Chapter 1

Introduction

This dissertation is the result of almost five years working in digital communications and signal processing. During all this time, multiple estimation problems have been addressed. The experience in these applications is materialized in this document. We thought that an attractive way of introducing this thesis is to explain its evolution from the original idea until the completion of this dissertation. In the following sections, the series of obstacles that were dealt with in the way of this thesis are briefly commented and contextualized. In short, this is the history of the thesis...

1.1 The nuisance unknowns in parameter estimation

In the beginning, our research activity was focused on *non-data-aided* (*blind)* digital synchronization [Gar88a][Men97] [Vaz00]. In this field, the receiver has to estimate some parameters from the received waveform in order to recover the transmitted data symbols. Basically, the receiver has to determine the symbol timing and, in bandpass communications, the carrier phase and frequency. With this aim, in most communication standards, a known training sequence is transmitted to assist the receiver during the signal synchronization. Once the training is finished, the synchronizer has to maintain the synchronism despite the parameters usually fluctuate due to the time-varying propagation channel and the terminal equipments nonidealities. In these conditions, the synchronizer has to cope with the thermal noise and, in addition, with the so-called *self-noise* –or pattern-noise– generated by the own unknown random data symbols. In fact, the random data symbols can be regarded as *nuisance parameters* that complicate the estimation of the parameter of interest. An special attention is given in this thesis to these nuisance parameters and the induced self-noise.

Most non-data-aided techniques in the literature are designed assuming a low SNR

[Men97][Vaz00]. This assumption is rather realistic in modern digital communications due to the utilization of sofisticated error correcting codes [Ber93]. In that case, it is well-known that the maximum likelihood (ML) estimator is *quadratic* in the received data in most estimation problems as, for instance, timing and frequency synchronization. The most important point is that, whatever the actual SNR, the ML estimator is known to attain the Cramér-Rao bound (CRB) if the observation time is sufficiently large [Kay93b]. Thus, the ML estimator is asymptotically the minimum variance unbiased estimator [Kay93b].

Unfortunately, the ML estimator is generally unknown for medium-to-high SNRs in the presence of nuisance parameters. Thus, the ML estimator is an unknown function of the observed data $\mathbf{y}$, that can be approximated around the true parameter $\boldsymbol{\theta}$ –*small-error approximation*– by means of the following N-th order polynomial:

$$\widehat{\boldsymbol{\theta}}_{ML} \simeq \sum_{n=0}^{N} \mathbf{M}_n(\boldsymbol{\theta})\, \mathbf{y}^n$$

where $\mathbf{y}^n$ is the vector containing all the n-th order sample products of $\mathbf{y}$, and $\mathbf{M}_n(\boldsymbol{\theta})$ the associated coefficients. Notice that the so-called small-error approximation can be achieved by means of iterative and *closed-loop* schemes (Chapter 2).

Regarding the last expression, we conclude that higher-order techniques (i.e., $N > 2$) are generally required to attain the CRB for medium-to-high SNR. For example, a heuristic fourth-order closed-loop timing synchronizer was proposed in [And90][Men97, Sec.9.4] for the minimum shift keying (MSK) modulation that outperforms –at high SNR– any existing second-order technique. This work was the motivation for deducing the optimal *fourth-order* estimator given by

$$\widehat{\boldsymbol{\theta}}_4 = \mathbf{M}_0(\boldsymbol{\theta}) + \mathbf{M}_2(\boldsymbol{\theta})\, \mathbf{y}^2 + \mathbf{M}_4(\boldsymbol{\theta})\, \mathbf{y}^4$$

where $\mathbf{M}_0(\boldsymbol{\theta})$, $\mathbf{M}_2(\boldsymbol{\theta})$ and $\mathbf{M}_4(\boldsymbol{\theta})$ are selected to minimize the estimator variance [Vil01b]. The proposed estimator became quadratic at low SNR ($\mathbf{M}_4 = \mathbf{0}$) and exploited the fourth-order component when the SNR was increased ($\mathbf{M}_4 \neq \mathbf{0}$). For more details, the reader is referred to the original paper [Vil01b]:

- "Fourth-order Non-Data-Aided Synchronization". J. Villares, G. Vázquez, J. Riba. *Proc. of the IEEE Int. Conf. on Accoustics, Speech and Signal Processing 2001* (ICASSP 2001). pp. 2345-2348. Salt Lake City (USA). May 2001.

