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OPTICAL SOLITONS IN QUADRATIC NONLINEAR MEDIA AND APPLICATIONS TO ALL-OPTICAL SWITCHING AND ROUTING DEVICES

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Chapter 1

Introduction

1.1 Optics, transmission and Optics for transmission

Little could attendants to John Tyndall’s lecture at the Royal Society of London in 1870 imagine that the simple experience they were witnessing in which Tyndall showed how the usual straight line paths followed by light rays could actually bend down to follow a stream of water flowing through a hole in an illuminated vessel, would ultimately set the scientific basis over which the today’s, more than a hundred years later, information highways and hence the whole concept of life and society is built up. The experience endowed with scientific category the phenomenon of interdielectrics light guidance very likely already familiar to Grecian and other ancient glassblowers who could have used it in fabricating their decorative glassware. As a matter of fact, some of the techniques used by the old Venetian glassblowers for making ‘millifiore’ have provided useful hints for the building up of the present fiber optics industry.

A rigorous theoretical description of the phenomenon, by Hondros and Debye, took still forty years to appear and still 17 years more were needed to find any practical application: in 1927, Baird in UK [1] and Hansell in the United States [2] applied for patents for image-transferring devices using fibres of silica. Following that suggestion in 1930, Lamm, in Germany [3], used a crude assembly of fibers to demonstrate the basic image and light transmission properties of fibers.

Before the great success attained by microwave technology as applied to remote sensing and both free space and guided transmission, some theoretical studies that praised the great
possibilities that optical carriers could offer, as greater transmission bandwidths, greater angular resolution and greater Doppler shift for moving target detection [4], were passed over. Fiber optics applications pointed therefore at a different direction. By the 30's the medicine industry already benefited from light piping around corners that allowed illumination of the hidden, of great utility for example in gastroscopy. However, since uncoated fibers were used, the efficiency was low, whereupon these ideas were not actively pursued and lay dormant until the early 1950s when Van Heel in Holland and Hopkins and Kapany in the UK reactivated the field with the basic purpose of developing an efficient 'flexible fiberscope'. From this renowned interest van Heel's idea of using solid coatings of lower refractive index to improve the light transmission efficiency was born giving rise to a burst of research activity for whose designation Kapany coined the today's familiar name of Fiber Optics. Even though image transmission through bunches of fibers (which were fabricated following similar techniques to those employed in Palestine to produce glass mosaics in the first century B.C.) was considered thus paving the path for application of fiber optics to transmission, by that time all the applications of fiber always concerned guidance of incoherent light with power losses in the 1000dB/Km range.

However amazing it may sound today, when the appearance of the laser in 1960 provided a powerful coherent source for modulation and transmission, the first optical transmission thoughts were for free space optics and complex systems were designed often using huge lens as repeaters [5].

Driven by the development of the transistor in 1947, the 60's are as well the years of the digital revolution: in the telephony network, the 64 Kbps voice channels were progressively introduced, and in 1969 ARPANET, a network connecting computers at 56 Kbauds began to operate. Better quality and variety of new services drove an exponential increase in the demand for connectivity that led to great congestion both in the radioelectric spectrum and in the cities underground ducts housing thick wire cables. The market demand for telecommunications services seemed to have overcome the possibilities offered by the microwave technology which together with the network maintenance problems mainly derived from the high failure rate of repeaters required every 2 Km, spurred the search for new transmission techniques and solutions.

After 10 years of research efforts devoted to find the link between optics and transmission,
the Corning Glass Works came about with the answer: following the suggestion of Kao and Hockam (1966) [6] the losses in the fiber were lowered down to $20\text{dB}/\text{Km}$ in the same year when room operation of a semiconductor laser was made possible. Optical transmission began to be considered an alternative to copper wires and the first fiber optics transmission links were constructed around 1973 which used LEDs or GaAlAs lasers and a carrier wavelength of $\lambda \sim 850\text{nm}$ for which the silica features a minimum losses spectral window with attenuations in the $4\text{dB}/\text{Km}$ range.

