaces. It has been developed to test the concept before assembling the whole PAU-16 instrument, and also to allow more flexibility when trying to gather field data, because of its compactness. The GNSS-R core, common to both instruments, has been assembled in PAU-OR to debug the reflectometer by acquiring real GPS data (Fig. 5.22a). The instrument was deployed in the roof of the department building, aiming to the Collserola range, to collect reflections of GPS signals.

Fig. 5.20 Comparison between the theoretical (obtained with MATLAB) and the FPGA DDM, using synthetic raw data emulating the PRN 9 code with a delay of 560 samples (at $f_s = 5.745$ MHz), with a delay resolution of (a) 1 sample and a Doppler resolution of 100 Hz, and (b) a delay resolution of 5 samples and a Doppler resolution of 500 Hz.
Fig. 5.21 (a) DDM obtained applying an FFT approach, and (b) DDM obtained applying the VHDL approach. Real data was captured and post-processed (PRN19, Garraf Cliffs, April 27th 2005)

from the mountain slope (Fig. 5.22b). Taking into account the position of the satellites at the time of the measurements, it was determined which one should be captured. In Fig. 5.23 one of these reflected DDMs is shown.

Fig. 5.22(a) Schematic view of the components that compose the PAU-OR demonstrator: The GNSS-R section, the radiometer section and the off-the-shelf GPS receiver (Trimble). (b) Experimental setup at the top of the D3 University building to capture GPS reflections from the Collserola range (left of the image). In the the picture a test acquiring direct signal was in progress. The test took place in May 2007.
To test the ability of the system to capture the actual GPS signal reflected over the sea surface an experimental test took place at the Garraf cliffs, 30 km South of Barcelona. It was a calm sea day, with SWH = 51 cm (the Significant Wave Height was introduced in section 1.2.4). To determine the GPS satellites with a suitable geometry for tracking their reflections (Fig. 5.24), a 45° incidence angle mask had to be considered due to the cliff steepness. The results for satellites 26 and 29 are shown in Fig. 5.18a and Fig. 5.18b in both operational modes (the four-satellite mode was processing twice each satellite). The coherent integration time was set to 1 ms, and the averaged number of views (incoherent) was of 100. Depending on the surface under observation (land or sea, for example), the maximum coherent integration time must be selected according to the surface’s correlation time, so that the coherent averaging of uncorrelated waveforms is not performed.

Fig. 5.24 Experimental measurements at Garraf Cliffs, on June 2007. On the left PAU-OR points towards the sea surface from above the cliff. On the right, the position of the available satellites can be seen in a polar diagram. The dashed red circle shows the used satellites. Results are shown in Fig. 5.18 for both modes of operation.
Additionally, in Fig. 5.25 the actual acquisition of the GPS satellites can be appreciated. At first the DDMs can be observed, since the antenna is aiming properly towards the specular reflection point. A change in the antenna orientation turns the DDMs into noise, until the proper antenna orientation is resumed. The 1-satellite operational mode with 1 s incoherent integration time can also be observed.

Fig. 5.25 Acquisition of the GPS satellites with the PAU-GNSS/R prototype during the experimental measurements at Garraf Cliffs, on June 2007 (click on image to play the video).

5.6 CONCLUSIONS

The implementation of a real-time VHDL reflectometer has been presented. So far it is able to acquire either four simultaneous satellites generating DDMs of 8x8 bins or 1 satellite with an associated DDM of 16x16 bins. The minimum coherent integration time is of 1 ms, but both the overall coherent and incoherent number of looks can be set by software. Additionally, the resolution in both the delay and Doppler domains can be freely configured. The design featured several in-house designed cores (DDM generator, delay offset estimator, buffer) to cope with the real-time nature of the system. There are three different data chains (reflected and direct signals and commercial GPS receiver data) that are combined together to obtain the system’s observable: the Delay-Doppler Map, or DDM. The system has been validated both in the laboratory and in the field.
6. **ALBATROSS 2008 MEASUREMENT CAMPAIGN AT GRAN CANARIA ISLAND**

6.1 **CAMPAIGN DESCRIPTION**

As mentioned in former chapters, the normalized DDM volume is envisioned as a single descriptor to account for the sea roughness when performing SSS retrieval by means of L-band radiometry. To test this assumption the Advanced L-Band emissivity and Reflectivity Observations of the Sea Surface 2008 campaign (ALBATROSS 2008) was planned. The aim was to acquire an exhaustive dataset of GNSS reflections and radiometric brightness temperatures over the sea surface under several sea-roughness conditions using ground-based sensors. Thus, the three main requirements of the test site were its geometry (the highest possible over the sea surface the better, because of a higher range of delays that can be covered), the variability of the sea state, and its orientation to North so as to be able to collect radiometric data during daytime conditions without Sun contamination. The Mirador del Balcón site is a scenic viewpoint located in the steep North-West coast of Gran Canaria island (Canary Islands, Spain) (Fig. 6.1). It is located at around 400 m over the mean sea level, with a slope of ~45° down to the sea. The area is driven by strong and moist North-component winds (Trade Winds) mainly in the summer. Whenever these winds reach the shores of the steepest islands (those younger from a geological point of view) they are blocked and generate mists and dense clouds (Fig. 6.2). Such phenomenon is locally referred to as ‘Mar de Nubes’ (Sea of Clouds), and has a key role in the ecology and water cycle of the islands, allowing the growth of tropical flora at the same latitude of the Sahara desert.