Although the focus shifted from fourth-order to second-order methods soon, this contribution was actually the basis of this thesis. In this work, the estimator coefficients were directly optimized for a given observation length and estimator order ($N = 4$). To carry out this optimization, the Kronecker product $\otimes$ and $vec(\cdot)$ operators were introduced in order to manipulate the n-th order observation $\mathbf{y}^n$. Moreover, during the computation of the optimal coefficients $\mathbf{M}_2$ and $\mathbf{M}_4$, we realized that some fourth-order moments of the MSK modulation were ignored in other well-known second-order ML-based approximations.

After this work, we wondered about the *best quadratic* non-data-aided estimator or, in other words, which are the optimal coefficients $\mathbf{M}_0(\boldsymbol{\theta})$ and $\mathbf{M}_2(\boldsymbol{\theta})$ in

$$\widehat{\boldsymbol{\theta}}_2 = \mathbf{M}_0(\boldsymbol{\theta}) + \mathbf{M}_2(\boldsymbol{\theta})\,\mathbf{y}^2$$

for a given estimation problem. At low SNR, the optimal second-order estimator is given by the low-SNR ML approximation, at least for sufficiently large data records. On the other hand, if the SNR increases, the optimal second-order estimator is only known in case of Gaussian data symbols. However, the *Gaussian assumption* is clearly unrealistic in digital communications because the transmitted symbols belong to a discrete alphabet. This intuition was confirmed for the MSK modulation in the following paper [Vil01a].

- "Best Quadratic Unbiased Estimator (BQUE) for Timing and Frequency Synchronization". J. Villares, G. Vázquez. *Proc. of the 11th IEEE Int. Workshop on Statistical Signal Processing* (SSP01). pp. 413-416. Singapore. August 2001. ISBN 0-7803-7011-2.

In this pioneering paper, the Gaussian assumption was found to yield suboptimal timing estimates at high SNR when the observation time is short. Quadratic estimators were improved considering the fourth-order cumulants or *kurtosis* of the MSK constellation. However, this fourth-order information was shown to be irrelevant if the number of observations is augmented. Thus, it was shown via Monte Carlo simulations that the Gaussian assumption is asymptotically optimal in the problem of timing synchronization. Additional simulations and remarks were given in the tutorial paper presented in the ESA workshop [Vaz01].

1.2 The Bayesian approach: the bias-variance dilema

Another contribution in the referred paper was the formulation of optimal second-order *open-loop* estimators. Open-loop schemes are very attractive in digital synchronization because they allow reducing the acquisition time of closed-loop synchronizers [Men97][Rib97]. To design open-loop estimators, the small-error approximation is abandonned and the parameter $\boldsymbol{\theta}$ is assumed to take values in a given interval Θ. In this *large-error* scenario, the N-th order expansion of the ML estimator depends on the *unknown* value of $\boldsymbol{\theta} \in \Theta$ and, consequently, the ML estimator cannot be generally implemented by means of a polynomial in $\mathbf{y}$.

To overcome this limitation, the *Bayesian formulation* was adopted and the parameter $\boldsymbol{\theta}$ was modelled as a random variable of know *prior distribution* $f_{\boldsymbol{\theta}}(\boldsymbol{\theta})$. Then, the prior was applied to optimize the estimator coefficients $\{\mathbf{M}_n\}$ *"on the average"*, that is, considering all the possible values of $\boldsymbol{\theta} \in \Theta$ and the associated probabilities $f_{\boldsymbol{\theta}}(\boldsymbol{\theta})$. Actually, the Bayesian approach encompasses both the small and large error scenarios since the small-error approximation can be imposed by considering an extremely informative prior.