Optical systems entered the telephony network through intercity links, i.e. the trunk telephony network, in the third voice channel multiplexing hierarchy comprising 64 voice channels and data rates around $34\text{Mbps}$. Existing technology by that time did not allow the fabrication of very thin fibers which would only permit one transmission mode and hence the repeaterless transmission length was severely limited by intermodal dispersion to about $10\text{Km}$.

Single mode fibers were made possible by 1976 and together with improvement of light sources and detectors, a new generation of optical transmission systems was born, called the *second generation* (which of course assumed the previous systems to belong to the *first generation*) working in the $\lambda \sim 1.3\mu\text{m}$ minimum losses spectral window with losses in the $0.5\text{dB}/\text{Km}$. These systems could carry third hierarchy streams ($34\text{Mbps}$) with repeater distances of $50\text{Km}$, or fourth hierarchy ($140\text{Mbps}$, 256 voice channels) with $15-20\text{Km}$ between repeaters. Still a third generation born around 1979, uses the third spectral window $\lambda \sim 1.5\mu\text{m}$ with losses around $0.2\text{dB}/\text{Km}$ and data rates around $565\text{Mbps}$, comprising about 1024 voice channels, and typical repeaterless distances of $50\text{Km}$.

Today the replacement of electronic repeaters which required the subsequent previous optical-electronic and ulterior electronic-optical transformations by optical Erbium Doped Fiber Amplifiers (EDFA) fed through laser diodes, as providing a fully optical point-to-point link through which any bit stream can be transmitted regardless of its data format, velocity or carrier wavelength, has added versatility to the system which in addition results more reliable due to the reduction in components. That further allows with traditional NRZ format, dispersion-shifted fibers and other techniques designed to compensate for dispersion, to increase the channel capacity in commercial optical links up to about $10\text{Gbps}$ and $10.000\text{Km}$ of repeaterless distance.

With the use of EDFA's, multiplication of the channel capacity through the use of several
carrier wavelengths, the so-called Wavelength Division Multiplexing (WDM) technique, is economically feasible. As allowing accumulation of nonlinear effects along the link, EDFAs further open the door to soliton based transmission. Concerning fiber transmission, optical solitons designate these pulses arising from an interplay between linear dispersion and the nonlinearity present in the fibre which due to its very special properties can propagate over very large distances without significant alteration of its temporal profile. One may say that when solitonic propagation takes place the linear dispersion is compensated through the nonlinearity [7].

In front of traditional NRZ format and linear dispersion-compensation, whose technology is already well-known and established, soliton-based RZ formats as taking advantage of the nonlinearities present in the system, not only remove the need for nonlinearity compensation techniques, but offer in addition bit unit robustness against perturbations and a suitable format for performing temporal switching. Accustomed to the theoretically simpler basic principles of operation of linear systems, the technology world is quite reluctant to introduce nonlinear systems into the market but ultimately ‘money talks’ and most likely the technology which proves more reliable and cheap while offering greater capacity will prevail.

Through the use of WDM it has been demonstrated in field experiments that the bit velocity and distance at which existing technology is capable of sending information over the optical fibre are not likely to be exceeded by the telecommunications market requirements. Specifically $1Tbps$ ($100Gbps \times 10$ WDM channels) transmission over a 40 $Km$ fiber loop has been reported [8]. Another issue is handling and routing of this information which still has to be done by electronic means ensuing data format change in every switching node which reduces the performance. If the increasing demand for a growing variety of telecommunications systems is to be satisfied, techniques that allow to remove this ‘electronic bottleneck’ as it is often referred, and pave the path to a future All-optical Global Network, in which not necessarily all parts in the network rely on optical technology, but the information travelling in the network has always optical format, both for transmission and routing, need to be searched for.
1.2 The information Era: new ideas needed

With the introduction of the digital technology into the telephony network, separation between voice and data networks no longer made any sense. Since all information had the same format in transmission it was not such a crazy idea to join all telecommunication services in a unique transmission network which would merely act as a data conveyor so that only emitter and receiver nodes would need to know the nature of the data for human interpretation while the rest of switching nodes would route and process the data regardless of the service they belonged to. That is the idea behind the Integrated Services Data Network, IDSN, standard developed by the mid 70's and designed to constitute the basis of a Global Network that would gather all communication services in a unique plug [9].