Fig. 6.1 (a) Location of the Canary Islands on the map, and the Trade Winds flow. (b) Map of Gran Canaria Island, with the location of the test site.
PAU-GNSS/R (described thoroughly in chapter 5) and the LAURA (L-band automatic radiometer, [18]) instruments were located at the viewpoint aiming towards the sea (Fig. 6.3). Oceanographic buoys gathering ground truth data were moored near the observation site (Fig. 6.4): There were two salinity buoys from Universidad de las Palmas de Gran Canaria (ULPGC) and from Universitat Politècnica de Catalunya (UPC), and a third directional spectrum buoy (TRYAXIS) from UPC right below the cliff, within the field of view of the instruments. In addition to that, data from a 4th, deep water buoy some 18 km offshore was available (SeaWatch from the Puertos del Estado Network). The field experiment lasted for 6 weeks since May 27th to July 4th, 2008, in an attempt to catch different sea state conditions.
The reflectometer acquired the GPS reflected signals and generated DDMs under two operating modes:

1. One-minute burst without averaging (1 complex DDM every 1 ms)
2. 50-minute continuous acquisition, with 1 s incoherent averaging (1000 looks to generate a single DDM)

The measurement plan was to perform two consecutive 1-minute burst captures followed by a 50 minute capture every hour, seven days a week from 10 am to 7 pm. The cliff morphology
imposed a mask for the useful field of view where GPS satellite reflections were found: elevations should be below 45° and azimuths between 270° and 30° North (0°: North direction, angles measured clockwise). The down-looking antenna was a 7 element array that resulted in a 25° -3dB beamwidth, whereas the antenna to track the direct signal was a single patch, offering a wider field of view.

Fig. 6.4 Several oceanographic buoys were deployed for ALBATROSS 08. (a & b) TRIAXYS buoy for measuring the directional wave spectrum. (c) Recovering the salinity buoy at the end of the campaign. (d) A fishing ship was used to operate the buoys. (e) View from the test site of one of the moored buoys. (f) Location of the Triaxys, Salinity, and SeaWatch buoys.
6.2 DATA PROCESSING

The real-time generated delay-Doppler maps have a resolution of 0.36 chips in delay (2 samples at the sampling frequency of the system) and 200 Hz in the Doppler domain. Note that the above values largely satisfy the Nyquist criteria since the sampling requirements are 1 chip in delay, and 500 Hz in Doppler. The overall size is of 16x16 points (Fig. 6.5). Each computed DDM (both in the burst and averaged modes) is stored along with additional parameters such as the estimated delay and Doppler used for its computation, the satellite identifier (ID), and the elevation angle. These DDMs are straightforwardly processed to extract the peak value, the peak-to-floor ratio (equivalent to the SNR), and the normalized volume as proposed in [51]. The units of this last parameter are Hz x chip or Hz x sample, since the 3rd dimension (the bin amplitude of the normalized DDM) has no units. For inter-comparison purposes throughout this chapter the volume value is normalized by the bin area (200 Hz x 0.36 chips); i.e., the volume is obtained as the addition of the DDM bin values above the defined threshold. Time-series of these derived parameters are then obtained. The additional parameters embedded with the DDM allow either setting a quality metric to discard an obviously faulty DDM or plotting the derived parameters (volume, SNR...) as a function of the elevation or the Doppler. The delay, for instance, is estimated from the direct signal. When acquiring low elevation satellites the signal quality can be so poor that even the direct signal chain loses track. In that case the estimated delay is useless, and so is the associated reflected signal DDM.

The reflectometer data is to be linked to the Sea State data collected by the buoys. The candidate parameters to act as Sea State descriptors are the Significant Wave Height (SWH), the Mean Squared Slope (MSS) and the Surface Wind Speed (WS). Usually the Wind Speed at 10 meters ($U_{10}$) is used to fully parameterize the Sea Roughness, even though it is known that
such a relationship does not hold properly at L-band, and that other parameters do play an important role. The Agaete buoy does provide WS, SWH, and time spectrum. The scatter plot between $U_{10}$ and SWH for this buoy is given in Fig. 6.6, where a weak dependence is exhibited ($R = 0.26$). On the other hand, the TRIAXYS buoy records SWH and directional spectrum, but keeps no record on the wind speed. In Fig. 6.7 it is seen that there is a pretty good correlation between the SWH recorded by the two buoys. This fact makes possible to work only with the data from the further-located Agaete buoy; since there is a data gap for the TRIAXYS buoy, due to sea water leaking inside, and the cast off and mooring that followed to fix the problem. This data gap is inconveniently collocated with the rougher sea days.