It can be shown that, in the small-error regime, it is equivalent to minimize the estimator *mean square error* (MSE) and the estimator *variance*. However, in the large-error regime, the minimum MSE estimator becomes usually biased with the aim of reducing the variance contribution. In fact, the more corrupted is the observation $\mathbf{y}$, the more biased is the minimum MSE solution. The reason is that, when the observation is severely degraded by the thermal and self noise terms, the minimum MSE estimator is not confident about the observation $\mathbf{y}$ and it resorts to the a priori knowledge on the parameters. In that way, the estimator reduces the variance induced by the random terms (noise and self-noise) although it becomes biased in return unless the value of $\boldsymbol{\theta}$ coincides with the expected value of the prior.

In this early paper, the main problem in second-order open-loop estimation was identified for the first time. In general, *unbiased* second-order open-loop estimators are not feasible. Even if the estimator variance can be usually removed by extending the observation time, there is always a residual bias that sets a limit on the performance of open-loop estimators. Despite this conclusion, the design of *almost* unbiased open-loop second-order estimators was addressed by imposing the *unbiasedness constraint* at L values of $\boldsymbol{\theta} \in \Theta$. Actually, the L *test points* were distributed regularly inside the parameter space Θ. The number of *unbiased* test points was in practice a function of the observation time and the oversampling factor.

This formulation was further improved by allowing the estimator to select automatically the best unbiased test points. In that way, the estimator can decide the number and position of the test points in order to minimize the overall estimator bias. This formulation was developed in the following conference paper for the problem of timing and frequency synchronization [Vil02b].

- "Sample Covariance Matrix Based Parameter Estimation for Digital Synchronization". J. Villares, G. Vázquez. *Proc. of the IEEE Global Communications Conference 2002* (Globecom 2002). November 2002. Taipei (Taiwan).

Another important advance in this paper was the closed-form derivation of the *kurtosis matrix* $\mathbf{K}$ for any linear modulations. This matrix contains all the fourth-order statistical information about the transmitted symbols that is relevant for second-order estimation. Actually, matrix $\mathbf{K}$ gathers all the statistical information about the digital modulation that is ignored when the Gaussian assumption is adopted.

The last two papers [Vil01a][Vil02b] are actually the foundation of Chapter 3 (open-loop estimation) and Chapter 4 (closed-loop estimation).

In the same year, the results obtained in the last two papers [Vil01a][Vil02b] were extended to estimate the timing and frequency parameters in the presence of multipath propagation. This work was actually motivated by the participation in the EMILY European project [Bou02a][Bou02b], in which advanced radiolocation techniques for wireless outdoor communication systems (e.g., GSM and UMTS) were studied. The results of this research were published in the following paper [Vil02a] and are included in Section 6.3.

- "Optimal Quadratic Non-Assisted Parameter Estimation for Digital Synchronisation". J. Villares, G. Vázquez. *Proc. of the Int. Zurich Seminar on Broadband Communications 2002* (IZS2002). pp. 46.1-46.4. Zurich (Switzerland). February 2002.

The Bayesian formulation adopted to design open-loop estimators requires in most cases to compute numerically the estimator coefficients. The reason is that, in most estimation problems, the average with respect to the prior $f_{\boldsymbol{\theta}}(\boldsymbol{\theta})$ does not admit an analytical solution. Exceptionally, closed-form expressions can be obtained for the frequency estimation problem if the prior is uniform. Thus, the exhaustive evaluation of open-loop second-order frequency estimators was carried out in the following paper [Vil03a].

- "Sample Covariance Matrix Parameter Estimation: Carrier Frequency, A Case Study". J. Villares, G. Vázquez. *Proc. of the IEEE Int. Conf. on Accoustics, Speech and Signal Processing* (ICASSP). pp. VI-725 - VI-728. Hong Kong (China). April 2003.

In this paper, it was shown that *unbiased* second-order open-loop estimators can be obtained by increasing the oversampling factor. In practice, unbiased open-loop estimators are feasible if the sampling rate is greater than *four times* the maximum frequency error (Section 3.4).

1.3 Noncircular nuisance unknowns

Thus far, the second-order framework was only applied to formulate NDA timing and frequency synchronizers. However, the problem of carrier phase synchronization was ignored because higher-order methods are usually required to estimate the signal phase. However, this is not true in case of *noncircular* modulations (e.g., PAM, BPSK, staggered formats and CPM). Remember that the transmitted symbols $\{x_i\}$ belong to a noncircular constellation if the expected value of $x_i x_k$ is different from zero for certain values of i and k.

