MODULATION AND CONTROL
OF THREE-PHASE PWM
MULTILEVEL CONVERTERS

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(ABSTRACT)

Three-phase diode-clamped multilevel topologies are studied in this dissertation. These static converters can generate three or more voltage levels in each output phase, and are generally applied to high-power applications because of their ability to operate with larger voltages than the classical two-level converter. The analysis is mainly focused on the three-level topology, although there are also some contributions for converters with a larger number of levels. The main objectives are to propose new computationally efficient space-vector PWM modulation algorithms, to analyze the imbalances in the DC-link capacitors and the compensation for their effects, and to study advanced control loops. The results are obtained from different models in order to guarantee conclusion reliability. Furthermore, most of the results have been checked experimentally. The main contributions are summarized in the following.

A new space-vector PWM scheme is presented. This algorithm takes advantage of symmetry in the space-vector diagram in order to reduce processing time. The low-frequency oscillation that appears in the neutral point of the three-level converter for some operating conditions is analyzed and quantified. The information provided will help for the calculation of the DC-link capacitors in a given specific application.

The modulation algorithm is extended to converters with more than three levels. DC current components appear in the mid points of the DC-link capacitors for some operating conditions that make the system unstable. The unstable operating area of the four-level converter is revealed.

A novel and efficient space-vector PWM feedforward algorithm in the three-level converter is presented. This modulation strategy can achieve balanced AC output voltages despite any imbalance in the neutral point.
The negative effects of unbalanced linear loads and nonlinear loads on the neutral-point voltage balance are analyzed. A direct sequence of fourth-order harmonics in the AC currents can produce instability. The maximum allowed amplitude of these harmonics is shown.

Significant voltage-balancing improvements can be obtained when two converters are connected back-to-back. The limits in which the low-frequency neutral-point oscillation in the three-level converter can be removed are revealed. A practical example of this connection is the AC/DC/AC conversion used in motor drive applications able to operate with unity power factor.

Finally, an optimal multivariable control loop is applied to a three-level boost rectifier. Since the task of balancing voltages of the DC-link capacitors is assigned to the modulator stage, the linear quadratic regulator has been simplified.
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A la meva dona Montse,
el seu suport i comprensió.

To my wife Montse,
for her support and understanding.
Table of Contents

CHAPTER 1. INTRODUCTION .................................................................................................1
  1.1. Motivation and Objectives ....................................................................................1
  1.2. Review of Previous Research .............................................................................3
  1.3. Major Results .....................................................................................................9

CHAPTER 2. MULTILEVEL TOPOLOGIES. PROTOTYPE DESCRIPTION AND MODELING .................................................................13
  2.1. Multilevel Topologies ..........................................................................................13
    2.1.1. Diode-Clamped Converter ..........................................................................14
    2.1.2. Floating-Capacitor Converter ...................................................................18
    2.1.3. Cascaded H-bridge Converter ....................................................................22
    2.1.4. General Data for Basic Multilevel Topologies .........................................23
  2.2. Prototype Description .........................................................................................25
    2.2.1. SMES System .........................................................................................25
    2.2.2. Digital Control Hardware .........................................................................26
    2.2.2.1. Description of the Controller Architecture ..........................................27
    2.2.2.2. Functions of the EPLD .........................................................................28
  2.3. Models of the Diode-Clamped Converter .........................................................31
    2.3.1. Multilevel Models ....................................................................................31
    2.3.1.1. Phase Model .......................................................................................32
    2.3.1.2. Line-to-Line Model .............................................................................34
    2.3.2. Three-Level Models .................................................................................35
    2.3.2.1. Phase Model .......................................................................................36
    2.3.2.2. Line-to-Line Model .............................................................................37
    2.3.3. Model for the Control .............................................................................38
    2.3.3.1. State-Space Model .............................................................................38
    2.3.3.2. Equivalent Circuit ..............................................................................42