![Graph: Fig. 6.7 Evolution in time of the SHW measured by both the Agaete and the Triaxys buoys.](image)

![Graph: Fig. 6.8 Computed MSS using several cut-off wavelengths.](image)

The derivation of the MSS from the time spectrum of the Agaete buoy is explained in Appendix B. The resulting Mean Squared Slopes are given in Fig. 6.8 for different cut-off wavenumbers. The yellow plot is the full MSS, the grey one the scaled MSS for GPS L1 (Appendix B) and the rest have been obtained using several cut-off wavelengths. It is seen that there is a pretty high correlation between MSS and SWH (Fig. 6.9a and Fig. 6.9b, with a correlation parameter $R = 0.91$), which is much higher than the correlation between $U_{10}$ and SWH (Fig. 6.6, with a correlation parameter $R = 0.26$).
To obtain the delay-Doppler maps first the data files are read, and the DDM is regenerated in 3000 s-long time series. From these complex Delay Doppler maps both the peak value and the normalized power DDM volume are computed. The first parameter is used to compute the SNR of the measurement, whereas the second is the observable to be linked to the Sea State. As explained in chapter 2, the amplitude DDM is a random process. Therefore, the DDM peak fluctuates in time (Fig. 6.10a). Additionally, some of the low-frequency fluctuations could be caused by multipath over the rough surface of the cliff and the surrounding peaks (Fig. 6.2). Other ancillary data such as the satellite elevation, the code delay or the Doppler shift are also retrieved. The oceanographic data has a time resolution of 1 h. Therefore it is necessary to resample it to match the resolution with the GNSS-R data. The next step is to discard faulty data points. When working with DDMs the Signal to Noise Ratio (SNR) is usually defined as the peak to noise floor ratio (section 2.2.3). Thus, if the noise floor remains constant throughout the measurement campaign, the SNR is directly proportional to the DDM peak value. However, the volume itself also depends on the signal quality for low SNR values (Fig. 6.10b). Therefore, it is necessary to choose a threshold for the minimum peak value necessary not to discard a volume value. For the ALBATROSS dataset, this threshold has been empirically determined to be $2 \cdot 10^5$. 

Fig. 6.9 (a) Evolution in time of both the SWH and the MSS (scaled x20), and (b) Correlation between SWH and MSS (correlation parameter $R = 0.91$).
6. ALBATROSS 08 CAMPAIGN AT GRAN CANARIA ISLAND

Fig. 6.10 (a) Time series of the DDM peak for several observations. The random nature of the scattering process and some multipath account for the peak fluctuations. (b) Relationship between DDM peak and normalized DDM volume. The minimum peak quality threshold is depicted with a blue dashed line.

The dependence of the Sea State descriptors with respect to the measured DDM volume is given in Fig. 6.11.

Fig. 6.11 Volume dependence on several Sea State descriptors: (a) wind speed, with $R = 0.37$, (b) SWH, with $R = 0.71$, and (c) MSS, with $R = 0.68$. The wind speed has the weakest correlation with respect to the DDM volume.
From Fig. 6.11 it is clear that the volume dependence on the WS is much weaker than the volume dependence on either the SWH or the MSS. These two parameters, in the ALBATROSS campaign, seem to be quite equivalent. Therefore, the SWH was chosen as the sea state descriptor because of having an slightly higher correlation with the DDM volume, even though MSS is acknowledged to be the parameter sensed by GNSS-R. As already stated, the 1 s incoherently averaged measurements are taken during 50 minutes. In such a time extend the GPS satellite changes significantly its position. For instance, the satellite elevation could increase up to 21°. Therefore, the time dependence of the volume can be straightforwardly translated to elevation dependence. For the geometry of the ALBATROSS campaign, the simulations performed show no relationship between these two parameters (Fig. 6.12).

![Simulated volume as a function of the incidence angle for the ALBATROSS 2008 geometry.](image)

To verify this extend with the collected data the volume measurements were assigned to 5°-wide elevation ranges, from 15° to 35°. Then, each VOL(SWH) dependence was fitted to a 1\textsuperscript{st} degree polynomial (Fig. 6.13).
Fig. 6.13 Volume dependence on SWH for several elevation angle ranges: (a) 15° – 20°, (b) 20° – 25°, (c) 25° – 30°, and (d) 30° – 35°. (e) Linear fitting coefficients for the 4 elevation ranges. There is virtually no dependence on the elevation angle.
No significant differences between the several elevation ranges was found. That means that, for the ground-based scenario of the ALBATROSS campaign, the volume observable is independent with respect to the satellite elevation. As explained in the following section, since the volume is a multiangular measurement a changing geometry has a limited impact on the retrieved measurements.

