MULTIDIMENSIONAL SPECKLE NOISE, MODELLING AND FILTERING RELATED TO SAR DATA

by

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Chapter 6

Interferometric Phasor Noise Reduction

6.1 Introduction

This chapter represents the beginning of the second part of the work presented in this thesis. The previous two chapters concerned the study of new speckle noise models for multidimensional SAR imagery. In particular, Chapter 4 is focused on the study of the interferometric phasor noise, both in the original and in the wavelet domain, whereas Chapter 5 concentrates on the definition of a speckle noise model for Polarimetric SAR (PolSAR) data, which has been extended to multidimensional SAR imagery. The aim of the present chapter is to exploit the results presented throughout Chapter 4, with the objective to estimate relevant information for SAR imagery. The results derived in this chapter will be shown to be fundamental, since they represent the basis to address the multidimensional SAR speckle noise problem.

As a result of the study carried out in Chapter 4, it has been possible to define, and to validate, two new noise models for the interferometric phasor. The first model, given by Eq. (4.30), is the interferometric phasor noise model in the original domain, whereas the second one, presented by Eq. (4.92), is the interferometric phasor noise model in the wavelet domain. Additionally, in Section 5.3 at Chapter 5, it has been demonstrated that these two noise models can be used beyond the InSAR data case, since they can be taken as models for the unit amplitude phasor whose phase is the Hermitian product phase difference between a pair of SAR images. As it has been stated in Chapter 4, the usefulness of the interferometric phasor noise model in the original domain is rather reduced since it is not possible, due to the signal characteristics, to extract useful information. Conversely, the interferometric phasor noise model in the wavelet domain has been shown very valuable, since it will allow the possibility to reduce phase noise, but also to retrieve coherence information through the parameter $N_c$.

The speckle noise reduction problem in SAR imagery is directly linked to the concept of spatial resolution. As shown in Chapter 2, the coherent processing of the scattered echo makes possible to obtain high spatial resolution imagery, but on the contrary, this coherent nature is also the origin of speckle noise. As a result, any speckle noise reduction technique has to be oriented to spatial resolution maintenance. As it will be presented below, this requirement will be largely fulfilled thanks to the use of the wavelet techniques presented in Chapter 3.

This chapter can be divided into two main parts. The objective of the first part is to exploit the concept of modulated coherence in the wavelet domain, in the frame of InSAR data, in order to retrieve relevant information. For this reason, a new algorithm for interferometric phase noise reduction and

\[ \text{The original domain refers to the spatial domain, in which the image is defined. In the following, original domain will refer to the spatial domain.} \]
coherence estimation in the wavelet domain is presented. At the same time, the usefulness of the wavelet
 techniques in order to retrieve valuable information, maintaining spatial properties, is studied. The
 second part of the chapter presents a series of results concerning interferometric phase noise reduction
 and coherence estimation obtained by means of the algorithm defined in the first part. This study will
 concentrate on different aspects as the properties of recovered data or the analysis of different aspects
 related with the wavelet transform.

6.2 State of the Art

Interferometric phase noise reduction has been extensively referenced in the associated literature. In the
same way, the calculation of the coherence parameter between a pair of SAR images, denoted by $|\rho|$, has
been also reported. The next lines pretend to give a general overview concerning the main interferometric
phase noise reduction techniques, as well as to present the principal problems that these techniques have
to solve. Additionally, the second part of this section is dedicated to present the main problems derived
from the classical definition of the complex correlation coefficient of a pair of SAR images. In this case, the
different references are basically focused on presenting alternative solutions to overcome the drawbacks
that the classical definition of the complex correlation coefficient between a pair of SAR images presents.

6.2.1 Interferometric Phase Noise Reduction

The necessity of an interferometric phase filter can be justified from two different points of view. As it has
been mentioned in Section 2.2, the angular diversity introduced by an InSAR system is the basis to obtain
topographic information from the Hermitian product phase of a pair of SAR images, i.e., it is inherent to
the image formation process. But this angular diversity is also a source of decorrelation between the pair
of SAR images, which introduces noise in phase. Besides, additional processes introduce decorrelation
between the SAR images, see Section 2.2.4 for details. Consequently, it is necessary to reduce noise
effects over the interferometric phase in order to improve the final quality of the Digital Elevation Models
or DEMs. On the other hand, since the interferometric phase is wrapped into the interval $[-\pi, \pi)$, it
is mandatory to unwrap it in order to derive relative height information. Phase unwrapping is also
affected by phase noise since, it introduces phase residues preventing a correct phase unwrapping [85].
For instance, it has been demonstrated that unwrapping processes based on linear estimators techniques
underestimate the phase slope in the presence of phase noise [45, 189]. Thus, phase noise filters are
convenient in order to facilitate the phase unwrapping process.

From what has been presented until now, it follows that one of the main SAR imagery characteristics
is the heterogeneity of both, the useful signal and the speckle noise components. Consequently, any
processing technique focused on speckle noise reduction should adapt to these particularities. Precisely,
the way to adapt to this inherent heterogeneity is the point which differentiates the different approaches
to reduce noise in the interferometric phase difference. The basic common point between the majority of
the proposed phase noise reduction techniques is to locally adapt to the heterogeneity through an $M$ by
$N$ pixel sliding window. Hence, the performance of all these techniques depend directly on the window’s
dimensions. In brief, small windows allow to maintain spatial properties, but with a poor noise reduction
as a result of the insufficient number of samples to estimate the filter’s parameters. On the contrary,
larger windows allow better noise reduction, but, at the expense of spatial resolution.

Let $S_1$ and $S_2$ be a pair of spatially uncorrelated SAR images acquired with a SAR system in an
interferometric configuration. Considering the useful signal and the noise components to be homogeneous,
the multilook interferometric phase $\hat{\phi}_{x,MLT}$ is defined as [58, 82]

$$
\hat{\phi}_{x,MLT} = \arg \left( \sum_{m=1}^{M} \sum_{n=1}^{N} S_1(m,n)S_2^*(m,n) \right)
$$ (6.1)
where \( m \) and \( n \) represent the spatial coordinates, and \( \arg(z) \) refers to the argument \( z \). As it can be observed, Eq. (6.1) corresponds to the phase of the sample, complex average of the Hermitian product of the pair of SAR images, which corresponds to the definition of the interferogram, Eq. (2.79). It has been demonstrated that Eq. (6.1) corresponds to the maximum likelihood (ML) estimation of the interferometric phase [190]. As it comes out, the average process mixes all the information within the \( M \) by \( N \) pixel window with the consequent loss of image details. But, since the average process of the multilook filter is performed in the complex domain, phase unwrapping is guaranteed as phase jumps are maintained. It can be concluded therefore, that it is more convenient to process the interferometric phase in the complex plane, since phase jumps do not represent a problem in this case. An improvement of the multilook approach has been presented in [191], where the authors proposed the introduction of a coherence-weight function in the complex average in order to give more importance to those pixels with high coherence.

The multilook filter, Eq. (6.1), is taken normally as a reference when a new interferometric phase noise filter is proposed, since the new filter should maintain the properties concerning noise reduction and should improve the spatial resolution maintenance capacities. A first group of techniques try to improve the properties of the multilook approach by adapting to the useful signal term, as well as to the noise characteristics, inside the analysis window in the original domain. The main exponent of this type of techniques has been presented in [94], termed \textit{interferometric phase Lee filter}. One of the main features of this approach is that it works over the phase difference in the real plane, since it is based on the additive phase noise model given by Eq. (2.125). Hence, the first step of this filter consists in a local phase unwrapping process, which allows to use the noise model given by Eq. (2.125). The second step consists in a process whose objective is to adapt to the useful signal morphology by means of a set of sixteen directional windows, from which the window presenting the most homogeneous signal is selected. In these windows, phase noise is reduced by means of the local statistics filter [192], taking into account the phase additive noise model, see Eq. (2.125). Additionally, the authors present a similar approach reducing noise in the complex plane in order to avoid possible errors due to the local phase unwrapping. In this case, the authors are unable to justify this approach theoretically. The results which have been provided show that the proposed filter outperforms clearly the multilook filter in terms of spatial resolution maintenance. In addition, it is proved that this approach reduces the influence of phase noise when phase is unwrapped.

In Sections 2.2.4 and 4.3.1, it was shown that locally, the pair of SAR images from which the interferogram is calculated, under the flat surface assumption, represent two shifted versions of the same terrain’s spectra. The local shift between both SAR images is called the Wavenumber Shift and has been shown to depend on the terrain’s topography. Consequently, the interferogram corresponds to a delta function located at the Wavenumber Shift frequency position in the Fourier domain. On the other hand, phase noise can be considered as a white noise component in the Fourier domain [193]. A second group of techniques is based on the signal model presented in the previous lines, reducing phase noise in the Fourier transformed interferogram. The main exponent of this type of filters has been presented in [194], which is denoted as the \textit{self-weighted interferometric phase filter}. Since the discrete fast Fourier transform (FFT) lacks of spatial resolution in the frequency domain, in order to adapt to the heterogeneity of the signal and noise, it is mandatory to calculate it over a sliding window. Let \( S_1 \) and \( S_2 \) be a pair of SAR images acquired with an InSAR system, if \( \tilde{S}_1 S_2^\ast(\omega_m, \omega_n) \) denotes the Fourier transform of the interferogram \( S_1 S_2[m, n] \), the estimated interferometric phase \( \tilde{\phi}_{x,SW} \) given by the self-weighted filter approach has the expression

\[
\tilde{\phi}_{x,SW} = \arg \left( \mathcal{F}^{-1} \left( |\tilde{S}_1 S_2^\ast(\omega_m, \omega_n)|^\alpha \tilde{S}_1 S_2^\ast(\omega_m, \omega_n) \right) \right) \tag{6.2}
\]

where \( \mathcal{F}^{-1} (\cdot) \) represents the inverse Fourier transform. The name self-weighted filter comes from the fact that the signal itself, in terms of power, is employed to filter noise. As observed in Eq. (6.2), this filter depends on the exponent \( \alpha \). The effect of the filter is to reduce the amplitude of those areas containing
only noise, since they present low amplitudes, whereas it maintains the areas with high amplitude, since they basically correspond to signal areas. For \( \alpha = 0 \) no filtering is performed, whereas a strong filtering is introduced for \( \alpha = 1 \). It must be mentioned that the performance of this type of approaches depend strongly on the window dimensions in which the Fourier transform is calculated. In the case of small windows, reduced speckle noise filtering is accomplished. On the contrary, large windows present the disadvantage that topography cannot be considered locally as a slope, with the consequent failure to adopt the interferogram signal model in the Fourier domain. In this case, due to the changing nature of the interferometric phase signal, it would be desirable to vary the window dimensions according to its local properties. A remarkable property of this approach is that for those areas which contain only noise, the filter does not introduce signal distortions. As reported in [193], this approach presents a good performance in the case of spaceborne systems.

In addition to the techniques presented in the previous paragraphs, further algorithms can be found in the literature. As mentioned, all of them share the fact that signal is processed by means of sliding windows. For instance, approaches based on morphological filters are presented in [195], whereas techniques making use of Markov random fields are given in [196].

### 6.2.2 Interferometric Coherence Estimation

In the case of multidimensional SAR imagery, the correlation coefficient amplitude between a pair of SAR images, defined in Eqs. (2.108) and (5.6), and known as coherence, plays an important role since it contains relevant information. For example, it gives information concerning the noise content of the interferometric phase for InSAR data. Additionally, coherence maps can be used for target classification [197, 74].

As given in Eqs. (2.108) and (5.6), the coherence \(|\rho|\) is defined statistically as an ensemble average. Since independent samples from a given pixel are not available, the way to estimate the complex correlation coefficient is to estimate the ensemble averages as spatial averages. The conditions under which this estimation is feasible are detailed in Section 2.3.5. Considering the processes within the complex correlation coefficient definition to be ergodic in mean and locally stationary, the coherence can be estimated as the amplitude of the sample, complex correlation coefficient

\[
|\hat{\rho}_{MLT}| = \frac{\sum_{m=1}^{M} \sum_{n=1}^{N} S_1(m,n)S_2^*(m,n)}{\sqrt{\sum_{m=1}^{M} \sum_{n=1}^{N} I_1(m,n) \sum_{m=1}^{M} \sum_{n=1}^{N} I_2(m,n)}}
\]

(6.3)

where \( I_k(m,n) = |S_k(m,n)|^2 \), \( k \in \{1, 2\} \). As one can deduce from Eq. (6.3), the performance of the sample coherence estimation \(|\hat{\rho}_{MLT}|\) depends on the number of samples, i.e., \(MN\), employed to estimate the coherence. It has been demonstrated that the sample covariance estimator \(|\hat{\rho}_{MLT}|\) overestimates the true coherence value \(|\rho|\) [113, 198], in such a way that the lower the coherence value, the larger the overestimation, and the lower the number of averaged pixels the larger the overestimation. In [113], several solutions reducing the effect of this overestimation have been proposed. Another drawback of the approach given by Eq. (6.3) is that image details are lost as a consequence of calculating the coherence value as the average over a window analysis.