The problem of carrier phase synchronization in case of MSK-type modulations was addressed in the following paper [Vil04b] and can be consulted in Section 6.2.

- "Self-Noise Free Second-Order Carrier Phase Synchronization of MSK-Type Signals", J. Villares, G. Vázquez, *Proc. of the IEEE Int. Conf. on Communications* (ICC 2004). June 2004. Paris (France).

1.4 Self-noise in multivariate problems: interparameter interference.

At high SNR, the dominant disturbance is enterely due to the randomness of the received symbols (i.e., the self-noise). In this high-SNR scenario, the self-noise variance is minimized if the kurtosis of the data symbols is taken into account. Otherwise, if the Gaussian assumption is imposed, the variance of the self-noise term increases. However, the self-noise contribution is normally negligible in digital synchronization and the Gaussian assumption is practically optimal. In order to test the Gaussian assumption, we decided to study other estimation problems in which the self-noise term was more critical.

With this purpose, the uniparametric formulation was generalized to encompass important multivariate estimation problems in the context of digital communications such as direction-of-arrival (DOA) and channel estimation. These problems were selected because the self-noise contribution was expected to degrade significantly the estimator performance at high SNR. Hence, these two problems were valuable candidates for examining the Gaussian assumption.

In the DOA estimation problem, the DOA estimator is faced with the self-noise caused by the user of interest and, in addition, by the other interfering users (*multiple access interference*). In the channel estimation problem, the received pulse is severely distorted by the unknown channel impulse response. Then, the intersymbol interference is enhanced at high SNR and hence the self-noise variance is amplified.

The formulation of the optimal second-order *multiparametric* open- and closed-loop estimator will appear in the IEEE Transactions on Signal Processing next July [Vil05]. The theoretical material in this article is presented in Chapter 3 (open-loop or large-error estimation) and Chapter 4 (closed-loop or small-error estimation).

- "Second-Order Parameter Estimation". J. Villares, G. Vázquez. *IEEE Transactions on Signal Processing.* July 2005.

As it was expected, the performance of second-order DOA estimators was severely degraded when the angular separation of the users was reduced because, in that case, the multiple access interference became the dominant impairment. In these singular scenarios, the Gaussian assumption yielded a significant loss for practical SNRs if the transmitted symbols were drawn from a constant-modulus constellation such as MPSK or CPM. The Gaussian assumption loss was a function of the angular separation as well as the the number of antennas. All these important results were presented in the following paper [Vil03b] and are included in Section 6.5.

- "Second-Order DOA Estimation from Digitally Modulated Signals", J. Villares, G. Vázquez, *Proc. of the 37th IEEE Asilomar Conf. on Signals, Systems and Computers*, Pacific Grove (USA), November 2003.

In this paper, the problem of *tracking* the DOA of multiple moving digitally-modulated users is considered. In this scenario, the tracking condition can be lost at low SNR when two users approach each other. In this paper, it is shown that this is usually the outcome if the tracker is forced to cancel out the multiple access interference. On the other hand, if the multiple access interference is incorporated as another random self-noise term in the tracker optimization, the optimal second-order tracker is able to maintain the tracking condition even if the users cross each other [Vil03b].

As it has been explained before, the problem of blind channel estimation was also a promising candidate for testing the Gaussian assumption. Some results are presented in Section 6.4 that confirm the interest of the optimal second-order estimator in the medium-to-high SNR range when the nuisance parameters have constant modulus. In that case, the Gaussian assumption cannot estimate the channel amplitude whereas the optimal solution yields self-noise free estimates even if the channel amplitude is unknown (Section 6.4). This channel estimation problem is currently being investigated in case of noncircular constant-modulus transmissions [LS04][LS05a][LS05b].

1.5 Informative priors: estimation on track

Thus far, all the second-order closed-loop estimators and trackers had been designed and evaluated in the *steady-state*, that is, assuming that all the parameters were initially captured during the *acquisition* phase. In fact, once the acquisition is completed, the estimator begins to operate in the small-error regime. The estimator coefficients were precisely optimized under the small-error assumption. However, the acquisition performance had never been involved into the estimator optimization.