### 6.3 ANTENNA PATERN IMPACT

The DDM peak value exhibits a strong variation with varying elevation angle (Fig. 6.14a). This is due, to a great extent, to the antenna pattern, which has a 25° -3 dB beamwidth. The DDM peak is related to the scattered power over the ocean surface at a given delay, Doppler, and incidence angle (complementary of the satellite elevation angle) values.

![Fig. 6.14 (a) DDM peak as a function of the elevation angle. (b) Normalized DDM volume as a function of the elevation angle. Each DDM volume value is computed after normalizing the measured DDM by its peak value.](image)

Other DDM bins are associated to surface points with different incidence angles. This explains that the antenna pattern affects much more the peak than the volume observable (Fig. 6.14b), since the latter is in fact a multiangular observation. This volume is computed after normalizing the power DDM (squared amplitude DDM) by defining a threshold to cut the grass level. Lower thresholds imply a larger section of the DDM being actually used to compute the volume, and thus result in higher volume values. However, the resulting standard deviation is also higher (Fig. 6.15) due to the presence of noise and multipath. Throughout ALBATROSS the SHW ranged between 1 m and 2.6 m. In this range the relationship between SHW and volume is pretty linear. It is possible to obtain an additional points for SWH = 0; i.e., for a perfectly flat Sea Surface. This is equivalent to use direct signal, which has not experienced any kind of scattering.
In doing so, it is seen how the dependence can no longer be considered linear throughout the whole SWH range (Fig. 6.16). This behavior is consistent with the volume dependence on the wind speed observed in section 4.3.

The following fitting function is proposed:

$$Vol = \alpha \cdot \text{SWH}^{1/\beta} + \gamma$$  \hspace{1cm} (5.1)
Table 6.1 Fitting parameters for the volume dependence on the SWH.

<table>
<thead>
<tr>
<th>Threshold</th>
<th>α</th>
<th>β</th>
<th>γ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>12.74</td>
<td>1.618</td>
<td>5.218</td>
</tr>
<tr>
<td>0.2</td>
<td>10.31</td>
<td>1.525</td>
<td>2.463</td>
</tr>
<tr>
<td>0.5</td>
<td>6.542</td>
<td>2.256</td>
<td>1.743</td>
</tr>
</tbody>
</table>

6.4 RETRIEVED CORRELATION TIMES

The dataset composed of 1-minute time series complex DDM generated every 1ms has been processed to infer the correlation time of the scattering surface. To do so the module of the autocorrelation of the time evolution of the peak is computed (Fig. 6.17). The obtained correlation times range between 7 ms and 12 ms. Unfortunately, the available ground truth is 1-hour averages, and therefore it is not possible to link the instantaneous Sea State that results in a given correlation time with the buoy data.

Fig. 6.17 Module of the autocorrelation of the DDM peak for the non-averaged dataset.
6.5 CONCLUSIONS

The ALBATROSS 08 campaign has been the first attempt to validate the volume of the normalized DDM as an observable to account for the L-band sea roughness. It has been shown that there is a strong correlation between the volume and the SWH. Additionally, the elevation angle or the antenna pattern has a limited impact on the proposed observable. Future editions of ALBATROSS will be focused on the relationship between the DDM volume and the radiometric brightness temperature.
7. CONCLUSIONS AND OPEN ISSUES

The present PhD. dissertation started emphasizing the importance of the SSS parameter in oceanography and climatology studies. It was stated that L-band radiometry offers a good potential to perform SSS retrieval, provided that the sea roughness impact on the radiometric observable is accounted for. Consequently, the use of GNSS-R signals to perform the sea state correction whenever retrieving SSS was introduced, and thus the PAU project was envisioned to demonstrate its feasibility. It considers the synergic operation of a L-band radiometer and GPS reflectometer, along with an IR radiometer.

As is many other GNSS-R studies, the Kirchhoff Model in its Geometric Optics approximation was used as a first approach to the actual scattering problem over the sea surface, and thus the Zavorotny-Voronovich model for the Delay Doppler Map was used to simulate the GNSS-R scenario. The retrieval of the sea surface roughness is possible since the scattering coefficient determines the extent of the glistening zone, and thus also the shape of the DDM. The sea was modeled as a random surface, where the MSS is used as the preferred roughness descriptor, even though the WS is also often used.