Alternative ways to estimate the coherence value have been presented in the literature. The coherence maximum likelihood (ML) estimator has been derived in [190]

\[
|\hat{\rho}_{ML}| = \frac{\sum_{m=1}^{M} \sum_{n=1}^{N} \sqrt{I_1(m,n)I_2(m,n)} \cos(\phi_1(m,n) - \phi_2(m,n) + \phi_x)}{\sqrt{\frac{1}{2} \sum_{m=1}^{M} \sum_{n=1}^{N} (I_1(m,n) + I_2(m,n))}}
\]

(6.4)

There exist two clear differences between the ML estimator \(|\hat{\rho}_{ML}|\) and the sample estimator \(|\hat{\rho}_{MLT}|\). Fist of all, the denominator in the case of the ML estimator consists of the arithmetic mean of the SAR images intensity, whereas it consists of the geometric mean in the case of the sample estimator. This difference
produces $|\hat{\rho}_{MLT}| \geq |\hat{\rho}_{ML}|$, but in [190] it was shown that the bias between both estimators is negligible for usual SAR processing parameters.

The second difference between the ML and the sample coherence value estimators appears in the phase information of these estimators. As it can be observed in Eq. (6.4), the phase term is compensated with the term $\phi_x$, whereas the sample estimator, Eq. (6.3), is not. The term $\phi_x$, in the case of InSAR data, consists in the deterministic phase, i.e., the interferometric phase. It has been shown in the literature [199], that if this term is not compensated, a biased coherence estimation is obtained. The reason behind this bias is explained by the fact that, even in the case the random processes involved in the coherence value estimation are ergodic and locally homogeneous, they differ by a deterministic term, i.e., the interferometric phase. As a result, in order to eliminate this dependence, the sample coherence estimator has to be compensated for the interferometric phase

$$|\hat{\rho}_{MLT+PH.C.}| = \frac{\sum_{m=1}^{M} \sum_{n=1}^{N} S_1(m,n)S_2^*(m,n) \exp(-j\hat{\phi}_x(m,n))}{\sqrt{\sum_{m=1}^{M} \sum_{n=1}^{N} I_1(m,n) \sum_{m=1}^{M} \sum_{n=1}^{N} I_2(m,n)}}$$

(6.5)

where the term $\hat{\phi}_x(m,n)$ represents an estimation of the interferometric phase component. This estimation can be obtained from the data itself or can be derived from an external DEM of the area under study. An intensity based, phase independent coherence estimator has been proposed in [199]

$$\gamma = \frac{\sum_{m=1}^{M} \sum_{n=1}^{N} I_1(m,n)I_2(m,n)}{\sqrt{\sum_{m=1}^{M} \sum_{n=1}^{N} I_1^2(m,n) \sum_{m=1}^{M} \sum_{n=1}^{N} I_2^2(m,n)}}$$

$$|\hat{\rho}| = \begin{cases} \sqrt{2\gamma - 1} & \gamma > .5 \\ 0 & \gamma \leq .5 \end{cases}$$

(6.6)

As reported by the authors in [199], this power based coherence estimation presents the limitation that a large number of samples are needed to achieve the same estimation accuracy than the interferometric phase compensated coherence estimator, Eq. (6.5).

6.3 Modulated Coherence Estimation: Theory

In Chapter 4, a complete theoretical study of the interferometric phasor has been presented. This phasor consists in a unit amplitude phasor, whose argument is the phase difference between a pair of SAR images acquired with an InSAR systems, i.e., the Hermitian product phase difference. Therefore, this phasor only contains information concerning the measured phase difference $\phi$. The results derived from the study presented in that chapter can be summarized into three main issues:

- Parameter $N_c$. As demonstrated in Section 4.2.1, it is possible to derive from the measured interferometric phase $\phi$ a new amplitude parameter, denoted by $N_c$, which has been shown to carry the same information as the coherence $|\rho|$, i.e., the amplitude of the complex correlation coefficient between a pair of SAR images. The expression of $N_c$ can be obtained in Eq. (4.11) and Appendix B, whereas plots as a function of $|\rho|$ are given by Figs. 4.1b and 4.11. The availability of $N_c$ allows to introduce the concept of modulated coherence, i.e., $N_c \exp(j\phi_x)$, where $\phi_x$ denotes the true interferometric phase. Despite $N_c$ is obtained from noise terms, the modulated coherence is considered as a useful signal component.

- Interferometric phasor noise model in the original domain. The study of the measured interferometric phasor has made possible to define a noise model for the interferometric phasor in the original domain. The expression of this model in the complex domain, is presented by Eq. (4.30) on page 81. The study of this model has shown its poor usefulness in order to extract directly relevant information.
· Interferometric phasor noise model in the wavelet domain. The noise model in the original domain has been transformed to the discrete wavelet domain, considering the theory exposed in Chapter 3. The expression of this noise model is given by Eq. (4.92). As demonstrated in Section 4.3, a very efficient use of this model can be performed within the wavelet domain, since the transformation process itself enhances the useful signal term, i.e., the modulated coherence term, whereas noise power is maintained.

The objective of this section is, hence, to define a new algorithm to estimate the modulated coherence term from the interferometric phasor, with high spatial resolution. This process will be based on the wavelet transform. The modulated coherence estimation process presents two different aspects. With respect to the phase information, it is pretended to find an estimation of the true interferometric phase, i.e., $\phi_x$. Thus, it can be seen as an interferometric phase noise filter. With respect to the coherence, it is pretended to estimate the parameter $N_c$, in order to derive the coherence value $|\rho|$. It is important to note here, that, departure data consist in the measured interferometric phasor, which contains only the measured interferometric phase $\phi$. On the contrary, it will be demonstrated that it is possible to estimate the true interferometric phase $\phi_x$, as well as the coherence $|\rho|$. The possibility to derive both informations only from a phase measurement is due, on the one hand, to the availability of a noise model for the interferometric phasor and, on the other hand, to the ability of the wavelet transform to make an efficient use of it.

6.3.1 Wavelet Domain Estimation

As it has been proved with different signal processing techniques and applications, the wavelet theory appears very effective in those situations in which signal resolution is an important issue to consider. Indeed, Chapter 4 has already demonstrated the usefulness of this theory in the case of SAR imagery, since it has been proved that the discrete wavelet transform (DWT) itself induces a noise reduction and makes possible to derive phase and coherence information from the measured interferometric phase. In addition to these features, the use of the DWT can be beneficial from other points of view.

Since the useful signal and the noise components of the interferometric phase difference are non-homogeneous, the majority of techniques designed to eliminate noise adapt to this nature through a windowing process. This process has been shown to introduce a loss in image resolution and image details. Some techniques try to overcome this disadvantage by means of directional or morphologic windows. Despite these approaches reduce the loss of spatial resolution, they can not adapt to any signal shape, since only a finite set of directional windows is available. These drawbacks can be solved efficiently employing the DWT. As it has been described throughout Chapter 3, the wavelet transform brings the signal from its original domain to a new domain characterized by presenting both, spatial and frequency resolution. Consequently, the measured interferometric phasor corresponding to a complete interferogram can be transformed to the wavelet domain, where a local analysis of the signal and the noise can be performed. Since the DWT is applied to the complete interferometric phasor, and not to a portion of it, the probable loss of spatial resolution due to a windowing process is avoided. In addition, any signal shape can be analyzed without restrictions, as the wavelet coefficients contain this information. From a point of view of signal processing, the use of the DWT makes not necessary a pre-processing to adapt to the signal morphology, since this is performed by the DWT itself.

The DWT allows to analyze the measured interferometric phasor in a space-frequency domain. As a consequence, this feature makes possible to combine the advantages of analyzing the interferometric phase in the original domain, as performed in Eq. (6.1) [94], with the advantages a frequency domain provides (6.2) [194]. In Section 4.3.3, it was demonstrated that the DWT process itself improves the quality of the useful signal component, i.e., the larger the wavelet scale $2^j$, the larger the modulated coherence signal to noise ratio. Of course, this improvement is accomplished at the expense of spatial resolution. As demonstrated theoretically in Section 3.3.1, and in Section 3.3.4 for the discrete case,
the wavelet transform is an invertible process. Hence, the important issue at this point is that this loss in spatial resolution can be recovered in the inverse DWT (IDWT). This point opens the possibility to obtain a phase estimation characterized by a remarkable maintenance of spatial resolution.

The signal in the wavelet domain is a multi-scale description of the original signal. In the case of a discrete two-dimensional signal, as for instance the measured interferometric phasor real and imaginary parts, the transformed signal in the wavelet domain, for a given number of wavelet scales $2^j$, is composed by two sets of coefficients. The first set, $a_j[m,n]$, corresponds to a coarse approximation of the original signal obtained through a low-pass filtering process, see Fig. 3.5. The second set \{d_H^j[m,n], d_V^j[m,n], d_D^j[m,n]\}_{0<j<J}, corresponds to the approximation coefficients. Since the coefficients $a_j[m,n]$ represent a low-pass version of the input signal, these coefficients, for the transformed measured interferometric phasor, can be considered as being the result of a multilook process. This parallelism can be considered since the multilook filter, Eq. (6.1), represents basically a low-pass filter of the interferogram. The multilook approach, Eq. (6.1), is penalized in terms of spatial resolution, since, only the low-frequency portion of the spectra is retained. This disadvantage is partially solved by the self-weighted filter approach, Eq. (6.2). The DWT represents, therefore, the union of the ideas behind the multilook and the self-weighted approaches, since it allows to analyze the interferometric phase in a space-frequency frame, and to retrieve signal components from all the range of spatial frequencies.

Since the transformed signal consists in the measured interferometric phasor, i.e., the interferometric phase is processed in the complex domain, it is not necessary to introduce a pre-processing step with the objective to unwrap, locally or globally, the interferometric phase. The interferometric phase noise will be reduced in the wavelet complex domain taking into account the noise model defined in Chapter 4. From a point of view of the estimation of the parameter $N_c$, to process the measured interferometric phasor in the complex wavelet domain opens the door to estimate coherence information without the necessity to compensate for the interferometric phase, which has been shown to introduce a bias when the coherence sample estimator is considered, Eq. (6.3).

### 6.4 Modulated Coherence Estimation: Algorithm

As indicated by this section’s title, the following lines shall describe an algorithm to estimate the modulated coherence term, i.e., $N_c \exp(j\phi_x)$, from the complex interferometric phase, based on the wavelet analysis theory. As it has been shown in the previous section, the use of the wavelet theory to estimate relevant information for multidimensional SAR imagery, and InSAR in particular, allows to collect ideas employed separately and to use them together in order to improve signal estimation.

Specifically, the algorithm which is going to be defined estimates the interferometric phase $\phi_x$ and the coherence $|\rho|$ from the measured interferometric phase $\phi$. The interferometric phase estimation process can be understood as a classical noise reduction process. This algorithm will be defined with two main premises. On the one hand, the objective is to estimate the interferometric phase with a minimum loss of spatial resolution and details. On the other hand, it is pretended that the algorithm does not process areas with a high noise content, i.e., low coherence areas, as it may introduce signal distortions, specially in the interferometric phase [193]. The coherence estimation process is based on the obtention of the parameter $N_c$ and its inversion.

### 6.4.1 Working Principles

As it is demonstrated by Eqs. (4.90) and (4.91) in Section 4.3.3, the DWT increases the SNR of the modulated coherence component in the wavelet domain. On the contrary, the noise power is maintained. Despite this improvement is accomplished at the expense of spatial resolution, this issue does not represent a problem since spatial resolution is recovered in the IDWT. The signal improvement introduced by the
DWT is lost when the IDWT is applied. Consequently, in order to maintain this improvement, the algorithm has to detect, in the wavelet domain, which coefficients contain useful information, i.e., the signal coefficients as defined in Section 4.4, and multiplies its real and imaginary parts by two every time a wavelet scale is inversely transformed, avoiding the loss of the improving factor introduced by the DWT [200]. Since noise coefficients (as defined in Section 4.4) are maintained, the final effect of this process is that the useful signal component is enhanced with respect to the noise.