Whether the great capacity offered by optical fibers has spurred the increase in the connectivity demand and with it, supported by the flexibility of the digital format, the urge for creation of new telecommunication services and facilities, or whether the degree of development achieved by optical transmission industry precisely responds to this growing necessity for connectivity, the truth is that presently telecommunication services have become an essential ingredient in our everyday life. That is so to the point that it is being said that after the two great Revolutions that marked radical changes in the Human Society concept, namely the Agricultural and the Industrial Revolutions, the third great radical change is being determined by the possibilities offered by telecommunications in what is called the Information Era. Certainly, telecommunication services are at the very basis of our society, hereof the importance of disposing of a reliable network that handles all the traveling information in an efficient and safe manner.

Before the foreseen great demand for telecommunication services and the feasibility of data transmission over the fiber at 10Gbps data rates and beyond [10], the fixed 64Kbps per channel defined by the ISDN standard appear ridiculous and evidence the necessity of new architectures and concepts for the Global Network management. The new Broadband ISDN standard designed for a better exploitation of the fiber optics bandwidth, considers flexible bandwidth channels in which every user occupies only the bandwidth that its transmission requires in every moment which is also the one for which he is charged for. To that aim, the standard foresees the use of Asynchronous Transfer Mode (ATM) with packet switching and establishment of
virtual connections that may be shared by packets belonging to different services and different virtual connections [11].

When these brilliant ideas are to be put into practice, one should consider that significant amounts of packets need to be routed throughout the network without significant delays noticeable to the user. To that aim, huge switching centers made out of ultrafast switchers capable of massive parallel switching are to be required [12]. Electronic switches based on VLSI technology are cheap and reliable and are believed to be able achieve switching velocities up to 40 Gbps or so but when arranged in large centers of bit processing, experience a significant reduction in performance due to the parasitic capacitances of connections [13]. There is in addition the network transparency issue. Shifting to electronics entails interpretation of the packet in every switching node which kills the flexibility sustained by the BISDN standard.

Packet switching and routing regardless of the packet meaning or the service it belongs to is therefore only achieved if the same data format is used for routing and transmission [14]. Design and building up of devices capable of optical data switching and routing is one of the goals pursued by the Photonics Technology which at present is vigorously developed at a research level and holds the potential of large switching velocities and large parallel arrangements inherent to photonic processes.

1.3 On the verge of a new generation of all optical devices

Presently, both market requirements and technology maturity have reached the point where all-optical operation in the network is cost-effective and therefore after over 30 years of preparations, the time seems to have come for all-optical devices to reach their place in the network [15]. Throughout these years the all-optical concept itself has experienced changes which responded to the telecommunications market evolution, optical materials industry development and the possibilities and impossibilities progressively found by basic phenomena research. Thus, from the initial all-optical concept as totally opposed to electronics entailing an almost complete elimination of all electronic parts in the network, the concept has evolved to approach eventual practical realizations to be equivalent to Network Transparency, meaning that even though many parts of the network may benefit from the advantages offered by electronics, the data
format is always optical, so that from a combination of optics and electronics, a more efficient network management is accomplished [16].

Other changes in the all-optical devices research mentality concern optical materials technology. A material with strong enough third-order nonlinear response and low losses which can bring into reality the many theoretical studies about all-optical switching in waveguides has yet to be discovered. Only fiber based devices which take advantage of the accumulated nonlinear phase shift over long propagated distances thus providing low-power temporal all-optical switching operation feature some chances of entering the commercial market even though still their technology is not mature enough so to allow for mass production.

