Appendix A

The Dijkstra Algorithm

The Dijkstra algorithm finds the path of minimum cost in a graph, from any initial node to a goal node. The algorithm has as a parameter a graph G and a goal node n_g, and returns the paths to go from any node to the goal node in a table S, indexed by the nodes, which contains for each node the identifier of the next node of the path. As an example Figure A.1 shows a simple graph whose solution table S associated to the goal node $n_g = 3$ is:

$$S = \{4, 1, -1, 3\} \tag{A.1}$$

where the value -1 means that the goal has been reached. Then, the path to reach the goal node $n_g = 3$ from node number 2 is:

$$\begin{aligned} S[2] &= 1 \\ S[1] &= 4 \\ S[4] &= 3 \end{aligned}$$

The variables used by the algorithm are:

V = list of visited nodes

N = list of non-visited nodes

n_c = identifier of the current node

$C[n_i]$ = cost to reach n_g from any node n_i of G

$C_{\{n_i,n_j\}}$ = cost of the arc connecting the nodes n_i and n_j

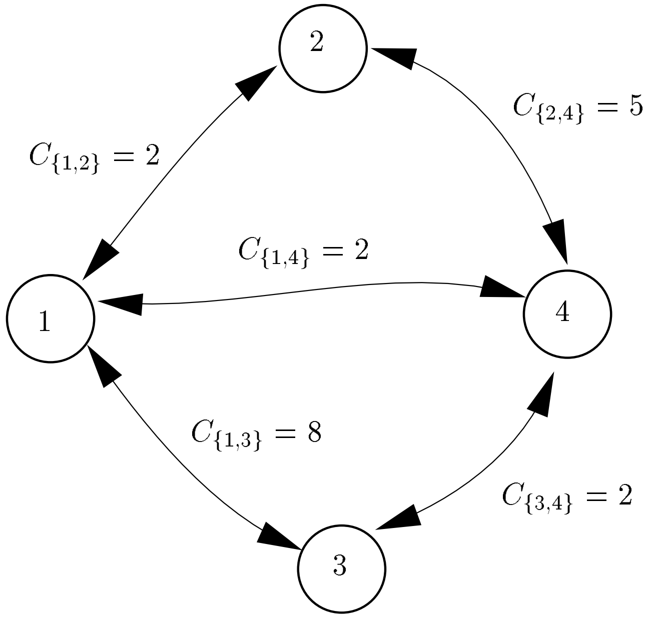

Figure A.1: *Graph composed of four nodes and five arcs.*

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Dijkstra(G, n_g)
    C[n_i] = ∞  ∀n_i ∈ G
    n_c = n_g
    Add n_g to V
    Add n_i to N ∀ n_i ≠ n_g
    C[n_c] = 0
    WHILE N ≠ ∅ DO
        FOR ALL n_i ∈ N
            IF n_i can be reached from n_c THEN
                IF C[n_i] > C[n_c] + C_{n_i,n_c} THEN
                    C[n_i] = C[n_c] + C_{n_i,n_c}
                    S[n_i] = n_c
        Remove n_j from N such that C[n_j] is minimum ∀n_j ∈ N
        n_c = n_j
        Add n_j to V
    RETURN S
END
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Appendix B

The Dual Representation of Forces

A reaction force $\vec{f} = [f_x \;\; f_y]^T$ resulting from a contact situation during a planar assembly task and producing a torque τ with respect to a reference origin O, is mapped, by the dual representation, into the point $F' = [f_y/|\tau| \;\; -f_x/|\tau|]^T$ and the sign of τ. Geometrically, F' lies on the normal to the force line through the reference origin O and at a distance $1/d$ from O, d being the distance between the force line and O.

$\vec{f}$ can be represented in a tridimensional force space $\mathcal{F}_3$ by a generalized reaction force $\vec{g} = [f_x \;\; f_y \;\; f_q]^T$, with $f_q = \frac{\tau}{\rho}$, ρ being the radius of gyration. The f_x and f_y coordinates of the intersection point P of the supporting line of $\vec{g}$ with the plane corresponding to a unitary torque (i.e. $f_q = \frac{1}{\rho}$), coincide with the coordinates of F' rotated $\pi/2$ clockwise around O (Figure B.1).