CHAPTER 3. SPACE-VECTOR PWM ..................................................................................45
  3.1. Space-Vector Modulation ....................................................................................45
    3.1.1. Three-Dimensional Vector Representation ..............................................45
    3.1.2. Two-Dimensional Vector Representation ................................................48
    3.1.3. Limiting Area .............................................................................................50
    3.1.4. Calculation of Duty Cycles ......................................................................51
    3.1.5. Calculation of Duty Cycles by Projections ..............................................52
  3.2. The Three-level Converter ..................................................................................54
    3.2.1. SVM under Voltage-Balanced Conditions ..............................................54
    3.2.2. Simplified Calculation of Duty Cycles ......................................................55
    3.2.3. The dq-gh Transformation ........................................................................58
    3.2.4. Defining Real Vectors ..............................................................................60
    3.2.5. Modulation Techniques .............................................................................60
    3.2.5.1. NTV Modulation .................................................................................60
    3.2.5.2. Symmetric Modulation .......................................................................63
    3.2.5.3. Simulated Results ..............................................................................67
    3.2.6. Limits of Control of the Voltage Balance .................................................73
    3.2.6.1. Limits of NTV Modulation ...................................................................74
    3.2.6.2. Limits of Symmetric Modulation ........................................................83
    3.2.7. Modulation Algorithm .............................................................................88
    3.2.8. Experimental Results ..............................................................................89
3.3. Conclusions of the Chapter