After this reflection, we were concerned with the optimization of closed-loop second-order estimators considering both the acquisition (large error) and steady-state (small-error) performance. With this aim, the Kalman filter formulation [And79][Kay93b] was adopted because it is known to supply the optimal transition from the large to the small error regime when the parameters and the observations are jointly Gaussian. Evidently, this assumption fails in most estimation problems in digital communications and the suboptimal *extended Kalman filter* (EKF) is only optimal in the steady-state [And79][Kay93b]. Despite this, the EKF provides a systemmatic and automatic procedure for updating the prior distribution $f_{\boldsymbol{\theta}}(\boldsymbol{\theta})$ every time a new observation is processed. In that way, it is possible to enhance the acquisition performance without altering the optimal steady-state solution.

The research in this direction yielded the so-called *quadratic* EKF (QEKF) that extended the classical EKF to deal with quadratic observations. The QEKF formulation was published in the following conference paper [Vil04a] and it has been included in Chapter 5.

- "On the Quadratic Extended Kalman Filter", J. Villares, G. Vázquez. *Proc. of the Third IEEE Sensor Array and Multichannel Signal Processing Workshop* (SAM 2004). July 2004. Sitges, Barcelona (Spain).

In this paper, the QEKF is designed and simulated for the aforementioned DOA estimation problem. The most important conclusion is that, at high SNR, the Gaussian assumption is also suboptimal during the acquisition phase if the data symbols are drawn from a constant-modulus constellation. In that way, the acquisition time can be notably reduced if the QEKF takes into account the kurtosis of the data symbols. Besides, the Gaussian assumption loss at high SNR is shown to persist in the steady-state even if the tracker observation time is increased to infinity (Chapter 5).

1.6 Limiting asymptotic performance

The last remark on the QEKF asymptotic performance persuaded us to study in detail the performance limits in second-order estimation. The objective was to determine the asymptotic conditions for the Gaussian assumption to apply. The asymptotic analysis confirmed that the Gaussian assumption was optimal at low SNR but it was suboptimal at high SNR if the nuisance parameters belonged to a constant-modulus alphabet. Finally, the performance of second-order closed-loop estimators was evaluated when the number of samples went to infinity. The conclusion was that the Gaussian assumption applies in digital synchronization, and in DOA estimation if the number of antennas goes to infinite. On the other hand, the Gaussian assumption fails for the medium-to-high SNR range in the problem of channel estimation and for DOA estimation in case of finite sensor arrays. All these asymptotic results were finally collected and are presented for the first time in Chapter 7.

1.7 Thesis Outline

The structure of the dissertation is presented next. The main contents and contributions are described chapter by chapter.

Chapter 2: Elements on Estimation Theory.

In this chapter, the most important concepts from the estimation theory are reviewed. The problem of parameter estimation in the presence of nuisance parameters is introduced and motivated. The maximum likelihood (ML) estimator is presented and the most important ML-based approaches in the literature are described. Special emphasis is put on the Gaussian ML (GML) estimator because it converges to the ML estimator at low SNR and yields the conditional ML (CML) solution at high SNR. The important point is that all these ML-based estimators are quadratic in the observation. The GML estimator is actually the ML estimator in case of having Gaussian nuisance parameters. However, the optimal second-order estimator is normally unknown if the nuisance parameters are not Gaussian. This was actually the motivation for this thesis.

The iterative implementation of the aforementioned ML-based estimators is considered and the utilization of closed-loop schemes motivated. Finally, a survey on estimation bounds is included for the interested reader.

Chapter 3: Optimal Second-Order Estimation.

In this chapter, the optimal second-order estimator is formulated from the known distribution of both the wanted parameters and the nuisance parameters. The Bayesian formulation and two different optimization criteria are considered. In the first case, the estimator mean square error (MSE) is minimized in the Bayesian sense, that is, averaging the estimator MSE according to the assumed prior distribution. In the second case, the estimator variance is minimized subject to the minimum bias constraint. Again, the variance and bias are averaged by means of the prior distribution.

The resulting large-error or open-loop estimators are evaluated for the problem of blind frequency estimation. The minimum MSE solution is shown to make a trade-off between the bias and variance terms. On the other hand, the minimum bias constraint is unable to completely eliminate the bias contribution although the observation time is augmented. Accordingly, the ultimate performace of quadratic open-loop estimators becomes usually limited by the residual bias.