The following paragraphs present a quantitative demonstration of the principle in which the algorithm to be proposed is based on. This demonstration concerns one-dimensional signals for simplicity, but the extension to the two-dimensional case is also considered. Let \( S_1[m] \) and \( S_2[m] \) be a pair of one-dimensional correlated SAR images whose complex correlation coefficient is \( \rho[m] \exp(j\phi_x[m]) \). Let the true interferometric phasor to follow the signal model given in Section 4.3.1, \( \exp(j\phi_x[m]) = \exp(j2\pi m/N) \), where \( 2\pi/N \in [-2^{-1}\pi, 2^{-1}\pi) \). Hence, the measured interferometric phasor \( \exp(j\phi[m]) \), where \( \phi[m] \) denotes the phase of the Hermitian product \( S_1[m]S_2^*[m] \), is described by the noise model Eq. (6.30)

\[
e^{j\phi[m]} = a_0[m] = N_c[m] \exp \left( j \frac{2\pi}{N} m \right) + (v_c[m] + jv_s[m]) (6.7)
\]

where \( a_0[m] \) indicates the signal in the original domain. The expression of the parameter \( N_c[m] \), as well as the statistical properties of the noise terms \( v_c[m] \) and \( v_s[m] \) can be found in Section 4.2. The information content in Eq. (6.7) is the noisy interferometric phase and a unit mean intensity which can be decomposed as \( N_c^2 + \sigma_v^2 + \sigma_s^2 = 1 \), see Section 4.2. The one-dimensional DWT is applied to the real and imaginary parts of Eq. (6.7) with one wavelet scale only, i.e., \( 2^j = 2^1 \), where the low-frequency coefficients \( a_1[p] \) are calculated as given by Eq. (3.58) and the high-frequency coefficients \( d_1[p] \) are given by Eq. (3.59). These coefficients, considering the noise model for the interferometric phasor in the wavelet domain, Eqs. (4.60) and (4.61), have the expressions

\[
a_1[p] = \sqrt{2}N_c[2p] \exp \left( j \frac{2\pi}{N} 2p \right) + (v_{c,a_1}^w[p] + jv_{s,a_1}^w[p]) (6.8)
\]

\[
d_1[p] = v_{c,d_1}^w[p] + jv_{s,d_1}^w[p]. (6.9)
\]

As observed, the signals \( a_1[p] \) and \( d_1[p] \) have half the original resolution and the useful term, i.e., the modulated coherence term, is concentrated on \( a_1[p] \) and is multiplied by the factor \( \sqrt{2} \). Given Eqs. (6.8) and (6.9), the algorithm which will be proposed detects that the coefficients \( a_1[p] \) contain useful signal and will multiply them by \( \sqrt{2} \), leading to the expressions\(^2\)

\[
a_1[p] = 2N_c[2p] \exp \left( j \frac{2\pi}{N} 2p \right) + (\sqrt{2}v_{c,a_1}^w[p] + \sqrt{2}jv_{s,a_1}^w[p]) (6.10)
\]

\[
d_1[p] = v_{c,d_1}^w[p] + jv_{s,d_1}^w[p]. (6.11)
\]

Eq. (3.60) gives the expression for the IDWT

\[
a_0'[m] = \sqrt{2}N_c[m] \exp \left( j \frac{2\pi}{N} m \right) + (v_c'[m] + jv_s'[m]) + (v_c''[m] + jv_s''[m]) (6.12)
\]

where \( v_c'[m] \) and \( v_s'[m] \) come from the noise terms \( \sqrt{2}v_{c,a_1}^w[p] \) and \( \sqrt{2}v_{s,a_1}^w[p] \), respectively for the low-frequency coefficients \( a_1[p] \), whereas \( v_c''[m] \) and \( v_s''[m] \) are due to the high-frequency coefficients \( d_1[p] \) noise terms \( v_{c,d_1}^w[p] \) and \( v_{s,d_1}^w[p] \), respectively. Eq. 6.12 can be expressed as

\[
a_0'[m] = \sqrt{2}N_c[m] \exp \left( j \frac{2\pi}{N} m \right) + (v_c'[m] + jv_s'[m]) (6.13)
\]

\(^2\)As shown in Chapter 4, the DWT introduces an improving factor of \( \sqrt{2} \), instead of 2, for one-dimensional signals.
where \( v_x^r[m] = v_x'[m] + v_x''[m] \) and \( v_x^i[m] = v_x'[m] + v_x''[m] \). It can be easily demonstrated that the different noise powers are

\[
\begin{align*}
\sigma_{v_x}^2 &= \sigma_{v_x'}^2 = \sigma_{v_x''}^2 \quad (6.14) \\
\sigma_{v_x'}^2 &= \sigma_{v_x''}^2 = \frac{1}{2} \sigma_{v_x}^2 \quad (6.15) \\
\sigma_{v_x''}^2 &= \frac{3}{2} \sigma_{v_x}^2 = \frac{3}{2} \sigma_{v_x''}^2. \quad (6.16)
\end{align*}
\]

If Eqs. (6.13) and (6.16) are considered with more attention, one can observe, for the real and the imaginary parts, that whereas the power of the useful signal term, i.e., the modulated coherence real and imaginary parts, is increased by a factor \( \Delta P_{signal} = 2 \), the noise power is only increased by a factor equal to \( \Delta P_{noise} = 3/2 \). Hence, the relative power increment between signal and noise is

\[
\frac{\Delta P_{signal}}{\Delta P_{noise}} = \frac{4}{3} \approx 1.333. \quad (6.17)
\]

The recovered phase from Eq. (6.13) is

\[
\hat{\phi}_x[m] = \arctan \left( \frac{\sqrt{2} N_c \sin(\phi_x[m]) + v_x^r[m]}{\sqrt{2} N_c \cos(\phi_x[m]) + v_x^i[m]} \right). \quad (6.18)
\]

Since the power of the useful signal term presents a larger increment than the noise power, the phase contained by the former, i.e., \( \phi_x[m] \), will dominate over the phase error introduced by the noise terms. Consequently, the noise effect is reduced without reducing spatial resolution. On the other hand, the intensity mean value of Eq. (6.13) has the expression

\[
E(|a_0'[m]|^2) = 2N_c^2 |m| + \frac{3}{2} \sigma_{v_x}^2 |m| + \frac{3}{2} \sigma_{v_x''}^2 |m|. \quad (6.19)
\]

As it can be observed, the effect of the term \( N_c \) is increased more than the noise effects. From the intensity, coherence will be recovered through the parameter \( N_c \). This demonstration has shown that useful information can be recovered without loss of resolution, despite the improvement is rather small. As shown in the following, this improvement is larger for the two-dimensional case and the DWT applied with more than one scale.

For two-dimensional signals, the coefficients detected as signal coefficients in the wavelet domain are multiplied by 2 instead by \( \sqrt{2} \), since the two-dimensional DWT enhances useful signal by this factor. Hence, in the original domain, the useful signal power is increased by a factor \( \Delta P_{signal} = 4 \), and the noise terms by a factor \( \Delta P_{noise} = 7/4 \). Hence,

\[
\frac{\Delta P_{signal}}{\Delta P_{noise}} = \frac{16}{7} \approx 2.286. \quad (6.20)
\]

That is, a larger increment than in the one-dimensional case. Eq. (6.20) gives the increment when only one scale is calculated. This improvement is increased when the DWT is calculated for more wavelet scales. Let \( \hat{\phi}_x[m,n] \) be a two-dimensional interferometric phase such that the corresponding modulated coherence is concentrated in the frequency interval \( \omega_m \in [-2^{-j'} \pi, 2^{-j'} \pi] \) and \( \omega_n \in [-2^{-j'} \pi, 2^{-j'} \pi] \), which corresponds to the coarse coefficients \( a_{j'}[p,q] \). Since, every time a wavelet scale is inversely transformed useful signal will be detected, the final factor for which it has been multiplied is \( 2^{j'} \), thus, \( \Delta P_{signal} = 4^{j'} \). On the contrary the final noise power is multiplied by a factor such that \( \Delta P_{noise} = 1 + 3j'/4 \). Fig. 6.1 shows the relative power increment as a function of the wavelet scale \( j' \). As one observes it presents an exponential behavior.

A qualitative representation of the interferometric phasor in the wavelet domain is depicted in Fig. 6.2. As it can be observed, the amplitude of this vector is multiplied by \( 2^j \) (for the two-dimensional
6.4.2 Signal Detection

The previous section has presented the principle of the algorithm which is going to be proposed. As mentioned, the algorithm is based on detecting which coefficients in the wavelet domain contain signal and which ones contain only noise. This section is devoted to present the process to detect these coefficients.

Before to present the detection process, it is worth to consider the decorrelation property of the wavelet transform [201]. Since the wavelet coefficients contain information concerning a particular area of the case), whereas the noisy coefficients, which correspond to the coefficients $d_j$, are not modified. Thus, the amplitude of $a_j$, which contains useful information, is increased with respect to the amplitude of the coefficients $d_j$. Since the amplitude of the signal coefficients $a_j$ is multiplied by two, for every wavelet scale, and the amplitude of the noise coefficients $d_j$ is maintained, the phase of the coefficients $a_j$, which contains useful information, is enhanced with respect to the phase of $d_j$, which is basically noise. Therefore, noise is reduced. It can be observed that in the wavelet domain, the problem of processing a phase signal has been reduced to process the amplitude of a complex coefficient. With respect to the amplitude information, since the term $N_c$ is recursively multiplied by two, this information is enhanced with respect to the amplitude of the noise coefficients. Finally, as $N_c$ is recovered, its value can be inverted in order to derive coherence information.
original image, they can be considered uncorrelated within the same wavelet scale. For two-dimensional signals, as every wavelet scale is composed by three spatial-frequency bands, the wavelet coefficients can be considered uncorrelated inside every band. This decorrelation does not exist between different scales since, the wavelet filters present common frequency bands. The smaller the frequency overlap, the smaller the correlation between the wavelet scales. Since the wavelet coefficients are uncorrelated inside the same scale and/or band, a coefficient can be processed independently of each other, allowing to use diagonal estimators theory [173].

The noise model for the measured interferometric phasor in the wavelet domain, given by Eq. (4.92), can be considered as a linear model [173]

\[ x = H \theta + w \]  

(6.21)

where \( x \) are the observed data, \( H \) is a known coefficient, \( \theta \) is the parameter to be estimated and \( w \) represents the random noise term. Under the assumptions that the useful signal term \( \theta \) and the noise \( w \) are uncorrelated, but also mutually uncorrelated, the Bayes linear minimum mean square error (LMMSE) estimator of \( \theta \) has the expression [173]

\[ \hat{\theta} = E \{ \theta \} + \sigma^2_w H (\sigma^2_\theta H^2 + \sigma^2_w)^{-1} (x - H E \{ \theta \}) \]  

(6.22)

where \( E \{ \theta \} \) and \( \sigma^2_\theta \) represent the mean and variance of \( \theta \), respectively. Under the assumptions that \( \theta \) can be characterized by a prior distributed as \( \mathcal{N}(\mu_\theta, \sigma^2_\theta) \) and \( w \) a noise distributed as \( \mathcal{N}(0, \sigma^2_w) \), Eq. (6.22) is the optimal linear estimator. Since the wavelet coefficients have a mean equal to zero, Eq. (3.17), Eq. (6.22) simplifies to

\[ \hat{\theta} = \frac{\sigma^2_w H}{\sigma^2_w H^2 + \sigma^2_w} x = \Gamma_{Wiener} x. \]  

(6.23)

\( \Gamma_{Wiener} \) is called the Wiener coefficient, which gives rise to the Wiener filter [173]. This coefficient filters input data in such a way that the larger the noise content, i.e., \( \sigma^2_w \), the lower the value of \( \Gamma_{Wiener} \). Therefore, if the Wiener filter is applied in the wavelet domain, it would reduce the amplitude of those coefficients dominated by noise.

The signals which are processed in the wavelet domain are the real and the imaginary parts of the wavelet coefficients which follow the noise model given by Eq. (4.92). In Eq. (6.21), the observation coefficient \( H \) introduces a variation on the useful information. Considering the interferometric phasor noise model given by Eq. (4.92), \( H = 2^j \). If \( H \) is considered equal to \( 2^j \), the Wiener filter eliminates the improving factor introduced by the DWT. Consequently, \( H \) will be assumed equal to one, and the improving factor \( 2^j \) is considered, thus, as a term to recover. The interferometric phase in the wavelet domain \( \phi^w_x \) can take any value in the interval \([-\pi, \pi]\). Hence, without loss of generality, it can be assumed that \( E \{ \cos^2(\phi^w_x) \} = E \{ \sin^2(\phi^w_x) \} = 1/2 \). Taking into account Eq. (4.29) for the noise term power expression, the Wiener coefficient for the real and imaginary parts of the measured interferometric phasor in the wavelet domain has the expression

\[ \Gamma_{Wiener} = \frac{2^{2j} N_c^2}{2^{2j} N_c^2 + (1 - |\rho|^2)^{0.685}} \]  

(6.24)

where the expression of \( N_c \) as a function of \( |\rho| \) is given by Eq. (4.11). Fig. 6.3a gives a plot of \( \Gamma_{Wiener} \) for several values of the wavelet scale parameter \( 2^j \).

The Wiener filter is not employed directly as it presents a main drawback for the modulated coherence estimation problem. As it can be deduced from Fig. 6.3a, the Wiener filter practically eliminates those wavelet coefficients which are mainly dominated by noise, introducing, hence, important distortions in the interferometric phase estimation. For this reason, the value of these coefficients should be maintained as they are. On the contrary, the Wiener coefficient \( \Gamma_{Wiener} \) can be employed as an indicator of the information content for a particular wavelet coefficient. That is, those wavelet coefficients whose coefficient \( \Gamma_{Wiener} \) is above a certain threshold can be classified as signal coefficient, whereas those below can be
classified as noise coefficients. The problem $\Gamma_{\text{Wiener}}$ presents in this case is, that, the selection of the threshold is very sensitive since, a small variation in the threshold is translated to a large variation in coherence. Additionally, two parameters have to be estimated, the noise powers $\sigma^2_\theta$ and $\sigma^2_w$.