CHAPTER 4. SPACE-VECTOR MODULATION IN HIGH-ORDER MULTILEVEL CONVERTERS

4.1. Introduction

4.2. Modulation Strategy for High-Order Multilevel Converters

4.2.1. Calculation of Duty Cycles

4.2.2. Voltage-Balancing Criteria

4.2.2.1. Method 1: Derivate Minimization

4.2.2.2. Method 2: Direct Minimization

4.2.2.3. Compensating for One-Period Modulation Delay

4.3. The Four-Level Converter

4.3.1. Calculation of Duty Cycles

4.3.2. Voltage-Balancing Control

4.3.3. Simulated Results

4.3.4. Limits of Voltage Balance

4.4. Conclusions of the Chapter

CHAPTER 5. FEEDFORWARD SPACE-VECTOR PWM

5.1. Introduction

5.2. SV Diagram under Unbalanced Conditions

5.3. The Feedforward SV-PWM

5.3.1. Bases of the Method

5.3.2. Simulated Results

5.3.3. Instability in the Rectifier Mode

5.3.4. Experimental Results

5.4. Conclusions of the Chapter

CHAPTER 6. IMBALANCES AND ADDITIONAL MODULATION ISSUES

6.1. Introduction

6.2. Different Values of DC-Link Capacitors

6.3. AC-Side Imbalances

6.3.1. Linear Imbalances

6.3.1.1. Symmetric Components

6.3.1.2. Effects of Linear Imbalances on the NP Balance

6.3.2. Nonlinear Loads

6.3.2.1. Effects of Nonlinear Loads on the NP Balance

6.3.3. Experimental Results

6.4. Back-to-Back Connection

6.5. Overmodulation

6.6. Conclusions of the Chapter

CHAPTER 7. MULTIVARIABLE OPTIMAL CONTROL

7.1. Applications of Multilevel Converters

7.2. Proportional-Integral dq Controller

7.3. LQR Controller

7.3.1. Introduction to the LQR

7.3.2. Control Actuation Delay

7.3.3. LQR with Integral Actuation

7.3.4. Three-Level Boost Rectifier with LQR

7.3.5. Simulated Results

7.3.5.1. Unity PF Operation
List of Figures

Fig. 2.1. Three-level diode-clamped converter .......................................................... 15
Fig. 2.2. Four-level diode-clamped converter ......................................................... 15
Fig. 2.3. Functional diagram of the n-level diode-clamped converter ..................... 16
Fig. 2.4. Example of voltage waveform generated by an n-level converter .......... 17
Fig. 2.5. (a) Three-level and (b) four-level floating-capacitor converters .......... 19
Fig. 2.6. Imbricate cell, base of the floating-capacitor converter ......................... 20
Fig. 2.7. Transitions between consecutive voltage levels ...................................... 21
Fig. 2.8. Three-level converter ............................................................................ 22
Fig. 2.9. Multilevel cascade converter .................................................................. 22
Fig. 2.10. Four-level cascade converter ............................................................... 23
Fig. 2.11. SMES prototype .................................................................................. 25
Fig. 2.12. General diagram of the SMES system .................................................. 26
Fig. 2.13. Architecture of the SMES digital controller ......................................... 27
Fig. 2.14. PWM signal-generation logic within the EPLD ...................................... 29
Fig. 2.15. General multilevel system to be modeled ............................................. 32
Fig. 2.16. Conventional signs of the variables of the three-level converter .......... 36
Fig. 2.17. Equivalent small-signal circuit ............................................................ 43
Fig. 2.18. Functional diagram of an n-level diode-clamped converter ................. 45
Fig. 2.19. Three-dimensional SV diagram ........................................................... 46
Fig. 2.20. Clark’s Transformation: (a) director vectors, and (b) example of spatial vector
for $v_{a0}=200$ V, $v_{b0}=300$ V, and $v_{c0}=-100$ V .................................................. 48
Fig. 2.21. SV diagram of the two-level converter ............................................... 49
Fig. 2.22. SV diagrams of (a) the three-level converter, and (b) the four-level converter .... 49
Fig. 2.23. Limiting area to generate the reference vector ($\vec{m}$) by using three vectors........ 50
Fig. 2.24. Boundary of the area, determined when $d_3=0$.................................. 51
Fig. 2.25. Projections of the reference vector $\vec{m}$ ($\vec{p}_1$ and $\vec{p}_2$) ...................... 52
Fig. 2.26. Three-level vector diagram divided into sextants and regions ............. 54
Fig. 2.27. Maximum length of the normalized reference vector in steady-state
conditions ............................................................................................................ 56
Fig. 2.28. Projections of the normalized reference vector in the first sextant ......... 56
Fig. 2.29. Projections for Regions 1, 2 and 3......................................................... 57
Fig. 2.30. Graphical representation of the dq-gh transformation ......................... 58
Fig. 2.31. Components gh from different reference vectors. Equivalences in the first
sextant ................................................................................................................ 59
Fig. 2.32. Example of control of the NP current by proper selection of the double
vectors. Vector 100 produces $i_1=i_{sw}$, whereas vector 211 produces $i_1+i_3=-i_{sw}$ ....... 61
Fig. 2.33. New regions for symmetric modulation. Example of vector sequences for
Region 1 .............................................................................................................. 63
Fig. 2.34. Sequence of modulation cycles ............................................................ 65
Fig. 2.35. NTV modulation with modulation period $T_m=50$ $\mu$s:
(a) $m=0.4$; (b) $m=0.6$; and (c) $m=0.8$ ................................................................. 69
Fig. 2.36. Symmetric modulation with modulation period $T_m=50$ $\mu$s:
(a) $m=0.4$; (b) $m=0.6$; and (c) $m=0.8$ ................................................................. 69
Fig. 2.37. NTV and symmetric modulation with modulation period $T_m=500$ $\mu$s and $m=0.6$:
(a) NTV; (b) NTV with compensation for one-period delay; and (c) symmetric
modulation .......................................................................................................... 70
Fig. 2.38. Voltages in the DC-link capacitors with symmetric modulation. Magnification
from Fig. 2.30(c) .............................................................................................. 71
Fig. 2.39. Normalized maximum (solid line) and minimum (dashed line) local averaged
NP current. Examples given for purely resistive load ($\varphi=0^\circ$):
(a) $m=1$; (b) $m=0.9541$; (c) $m=0.9$; (d) $m=0.7$; (e) $m=0.5$; and (f) $m=0.3$ ......... 72
Fig. 3.23. Normalized maximum (solid line) and minimum (dashed line) local averaged NP current. Examples given for purely inductive load ($\varphi=-90^\circ$): (a) $m=1$; (b) $m=0.9$; (c) $m=0.7$; (d) $m=0.5774$; (e) $m=0.5$; and (f) $m=0.3$...........................77