Chapter 4: Optimal Second-Order Small-Error Estimation.

In this chapter, the design of closed-loop second-order estimators is addressed. Assuming that all the parameters have been previously acquired, closed-loop estimators are due to compensate for small errors. In this context, the optimal second-order *small-error* estimator is derived from the minimum variance estimator in Chapter 3 by considering an extremely informative prior. The resulting estimator is the best quadratic unbiased estimator (BQUE) and its variance is the (realizable) lower bound on the variance of any sample covariance based parameter estimator. The BQUE is proved to exploit the kurtosis matrix of the nuisance parameters whereas the Gaussian ML estimator ignores this information.

Later, the conditions for second-order identifiability are analyzed and some important remarks are made about the so-called interparameter interference in multiuser scenarios. The frequency estimation problem is chosen once again to illustrate the main results of the chapter. Some simulations are selected to illustrate how the Gaussian assumption fails at high SNR.

Chapter 5: Quadratic Extended Kalman Filtering.

In this chapter, the well-known extended Kalman filter (EKF) is adapted to deal with quadratic observations. The coefficients of the quadratic EKF are calculated from the actual distribution of nuisance parameters. The optimal tracker is shown to exploit the kurtosis matrix of the nuisance parameters.

The Gaussian assumption is evaluated during the acquisition and the steady-state for the problem of DOA estimation. It is shown that the acquisition time and the steady-sate variance can be reduced at high SNR if the transmitted symbols are drawn from a constant-modulus alphabet (e.g., MPSK or CPM) and this information is incorporated.

Chapter 6: Case Studies.

In this chapter, the optimal second-order small-error estimator deduced in Chapter 4 is applied to some relevant estimation problems. In the first section, some contributions in the field of non-data-aided sychronization are presented. Specifically, Section 6.1 is devoted to the global optimization of second-order closed-loop synchronizers and the design of open-loop timing sycnronizers in the frequency domain. In Section 6.2, the problem of second-order carrier phase synchronization is addressed in case of noncircular transmissions. In this section, the ML estimator is shown to be quadratic at low SNR for MSK-type modulations. Moreover, second-order self-noise free estimates are achieved at high SNR exploiting the non-Gaussian structure of the digital modulation.

In Section 6.3, the problem of time-of-arrival estimation in wireless communications is studied. The frequency-selective multipath is shown to increase the number of nuisance parameters and the Gaussian assumption is shown to apply in this case study. In Section 6.4, the classical problem of blind channel identification is dealt with. The channel amplitude is shown to be not identifiable unless the transmitted symbols belong to a constant-modulus constellation and this information is exploited by the estimator.

Finally, the problem of angle-of-arrival estimation in the context of cellular communications is addressed in Section 6.5. The Gaussian assumption is clearly outperformed for practical SNRs in case of constant-modulus nuisance parameters and closely spaced sources. In this section, the importance of the multiple access interference (MAI) is emphasized and MAI-resistant second-order DOA trackers are derived and evaluated.

Chapter 7: Asymptotic Studies.

In this chapter, analytical expressions are obtained for the asymptotic performance of the second-order estimators presented in Chapter 3 and Chapter 4. Firstly, the low SNR study concludes that the nuisance parameters distribution is irrelevant at low SNR and, therefore, the Gaussian assumption is optimal. On the other hand, the high SNR study states that the Gaussian assumption does not apply in case of constant-modulus nuisance parameters. This conclusion is related to the eigendecomposition of the nuisance parameters kurtosis matrix. Finally, the large sample study confirms that the Gaussian assumption is optimal in digital synchronization if the observation time goes to infinity. Likewise, the Gaussian assumption applies in DOA estimation if the number of antennas goes to infinity. However, the Gaussian assumption cannot be applied –even if the number of snapshots is infinite– in case multiple constant-modulus signals impinge into a finite array. Regarding the channel estimation problem, the asymptotic study indicates that second-order estimates could be improved by considering the actual distribution of the nuisance parameters.

Chapter 8: Conclusions.

This chapter concludes and summarizes the main results of this thesis. To finish, some topics for further research are proposed.