Second-order nonlinear materials on their side, whose technology, due in great part to the electrooptical external laser modulators industry development, has reached a high degree of maturity presently are in a good position to constitute an alternative for some future all-optical devices. This is so specially for the ubiquitous Lithium Niobate, LiNbO₃, to the point that its key role in optical technology is somehow compared to that of silicon for the electronics industry, although significant improvements coming from intensive research are still required to approach switching performance market requirements.

Another of the drives of the all-optical devices research change of mentality is the telecommunications services market incredible grow which has led to the conviction that in the near future all-optical network operation will become necessary to respond to the demand for connectivity. At present, the existing options which have been considered for all-optical operation deployment do not appear still as definitive ones and hence till the time when the telecommunications demand urges for all-optical network management, new ideas and solutions need to be searched for. Since all-optical light control with applicability to switching and routing operation is based on nonlinear optical processes, the new ideas should come through vigorous research in the Nonlinear Optics research field.
1.4 Nonlinear Optics research: from intriguing phenomena to useful applications

Nonlinear effects in light propagation were identified as early as 1875 when J. Kerr observed double light refraction in an isotropic liquid which had quadratic dependence upon the incident field. Another double refraction effect but with linear dependence upon the incident field was observed later on, in 1906, by F. Pockels in several crystals. Although some applications of these nonlinear effects, named after their discoverers Kerr and Pockels effects respectively, were envisioned [17], for any practical realization to be considered seriously a reliable enough, stable, powerful light source was required. The new light source developed by 1960, the laser, came to meet all these and still more requirements allowing for construction of both Kerr and Pockels cells which served as high speed optical modulators, and optical frequency doublers, among others. The avalanche of new discoveries made possible with the advent of the laser gave birth to a new research activity field taken to be called generically Nonlinear Optics.

Nonlinear Optics applications have been traditionally divided into two main research areas. On one side there were the processes involving more than one frequency, such as optical modulation and rectification and frequency doubling, mainly relying on second-order phenomena, i.e. the polarization depends of the square of the electric field [18], [19], [20]. On the other side, third order processes in which the polarization depends on the electric field to the cube so that the nonlinear field contribution has the same frequency as the input resulting in a net nonlinear phase shift experienced by the input field as a consequence of the material's nonlinearity [21], [19], [20]. This power dependent nonlinear phase shift was meant to taken advantage of for all-optical switching in interferometric devices and directional couplers, and in general for functions related to handling and processing of information for the sake of more efficient telecommunications networks management.

While the results in the first research activity have moved to the commercial arena, so that efficient external laser modulators relying on the electrooptic or Pockels effect and laser frequency doublers were ready to meet the market requirements in practical applications, the second research field was mainly developed at a theoretical level due to the lack of suitable materials with strong enough third order nonlinear response free from absorptive losses which
would allow for obtention of the required nonlinear phase shift.

When, by 1990, it seemed that the lack of suitable materials with a high enough third-order nonlinearity might have led the topic of all-optical switching by exploitation of nonlinear phase shifts to a stand-by status [22]-[23], large phase shifts steaming from a second-order nonlinearity in Potassium Titanil Phosphate were observed by DeSalvo and co-workers at the Center for Research and Education in Optics and Lasers, CREOL, in Florida [24]-[25]. This observation opened a new field for exploration [26] which was generically known as Cascading. Although their relevance was not fully acknowledged at the time, phase shifts in excess of $\pi$ had already been observed by Belanshenkov et al. [27] in organic CDA. The phase distortion effect occurring in second harmonic generation processes had been indeed a matter of concern [28]-[46] but merely as a limiting factor of the efficiency of frequency doublers, being not identified as a new way to achieve all-optical operation. As a matter of fact, the existence and relevance of the $\chi^{(2)}$ induced nonlinear phase shift, was recognized as early as 1967 by L. A. Ostrovskii [47] although at that time the idea was not actively pursued due to the lack of suitable materials.