Fig. 3.24. (a) Minimum local averaged values and (b) whole-line-period averaged values of the upper NP current waveforms..........................................................79

Fig. 3.25. NP current availability for different PF values........................................................................80

Fig. 3.26. Normalized NP voltage ripple for NTV.............................................................................82

Fig. 3.27. Normalized maximum (solid line) and minimum (dashed line) local averaged NP current for symmetric modulation. Examples given for a purely resistive load ($\varphi=0^\circ$): (a) $m=1$; (b) $m=0.9541$; (c) $m=0.9$; (d) $m=0.7$; (e) $m=0.5$; and (f) $m=0.3$..................85

Fig. 3.28. Normalized maximum (solid line) and minimum (dashed line) local averaged NP current for symmetric modulation. Examples given for a purely inductive load ($\varphi=-90^\circ$): (a) $m=1$; (b) $m=0.9$; (c) $m=0.7$; (d) $m=0.5$; (e) $m=0.3$; and (f) $m=0.1$..................88

Fig. 3.29. (a) Minimum local averaged values and (b) whole-line-period averaged values of the upper NP current waveforms..........................................................86

Fig. 3.30. Normalized NP voltage ripple for symmetric modulation..................................................87

Fig. 3.31. General diagram of the SVM algorithm .............................................................................88

Fig. 3.32. Line-to-line voltage ($v_{ab}$) and output currents ($i_a$ and $i_b$), for modulation index $m=0.9$.................................................................90

Fig. 3.33. Line-to-line voltage ($v_{ab}$) and output currents ($i_a$ and $i_b$), for modulation index $m=0.4$.................................................................91

Fig. 3.34. DC-link voltages ($v_{C1}$ and $v_{C2}$), filtered line-to-line voltage ($v_{ab}$) and output phase current ($i_a$). The NP voltage is released to be controlled by the modulation itself..................................................92

Fig. 4.1. Possible triangular regions: (a) up triangle and (b) down triangle.........................................96

Fig. 4.2. Distribution of the MP currents in the capacitors....................................................................98

Fig. 4.3. Distribution of currents in the DC-link capacitors of a four-level converter.....................102

Fig. 4.4. Four-level vector diagram divided into regions.....................................................................103

Fig. 4.5. Maximum amplitude of the normalized reference vector in steady-state conditions..........................................................104

Fig. 4.6. Examples of reference vector lying in (a) up-triangle region and (b) down-triangle region........104

Fig. 4.7. Analysis of voltage balancing strategies operating with unity PF.
Left graphics: method 1 (derivate minimization).
Right graphics: method 2 (direct minimization)..........................................................108

Fig. 4.8. Analysis of voltage balancing strategies operating with 0.5 inductive PF.
Left graphics: method 1 (derivate minimization).
Right graphics: method 2 (direct minimization)..........................................................109

Fig. 4.9. Limits of voltage balance in the four-level diode-clamped converter.................................111

Fig. 5.1. Distortion in the line-to-line output voltages due to low-frequency oscillation in the NP..........................................................114

Fig. 5.2. Vector diagram in the case of unbalanced voltages in the DC-link capacitors:
(a) $v_{C1}>v_{C2}$ and (b) $v_{C1}<v_{C2}$....................................................................................115

Fig. 5.3. Symmetries in the first sextant in the case of unbalanced voltages in the DC-link capacitors ($v_{C1}>v_{C2}$) .................................................................................................118

Fig. 5.4. Shape of the regions depends on which short vectors are selected.................................118

Fig. 5.5. Different projections of the reference vector in the normalized first sextant ................119

Fig. 5.6. Simulated results. Averaged variables for different load current angles ($\varphi$).
Left graphics: without feedforward compensation.
Right graphics: with feedforward compensation........................................................124

Fig. 5.7. Voltage ripple in the NP for different load current angles ($\varphi$):
Discontinuous lines: no compensation.
Continuous lines: with feedforward compensation..................................................125

Fig. 5.8. Stability limits of feedforward modulation in the rectifier operation mode ..................127