To perform DDM simulations the usual approach of straightforwardly evaluating the Zavorotny-Voronovich expressions to obtained either amplitude or power correlation was implemented. However, spaceborne scenario simulations increase the resource consumption (time and memory) as the surface gets larger, until becoming not affordable at all. This dissertation introduced the use of explicit expressions for the delay and Doppler values that allow fast computation of the DDMs using the Jacobians of the change of variables between the space and delay-Doppler domains. By doing so the number of points to be evaluated significantly reduces from all the surface points (hundreds of thousands or millions) to just the number of bins of the desired DDM. The obtained DDM differs less than 1 % with respect to the reference one (obtained without any approximation) for the flat Earth scenario. However, for the spaceborne case the flat Earth approximation no longer holds. Because of that an iterative approach to compute the jacobians is introduced. The retrieved DDM differs less than 7 % (peak difference) with respect to the reference one. The significant speed improvement of the new approach makes affordable simulations of satellite reflections that were too slow or not feasible at all. The described Jacobian approach is understood as a general reference frame, where closed-form expressions for other scenarios apart from those described in the present work can be derived and used in a similar way. Both simulation approaches were implemented into a simulator to readily obtain DDMs, compare the different approaches, and allow validating the DDM measured in field campaigns.
In chapter 4 two semi empirical approaches to the DDM parameterization were proposed. The first one allows regenerating a reflected DDM coded with six scalar values, but it is not useful to derive a relationship with respect to the sea state. The second approach integrates the normalized DDM to obtain the area and/or volume under it. These normalized area and volume descriptors were related to the wind speed, and are meant to be used as single sea state descriptor. A simulation step further was made to link the brightness temperature correction due to the sea state with the area or volume under the normalized DDM. This was considered an intermediate step, since the ultimate goal is to directly relate the area or volume under the DDM to the brightness temperature correction, when measurements are available in the future. As in [30], simulation showed that both the area and the volume as a function of the wind speed exhibit the saturation trend for higher wind speeds. For the particular scenario of a spaceborne GNSS-R receiver at 680 km height and a 25º beamwidth antenna the DDM requirements to accurately correct for the sea state impact on the L-band brightness temperature were determined to be the following: DDM size 220 chips x ± 11 KHz, DDM resolution = 1 chip x 1 KHz, DDM quantization of at least 9 bits, and $T_i = 1$ ms coherent integration followed by 100 ms incoherent integration. These results could be applied to the proposed PAU in SEOSat secondary payload mission.

The design, implementation, and validation of the PAU-GNSS/R instrument have been one of the main contributions of this work. It has been the first real-time GNSS-R receiver to generate full DDMs [62]. So far it is able to acquire either four simultaneous satellites generating DDMs of 8x8 bins or 1 satellite with an associated DDM of 16x16 bins. The minimum coherent integration time is of 1 ms, but both the overall coherent and incoherent number of looks can be set by software. Additionally, the resolution in both the delay and Doppler domains can be freely configured. The design featured several in-house designed cores (DDM generator, delay offset estimator, buffer) to cope with the real-time nature of the system. There are three different data chains (reflected and direct signals and commercial GPS receiver data) that are combined together to obtain the system’s observable: the Delay-Doppler Map, or DDM. The system was validated both in the laboratory and in the field.

The ALBATROSS 08 campaign was the first attempt to validate the volume of the normalized DDM as an observable to account for the L-band sea roughness. It has been shown that there is a strong correlation between the volume and the SWH/MSS. It was also noted that the elevation angle or the antenna pattern have a limited impact on the proposed observable. Future editions of ALBATROSS will be particularly focused on the relationship between the DDM volume and the radiometric brightness temperature. The completion of the full PAU-16 instrument will allow to obtain truly co-located DDM and $T_b$ measurements, and thus the direct correlation between volume and $T_b$ will be studied.
The implementation of an enhanced version of the PAU-GNSS/R instrument within the research team is already on its way. The goals are both to solve some detected minor problems as well as to add new functionalities: enhance the user interface, increase the number of bins, automate the tracking of the specular point during long acquisitions, add a fully collocated L-band radiometer, or ruggedize the system are the most notable ones.

With respect to the DDM simulation, the newly introduced Jacobian approach will be further tested in a wide range of scenarios to assess its validity boundaries. It would also be interesting to develop other analytical expressions for the change of variables, i.e., for a ground-based receiver, and to test them. Additionally, the SSA approach would be used instead of the KM.