In order to solve the previous drawbacks, a new parameter for the selection of the wavelet coefficients, based on $\Gamma_{\text{Wiener}}$, is introduced. The new parameter solves them, since the selection of the wavelet coefficients is much less sensitive to the selection of the threshold and only the noise power has to be estimated. The proposed coefficient $\Gamma_{\text{sig}}$ has the expression

$$\Gamma_{\text{sig}} = I_w - 2^{2j'} \sigma^2_w I_w$$

where $2^{j'}$ is the maximum wavelet scale. The term $I_w$ represents directly the intensity of a complex coefficient in the wavelet domain. Since wavelet coefficients are the result of a filtering process, it can be assumed that intensity has the expression given by Eq. (4.93). The parameter $\sigma^2_w$ denotes the noise power, whose expression can be found in Eq. (4.29). Substituting these values, Eq. (6.25) takes the form

$$\Gamma_{\text{sig}} = 1 - 2^{2j'} \frac{1}{2^{2j} N_c^2 + (1 - |\rho|^2)^{0.685}} (1 - |\rho|^2)^{0.685}.$$  

A representation of $\Gamma_{\text{sig}}$ in the case the maximum wavelet scale equals $2^{j'} = 2^3$ is given by Fig. 6.3b. As it can be observed, for a fixed threshold, the selection of coefficients will depend on the wavelet scale. As it has been already mentioned in Chapter 4, the low wavelet scales (high-frequency scales) contain basically noise as a consequence of the interferometric phase and the noise natures. Besides, the improvement introduced by the DWT in these scales is rather small. Consequently, as shown in Fig. 6.3b, only those coefficients with very high quality, i.e., coefficients corresponding to high coherence areas, will be selected in these wavelet scales. On the contrary, the larger the wavelet scale, the larger the improvement of the useful signal term. Hence, coefficients corresponding to low coherence areas can be selected with a minimum risk to amplify noise when multiplying the intensity by 2. If one compares $\Gamma_{\text{Wiener}}$, Fig. 6.3a, with $\Gamma_{\text{sig}}$, Fig. 6.3b, it can be observed that both parameters are qualitatively equal. The advantages of $\Gamma_{\text{sig}}$ are, on the one hand, that the selection of a given threshold to classify wavelet coefficients does not need to be very accurate, and, on the other hand, only the noise power needs to be estimated from data.

---

**Figure 6.3**: Values of $\Gamma_{\text{Wiener}}$ and $\Gamma_{\text{sig}}$ for the first three wavelet scales, $j \in \{1, 2, 3\}$. In the case of $\Gamma_{\text{sig}}$, the maximum wavelet scale is $2^{j'} = 2^3$. 

![Figure 6.3: Values of $\Gamma_{\text{Wiener}}$ and $\Gamma_{\text{sig}}$ for the first three wavelet scales, $j \in \{1, 2, 3\}$. In the case of $\Gamma_{\text{sig}}$, the maximum wavelet scale is $2^{j'} = 2^3$.](image-url)
6.4.3 Algorithm Description

This section presents a step-by-step description of the algorithm designed to estimate the modulated coherence term on the basis of the principles detailed in Section 6.4.2. An scheme of this algorithm is given first in Fig. 6.4.

![Scheme of the algorithm defined to estimate the modulated coherence term in the wavelet domain.](image)

The different steps of the algorithm are:

**Step 1:** *Wavelet transform.* As indicated, the first step consists in the calculation of the two-dimensional DWT corresponding to the real and imaginary parts of the measured interferometric phasor \(\exp(j\phi)\), where \(\phi\) represents the measured interferometric phase. The DWT will be calculated with the two-dimensional Mallat algorithm, see Section 3.3.5. At this point, the number of wavelets scales \(2^j\) and the type of wavelet filters have to be specified. With respect to the former, in Section 4.4 it was demonstrated that noise effects are negligible with three wavelet scales, i.e., \(2^j = 2^3\). Hence, three wavelet scales will be calculated. A larger number of wavelet scales can be calculated for a larger reduction of noise effects. The selection of the wavelet filter is explained in detail within the next section.

As it has been mentioned, the DWT introduces a factor 2 for each wavelet scale. Hence, if three scales are calculated, the final improving factor is equal to: 8 for the spatial frequency range \(\omega_m \in [-\pi/4, \pi/4]\) and \(\omega_n \in [-\pi/4, \pi/4]\), 4 for the range \(\omega_m \in [-\pi/2, -\pi/4] \cup [\pi/4, \pi/2]\) and \(\omega_n \in [-\pi/2, -\pi/4] \cup [\pi/4, \pi/2]\), and 2 for \(\omega_m \in [-\pi, -\pi/2] \cup [\pi/2, \pi]\) and \(\omega_n \in [-\pi, -\pi/2] \cup [\pi/2, \pi]\). Despite the algorithm does not eliminate any wavelet coefficient, it can be considered that useful signal is mainly concentrated on the frequency range \(\omega_m \in [-\pi/2, \pi/2]\) and \(\omega_n \in [-\pi/2, \pi/2]\), whereas the range \(\omega_m \in [-\pi, -\pi/2] \cup [\pi/2, \pi]\) and \(\omega_n \in [-\pi, -\pi/2] \cup [\pi/2, \pi]\) contains mainly noise. Nevertheless, as it will be shown, the algorithm will be able to recover information contained in \(\omega_m \in [-\pi, -\pi/2] \cup [\pi/2, \pi]\) and \(\omega_n \in [-\pi, -\pi/2] \cup [\pi/2, \pi]\). Consequently, in order to have an homogeneous improving factor equal to 8 in \(\omega_m \in [-\pi/2, \pi/2]\) and \(\omega_n \in [-\pi/2, \pi/2]\) the discrete wavelet packet transform (DWPT) is employed. In this case the selection of the band-splitting process is deterministic. Fig. 6.5 presents a plot of the scheme to perform the DWPT. After transforming the real and imaginary parts of the measured interferometric phasor with two wavelet scales, they are divided into one set of coarse coefficients \(a_2\) and six sets of detail coefficients, \(d_H^m, d_V^m, d_D^m\) for the first wavelet scale and \(d_H^n, d_V^n, d_D^n\) for the second one. The DWT would split only the coarse coefficients \(a_2\) into four additional bands. On the contrary, the DWPT splits into four additional bands each one of the following bands: \(a_2, d_H^2, d_V^2, d_D^2\). Therefore, the constant increasing factor of 8 is accomplished.

**Step 2:** *Detection.* Once the measured interferometric phasor is transformed to the wavelet domain, those coefficients containing useful information have to be detected. To do this task, the parameter \(\Gamma_{\text{sig}}\), Eq. (6.25), is employed.
Step 3: Signal Increase. The previous step has detected those coefficients containing useful information. Consequently, the real and imaginary parts (or the amplitude) of the signal wavelet coefficients are multiplied by 2. This will avoid to loose the improving factor introduced by the DWT in the IDWT. It can be considered that noise reduction is performed at this point.

The idea behind this processing is to improve those coefficients which contain useful information, enhancing its information content in front of noise. Since noise coefficients are not reduced or eliminated, the possible information contained in these coefficients, which is difficult to detect as a consequence of noise, is incorporated in the inversion process.

Step 4: Inverse wavelet transform. At this point the IDWT is applied, but only a wavelet scale is inversely reduced. From every set of four wavelet bands at any scale $2^j$, the band at the wavelet scale $2^{j-1}$ is obtained. This inversion process can be observed in Fig. 6.5.
Step 5: Mask growing. In order to obtain a mask locating signal coefficients for lower wavelet scales, i.e., higher frequency scales, a new mask is derived from the one generated within Step 2. Each four bands at the scale $2^j$ are derived from a single band in the previous wavelet scale $2^{j-1}$, where a 1 to 4 space relation is established between wavelet coefficients. First, the masks of the four bands at the scale $2^j$ are merged through a logical OR operation. Then, the dimensions of the merged mask are doubled to fit the $2^{j-1}$ band dimensions. In this case, if a pixel of the merged mask is classified as signal, the four pixels referring to the same spatial area, but in the band of the scale $2^{j-1}$, are also classified as signal, otherwise, they are classified as noise. This sequence of mask growing allows to obtain a mask locating useful signal in the original domain.

At this point, the algorithm returns to Step 2, to start the process of signal detection in a new wavelet scale. The mask generated at Step 2, and whose dimensions have been increased in Step 4, is stored and combined with the mask derived in the new iteration. This issue ensures that those coefficients selected as signal coefficients will be classified as signal in the new iteration. Since the DWT has been calculated with three wavelet scales, this iteration process has to be performed six times. Four to undo the DWPT, and two iterations to calculate the IDWT with two wavelet scales.

Step 6: Interferometric phase estimation. Once the complex signal is brought to the original domain, the argument of the recovered signal is the interferometric phase estimation $\hat{\phi}_x$. This estimation is renamed $\hat{\phi}_{WLT}$. The algorithm to estimate the interferometric phase has been named Wavelet interferometric phase Filter (WInPF).

Step 7: Coherence estimation. The amplitude of the recovered complex number is the estimation of the parameter $N_c$. Therefore, Eq. (4.11) has to be inverted to estimate coherence information, $|\hat{\rho}_{WLT}|$. Due to the complex expression of Eq. (4.11), which contains an infinite series, a look-up-table is employed for the inversion process. As it can be observed in Eq. (6.19), the amplitude recovered in the original domain is multiplied by the total increasing factor introduced by the algorithm. Therefore, in order to have the parameter $N_c$ in the interval $[0,1]$, and to invert it through Eq. (4.11) to derive $|\rho|$, it is necessary to divide the recovered amplitude by $2^j = 2^3$. For a different number of wavelet scales, it has to be divided by the corresponding factor.

As shown, with this algorithm one can estimate the complex correlation coefficient between a pair of SAR images, that is $\rho$. Therefore, the complete algorithm is called Wavelet Correlation Coefficient Estimator (WCCE).

6.5 Modulated Coherence Estimation: Results on Interferometric Phase

The following section is devoted to present the performance of the algorithm, presented above, to estimate the interferometric phase $\phi_x$. The evaluation of this algorithm, or any other approach, has to deal with the drawback that it is not possible to have access to the original signal to recover. For this reason, this section is divided into two parts. A series of simulated interferograms will be employed in the first part to present a quantitative performance analysis. The second part is focused on the analysis of real interferograms. A fairly important part of this section is focused on the analysis of different aspects concerning the maintenance of spatial properties of the original signal.

The analysis of real interferometric phases can be considered from two sides depending on the information content. On the one hand, the algorithm has to deal with interferograms presenting mainly topography. In this case, the main task of the algorithm is to maintain the fringes information. On the other hand, the algorithm has to deal with interferograms containing man-made structures. In this case,
spatial resolution maintenance acquires a main role. As it will be shown, the WinPF algorithm is able to handle both types of interferograms.

### 6.5.1 Interferometric Phase Estimation: Simulated InSAR Data

Within this section, the performance of the WinPF algorithm is evaluated and compared with the approaches which have been presented in Section 6.2. In order to perform this task, two types of simulated interferograms are employed: a ramp and a cone. In all the cases, different levels of noise, i.e., different levels of coherence between the SAR images from which the interferogram is constructed, are considered. These simulated data are obtained with the algorithm presented in Section 4.2.3.

This first analysis consists in a global comparison between the WinPF algorithm based on the wavelet transform and the approaches presented in Section 6.2. The multilook filter has been applied with a 5 by 5 pixel window in order to minimize spatial resolution losses. The same argument leads to the self-weighted filter to be applied in a 6 by 6 pixel window, maximizing noise reduction with $\alpha = 1$. In the case of the WinPF approach, $t_h = -1$ and the 81 coefficient truncated Shannon wavelet filter is considered to calculate the DWT, see Section 3.4.2. The simulated interferometric phase consists in a 256 by 256 pixel interferogram representing a cone with a 6 pixel fringe period. Table 6.1 presents the performance analysis comparison between the different approaches, for different levels of coherence. Fig. 6.6 depicts the results for the lowest coherence, i.e., $|\rho| = 0.4$. From the results presented in Table 6.1, one can observe that the WinPF algorithm overcomes the other filters in error terms. In this case, two mean square errors (MSE) have been calculated. The MSE calculated in the real plane, between the true and the filtered interferogram, accounts mainly for errors in $2\pi$ phase jumps areas providing, therefore, information about how the filter maintains phase jumps. The MSE in the complex plane is calculated from the phase of the complex Hermitian product of the phasors containing the real and the processed phases. In this case, phase jumps are not present. The third row in Table 6.1 accounts for the number of phase residues, which for the true interferometric phases (as % of the total number of pixels of the image) are: 4.2% for $|\rho| = 0.9$, 14.5% for $|\rho| = 0.7$, 23.4% for $|\rho| = 0.5$ and 27.1% for $|\rho| = 0.4$. Since, the lower the number of residues, the easier the phase unwrapping, it can be concluded that the WinPF approach presents the easier interferogram to unwrap, since the residues reduction ranges from a 100% for $|\rho| = 0.9$ to 90% for $|\rho| = 0.4$. Finally, execution times, obtained with a computer based on

<table>
<thead>
<tr>
<th>WinPF algorithm</th>
<th>MLT filter</th>
<th>SW filter</th>
<th>LEE filter</th>
</tr>
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<tbody>
<tr>
<td>$</td>
<td>\rho</td>
<td>= 0.9$</td>
<td></td>
</tr>
<tr>
<td>MSE r.p. (dB)</td>
<td>-1.034</td>
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<td>0.511</td>
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<td>-10.518</td>
<td>-12.518</td>
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<tr>
<td>Num. residues</td>
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<td>16</td>
</tr>
<tr>
<td>Exec. time (s)</td>
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</tr>
<tr>
<td>$</td>
<td>\rho</td>
<td>= 0.7$</td>
<td></td>
</tr>
<tr>
<td>MSE r.p. (dB)</td>
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<td>2.600</td>
<td>0.449</td>
</tr>
<tr>
<td>MSE c.p. (dB)</td>
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<td>-5.278</td>
</tr>
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<td>Num. residues</td>
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<td>798</td>
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<tr>
<td>Exec. time (s)</td>
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</tr>
<tr>
<td>$</td>
<td>\rho</td>
<td>= 0.5$</td>
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<tr>
<td>MSE r.p. (dB)</td>
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<tr>
<td>Exec. time (s)</td>
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<td>0.6</td>
<td>4.8</td>
</tr>
</tbody>
</table>

6.5. MODULATED COHERENCE ESTIMATION: RESULTS ON INT. PHASE

Figure 6.6: Interferometric phase filtering result. In (b) noise corresponds to a coherence $|\rho| = 0.4$.