Fig. 5.9. Connection used for the experimental results............................................................128
Fig. 5.10. DC-link voltages ($v_{C1}$ and $v_{C2}$), filtered line-to-line voltage ($v_{ab}$) and output phase current ($i_a$): (a) no compensation, and (b) with feedforward modulation........ 129

Fig. 5.11. Experimental and simulated results with significant NP voltage ripple for the uncompensated modulation................................................................. 130

Fig. 5.12. Experimental and simulated results with significant NP voltage ripple for the feedforward modulation................................................................. 131

Fig. 6.1. Dynamic model of the NP in the three-level converter.............................. 134

Fig. 6.2. Example of symmetrical component decomposition............................... 136

Fig. 6.3. Averaged variables for different loading conditions:
(a) balanced linear load,
(b) positive and negative sequences of currents, and
(c) only negative sequence of currents.................................................................. 138

Fig. 6.4. Low-frequency NP voltage oscillation produced by some h-order current harmonics........................................................................................................... 140

Fig. 6.5. Direction of the currents in the SV diagram as a consequence of a set of fourth-order current harmonics................................................................. 141

Fig. 6.6. Maximum amplitude of the fourth-order harmonic in the steady-state condition... 142

Fig. 6.7. Connection used for the analysis of linear imbalances and nonlinear loads.... 144

Fig. 6.8. Balanced linear load.................................................................................... 144

Fig. 6.9. Unbalanced linear load................................................................................. 145

Fig. 6.10. Nonlinear load (case 1)................................................................................ 146

Fig. 6.11. Nonlinear load (case 2)............................................................................... 147

Fig. 6.12. Three-level motor drive application with active front end....................... 148

Fig. 6.13. Limits of the NP current control for constant output current ($I_{RMSo}=ct.$), unity input PF ($\varphi=0^o, 180^o$) and 100% efficiency ($\eta=1$)........................................... 149

Fig. 6.14. Limits of the NP balance versus the DC-link voltage............................... 151

Fig. 6.15. Local averaged variables: NP voltage ($v_{C1}$), and line-to-line output voltages for the case $\varphi_o=30^o$: (a) $m=0.7$ and (b) $m=0.8$ with active front end, and (c) $m=0.8$ with passive front end................................................................. 152

Fig. 6.16. Examples of overmodulation: (a) general case and (b) maximum amplitude ...... 154

Fig. 6.17. Amplitude of the output voltage fundamentals versus the modulation index.... 155

Fig. 6.18. Line-to-line local averaged output voltages and spectra for: (a) $m=1$;
(b) $m=1.08$; and (c) $m=1.1547$............................................................................. 156

Fig. 6.19. Low-frequency THD in overmodulation.................................................... 157

Fig. 7.1. Multilevel converter applied to renewable energy...................................... 160

Fig. 7.2. Decoupled PI-dq controller........................................................................ 161

Fig. 7.3. Basic LQR diagram..................................................................................... 164

Fig. 7.4. LQR diagram with integral action............................................................... 165

Fig. 7.5. Three-level system to be controlled........................................................... 168

Fig. 7.6. Simulated results for unity PF operation..................................................... 171

Fig. 7.7. Simulated results for non-zero reactive current.
Left graphics: with non-compensated NTV modulation.
Right graphics: with feedforward modulation....................................................... 173

Fig. 7.8. Voltages and currents in the three-level system........................................ 174

Fig. 7.9. System considered for the experimental results......................................... 177

Fig. 7.10. DC-link voltage ($v_{dc}$), a utility phase voltage ($e_{u1}$) and a phase current ($i_{u1}$) during the transitory process of turning the switch on........................................ 178

Fig. 7.11. Line-to-line voltage of the converter ($v_{ab}$) during a process similar to that shown in Fig. 7.10.......................................................... 179

Fig. 7.12. Voltages of the DC-link capacitors ($v_{C1}$ and $v_{C2}$) and dq-transformed currents ($i_d$ and $i_q$) during the connection and disconnection of the load........................................... 180