As to conclude, some of the presented contributions (the PAU-GNSS/R instrument, or the Jacobian DDM simulation) could also be use for the GNSS-R study on other scenarios, such as ground or ice.
CONCLUSIONS AND OPEN ISSUES
APPENDIX A: EXPLICIT EXPRESSIONS FOR THE CHANGE OF VARIABLES BETWEEN THE XY AND DELAY-DOPPLER DOMAINS IN THE FLAT EARTH SCENARIO

The positions of the receiver, transmitter, and an arbitrary surface point for the flat Earth approximation are:

\[
\vec{R}_i = \left(0, \frac{h_0}{\tan \gamma}, h_0 \right), \quad (A.1)
\]

\[
\vec{R}_r = \left(0, -\frac{h}{\tan \gamma}, h \right), \quad (A.2)
\]

and

\[
\vec{r} = (x, y, 0), \quad (A.3)
\]

Additionally, the absolute delay associated to a surface point is:

\[
\tau_{xy, abs} = \left| \vec{r} - \vec{R}_i \right| + \left| \vec{R}_r - \vec{r} \right| . \quad (A.4)
\]

Then, substituting Eqns. A.1, A.2 and A.3 in Eqn. A.4 results in:

\[
\tau_{xy, abs} = \sqrt{x^2 + \left(y - \frac{h_0}{\tan \gamma} \right)^2 + h_0^2} + \sqrt{x^2 + \left(y + \frac{h}{\tan \gamma} \right)^2 + h^2} . \quad (A.5)
\]

The first square root can be further simplified considering that \(h_0 \gg x, y\) (assumption of very far away transmitter), and keeping the terms up to first order only:

\[
\tau_{xy, abs} = \sqrt{x^2 + \left(y + \frac{h}{\tan \gamma} \right)^2 + h^2} + \frac{h_0}{\sin \gamma} - y \cos \gamma . \quad (A.6)
\]
It is usual to express the code delay with respect to the specular point delay (surface point with \( x = y = 0 \)); then:

\[
\tau_{xy} = \tau_{xy,\text{abs}}(x,y,h,b,\gamma) - \tau_{xy,\text{abs}}(0,0,h,b,\gamma) = \sqrt{x^2 + \left(y + \frac{h}{\tan \gamma}\right)^2 + h^2} - \frac{h}{\sin \gamma} \cos \gamma . \quad (A.7)
\]

The Doppler shift of each point is derived in an analog way (very far away transmitter approximation):

\[
f_{d,xy} = \vec{V}_t \cdot \hat{n}_i - \vec{V}_r \cdot \hat{n}_s = -V_t \cdot \cos \gamma - V_x \cdot \sin \gamma + \frac{V_x \cdot x + V_y \left(y + \frac{h}{\tan \gamma}\right) - V_z \cdot h}{\sqrt{x^2 + \left(y + \frac{h}{\tan \gamma}\right)^2 + h^2}}, \quad (A.8)
\]

where \( \vec{V}_t \) and \( \vec{V}_r \) are the transmitter and receiver velocities, respectively. To compute the Jacobians it is necessary to express the \( x-y \) coordinates as a function of \( \tau_{xy}, f_{d,xy} \):

\[
\begin{align*}
x_1 &= f_1(\tau_{xy}, f_{d,xy}, h, \gamma, \vec{V}_t, \vec{V}_r), \\
y_1 &= g_1(\tau_{xy}, f_{d,xy}, h, \gamma, \vec{V}_t, \vec{V}_r), \\
x_2 &= f_2(\tau_{xy}, f_{d,xy}, h, \gamma, \vec{V}_t, \vec{V}_r), \\
y_2 &= g_2(\tau_{xy}, f_{d,xy}, h, \gamma, \vec{V}_t, \vec{V}_r),
\end{align*}
\]

(A.9)

The explicit expressions are:

\[
x_{1,2} = \left(-\frac{\sqrt{\alpha} \cos \gamma + \sin \gamma \cdot V_x (\tau + h \cdot \sin \gamma)}{V_x \cdot \beta} \right) \left( f_x \cdot \lambda + V_x \cos \gamma + V_y \cdot \sin \gamma \pm \frac{\sqrt{\alpha} \cdot V_y \cdot h \cdot \sin \gamma - V_x \cdot \cos \gamma \cdot (\tau + h \cdot \sin \gamma)}{V_x \cdot \beta} \right),
\]

(A.10)

\[
y_{1,3} = -\frac{1}{\beta} \left( -\frac{\sqrt{\alpha} + 2 \cdot V_x \cdot f_x \cdot \lambda \cdot \cos \gamma + 2 \cdot V_x \cdot \sin \gamma \cdot f_x \cdot \lambda \cdot \cos \gamma + V_y \cdot \sin \gamma \cdot V_y \cdot \cos \gamma + 2 \cdot V_y \cdot \sin \gamma \cdot V_y \cdot \cos \gamma \right.
\]

\[
- V_x \cdot \cos \gamma \cdot V_x \cdot \cos \gamma - V_y \cdot \cos \gamma \cdot V_y \cdot \cos \gamma - V_y \cdot V_y \cdot \cos \gamma \cdot V_y \cdot \cos \gamma - V_x \cdot V_x \cdot \cos \gamma \cdot V_x \cdot \cos \gamma
\]