The nonlinear phase mismatch in second-order processes is the result of the contribution to the incident field at fundamental frequency of consecutive up-conversion to second harmonic and down-conversion back to fundamental processes in such a way that it is proportional to the input field intensity $|E_0|^2$ and the square power of the corresponding nonlinear coefficient $(\chi^{(2)})^2$, hereof the name of Cascading. Significant phase shifts in short propagated distances with moderate power levels are therefore achieved thanks to stronger $\chi^{(2)}$ nonlinear response of existing materials.

Waveguide structures confine the light beam in very small regions allowing to reach high power levels that thus improve the nonlinear phase shift and ease integration. The first experiments demonstrating the $\chi^{(2)}$ nonlinear phase shift in a waveguide were carried out again at CREOL in KTP samples and Lithium Niobate structures by Sundheimer and co-workers [48]-[49] and Schiek et al. respectively [50]. The effect has been demonstrated in a wide variety of materials: $\beta$-barium borate, potassium niobate, semiconductors such as gallium arsenide, organics as MBA-NP, DAN, NPP and DAST. Some of the cited materials exhibit impressively huge nonlinear responses [51]-[60] that hold promise for applications provided long enough, low-loss samples can be fabricated in geometries where the appropriate elements of the $\chi^{(2)}$ tensor
are used. Research is in progress in this area [57].

To the significant phase shift observations at CREOL, suggestions for its use in all-optical devices soon followed [61]-[64], some of them being even tested in laboratory setups [65]-[67].

With these discoveries, second-order nonlinear processes, till 1990 only thought useful for frequency doubling and optical modulation via the electrooptic (Pockels) turned out to be promising as well for the practical realization of some all-optical switching devices [68].

1.5 Solitons

About the same year that the first operative fiber transmission systems were deployed, Hasegawa predicted that short enough optical pulses in the subpicosecond regime at the wavelengths for anomalous dispersion should propagate in the fiber as optical solitons [69]. The word soliton alludes to the particle-like properties of certain localized structures which propagate undistorted over long distances thanks to the nonlinear compensation of linear effects such as dispersion or diffraction, and undergo elastic collisions [70]. Chance provided the first observation of such structures in the form of water waves in a Scottish narrow barge channel in 1838. J. Scott Russell's report in ‘Reports of the Meetings of the British Association for the Advancement of Science’ in 1844 talks about 'a large solitary elevation, a rounded, smooth and well defined heap of water, which continued its course along the channel, apparently without change of form of diminution of speed' and which emerged in the channel when a boat suddenly stopped. The mathematical formulation of the phenomenon is owed to Korteweg and deVries who in 1895 published their famous KdV equation. Zabusky and Kruskal in 1965 rediscovered the phenomenon numerically and were the first to use the word soliton [71]. Starting from Zabusky and Kruskal's studies the analytic theory leading to formulation of the inverse scattering method for mapping the nonlinear solution to the KdV on solutions of a linear system of equations was developed by Gardner et al. [72]. In 1972, [73] Zakharov and Shabat showed that the nonlinear Schrödinger equation describing intense pulse propagation in a fiber supports solitons that can be derived using inverse scattering theory at the same time of Hasegawa prediction [69]. Observation of optical solitons had to wait development of an appropriate laser which could provide short enough pulses at the wavelength of anomalous dispersion in the fiber, around
λ ~ 1.5μm, a task that was covered by L. F. Mollenauer and co-workers at the Bell Laboratories so that they finally could experimentally observe optical soliton propagation in a fiber by 1980 [74].

With the advent of the EDFA, the idea of using solitons as the bit unit for long distance optical transmission began to take shape. Key field demonstrations of the feasibility of this idea were performed by L. F. Mollenauer, at Bell Laboratories. At the same place and almost at the same time, M. N. Islam and co-workers demonstrated the soliton dragging and soliton trapping properties of birefringent fibers allowing for the switching and routing of data streams in the temporal domain [75].