...a 600 MHz processor, are presented. The multilook and the self-weighted approaches present the lower execution times since their algorithms are the most simple. In the case of the WInPF algorithm, low times are obtained by using the Mallat algorithm, which can be consider as a fast DWT [23].

If now, the results of Fig. 6.6 are considered, it can be seen that the WInPF phase estimator is able to process any fringe direction in a very effective way, despite the algorithm does not explicitly adapt to the signal morphology. As explained in the first part of this chapter, the wavelet transform provides the indirect mean to adapt to the fringes direction. The result obtained with the self-weighted filter, Fig. 6.6d, deserves a special comment. In this case, due to the high regularity of the interferometric phase, the signal model in which the filter is based on is completely fulfilled. Hence, a relative good result is obtained.

The properties of the interferometric phase noise depend on the interferometric coherence, see Section 2.2.3, in such a way that the lower the coherence, the larger the phase noise variance. The next analysis studies the WInPF algorithm’s behavior as a function of the interferometric coherence $|\rho|$. In this case, two simulated interferometric phases have been employed. The first one corresponds to a 256 by 256 pixel ramp with a slope producing 6 pixel fringes. The second represents a 256 by 256 pixel, 6 pixel fringe cone. For these analyses, the multilook and the self-weighted filters are applied with the same parameters as in the previous case, whereas in the WInPF algorithm a 40 coefficient Daubechies filter, see Section 3.4.2, instead the Shannon filter, is employed. Fig. 6.7 presents the evolution of the MSE error, in the complex plane case, as a function of coherence. In this case, two MSE errors are considered: the first is obtained as the difference between the estimated ($\hat{\phi}_x$) and the true interferometric ($\phi_x$) phases and the one interchanging the true by the noisy interferometric ($\phi$) phase. These two errors allow to analyze very precisely the behavior of each algorithm.

If Fig. 6.7a is compared with 6.7b, one can observe that the WInPF approach clearly overcomes the multilook filter and the self-weighted approach. Despite the multilook is applied with a 5 by 5 pixel window, it destroys the interferograms due to the high value of the fringes frequency, see Fig 6.6d. Of course, the higher the noise content the worst the recovered interferometric phase in all the cases. The
Figure 6.7: Mean square error (MSE) as a function of coherence, for different filtering approaches, WInPF: Wavelet based algorithm. MLT: 5 by 5 pixel multilook. SW: 6 by 6 self-weighted. The first column refers to the MSE obtained between the true and the estimated interferometric phases, whereas the second column refers to the error obtained with respect to the noisy interferometric phase. As indicated, the interferometric phases represent a cone and a ramp.

second column of Fig. 6.7, where the MSE considers the error between the processed and the noisy phase, gives relevant information as it indicates the amount of filtering. The shape of these curves show the final behavior of the interferometric phase estimators. In the case of the self-weighted filter and the WInPF approach, the maximum error is obtained for middle coherences. In the range from $|\rho| \approx 0.2$ to $|\rho| \approx 0.7$, the WInPF approach presents larger errors than the self-weighted filter, indicating a larger noise reduction. Additionally, if one looks to Figs. 6.7a and 6.7c, the lowest MSE with respect to the original signal is obtained in the case of the WInPF approach. For high coherences, since noise variance is small, the filters adapt to this feature reducing the amount of filtering in order to avoid the loss of spatial details. The same behavior is observed for low coherences, and specially in the case of the WInPF approach. As mentioned in Section 6.4 the WInPF filter was designed to do not filter low coherence areas in order to avoid the introduction of geometric distortions.

Fig. 6.8 presents, for the set of interferometric phases simulated and analyzed in the previous paragraph, the number of residues for the estimated interferometric phases. For high coherence areas, all the approaches, except the multilook filter, practically eliminate all noise induced residues. The lower the coherence, the lower the residue reduction. But, as it can be observed in Fig. 6.8, the WInPF approach
reduces a higher number of them.

![Ramp Interferogram](image1)

![Cone Interferogram](image2)

**Figure 6.8:** Residues reduction. The number of residues is expressed as a percentage of the total number of the image pixels. (a) Number of residues for the Ramp interferogram. (b) Number of residues for the Cone interferogram.

### 6.5.2 Interferometric Phase Estimation: Real InSAR Data

The quantitative analysis presented in the previous section has shown that the WInPF algorithm is able to estimate interferometric phases correctly, overcoming existing approaches. The drawback of this analysis comes from the fact that the simulated interferometric phases are highly regular. Despite the study is valid to compare quantitatively the different approaches, simulated interferograms are rather far away from real interferograms. Due to this reason, but also to the fact the wavelet based approach overcomes other interferometric phase filters, this section is concerned basically to the study of the WInPF approach with real data, despite in some cases, comparison with other estimation techniques is provided.

Since, due to the interferometric phase noise, access to the original interferometric phase is not possible, this section will present basically qualitative measurements, despite when possible, quantitative results are also presented. The comparison is performed with four real InSAR datasets. The first dataset corresponds to InSAR data of Mt. Etna (Italy), already employed in Section 4.2.3 to validate the interferometric phasor noise model. The second dataset corresponds also to Mt. Etna, but this time data were acquired with the spaceborne sensor SIR-C/X-SAR. The third dataset, acquired with the spaceborne ERS-1/2 sensor, corresponds to Serra de Cardó, a mountainous region on the northeastern coast of Spain, whereas the final dataset has been acquired with the E-SAR system over the Oberpfaffenhofen test site in Germany. Each of these datasets will allow to analyze the WInPF algorithm over a wide range of different signal conditions.

**E-SAR InSAR Data: Mt. Etna (Italy)**

The first interferometric phase which has been analyzed, corresponding to Mt. Etna, was acquired with the E-SAR system \[186,202\] at X-band (9.6 GHz), in a single-pass system configuration with a baseline of approximately 1.3 m. Fig. 4.9, on page 85, presents the noisy interferometric phase. This interferometric phase has been processed with the WInPF approach, in which the threshold \(th_w\) equals -3 and the 10-coefficient Daubechies filter has been employed. Fig. 6.9 presents the results which have been obtained in this case.
As it can be observed in Fig. 6.9a, the WInPF algorithm is able to eliminate phase noise without the introduction of geometric distortions or false information. Despite the DWT has been applied in a global scale to the whole interferometric phase, as a consequence of the local analysis performed within the wavelet domain, the WInPF algorithm is able to eliminate phase noise with the same performance in areas with a smooth surface (low-frequency fringes) and in areas with a steep slope (high-frequency fringes). As designed, the WInPF algorithm does not filter areas containing only noise, as observed in the phase difference image, Fig. 6.9b. The original interferometric phase presents 178036 phase residues, whereas the processed contains only 92430. As Fig. 6.9d shows, in the case of the processed phase image, these residues are located basically within those areas which contain only noise, that is, the areas the filter has not processed.

One important issue to consider when employing techniques based on the wavelet analysis theory is the selection of the wavelet filter to perform the DWT. In the theoretical development introduced in Chapter 4, in order to derive the interferometric phasor noise model in the wavelet domain, the wavelet filter was supposed ideal from a point of view of its frequency response. In this sense, it is possible to be close to this behavior by employing a truncated and discrete version of the Shannon wavelet filter, see Section 3.4.2 for details. Indeed, the estimation results presented in Table 6.1 have been obtained by means of a truncated version of 81 coefficients. It is worth to mention that, despite a wavelet with a long space response is employed in this case, low execution times are obtained and hence, the use of long response wavelet filters does not represent a main drawback.

In the following, the selection of the wavelet filter is addressed from the point of view of interferometric
phase noise reduction. From the Mt. Etna interferometric phase acquired with the E-SAR system, Fig. 4.9, a 512 by 512 pixel area is selected to test the WInPF performance as a function of the wavelet filter. Fig. 6.10a presents the execution times of the WInPF algorithm employing a three scale DPWT with the Daubechies family of wavelet filters, whose number of coefficients ranges from 2 to 40. As observed, the difference in execution times between the shortest and the longest filter is not very large. As a conclusion, the algorithm’s execution time is not a decisive factor which determines the selection of the wavelet filter. The second figure, Fig. 6.10b, represents the number of residues within the processed interferometric phase as a function of the Daubechies filter length and the threshold $\Gamma_{\text{sig}}$. As one can see in this case, the wavelet filter length is not a decisive factor either concerning the reduction of phase residues. This reduction has a more important dependence on the threshold $th_w$, since a reduction of its value implies that the algorithm filters areas with lower coherence, with the consequent reduction of the number of residues. Consequently, from this results, the 10 coefficient Daubechies filters presents the best compromise.

The impact of the wavelet filter length can be specifically studied from a local point of view. Fig. 6.11 corresponds to results obtained with the WInPF algorithm over a 200 by 200 pixel area of the Mt. Etna interferogram with different Daubechies filters. These results present clear differences.

As it can be observed, Fig. 6.11a is clearly affected by a square effect. This case has been processed

![Figure 6.11: Estimated interferometric phase detail (200 by 200 pixel). The results correspond to the Daubechies filters of (a) 2 coefficients (Haar wavelet), (b) 10 coefficients and (c) 30 coefficients.]
with the 2 coefficient Daubechies filter, which corresponds to the Haar wavelet. As mentioned in Section 3.4.2, this wavelet is characterized by presenting only one vanishing moment, producing the wavelet to be a piecewise function, i.e., the wavelet is not regular. This effect disappears if wavelet filters with longer responses are employed, Figs. 6.11b and 6.11c. This is due to the fact that in the Daubechies family case, the longer the wavelet the higher the number of vanishing moments [135]. Additionally, from Fig. 6.11 it can be observed that the longer the Daubechies filter, the more areas are filtered. The explanation of this effect has to be found on the frequency behavior of the wavelet filters. Fig. 6.12 gives the frequency response of the low pass filter \( h \) for the filters employed in Fig. 6.11, in the one-dimensional case. As observed in Fig. 6.12, the enhancement introduced by the wavelet transform depends on the filter itself. Therefore, the longer the filter length, the wider the frequency range in which coefficients are enhanced by a factor 2 in the two-dimensional case (\( \sqrt{2} \) in the one-dimensional case). Short filters produce the coefficients to be enhanced by lower factors, therefore less coefficients are detected as signal coefficients. But as observed, the variation between the filters of 10 and of 30 coefficients is not very high. Consequently, it can be concluded that the 10 coefficient Daubechies filter presents the best compromise for interferometric phase noise reduction, despite minor improvements occur for longer filters.

The algorithm presented in Section 6.4.3 has been defined on the basis of the interferometric phasor’s behavior in the wavelet domain, as it has been described in Chapter 4. Therefore, whenever the selected wavelet filter has a frequency response close to the ideal case, i.e., the frequency response of the wavelet Shannon filter, the WinPF algorithm will be able to reduce interferometric phase noise efficiently. Indeed, whenever the selected wavelet filter has a frequency response rather close to the ideal case, the selection of a particular family of wavelet filters, from which some examples employed in this work are detailed in Section 3.4.2, does not introduce large differences in the characteristics of the estimated phase. Consequently, it can be affirmed, without the slightest doubt, that the results which can be obtained with the WinPF algorithm are rather independent of the wavelet family of filters employed to calculate the wavelet transform.