Fig. 8.1. Concerned areas for small duty cycles of vectors in the first sextant............ 188

Fig. 8.2. Displacement of the reference vector due to small duty-cycle correction ($d, <D_{max}$)......................................................................................... 189

Fig. 8.3. First-sextant regions:
(a) standard nearest-vector distribution and
(b) new regions based on zero-NP-current vectors................................................ 191
List of Tables

Table 2.1. Possible states of the switches in (a) the three-level, and (b) the four-level floating-capacitor converters ................................................................. 20
Table 2.2. Main characteristics of multilevel topologies ................................................................. 24
Table 3.1. Summary of information for the SVM ........................................................................... 57
Table 3.2. Determination of the sextant and the equivalent components $m_1$ and $m_2$ in the first sextant .......................................................................................... 59
Table 3.3. Interchanges of the output states depending on the sextant in which the reference vector lies (after making calculations in the first sextant) .................... 60
Table 3.4. (a) Criteria for selection between vectors 100 and 211, and vectors 110 and 221; and (b) equivalences of currents to process calculations in the first sextant .................................................................................................................. 61
Table 3.5. Sequences of vectors in the first sextant by NTV modulation ........................................ 62
Table 3.6. Sequences of vectors in the first sextant by symmetric modulation ............................ 64
Table 3.7. Equivalences of currents ............................................................................................... 67
Table 4.1. Regions and duty cycles of vectors in the first sextant .................................................. 105
Table 5.1. Dependence of vectors in the first sextant with the voltages of the DC-link capacitors ........................................................................................................... 116
Table 5.2. Region set condition and duty-cycle calculation ............................................................. 120
Table 8.1. Duty cycles that may produce narrow pulses in the devices ......................................... 188
List of Abbreviations and Acronyms

A/D: analog-to-digital (converter)
AC: alternating current
AVG: average
CLK: clock
CPES: Center for Power Electronics Systems
CMOS: complementary metal-oxide-silicon (transistor)
D/A: digital-to-analog (converter)
DC: direct current
DEE: Department of Electronic Engineering
DSP: digital signal processor
EMI: electromagnetic interference
EPE: European Conference on Power Electronics and Applications
EPLD: erasable and programmable logic device
GTO: gate turn-off thyristor
HVDC: high-voltage direct current
IEEE: Institute of Electrical and Electronics Engineers
IGBT: insulated gate bipolar transistor
IRQ: interrupt request
LQR: linear quadratic regulator
MAX: maximum
MIN: minimum
MIMO: multi-input multi-output (system)
MIPS: million of instructions per second
MP: mid point
MSPS: million of samples per second
MUX: multiplexor
NP: neutral point
NPC: neutral-point-clamped (converter)
NTV: nearest-three-vector (modulation)
PEBB: power electronics building block
PEDS: Power Electronics and Drive Systems
PEMC: Power Electronics and Motion Control
PESC: Power Electronics Specialists Conference
PF: power factor
PH: phase
PI: proportional integral
PIEMC: Power Electronics and Motion Control conference
PLL: phase-locked loop
PROM: programmable read only memory
PWM: pulsewidth modulation
RMS: root mean square
SAAEI: Seminario Anual de Automática, Electrónica industrial e Instrumentación
S/H: sample and hold
SMES: superconducting magnetic energy storage
SPWM: sinusoidal pulsewidth modulation
SV: space vector
SVM: space-vector modulation
SV-PWM: space-vector pulsewidth modulation
UPC: Universitat Politècnica de Catalunya
UPS: uninterruptible power system
USA: United States of America
VA: Virginia
VAR: volt-ampere reactive
VSI: voltage-source inverter
ZCT: zero-current transition
Terminology

