Appendix A

Notation

In general, uppercase boldface letters (\(A\)) denote matrices, lowercase boldface letters (\(a\)) denote (column) vectors and italics (\(a, A\)) denote scalars. In some occasions, matrices are also represented with calligraphic fonts (\(\mathcal{A}\)).

\(A^T, A^*, A^H\) Transpose, complex conjugate and transpose conjugate of matrix \(A\), respectively.

\(A^{-1}, A^#\) Inverse and Moore-Penrose pseudoinverse of matrix \(A\), respectively.

\(A^{1/2}\) Positive definite Hermitian square root of matrix \(A\), i.e. \(A^{1/2}A^{1/2} = A\).

\(A(B)\) Matrix \(A\) is a function of the entries in matrix \(B\).

\(\text{det}(A)\) Determinant of matrix \(A\).

\(\text{Tr}(A)\) Trace of matrix \(A\).

\(\text{vec}(A)\) Column vector formed stacking the columns of matrix \(A\) on top of one another.

\(\text{diag}(a), \text{diag}(A)\) Following the Matlab notation, \(\text{diag}(a)\) is the \(N \times N\) diagonal matrix whose entries are the \(N\) elements of the vector \(a\), and \(\text{diag}(A)\) is the column vector containing the \(N\) diagonal elements of matrix \(A\).

\(\text{Dg}(A)\) An \(N \times N\) diagonal matrix whose entries are the \(N\) elements in the diagonal of matrix \(A\), i.e., \(\text{diag}([A]_{1,1}, \ldots, [A]_{N,N})\) or, equivalently, \(\text{diag} (\text{diag} (A))\).

\(\|a\|\) Euclidean norm of \(a\), i.e. \(\|a\| = \sqrt{a^H a}\).

\(\|a\|_W\) Weighted norm of \(a\), i.e. \(\|a\|_W = \sqrt{a^H W a}\) (with Hermitian positive definite \(W\)).

\([A]_{i,j}\) The entry of matrix \(A\) in the \(i\)-th row and the \(j\)-th column.

\([A]_i\) The \(i\)-th column of matrix \(A\).
APPENDIX A. NOTATION

\[ [v]_i \] The \( i \)-th element of vector \( v \).

\( A \otimes B \) Kronecker product between \( A \) and \( B \). If \( A \) is \( M \times N \),

\[
A \otimes B = 
\begin{bmatrix}
[A]_{1,1} B & \cdots & [A]_{1,N} B \\
\vdots & \ddots & \vdots \\
[A]_{M,1} B & \cdots & [A]_{M,N} B
\end{bmatrix}.
\]

\( A \odot B \) Elementwise (Schur-Hadamard) product between \( A \) and \( B \) (they must have the same dimensions).

\( A \geq B, A > B \) The matrix \( A - B \) is positive semidefinite and positive definite, respectively.

\( I_N, I \) The \( N \times N \) identity matrix and the identity matrix of implicit size.

\( 0_{M \times N}, 0_M, 0 \) An \( M \times N \) all-zeros matrix, an \( M \)-long all-zeros vector and, an all-zeros matrix or vector of implicit size.

\( 1_{M \times N}, 1_M, 1 \) An \( M \times N \) all-ones matrix, an \( M \)-long all-ones vector and, an all-ones matrix or vector of implicit size.

\( d_N \) The vector defined as \( d_N = [0, \ldots, N-1]^T \).

\( e_i \) Vector that has unity in its \( i \)-th position and zeros elsewhere.

\( \mathbb{R}^{M \times N}, \mathbb{C}^{M \times N} \) The set of \( M \times N \) matrices with real and complex valued entries, respectively.

\( j \) Imaginary unit \( (j = \sqrt{-1}) \).

\( \text{Re} \{ A \}, \text{Im} \{ A \} \) The matrices containing the real and imaginary parts of the entries of \( A \) respectively.

\( \arg \{ a \} \) Angle of the complex number \( a \), i.e., \( \arg \{ a \} = \arctan \left( \frac{\text{Im}(a)}{\text{Re}(a)} \right) \).

\( |a|, \text{sign} \{ a \} \) Absolute value and sign of a real valued \( a \).

\( \lceil a \rceil \) Smallest integer bigger than or equal to \( a \).

\( \hat{A} \) Estimator or estimate of the matrix \( A \).

\( f_A (A) \) Probability density function of the random matrix \( A \).

\( E \{ A \} \) Expectation of a random matrix \( A \).

\( E_B \{ A \} \) Expectation of a random matrix \( A \) with respect to the statistics in \( B \).

\( \arg \min_B f(B) \) Matrix \( B \) minimizing the scalar function \( f(B) \)

\( \arg \max_B f(B) \) Matrix \( B \) maximizing the scalar function \( f(B) \)
\( \frac{\partial A}{\partial B} \)  
If \( B \) is \( M \times N \), \( \frac{\partial A}{\partial B} \) is a matrix formed as
\[
\frac{\partial A}{\partial B} = \begin{bmatrix}
\frac{\partial A}{\partial (B)_{1,1}} & \cdots & \frac{\partial A}{\partial (B)_{1,N}} \\
\vdots & \ddots & \vdots \\
\frac{\partial A}{\partial (B)_{M,1}} & \cdots & \frac{\partial A}{\partial (B)_{M,N}}
\end{bmatrix}.
\]

In addition, for a given scalar \( b \), \( \frac{\partial A}{\partial b} \) is the matrix containing the derivatives of the entries of \( A \) with respect to \( b \). If \( b \) is complex, we have \( \frac{\partial}{\partial b} = \frac{\partial}{\partial \text{Re}\{b\}} - j\frac{\partial}{\partial \text{Im}\{b\}} \), see [Bra83].

\( \delta (i_1, \ldots, i_N) \)  
Multidimensional Kronecker delta defined as
\[
\delta (i_1, \ldots, i_N) = \begin{cases} 
1 & i_1 = \ldots = i_N \\
0 & \text{otherwise}
\end{cases}.
\]

\( \delta (x) \)  
Vectorial Dirac’s delta defined as
\[
\delta (x) = \begin{cases} 
1 & x = 0 \\
0 & \text{otherwise}
\end{cases}.
\]

\( \mathcal{F}\{\cdot\}, \mathcal{F}^{-1}\{\cdot\} \)  
Direct and inverse Fourier transform for both the analog and discrete cases defined as \( \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \) and \( \mathcal{F}\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]e^{-j2\pi fn} \), respectively.

\( * \)  
Analog or discrete convolution defined as \( x(t) * y(t) = \int_{-\infty}^{\infty} x(\tau)y(t-\tau)d\tau \) or \( x[n] * y[n] = \sum_{k=-\infty}^{\infty} x[k]y[n-k] \), respectively.

\( x \in (A, B) \)  
The scalar \( x \) belongs to the interval given by \( x > A \) and \( x \leq B \)

\( \text{sinc} (x) \)  
Function defined as \( \text{sinc} (x) \triangleq \begin{cases} 
\sin (\pi x) / (\pi x) & x \neq 0 \\
1 & x = 0
\end{cases}.
\]

\( \sup \)  
Supremum (lowest upper bound). If the set is finite, it coincides with the maximum (\( \max \)).

\( \limsup \)  
Limit superior (limit of the sequence of suprema).

\( O(\cdot), o(\cdot) \)  
Landau symbols for order of convergence.

\( \triangleq \)  
Symbol used to define a new variable.

\( \propto \)  
It stands for “proportional to” or sometimes “equivalent to”.

\( \ln (\cdot) \)  
Natural logarithm.