**SIR-C/X-SAR Data: Mt. Etna (Italy)**

The performance of the WinPF algorithm proposed in the first part of this chapter is now tested with a low coherence interferogram. As mentioned, the proposed algorithm has two main parameters controlling the capability to process these areas. On the one hand, the lower the threshold parameter \( \text{th}_w \), the larger the filter ability to enhance wavelet coefficients containing information concerning low coherence areas. On the other hand, the number of wavelet scales \( 2^j \). For interferograms characterized by presenting medium
and high coherence values, it has been shown that to calculate the DWT with three wavelet scales is a good compromise concerning the signal enhancement introduced by the DWT. Despite the algorithm described in Section 6.4.3 has been defined with a three-scale DWT transform, where the DWPT is employed to calculate the third wavelet scale, the interferometric phase noise reduction algorithm is completely valid if more wavelet scales are calculated.

The WInPF is now applied to a X-band (9.6 GHz) interferometric phase acquired with an spaceborne SAR sensor during the mission SIR-C/X-SAR [203], in October 1994. This interferogram belongs also to Mt. Etna, Sicily (Italy), and was acquired in a repeat-pass configuration with a baseline of 60 m and a temporal separation between both acquisitions of 1 day. From this interferogram, the 1024 by 1024 pixel area containing the volcano’s mouth has been selected to test the WInPF phase noise reduction algorithm. The original interferometric phase presents 320675 phase residues, i.e., the 30.58% of the total number of pixels. Fig. 6.13 shows the original single-look interferometric phase, as well as the coherence histogram, whose mean value equals 0.278.

The interferometric phase presented in Fig. 6.13a has been processed with the WInPF algorithm. In order to be able to reduce phase noise from the most of the image, the threshold \( th_w \) equals -32. With this choice, one can ensure that only those coefficients belonging to areas with a coherence practically zero are not enhanced. In order to study possible effects due to the wavelet filter choice, the interferometric phase has been processed with two different filters, the Daubechies filters of 10 and 40 coefficients. Fig. 6.14 presents the estimated interferometric phase from data given in Fig. 6.13a with the two Daubechies filters.

The first conclusion which can be extracted from the results presented in Fig. 6.14 is, that, despite input data is characterized by a very low coherence, the WInPF phase estimator does not introduce any artifact or geometric distortion in the estimated phases. A qualitative comparison between the estimated interferometric phases with both Daubechies filters does not show relevant differences between them. In both cases, the algorithm reduces considerably the number of phase residues. But, whereas in the case in which the DWT is calculated with the 10 coefficient Daubechies filter the estimated interferometric phase presents 45230 residues (4.31% of image pixels), this figure is reduced to 37977 (3.62% of image pixels) when the 40 coefficient Daubechies filter is applied. Therefore, the selection of the wavelet filter to calculate the DWT does not represent, as shown above, a main drawback.

The interferometric phasor noise model in the wavelet domain developed in Chapter 4, as well as the modulated coherence estimator presented in the first part of this chapter, are based on the properties of
the DWT. It has to be noticed that at any point, neither the noise model nor the estimation algorithm assume a particular wavelet family of filters. The only assumption which has been taken is to regard the wavelet transform as an ideal filter bank considering the Shannon family of wavelet filters, see Section 4.3. The noise model and the WInPF algorithm rely on the DWT capability to split the input signal into different hierarchically related frequency scales, leading to a multiscale representation. To consider an ideal DWT means that the wavelet scales do not overlap in frequency. On the contrary, to consider a particular wavelet filter family has to be seen as the fact that the filters do not present an ideal frequency response, producing the different wavelet bands to overlap in frequency. Wavelet filters with a low number of vanishing moments (VM) produce the wavelet bands to have a large frequency overlapping. On the contrary, the larger the number of VM, the smaller the frequency overlap between wavelet scales. Therefore, as demonstrated in the previous section, as well as by results presented here in Fig. 6.14, the only characteristic a wavelet filter family has to present is a relatively high number of VM to ensure sufficient frequency separability. In order to give a number to this figure, it can be concluded that the WInPF algorithm is able to estimate the interferometric phase correctly when the DWT is calculated with Daubechies filers of order equal or higher than 10, which corresponds to 5 VM. Above this order, the larger the number of VM the larger the reduction of phase residues, despite this reduction is not very significant.

The employment of the DWT to remove noise presents the advantage to enhance the spatial resolution properties of the estimated interferometric phase. Fig. 6.15 presents 256 by 256 pixel detail images from the original interferometric phase, as well as from the estimated interferometric phases with the multilook approach, with a 5 by 5 pixels window, and from the proposed algorithm, in which the DWT is calculated with the Daubechies wavelet filters with 10 and 40 coefficients.

As it can be noticed with the detail images, Fig. 6.15, the estimated interferometric phases with the WInPF algorithm are clearly smoother than the phase estimated with the multilook approach. One observes that the later is clearly affected by the 5 by 5 pixel analysis window, whereas the ones estimated with the WInPF approach do not present any signal artifact. The result is due, in part, to the fact that the wavelet approach does not segment the image, since the local analysis of the signal is performed in the wavelet domain.

As observed from the interferometric phase images which have been presented until now, the height information is wrapped within this phase in a $2\pi$-modulus. Consequently, the interferometric phase has to be unwrapped in order to recover the terrain’s relative height. In absence of phase residues, this process would be trivial. But the presence of these hinder the unwrapping process, in such a way that the larger
the number of phase residues, the more difficult the unwrapping [189]. Therefore, one of the reasons behind the use of an interferometric phase filtering process, apart from the phase noise reduction itself, is to decrease the number of phase residues. There exist different approaches which can be employed for the task of phase unwrapping [204, 205, 206, 207, 208], whereas a comprehensive treatise is given in [85]. It exists a category of relatively simple procedures to unwrap the interferometric phase which are based on deriving the least squares solution of the problem. The weighted least squares unwrapping process, in which the unwrapping process is determined in the basis of coherence information, has been employed to unwrap the filtered phase given by Fig. 6.14b. The unwrapped phase is presented in Fig. 6.16.

ERS-1/2 InSAR Data: Serra de Cardó (Spain)

Within the previous sections, it has been demonstrated that the WInPF algorithm is able to reduce interferometric phase noise, overcoming already existing techniques in terms of MSE and number of phase residues. Additionally, its performance to reduce noise in problematic situations, as for instance steep slopes producing high-frequency fringes or low coherence interferograms has been illustrated with different types of InSAR data. Another important feature of the WInPF algorithm is its capability to estimate
the interferometric phase with a high spatial resolution. This property has been already analyzed at the end of the previous section.

The capability of the WInPF technique to maintain spatial resolution is a property which can not be measured quantitatively in a simple and effective way. In Section 6.5.1, the spatial resolution properties of the WInPF algorithm have been illustrated with simulated interferometric phases, showing that it is able to recover close fringes. This ability has been also demonstrated with real data. But usually, the real interferometric phases present spatial details with a morphology completely different from the one of the phase fringes. The following two sections focus specifically on the analysis of the spatial properties of the WInPF algorithm employing real interferograms. In this section, spatial details are analyzed over interferograms containing basically topographic information, whereas in the next one this property is analyzed over an interferogram containing man-made structures.

In this case, InSAR data correspond to a C-Band (5.3 GHz) interferogram acquired with the ERS-1/2 tandem mission in a repeat-pass configuration with an approximate baseline of 160 m. The area contained by this interferogram belongs to the mountainous region of Serra de Cardò, on the northeastern coast of Spain. Fig. 6.17a contains a 1024 by 1024 pixel image of this interferometric phase, where from top to bottom it can be observed: the Mediterranean sea, a flat area and a mountainous region with a maximum height of 1200 m. This image is characterized by presenting severe geometric distortions due to layover and foreshortening. This interferometric phase presents 159939 phase residues (14.39% of image pixels), whose location is given by Fig. 6.17c.
The interferometric phase presented in Fig. 6.17a has been processed with the WInPF algorithm. In this case, since the interferogram presents only low coherence values in the sea area, the threshold $th_w$ is equal to -1. In order to perform the DWT, the 10 coefficient Daubechies filter has been employed. Fig. 6.17b presents the estimated phase, whereas Fig. 6.17d gives the location of the phase residues. The estimated interferometric phase contains a total number of 28049 phase residues which represent only the 2.6% of the image pixels. As it can be observed in Fig. 6.17d, the majority of the phase residues are located in the sea area, since as designed, the proposed algorithm does not process low coherence areas. A small amount of phase residues are observed in the mountainous region in almost horizontal lines. These residues are not due to noise but to the interferometric distortions introduced by aliasing due to layover and foreshortening. As it can be noticed by comparing the original and the estimated interferometric phase in Fig. 6.17, it can be concluded that the proposed algorithm is able to recover the correct interferometric fringes, despite its irregular forms. One can observe, that this interferogram does not present so regular fringes as the Mt. Etna interferograms analyzed in the previous regions as a consequence, basically, of the spatial resolution.

The properties to recover spatial details of the different interferometric noise reduction algorithms are analyzed in the following, over the 200 by 200 pixel red area which has been indicated over Fig. 6.17a. This interferogram has been also processed with the multilook approach on a 5 by 5 pixel window and with the self-weighted filter, calculating the Fourier transform over a window of 8 by 8 pixels, and a coefficient $\alpha$ equal to 1. The estimated interferometric phases, as well as the original one are presented in Fig. 6.18.

![Image](image.png)

**Figure 6.18:** ERS-1/2 Interferometric details phase over Serra de Cardò. (a) Original phase. (b) Estimated phase with the multilook approach. (c) Estimated phase with the self-weighted approach. (d) Estimated phase with the WinPF algorithm.

As one can notice from the results presented in Fig. 6.18, there exist clear differences between the three estimated interferometric phases. This small area is characterized by presenting some areas where the interferometric phase is relatively smooth and some areas where the fringe’s shape is irregular. Important details are the clear areas of layover and foreshortening which can be identified as horizontal lines crossing the image. In this case, the mountain side has a slope very close to the look angle of the SAR system. This fact produces all the mountain slope to be compressed in one or two pixels with the consequent signal aliasing, which as a result creates low coherence areas.

The interferometric phase obtained with the multilook approach, Fig. 6.18b, is able to reduce phase noise over the smooth areas, despite a clear square effect appears as a consequence of the averaging window. The effect of the analysis window is clearly visible on the layover and foreshortening areas. Despite these areas are one or two pixels wide, the 5 by 5 averaging window extends them all along the window dimension. The second result, Fig. 6.18c, has been derived with the self-weighted filtering approach. This filter, as explained in Section 6.2, process the interferometric phase in the Fourier transformed domain on the basis that the interferometric phase can be locally considered as an slope. This signal model is not valid in the areas where geometric distortions are present. Therefore, as the estimated phase image shows, this filtering approach is not able to give a correct estimation of the interferometric phase in these areas.
The third result, presented by Fig. 6.18d, is derived with the WInPF algorithm presented in Section 6.4.3. The basis of this noise reduction algorithm are, on the one hand, the interferometric phasor noise model developed in Chapter 4, and, on the other hand, on the wavelet analysis theory. It is worth to mention that the interferometric phasor noise model derived in the wavelet domain, is based on the same signal noise model as the self-weighted filter. Despite they share the same basis, the estimated phase with the WInPF algorithm, Fig. 6.18d, is able to recover interferometric phase with a higher quality and with a lower loss of spatial details. The reason behind the difference between both results has to be found on the wavelet analysis theory itself. The interferometric phasor is locally analyzed within the wavelet domain. Despite the useful signal has been assumed a priori to be locally a ramp, the wavelet coefficients capability to retain very accurate spatial information makes possible to keep, to analyze and to process the difference between the signal model and the real signal to be modelled, therefore, maintaining the spatial resolution.

The previous series of results demonstrate that the WInPF algorithm is able to reduce noise effects over the interferometric phase overcoming existing approaches in terms of spatial resolution maintenance. This is not a consequence of the noise model for the interferometric phase noise model directly, but to the use of this model inside the wavelet domain.

**E-SAR InSAR Data: Oberpfaffenhofen (Germany)**

In all the previous sections, the interferometric phases which have been employed to test the different interferometric phase noise reduction algorithms were characterized by containing basically information concerning terrain topography. As a result, these phase images were dominated by phase fringes. A completely different type of interferometric phase appears when the SAR sensor images inhabited areas. These areas are characterized, quite often, by presenting a very smooth topographic component. Consequently, topography does not play an important role since, man-made structures give most of the information. This situation is more severe when the SAR sensor is placed in an airplane, since these are characterized by presenting larger spatial resolutions than spaceborne systems. Therefore, more image details will appear on the interferometric phase as: buildings, roads or other man-made constructions. In this situation, it is extremely important to maintain the spatial resolution of the estimated interferometric phase in order to avoid the loss of spatial details and eluding to mix contributions from different scatterers.

The InSAR dataset which has been employed in this section belongs to an L-band (1.3 GHz) interferogram acquired with the E-SAR sensor [186, 202] over the Oberpfaffenhofen test site, on the west of the

![Figure 6.19: E-SAR interferometric phase from the Oberpfaffenhofen test site. (a) Original single-look phase. (b) Estimated phase with the WInPF algorithm.](image)
German city of Munich. The interferogram was acquired in a repeat-pass mode, with an approximate baseline of 10 m and a time delay of 10 minutes. Hence, time decorrelation effects can be neglected. From the full dataset, an area of 1024 by 1024 pixel is selected in order to test and compare the different approaches to filter the interferometric phase. The single-look interferometric phase is presented in Fig. 6.19a.