Glossary of Generic Terms

d_\text{x}: duty cycle
\hat{i}_{hx}: current amplitude of the x-order harmonic
i_{Cx}: current thought a capacitor
\text{Im}(\hat{x}): imaginary part of \hat{x}
i_\text{x}: mid current in the x-point of an n-level converter (x={1, 2, 3, …, n-1})
\bar{p}_x: vector projection
\text{Re}(\hat{x}): real part of \hat{x}
U_\text{x} = [1 \ 1 \ 1 \ \cdots \ 1]^T; in which x is the number of ones
v_{Cx}: voltage of a DC-link capacitor (x={1, 2, 3, …, n-1})
\hat{x}: generic vector
|\hat{x}|, x: vector norm or length of vector \hat{x}
x_\text{i}: input variables of the back-to-back connection
x_\text{o}: output variables of the back-to-back connection
\bar{x}: local-averaged variable
\bar{x}: small-signal variable
\hat{X}: vector (or matrices) of a state-space-formulated system that includes the control loop
X^{-1}: inverse of square matrix X
X^T: transpose of matrix X
X_{(i,j)}: i x j matrix
X^*: reference value
\hat{X}: amplitude of a sinusoidal variable
X_{\text{RMS}}: RMS value of a periodic waveform
x_{(k)}: sampled variable at period k
x_{(k)}: sampled vector at period k
x_{(t)}: time-dependent variable
x_{(t)}: time-dependent vector
x_{ss}: steady-state value of a variable
x_\text{r}: rotating coordinate variables
X_\text{r}: rotating coordinate matrices
[X_1, X_2]: closed interval \ X_1 \leq x \leq X_2
\theta: angle
Glossary of Particular Terms and Definitions

\[ \tilde{a} = e^{\frac{2\pi}{3}} \]

**A, B, C and D:** matrices of the linear state-space representation of a system

**A_d, B_d, C_d and D_d:** matrices of the discrete state-space representation of a system

**C:** capacitor

**C_f:** floating capacitor

**e_a, e_b** and **e_c:** utility phase voltages

**e_d, e_q** and **e_o:** dq transformed utility phase voltages

\[ e_{ph} = [e_a, e_b, e_c]^T \]

\[ e_{LL} = [e_{ab}, e_{bc}, e_{ca}]^T = [e_a - e_b, e_b - e_c, e_c - e_a]^T \]

**E_L:** RMS line-to-line utility voltage

**f:** line frequency

**f_m:** modulation frequency

**f_s:** sample frequency and switching frequency

**f_s mean:** mean switching frequency. Complete turn-on and turn-off cycles in the total switches of the NPC divided by 12.

**G:** parameter related to the quadratic error of the voltages in the DC-link capacitors

**I:** unit matrix

**i_a, i_b** and **i_c:** phase currents in the AC side of a converter

**i_{ap}, i_{bp}** and **i_{cp}:** positive sequence of currents

**i_{an}, i_{bn}** and **i_{cn}:** negative sequence of currents

**i_{ao}, i_{bo}** and **i_{co}:** zero sequence of currents

\[ i_{ph} = [i_a, i_b, i_c]^T \]

\[ i_{LL} = [i_{ab}, i_{bc}, i_{ca}]^T = [i_a - i_b, i_b - i_c, i_c - i_a]^T \]

**i_d, i_q** and **i_o:** dq transformed phase currents

\[ i_{dq} = [i_d, i_q]^T \]

\[ i_{MP} = [\tilde{i}_{n-2}, \tilde{i}_{n-3}, \ldots, \tilde{i}_1]^T ; \] local-averaged mid-point currents

**\tilde{i}_{min}:** minimum local-averaged NP current

**\tilde{i}_{avg}:** average NP current over a line period

\[ \hat{i}: \] current amplitude (fundamental)

**I_{RMS}:** RMS value of the phase currents of the converter (fundamentals)

**J:** LQR quadratic performance index

**K:** LQR optimal solution matrix

**L:** inductance

**m:** modulation index (in the case of linear modulation \( m = \frac{\hat{V}_{LL}}{V_{DC}} \) and \( 0 \leq m \leq 1 \))
\((m_g, m_h)\): normalized components of the reference vector in the non-orthogonal \(gh\) axes
\((m_1, m_2)\): normalized components of the equivalent reference vector in the first sextant in the non-orthogonal \(gh\) axes
\((m'_1, m'_2)\): components of the equivalent reference vector in the first sextant in the non-orthogonal \(gh\) axes
\((m_{OV1}, m_{OV2})\): normalized components of the equivalent reference vector in the first sextant in the non-orthogonal \(gh\) axes for overmodulation mode
\(m_{12} = 2 - m_1 - m_2\)
\(\hat{m}\): reference vector
\(\hat{m}_n\): normalized reference vector
\(m_n = \frac{\sqrt{3}}{2} (n - 1) m\); amplitude of the normalized reference vector