Just in the same way that optical solitons in the temporal domain manage to preserve their shape on propagation through the power dependent nonlinear phase shift compensating linear dispersion, spatial solitons may be formed from nonlinear compensation of diffraction so that the spatial beam distribution remains unaltered on propagation. Spatial solitons were observed for the first time by Barthelemy and co-workers [76] in a planar waveguide made of liquid CS$_2$ by 1985, while their excitation in a solid-state waveguide was achieved, once again, at Bell Laboratories by Atchinson et al. [77]. These spatially localized structures might be convenient for the design of spatial routers and other switching devices but for a third order nonlinearity only are stable in the 1+1 case, namely monochromatic wave propagation in planar waveguides.

1.6 Solitons in nonlinear quadratic structures and the subject of the Thesis

If the nonlinear phase shift is the effect balancing linear dispersion/diffraction for temporal/spatial soliton formation in third-order nonlinear media, can the nonlinear phase shift induced through a second-order nonlinearity under some conditions achieve as well the equilibrium so that a solitary entity made up of fundamental and second harmonic waves propagating together is formed?

Some clues as to a positive answer can be adduced. On one side there is the fact that under some conditions the coupled normalized equations governing the propagation of first and second harmonic beams reduce to the Nonlinear Schrödinger Equation which is known to
support exact solitary wave solutions, and on the other one has the existence of an analytic solitary wave solution to the normalized equations for certain fixed values of all parameters in the system [79]. This solution is seen to constitute a mere sample of a broader family of stationary solutions whose transverse profiles can not be cast into analytic form.

Although the potential of second-order nonlinear processes to support solitonic propagation was recognized as early as 1974 by Karamzim and Sukhorukov [78]-[79], it was not until the boom of cascading in the 90s that the possibility of obtaining steady solitary waves structures in quadratic nonlinear media aroused the curiosity of the scientific community and along with new findings some of the earlier results were rediscovered [80]-[84].

The numerical experiments simulating propagation in $\chi^{(2)}$ media [85]-[88] evidenced the existence of solitary wave structures made up of fundamental and second harmonic beams propagating locked together and emerging from a wide variety of input and system conditions. These solitary wave structures featured an oscillating behavior around stationary states.

The families of stationary bright solutions were found numerically by Buryak and Kivshar [89]-[91] and Torner [92]-[93]. They were shown to be actually the ones around which the solitary wave structures slightly oscillated in the simulations and their stability confirmed through several theoretical works [94]-[100] lead ultimately to their actual experimental observation in both $LiNbO_3$ planar waveguides and bulk $KTP$ by Schiek et al. [101] and Torruellas et al. [102] respectively.

During the last few years while this Thesis work advanced, a vigorous research activity about quadratic solitons has taken place. Initial work focused in the so-called Type I of wave interaction, but Type II geometries have as well been considered in [103]-[109].

In addition to single solitary waves, multihumped solutions were also shown to verify the stationary equations but in all cases presented to date they turned out to be unstable [110]-[112].

Besides being excited with significantly lower power levels, the new $\chi^{(2)}$ solitons were found to be stable not only in 1+1 arrangements as the old-known $\chi^{(3)}$ solitons were, but also in 2+1 and 3+1 structures thus opening the door to two dimensional routing of bits and to the practical realization of light bullets. Modulational instabilities of (1+1) and (2+1) solitons yielding (3+1) bullets have also been examined [113]-[115], as well as competition between $\chi^{(2)}$ and $\chi^{(3)}$ effects [116]-[117].
The mutual dragging and trapping of beams taking place in the presence of walk-off was experimentally verified by Torruellas et al. [118]-[119] demonstrating the potential of $\chi^{(2)}$ materials for construction of space switches and routers in analogy with the temporal devices built up in optical fibers. Analytic work and numerical simulations assessing such a potential in a variety of setups is found in [120]-[125], and the stationary solutions that are susceptible of being excited in the presence of walk-off and their properties are discussed in [126]-[131], some of the works cited gathering results of this thesis.

A complete review of the advances in all research areas studying effects that exploit large nonlinear phase shifts in second-order media up to summer 1996 is given in [132] and [68].