The original interferometric phase depicted in Fig. 6.19a has been processed with the WInPF algorithm. In this case, the threshold $t_{th}$ equals -1 and the 40 coefficient Daubechies filter has been employed to perform the DWT in order to minimize possible spatial resolution losses. The estimated interferometric phase is given in Fig. 6.19b. As it can be noticed, the WInPF algorithm is able to reduce noise without the introduction of any type of artifact in the processed image. As it can be observed from Fig. 6.19a, the interferometric phase reflexes a flat region. Consequently, only two phase fringes due to topography are observed. On the contrary, the details of this phase are mainly originated by man-made structures such as buildings, roads or other man-made constructions. In this case, the maintenance of the spatial resolution, as well as image details, is one of the key factors to consider. Hence, any interferometric phase filter has to face this new type of signal morphology.

It is worth to make a reflection at this point concerning the nature of interferometric data and the way to reduce noise effects. As it can be observed in the different InSAR datasets employed throughout this section, the interferometric phase contains information from a wide range of details about the surface being imaged. On the one hand, the interferometric phase is sensitive to surface topography, but, on the other hand, it is also sensitive to all kind of details not being originated by the terrain’s topography. Hence, any interferometric phase noise filter has to adapt, somehow, to this signal heterogeneity. A first group of approaches, in which the self-weighted filter, Eq. (6.2), is a good exponent, adapt to heterogeneity through a signal model. Whenever the proposed signal model is valid, the filter will be able to reduce noise. But, in all those locations where the model is not valid, wrong results are expected. Since a simple signal model would not be able to efficiently characterize the useful signal, an algorithm based on it can not be employed as a general interferometric phase noise filter. A second group of techniques are not based on a signal model, but try to adapt to the signal morphology through a set of pre-defined signal shapes. Again, the heterogeneity inherent to the interferometric phase signal produces these algorithms to fail whenever the analyzed signal is not present on the set of analyzed shapes.

The WInPF algorithm, defined in Section 6.4.3, is able to unify the ideas detailed in the previous paragraph. First, the noise reduction capability of this algorithm is based on the interferometric phasor noise model described in Chapter 4. This model describes, basically, the InSAR data statistical behavior for distributed scatterers. Consequently, it fails to describe data when they can not be considered as originated by a distributed scatterer. Second, the proposed algorithm is based on the wavelet analysis theory, allowing to perform an accurate local analysis of a given signal. Hence, this property permits to make an efficient use of the interferometric phasor noise model in the wavelet domain. On the one hand, the algorithm does not try to adapt to the signal morphology since this process is directly performed by the DWT. Indeed, the use of the DWT allows to adapt to any type of signal, i.e., it allows to adapt to the signal heterogeneity. On the other hand, the wavelet transform allows to make an efficient use on the noise model for the interferometric phasor. In the following, the properties of the WInPF algorithm with respect to spatial resolution are analyzed and compared with other filtering approaches.

The interferometric phase presented in Fig. 6.19a has also been processed with the multilook filter employing a 5 by 5 pixel window and by the self-weighted approach, where in this case the FFT is applied over 8 by 8 pixel window and the exponent $\alpha$ equals 1. In order to study the spatial resolution maintenance capabilities of these approaches, as well as in the case of the WInPF algorithm, a set of three image details have been selected. Fig. 6.20 contains the original details and the estimated phases with the three approaches.

The detail in the first row of Fig. 6.20 corresponds to the response from a point scatterer. As it can be observed, the WInPF algorithm is able to recover practically the original shape and filtering
Figure 6.20: Interferometric phases for three image details. (a) Original phase. (b) Estimated phase with the multilook approach. (c) Estimated phase with the self-weighted filter. (d) Estimated phase with the WinPF algorithm.

In the light of the previous results, it can be concluded, that the WinPF algorithm filters the interferometric phase details without practically reducing the spatial resolution of the input image. It can be affirmed that this approach reduces interferometric phase noise, as much as possible, but until the limit this filtering does not introduce signal distortions. Hence, one can conclude that the algorithm is able to differentiate between distributed and point scatterers. In the former case, the algorithm reduces noise whereas in the later it maintains the shape. On the contrary, the multilook filter can be interpreted as a brute force method, filtering everything without considering signal properties. The self-weighted filter deserves a special attention. From the results given in the previous sections, it can be said that this approach perform relatively well for those cases in which the interferogram contains only topographic information. This filter relies on a signal model considering the interferometric phase to be locally a ramp. Since this model fails to describe image details, the filter introduces distortions on the estimated image.

Application: Urban Areas Processing

One of the main SAR imagery features is the high spatial resolution which allows to be sensitive to small details. One of the main drawbacks of SAR imagery is speckle noise, as a consequence of the coherence nature of SAR systems. Both aspects can not be separated since high spatial resolution is obtained at
the expense of processing the radar signal in the complex domain, where speckle noise has its origin. This two aspects converge when urban areas are considered in SAR imagery, and specially, in InSAR. As it has been shown in the previous section, the WInPF algorithm is characterized by maintaining the spatial resolution of the original image. Consequently, an important field of application of the WInPF algorithm is to reduce phase noise over urban areas.

From the InSAR dataset of the Oberpfaffenhofen test site presented in the previous section, an urban area containing several buildings have been selected. This area has been processed with the multilook filter and with the proposed algorithm. In this case, the threshold \( t_h \) has a value equal to -1, and the Daubechies filter of 40 coefficients has been employed to perform the DWT, in order to determine possible a dependence on the wavelet filter. Fig. 6.21 presents the urban area image employed to test the algorithm.

![Urban area test site](image)

(a) \( \phi \)

(b)

**Figure 6.21:** Urban area test site. (a) Original interferometric phase. (b) Optic image.

As it can be observed, two image cuts have been marked in Fig. 6.21a. The results of the two filters are presented in Fig. 6.22.

![Urban area image cuts](image)

(a) Building cut 1

(b) Building cut 2

**Figure 6.22:** Urban area image cuts where: red line is the original phase, dashed line corresponds to the multilook filter and black line corresponds to the WInPF filter. (a) First image cut. (b) Second image cut.

The image cuts presented in Fig. 6.22 demonstrate that the WInPF algorithm is able to maintain the structure of the interferometric phase, whereas the multilook approach, as a consequence of the processing window, destroys it. An important aspect which can be clearly observed in the first image cut, Fig. 6.22a, is the ability of the WInPF algorithm to face the different processes producing the signal. As it can be seen in the ground-truth image, Fig. 6.21b, the first part of the first image cut, from the first to the tenth pixel, belongs to a flat area composed by vegetation and a car park. The original phase image cut presents a noise-like signal, whereas the cuts from processed data give, approximately a flat area. Consequently, the filters remove noise in these areas. The next part of the image cut belongs to the buildings area, where it can be observed that the WInPF algorithm is able to follow the exact structure of the original signal maintaining the correct phase values. The second image cut, Fig. 6.22b, presents a similar behavior, despite clear differences between the original and the processed interferometric phases are visible. These results confirm the capability of the WInPF approach to maintain the properties of
the signal over urban areas.

6.6 Modulated Coherence Estimation: Results on Int. Coherence

Eq. (2.108) on page 30 corresponds to the formal definition of the interferometric coherence $|\rho|$. Under proper conditions, the expectation operator can be substituted by an spatial average. As shown in Section 6.2.2, the resulting estimator obtained from this substitution, Eq. (6.3), presents two important deficiencies. On the one hand, the estimation process itself, as a direct consequence of the spatial average process, introduces a loss of spatial resolution. On the other hand, unbiased coherence estimation needs the interferometric phase to be compensated.

One of the results derived from the study of the interferometric phasor noise model carried out in Chapter 4, has been the definition of the parameter $N_c$. As demonstrated, this parameter contains the same information as the coherence term $|\rho|$ and has been shown to be a fundamental piece to define the series of multidimensional SAR imagery noise models presented in Chapters 4 and 5. On the other hand, as demonstrated in the first part of this chapter, the wavelet analysis theory allows to estimate $N_c$ and, consequently, to estimate $|\rho|$. The next sections will analyze the coherence estimator, based on $N_c$, detailed in Section 6.4.3, together with the advantages over existing estimators [175].

6.6.1 Interferometric Coherence Estimation: Simulated InSAR Data

The performance of the WCCE algorithm is now analyzed and compared with the approaches presented in Section 6.2.2 by means of simulated InSAR data. On the one hand, it is demonstrated that the proposed algorithm estimates the right coherence value. On the other hand, it is proved that the estimation is independent of the interferometric phase term. That is, it is not necessary to estimate it from data or from external sources, in order to compensate for it.

The WCCE algorithm independence of the interferometric phase is demonstrated by comparing the estimated coherence values $|\hat{\rho}_{WLT}|$ with those obtained with the sample estimator $|\hat{\rho}_{MLT}|$, Eq. (6.3), and the sample estimator with interferometric phase compensation $|\hat{\rho}_{MLT+PH.C.}|$, Eq. (6.5). The WCCE algorithm has been applied by considering the 40 coefficient Daubechies wavelet filter and a threshold $th_w = -1$. The sample estimators have been applied over 5 by 5 pixel windows. The test consists in

![Figure 6.23: Coherence estimation with simulated interferometric phases. (a) Phase ramp with 40 pixel fringes. (b) Phase ramp with 12 pixel fringes. WCCE: Wavelet based estimation. MLT: Multilook estimation. MLT + PH. C.: Multilook estimation with phase compensation.](image-url)
applying the three estimators over two interferograms. This first one contains a ramp with a 40 pixel fringes slope, whereas in the second, the ramp produces 12 pixel fringes. The first case can be considered as a smooth slope, hence the sample estimators have to obtain good estimations since the interferometric phase is almost constant in the 5 by 5 pixels window. This is not the situation in the second case. Fig. 6.23 presents the results of both simulations.

As observed in Fig. 6.23a, the three coherence estimators obtain the same coherence value for the 40 pixel fringe ramp case. A small bias is observed in the case of the sample coherence estimator which is eliminated when the interferometric phase is compensated. The cause of this bias is that the interferometric phase does not fulfills the homogeneity condition inside the 5 by 5 pixel window. The WCCE estimator gives the same coherence value as the sample estimator in which the interferometric phase has been compensated. The second simulation, presented by Fig. 6.23b, shows a completely different situation. In this case, the interferometric phase presents an approximate variation of 2.6 rad in 5 pixels. This large difference produces the estimated coherence values by the multilook estimator to present a large bias with respect to the actual values. This bias is removed when the interferometric phase is compensated. On the contrary, the WCCE algorithm, even in the case of a steep slope topography, provides the correct coherence value.

The reason why the proposed algorithm is able to estimate the actual coherence value is found in Eqs. (4.5) and (4.6) on page 76, and on the use of the DWT. To consider the interferometric phasor in the complex plane allows to separate the interferometric phase component $\phi_x$ from the phase noise term $v$. $N_c$ is defined as the mean value of $\cos(v)$, see Eq. (4.11), and therefore, is independent of $\phi_x$. Until this point, there are no conceptual differences between the interferometric phasor and the Hermitian product of two SAR images, from which coherence is calculated. On the contrary, the DWT allows to exploit this independence in the transformed domain. The proposed algorithm, as shown in Section 6.4.3, enhances the wavelet coefficient amplitude independently of the coefficient phase content. As a result, $N_c$, and therefore $|\rho|$, is estimated independently of phase.

6.6.2 Interferometric Coherence Estimation: Real InSAR Data

The following two sections focus on the analysis of the WCCE algorithm over real InSAR data. Special attention will be placed on the analysis of the spatial properties of the estimated coherence value.

E-SAR InSAR Data: Mt. Etna (Italy)

The WCCE algorithm is now applied to the Mt. Etna interferogram acquired with the E-SAR system. Again, an area of 512 by 512 pixel from the image presented by Fig. 4.9 is analyzed.

As it has been demonstrated with simulated data, a way to remove the coherence bias introduced by the multilook coherence estimation is to compensate of the interferometric phase. Hence, the interferometric phase is needed in advance. There exist various ways to derive this information. The most reliable is to obtain it from an existing DEM of the terrain being imaged. Since, quite often, this model is not available, a way to proceed is to estimate the interferometric phase directly from data. In this case, the coherence information of Mt. Etna interferogram is estimated with three different algorithms. The first estimator consists in the multilook approach whereas, the second one is the same estimator but with an interferometric phase compensation. In this case, the estimated interferometric phase derived with the WInPF algorithm is employed for the compensation. In both cases, the average is applied over 5 by 5 pixel window because of the steep slope areas. The coherence is also estimated with the proposed algorithm in which the threshold $th_w$ is equal to -1 in order to process low coherence areas and the 40 coefficient Daubechies filter has been employed to perform the DWT. Fig. 6.24 presents the estimated interferometric phases, as well as the estimated coherences.
As observed in Fig. 6.24, with special attention to the difference plots, Figs. 6.24g and 6.24h, it can be observed that the three approaches estimate the same coherence values in the upper right-hand corner of the interferometric phase image. This is because this region contains the smoother slope in the image and it is also characterized by presenting the highest coherence values. This is not the situation in the middle band which goes from the upper left-hand to the lower right-hand corner of the image. Since this area contains steeper slopes, as seen before, it affects the multilook coherence estimation. If the interferometric phase is compensated, Fig. 6.24h, one observes that the differences between the coherence estimation provided by the multilook approach and the wavelet approach are much smaller and clearly centered around zero. Therefore, it can be concluded that the coherence estimator based in the parameter $N_c$ provides the true coherence values without being affected by the interferometric phase.