\(M, Q \) and \(R\): hermitic weighting matrices positively defined for evaluation of the parameter \(J\) in the LQR

\(n\): number of available voltage levels in each leg of a multilevel converter. This number typifies an \(n\)-level converter

\(N\): neutral-point of a three-phase star-connected load. Also, last sample of a discrete sequence in the definition of the parameter \(J\) in the LQR

\(p\): instantaneous power

\(R\): electrical resistance

\(s_{ij}\): switching function, \(s_{ij} = \begin{cases} 1 & \text{if } i \text{ is connected to } j, \text{ and} \\ 0 & \text{otherwise.} \end{cases}\)

\(s_x\): sextant function, \(s_x = \begin{cases} 1 & \text{if the reference vector lies in sextant } x, \text{ and} \\ 0 & \text{otherwise.} \end{cases}\)

\(S\): interchanging matrix that depends on sextant functions

\(S_T = T_{dq} S^T\); transformed interchanging matrix that depends on sextant functions

\(t\): time

\(T\): line period

\(T_m\): modulation period

\(T_{dq}\): dq transformation or park transformation

\(u\): input vector or control variables in the state-space representation

\(v_{a0}, v_{b0} \) and \(v_{c0}\): output voltages of a multilevel converter refereed to the lower DC-link voltage

\(v_{ph} = [v_{a0} \ v_{b0} \ v_{c0}]^T\)

\(v_C = [v_{C(n-1)} \ v_{C(n-2)} \ \cdots \ v_{C1}]^T\)

\(v_{C3L} = [v_{C2} \ v_{C1}]^T\)

\(v_{DC}\): total DC-link voltage as a variable (in modeling and controlling sections)

\(V_{DC}\): constant or rated total DC-link voltage

\(V_{DC1}\): voltage source applied to the lower DC-link capacitor
\( V_{DC2} \): voltage source applied to the upper DC-link capacitor
\( \hat{V}_{LL} \): line-to-line voltage amplitude
\( v_{N0} \): neutral voltage of a star-connected load refereed to the lower DC-link level
\( \mathbf{x} \): state vector in the state-space representation
\( \mathbf{y} \): output vector in the state-space representation
\( \alpha, \beta \): orthogonal axes in a two-dimensional representation

\[ \beta = \frac{l_{RMS}}{fCV_{DC}} \]; nondimensional balancing parameter

\[ \gamma_1 = \frac{V_{C1}}{V_{DC}/2} \]; parameter for the imbalance in the lower DC-link capacitor

\[ \gamma_2 = \frac{V_{C2}}{V_{DC}/2} \]; parameter for the imbalance in the upper DC-link capacitor

\( \Delta \rho_C = \frac{d\epsilon_C}{dt} \); total instantaneous power in the DC-link capacitors

\( \Delta v_C = v_C - \frac{V_{DC}}{n-1} \); voltage error in a DC-link capacitor

\( \Delta v_{NP} \): low-frequency peak-to-peak NP voltage ripple

\( \Delta v_{NPn} = \frac{\Delta v_{NP}}{l_{RMS}/fC} \); normalized low-frequency peak-to-peak NP-voltage ripple

\( \epsilon_C \): total electric energy stored in the DC-link capacitors
\( \theta_n \): angle of the normalized reference vector (first sextant)
\( \theta_0 \): initial angle
\( \theta_r \): rotating coordinated angle of the dq transformation
\( \eta \): efficiency
\( \varphi, \varphi_1 \): current phase angle refereed to the voltage phase angle (fundamentals)
\( \omega = 2\pi f \); angular frequency