The goal in this Thesis is to carry out a comprehensive study of the properties of solitons existing in second-order media both in configurations with negligible Poynting vector walk-off and in those in which a moderate walk-off is present, commonly referred to as walking solitons. The study comprises the details of determination of the solitons transverse phase and amplitude profiles in planar waveguides, i.e. the 1+1 case. The focus is on type I wave interaction setups which assumes fundamental ordinary and second harmonic extraordinary waves. Also discussed are stability against perturbations, the fields dependence in the soliton tails and the dynamics of excitation. Relying in extensive numerical simulations, emphasis is lied in assessing the potential of these solitary wave structures for constituting the basis of efficient all-optical switching and routing devices in several configurations of practical relevance.

Attending to the contents, the Chapters have been organized in two parts, entitled respectively: ‘Optical Solitons in Quadratic Nonlinear Media’ and ‘Soliton Beam Steering, Pointing and Switching’.

The first part comprising two chapters (2 and 3) deals with properties and analysis of bright solitons existing in the absence of Poynting vector walk-off, while the second part which gathers chapters 4, 5, 6 and 7 is devoted to optical steering issues, presenting the walking solitons and extensive simulations showing possibilities for steering control.

Chapter 2 features a revision of the basic tools and concepts about light propagation, with special emphasis on those related to second order nonlinear media. Derivation of the basic equations that formally govern either pulse propagation in dispersive media or diffractive beam propagation in the presence of the quadratic nonlinearity clears up the premises under which
they are physically relevant. Also it gives insight into the significance of the phenomena described and the role played the normalized parameters. The governing equations are shown to constitute a hamiltonian dynamical system and conserved quantities as the hamiltonian, beam momentum and energy flow are identified. Together with the characteristic lengths of the evolution, some relevant physical values of the normalized parameters are provided and finally an intuitive picture of the characteristics of the propagation is derived from the equations.

The properties and characteristics of bright solitons existing in the absence of Poynting vector walk-off is the subject of Chapter 3 which also features a general treatment of the problem of finding solitary wave solutions to the normalized governing equations for propagation in $\chi^{(2)}$ media and a brief revision of the shooting method. Thanks to the similarity rules enjoyed by the stationary equations, a complete description of the soliton families requires only characterization of three soliton families for different values of the wavevector mismatch: zero, a positive value and a negative value. Stability issues are addressed in the fifth section and sixth section features an analysis of the solitons tails. The chapter is completed with an intuitive picture found useful to comprehend the characteristics displayed by the solitons.

Chapter 4 deals with the basic concepts and tools related to soliton beam steering as for example the energy centroid in both type I and type II configurations. With eyes to an eventual connection with the phenomena taking place inside the $\chi^{(2)}$ crystal, a brief revision of soliton dragging and soliton trapping concepts in birefringent fibers has also been included.

Chapter 5 presents the numerically found solitary wave solutions which propagate in the presence of walk-off, the so-called walking solitons. Because of the complicated phase transverse profiles featured by the walking solitons, the appropriate searching method was found to be a Newton-Raphson method whose basis are briefly revised. The basic properties of the walking solitons families along with stability issues, analysis of soliton tails and dynamical excitation investigation are also presented.

Chapter 6 features a first section devoted to analysis of fundamental and second harmonic beam mutual trapping and dragging taking place in the presence of walk-off for achieving power controlled steering. In the second section the many possibilities for steering control when providing tilted input beams are explored. The third section discusses steering control by means of an input phase grating and in last section the input beam break up into several
solitons occurring under appropriate conditions is investigated.

In chapter 7 we briefly explore bulk (2+1) geometries. Specifically the chapter is devoted to study of the splitting of input optical vortices in bulk $\chi^{(2)}$ materials, together with discussion of their relevance to soliton steering and routing. A brief description of the simple laboratory setup required to observe the phenomenon is given.

Finally in chapter 8 the main conclusions extracted from the Thesis work are summarized.