From the results presented in Fig. 6.24, the interferometric phase estimated by the WInPF approach, Fig. 6.24c, has been employed for its compensation in the multilook coherence estimator. But, as it can be observed in Fig. 6.24e, the interferometric phase compensation does not guarantee a right coherence estimation in low coherence areas. This is because a reliable interferometric phase cannot be estimated.
6.6. MODULATED COHERENCE ESTIMATION: RESULTS ON INT. COHERENCE

in these areas. Since, the interferometric phase estimated with the wavelet approach does not process low coherence areas, when compensated, a zero phase is obtained, see Fig. 6.24b, producing an artificial increase of the coherence values. For instance, in Fig. 6.24d, a square red area has been marked. The average coherence values for each one of the three coherence estimators in this area are: 0.397 for $|\hat{\rho}_{MLT}|$, 0.668 for $|\hat{\rho}_{MLT+PH.C.}|$ and 0.372 for $|\hat{\rho}_{WLT}|$. It can be concluded, therefore, that a correct coherence estimation is only possible with the multilook approach when a very reliable interferometric phase estimation is available or the estimation itself is independent of this phase.

ERS-1/2 InSAR Data: Serra de Cardò (Spain)

The results presented within the previous two sections demonstrate that the WCCE algorithm is able to estimate the right coherence values without the interferometric phase interference. As mentioned, the multilook coherence estimator with or without phase compensation, as a consequence of the spatial average, introduces a loss of spatial resolution in the coherence image with respect to the original signal. As it is demonstrated in the following two sections, the WCCE algorithm is able to estimate coherence information with a higher spatial resolution. One of the key consequences of this property is that more spatial details and information will be available from the coherence image.

This section focus on the analysis of the spatial resolution properties of the WCCE coherence estimator which has been presented in Section 6.4.3, by means of an interferometric phase containing basically topographic information. The ERS-1/2 dataset, which has been processed with the wavelet based algorithm, and whose results concerning interferometric phase estimation have been presented in Section 6.5.2, is now analyzed concerning, on the contrary, coherence estimation.

Figure 6.25: Coherence detail images of 200 by 200 pixels. (a) Multilook coherence estimator (MLT). (b) Proposed coherence estimator (WLT). (C) Coherence images cut.
The spatial resolution properties are considered over the same 200 by 200 pixel detail area analyzed in Section 6.5.2. Both, the multilook approach and the WCCE algorithm are applied considering the same parameters as in Section 6.5.2. Fig. 6.25 presents the plots of the estimated coherence values, as well as the coherence values plot over the red line. The histogram of the two estimated coherences, with the multilook and the WCCE approach, is presented by Fig. 6.25. The histogram of the differences between both values of the coherence estimators is given in Fig. 6.26.

First of all, from Fig. 6.25, it can be concluded that both estimators $|\hat{\rho}_{MLT}|$ and $|\hat{\rho}_{WLT}|$ estimate, in average, the same coherence values. This issue can be also confirmed with the image cut which has been presented in Fig. 6.25c. This coherence cut displays that both estimators give practically the same coherences values, independently of if the coherence value itself is high or low. The coherence estimation with the multilook approach has been derived without interferometric phase compensation. The phase compensation has been not applied since on the one hand, the type of topography suggested that the compensation could be superfluous. On the other hand, it was pretended to do not alter, in any sense, the spatial properties of the input signal. The small bias from zero of the mean value of the histogram given by Fig. 6.25, insinuates that interferometric phase estimation may be necessary.

Despite both approaches estimate the same coherence values, it can be observed clearly in the results of Figs. 6.25a and 6.25b, that the wavelet approach is able to give a coherence estimation with a higher spatial resolution. As it has been mentioned in Section 6.5.2, the dark, horizontal lines crossing the image correspond to layover and foreshortening areas, which are characterized by a low coherence. In the case of $|\hat{\rho}_{MLT}|$, the shape of these areas are clearly determined by the dimensions of the sliding windows, which in this case have a 5 by 5 pixel dimensions. On the contrary, these areas in the case of the estimator $|\hat{\rho}_{MLT}|$, are clearly narrower as it can be observed in the coherence image, as well as in the image cut, specially in those areas around pixels number 50 and 150. For instance, if one focus on the upper left-hand corner of the detail coherence images in Figs. 6.25a and 6.25b, it can be observed, in the case of $|\hat{\rho}_{MLT}|$, a series of horizontal lines crossing the area. The narrowness of these lines makes them to disappear under the 5 by 5 pixel averaging window employed in $|\hat{\rho}_{MLT}|$.

The basis of the coherence estimator proposed in Section 6.4.3, and employed here to estimate $|\rho|$ information is the parameter $N_c$ and the use of the wavelet analysis theory. The spatial resolution maintenance properties are provided by the DWT, since it allows to perform a multiscale estimation of the coherence information. High wavelet scales, characterized by containing low-frequency information, are employed basically to obtain the correct coherence value. Indeed, this is almost what the multilook approach does, as it can be considered as a low-pass frequency filter. The advantage of the DWT is that it allows to combine this information with that of lower wavelet scales. These contain high-frequency
information, hence, they are the main responsible to provide spatial details to the coherence information. It can be concluded that the DWT allows to estimate the coherence value employing the complete spectra of spatial frequencies.

**E-SAR InSAR Data: Oberpfaffenhofen (Germany)**

As demonstrated, the use of the DWT allows to estimate coherence from the amplitude of the modulated coherence term, i.e., $N_c$, with more spatial resolution than the multilook approach. Additionally, the WCCE algorithm estimates the true coherence values. The spatial resolution properties of the proposed coherence estimator makes possible to extract more information from coherence. This new information is basically concerned with the morphology of the scene being imaged. Until this section, the WCCE estimator has been tested over interferograms containing basically topographic information. But, this new estimator can be very useful in those cases where the interferogram contains much more spatial details. This is the case when the interferogram belongs to inhabited areas, since buildings, roads or other man-made constructions are present. This section focus specifically on this topic.

The dataset employed is this section consists in the L-band (1.3 GHz) interferogram of the Oberpfaffenhofen test site, presented in Section 6.5.2. The coherence images have been calculated with two difference methods. On the one hand, the multilook approach over a 5 by 5 pixel window, and, on the other hand, the WCCE algorithm. In this case, the wavelet transform is calculated with the 40 coefficient Daubechies filter and the threshold $t_{w}$ equals -1. The results concerning coherence estimation are shown in Fig. 6.27. The amplitude of the master SAR image is given in Fig. 6.27a. Fig. 6.27b consists in a ground truth, optical image of the car park located on the lower left-hand corner of this dataset. Fig. 6.28 presents the histogram of the difference between the coherence estimations presented in 6.27c and 6.27d, whereas Fig. 6.28b contains the cuts along red lines of the different images of Fig. 6.27.

First of all, as it can be concluded from the histogram given by Fig. 6.28a, both estimators, i.e., $|\hat{\rho}_{MLT}|$ and $|\hat{\rho}_{WLT}|$, provide the same coherence values, in average. It is worth to notice, that the difference mean equals 0.026, i.e., the difference histogram is centered around zero. This fact is confirmed when the coherence images given by Figs. 6.27c and 6.27d are compared visually. From this comparison, one can clearly observe that the coherence image obtained through the multilook approach is affected by a square effect as a result of the averaging window. For instance, the central area of these images contain some building and constructions which normally provide high coherence values, since they can be considered as point scatterers. When the multilook approach is applied, the response of these scatters is so high, compared with surrounding areas, that the average over the 5 by 5 pixel window accounts only for these point scatterers. This reason explains why the estimation in 6.27c presents a square effect. On the contrary, the proposed algorithm makes a more intelligent use of information in the wavelet domain. On the one hand, it takes the information from the high wavelet scales (low-frequency scales) to estimate the actual coherence value, but, on the other hand, it takes the information from the high-frequency scales to improve the spatial resolution of the coherence image. Going back to the building examples, it can be affirmed that, the WCCE algorithm is able to perform an intelligent average process. Therefore, it avoids to mix building contributions with those of surrounding areas.

A special attention deserves the car park located on the lower left-hand corner of the image. Fig. 6.27b presents a ground-truth of this area. As it can be observed, the car park consists of a series of line-wise parked cars separated by a series of asphalt roads. From the cars, it is expected to have a high backscattering producing high coherence values, since the cars can be considered as point scatterers. On the contrary, a lower scattering is expected from the road lines as they can be assumed to be smooth rough surfaces where, approximately specular reflection occurs. Since, in these areas the response is low as they are distributed scatterers, but also to the effect of thermal noise which has a large influence, low coherence values should be expected. Fig. 6.28b presents the cut plot over the red lines in the coherence images, but also over the master image amplitude. The last one has been conveniently scaled to fit the
coherence dynamic range. The central part of the graphic, between pixels 50 and 200, belongs to the car park. The dotted line corresponds to the amplitude image, where the lower values indicates the roads separating the car lines. The positions where the amplitude line equals one indicate the return from cars. As it is observed, the multilook estimator (blue line), gives a coherence practically constant over all the car park. On the contrary, it can be clearly noticed that the proposed WCCE algorithm (red line) gives low coherence values in those positions with low backscattering, which belong to the roads separating the cars. For instance, around the 100th pixel it can be observed that the low coherence area is wide enough for the multilook to be sensitive to it.

From the previous results, it can be concluded that the proposed algorithm is able to estimate the coherence information with higher spatial resolution. This increment of spatial resolution has to be understood as the ability of this algorithm to estimate coherence without mixing contributions from different scatterers. Some references in the literature have shown that the coherence images can be employed to extract valuable information [197, 74]. Consequently, the proposed algorithm is able to increase the information content of the coherence itself, specially from a morphological point of view.

Figure 6.27: Oberfaphenhoffen test site data. (a) Master SAR image amplitude. (b) Optic image detail of the parking located in the lower right-hand corner (ground truth). (c) Interferometric coherence estimated with the multilook approach. (d) Interferometric coherence estimated with the WCCE algorithm.
6.7 Summary

This chapter presents the practical application of the concepts and ideas which have been presented in Chapter 4. In summary, a new algorithm to estimate the true interferometric phase, as well as the interferometric coherence, is proposed and tested. This algorithm exploits the capability of the discrete wavelet transform to represent the image spatial details and the fact that the transform itself increases the useful signal quality, with the objective to obtain high spatial resolution estimations.

The first part of this chapter concerns the definition of the algorithm to estimate the interferometric phase and the interferometric coherence. The algorithm is based on avoiding the loss, in the inverse transformation process, of the improving factor introduced by the discrete wavelet transform in those complex wavelet coefficients which contain useful information. As demonstrated theoretically, the maintenance of this factor allows to estimate the true interferometric phase and the parameter $N_c$ without the loss of spatial resolution. The coherence information is obtained from $N_c$.

The second part of the chapter presents the results of the algorithm with respect to the true interferometric phase estimation. A set of simulated interferograms are employed to perform a quantitative comparison between this algorithm and other alternatives proposed in the literature, from which one can conclude that the proposed algorithm outperforms already existing techniques. In a second stage, the algorithm is tested with real interferometric SAR data representing basically topographic details. As it can be concluded from the results, the proposed algorithm is able to estimate the true interferometric phase with an imperceptible loss of spatial resolution. In the light of the obtained results, it has been possible to establish the characteristics a wavelet filter needs to fulfill in order to be suitable to reduce interferometric phase noise. In a final stage, the algorithm is tested with interferometric phases in which the important details are due to man-made objects. In this case, it is proved that the proposed approach, based on the wavelet transform, is able to retrieve all the spatial details.

In the last part of this chapter, the algorithm is tested with respect to the interferometric coherence estimation. A test with simulated interferometric data shows that the algorithm is able to estimate the correct coherence value. The most important feature of this approach is that the coherence estimation does not need a compensation for the interferometric phase. Finally, the algorithm is tested with real interferometric SAR data. As the results obtained in these cases show, the use of the wavelet transform allows to estimate the coherence with a higher spatial resolution than the standard multilook approaches. Therefore, the coherence maps derived in this case contain much more spatial details, and hence, more information concerning the different details of the data.

Figure 6.28: Histogram corresponding to $|\tilde{\rho}_{MLT}| - |\tilde{\rho}_{WLT}|$ for the coherence images shown in Fig. 6.27. The difference mean is equal to 0.026.