\[
+ V_x \cdot \cos \gamma \cdot V_y \cdot \cos \gamma + V_y \cdot \cos \gamma \cdot V_x \cdot \cos \gamma + V_x \cdot \cos \gamma \cdot V_x \cdot \cos \gamma
\]

\[
- V_y \cdot \cos \gamma \cdot V_y \cdot \cos \gamma + V_x \cdot \cos \gamma \cdot V_x \cdot \cos \gamma - V_x \cdot V_x \cdot \cos \gamma \cdot V_x \cdot \cos \gamma
\]

\[
+ f_x \cdot \lambda \cdot \cos \gamma \cdot \sin \gamma \cdot V_x \cdot h \cdot \cos \gamma \cdot V_y \cdot \sin \gamma \cdot V_y \cdot h \cdot \cos \gamma \cdot \tau \cdot V_y
\]

(A.11)
where $\alpha$ and $\beta$ are defined as:

$$\alpha = \nu' \left( -1 + \cos \gamma \right) \left( 2 f_\nu \cdot \lambda \cdot h \cdot V_\nu \cdot \sin \gamma - 2 f_\nu \cdot \lambda \cdot h \cdot V_\nu \cdot \cos \gamma + f_\nu \cdot \lambda \cdot h \cdot V_\nu \cdot \sin \gamma + 2 f_\nu \cdot \lambda \cdot \tau \cdot V_\nu \cdot h \right)$$
$$+ 2 f_\nu \cdot \lambda \cdot h \cdot \tau \cdot \sin \gamma + 2 V_\nu \cdot \cos \gamma \cdot h^2 \cdot V_\nu \cdot \sin \gamma + 2 V_\nu \cdot \cos \gamma \cdot h^2 \cdot V_\nu \cdot \sin \gamma + 2 V_\nu \cdot \cos \gamma \cdot \tau \cdot V_\nu \cdot h \cdot \cos \gamma \cdot \tau$$
$$+ 2 V_\nu \cdot h^2 \cdot \cos^2 \gamma - 2 V_\nu \cdot \sin \gamma \cdot h^2 \cdot V_\nu \cdot \cos \gamma + 2 V_\nu \cdot \sin \gamma \cdot h^2 \cdot V_\nu \cdot \cos \gamma + 2 V_\nu \cdot \tau \cdot \sin \gamma \cdot \tau \cdot V_\nu \cdot h \cdot \cos \gamma \cdot \tau$$
$$+ 2 \cdot \cos \gamma \cdot \tau \cdot V_\nu \cdot f_\nu \cdot \lambda \cdot + 2 \cdot \cos^2 \gamma \cdot \sin \gamma \cdot \tau \cdot V_\nu \cdot h + 2 V_\nu \cdot \sin \gamma \cdot V_\nu \cdot h \cdot \cos \gamma \cdot \sin \gamma \cdot \tau \left( V_\nu \cdot \sin \gamma + f_\nu \cdot \lambda \right)$$
$$+ V_\nu \cdot \cos^2 \gamma \cdot \left( h^2 + \tau^2 \right) + 4 V_\nu \cdot f_\nu \cdot \lambda \cdot h \cdot \tau - V_\nu \cdot \tau \cdot \cos^2 \gamma \cdot \sin \gamma \cdot \tau \cdot V_\nu \cdot h \cdot \cos \gamma \cdot \tau$$
$$+ 2 V_\nu \cdot h^2 \cdot \cos^2 \gamma \cdot \sin \gamma \cdot \tau - 2 \cdot \cos \gamma \cdot \tau \cdot V_\nu \cdot h \cdot \cos \gamma \cdot \tau$$
$$+ 2 V_\nu \cdot h^2 \cdot \cos^2 \gamma \cdot \sin \gamma \cdot \tau - \nu' \cdot \cos \gamma \cdot \tau - V_\nu \cdot h^2 \cdot \sin^2 \gamma \right),$$

(A.12)

$$\beta = \sin \gamma \cdot \left( V_\nu \cdot \sin^2 \gamma + V_\nu \cdot f_\nu \cdot \lambda \cdot \cos \gamma \left( -2 V_\nu \cdot \sin \gamma \cdot \cos \gamma \right) + 2 \cdot V_\nu \cdot \cos \gamma \left( -2 V_\nu \cdot \sin \gamma \cdot \cos \gamma \right) + V_\nu \cdot \cos \gamma \left( 2 \cdot \cos \gamma \cdot V_\nu \cdot \sin \gamma + V_\nu \cdot \cos \gamma \cdot 2 \cdot \cos \gamma \cdot f_\nu \cdot \lambda - 2 V_\nu \right) \right)$$

(A.13)
APPENDIX B: OBTENTION OF THE SIGNIFICANT WAVE HEIGHT FROM THE TIME SPECTRUM OF THE AGAETE BUOY

B.1 AGAETE BUOY FILE FORMAT

The format of the Puertos del Estado Agaete buoy data file is as follows:

- The energy between 0.025 Hz and 0.635 Hz is computed. The squared root of that magnitude is stored in \( \sqrt{m_0} \).
- The energy is computed by integrating the original spectrum. Since the units are centimeters, it is important to divide by 100 to correct the units.
- The energy spectrum is divided in bands, each of them having a predetermined amount of energy:

<table>
<thead>
<tr>
<th>Band</th>
<th>Base</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5-10</th>
<th>11</th>
<th>12</th>
<th>Rem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy Fraction</td>
<td>1/512</td>
<td>1/256</td>
<td>1/128</td>
<td>1/64</td>
<td>1/32</td>
<td>1/16</td>
<td>1/8</td>
<td>1/16</td>
<td>1/32</td>
<td>Remaining of ( m_0 )</td>
</tr>
</tbody>
</table>

- Base Band is defined between 0.025 and \( F_{upper} \).
- Remaining Band is defined between the last incremental frequency and 0.635 Hz. This band usually contains more than 1/30 of \( m_0 \), and thus it is quite wide. It is useful to consider that energy as if it was centered and at medium point between the last incrementally computed frequency and 0.4.
- For each band the following parameters are defined:

\[
\begin{align*}
  f_{0,k} & \quad \text{Width of the } k\text{-th band} \\
  m_{Dir,K} & \quad \text{Mean direction of the } k\text{-th band} \\
  spread_K & \quad \text{Angular spread of the } k\text{-th band}
\end{align*}
\]

- To retrieve the spectral value at each band:
For instance, for band 0:

$$S(f_0) = \frac{f_{P,K}}{256} \left( \frac{\sqrt{m_0}}{100} \right)^2.$$  \hspace{1cm} (B. 2)

- Due to the satellite transfer resolution limit, it could happen that $f_{P,K} = 0$ for some band. In that case, the value $f_{P,K} = 0.0003 \, \text{Hz}$ can be used. It is half of the minimum transferable resolution.

B.2 SPECTRUM PROCESSING

First, the time spectrum is assembled from the instructions given above. The result is shown in Fig. B.1.

Then the following relationship (deep water and gravitational waves assumptions) is used to obtain the omnidirectional wavenumber spectrum [Ulaby et al., 1982], given in Fig. B.2:
\[
\begin{align*}
\Psi(\omega) &= \left( \frac{2\omega^2}{g^2} \right) S \left( \frac{\omega^2}{g} \right), \\
k &= \frac{\omega^2}{g}.
\end{align*}
\]

(B.3)

To perform the integration of the spectrum it is necessary to resample \( S(K) \) to a higher sampling rate (Fig. B.3).

It is known that the spectrum decreases as \( K^4 \) for high wavenumbers. This allows to correctly extrapolating the spectrum:

Fig. B.2 Wavenumber Spectrum from the Agaete buoy.

Fig. B.3 Wavenumber Spectrum resampled in a rectangular grid. (a) Linear and (b) log-log.
\[ S(K) = \alpha \cdot K^{-\alpha} \quad . \]  

(B. 4)

To seamlessly connect both the extrapolated \( S_{\text{extrap}}(K) \) and the read \( S_{\text{read}}(K) \) spectra the parameter \( \alpha \) is obtained as:

\[ \alpha = S_{\text{read}}(K_{\text{max}}) \cdot K_{\text{max}}^{-\alpha} \quad . \]  

(B. 5)

where \( K_{\text{max}} \) is the upper value of the \( K \) range for the sea spectrum read from the buoy. The omnidirectional MSS is computed as:

\[ \text{MSS} = \int_0^{\infty} K^2 S(K) dK \quad . \]  

(B. 6)

However, often a \( K_{\text{cut}} \) parameter is defined to derive the MSS by low-pass filtering the wavenumber spectrum:

\[ \text{MSS}_{\text{cut}} = \int_0^{K_{\text{cut}}} K^2 S(K) dK \quad . \]  

(B. 7)

Another approach found in bibliography [36] is to down-scale the full-integral MSS:

\[ \text{MSS}_{\text{GPS}} = 0.45 \cdot \text{MSS} \quad . \]  

(B. 8)

Several MSS are plotted in Fig. B.4. The yellow plot is the full MSS, the grey one the scaled MSS, and the rest have been obtained using several cut-off wavelengths\((K_{\text{GPS}}/10^4, K_{\text{GPS}}/10^3, K_{\text{GPS}}/10^2, K_{\text{GPS}}/10, K_{\text{GPS}}, \text{and } \infty)\).
It is appreciated that, from $K_{\text{cut}} = K_{\text{GPS}}/10^3$ onwards (green plot), there are no significant differences in the derived MSS.
REFERENCES


LIST OF PUBLICATIONS

JOURNAL PAPERS


CONFERENCE PAPERS