Properties of the dual representation

Property 1: A line of force $ax + by + c = 0$ maps into a point $(\frac{a}{c}, \frac{b}{c})$ (Figure B.2a).

Property 2: The lines of forces passing through a point map into the points of a line, called the dual line of the point.

Property 3: The lines of forces lying inside a cone[1] $\widehat{r_1 r_2}$ map into the points inside a cone $\widehat{r_1' r_2'}$ with vertex on the origin, r_1' and r_2' being orthogonal to r_1 and r_2 respectively (Figure B.2b).

Property 4: The lines of force passing through a point P and lying inside a cone $\widehat{ab}$, map into points of a segment $\overline{A'B'}$ of the dual line p' of point P, A' and B' being the dual points of lines a and b, respectively (Figure B.2c).

Property 5: The lines of forces crossing a segment $\overline{AB}$ of a line p map into points of a cone $\widehat{a'\,b'}$ with vertex at P', the dual point of p, a' and b' being the dual lines of points A and B.

[1] A cone on the plane, represented by $\widehat{a\,b}$, a and b being two straight lines, refers to the sector swept by line a when it is rotated counterclockwise around the intersection point of a with b until b is reached.

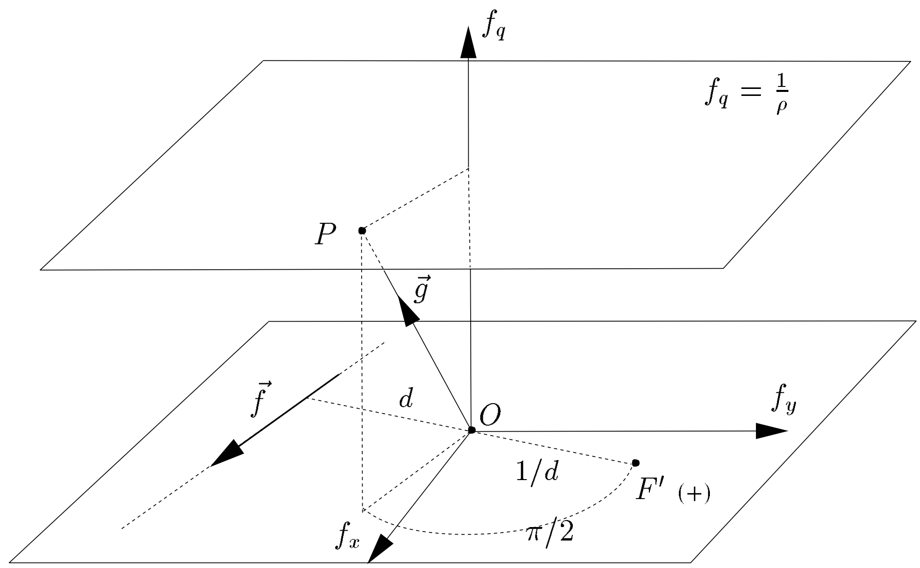

Figure B.1: *Dual representation F' of a force $\vec{f}$.*

Property 6: The lines of forces tangent to a conic C map into the points of a conic C'; the lines crossing C map into the points exterior to C'. The type of conic of C and the relative position of the reference origin and C, determine the type of conic of C' (Figure B.2d shows a case where C is a circumference that does not contain the origin and C' is an hyperbola).

Property 7: The lines of forces that are a non-negative linear combination of a set of forces, map into the points of the convex hull[2] defined by the dual points of the supporting lines of the forces in the set (Figure B.2e shows a *closed* polygon representing the convex hull of three dual forces with the same sign and Figure B.2f shows an *open* polygon representing the convex hull of three dual forces one of which is of different sign from the others).

[2] If two consecutive vertices of the convex hull correspond to two forces that produce a torque of the same sign, then the edge that links them is the interior line segment; otherwise, it is the part of the line outside the two forces. Therefore, if all the vertices have the same sign the convex hull is a *closed* polygon; otherwise it is an *open* polygon.

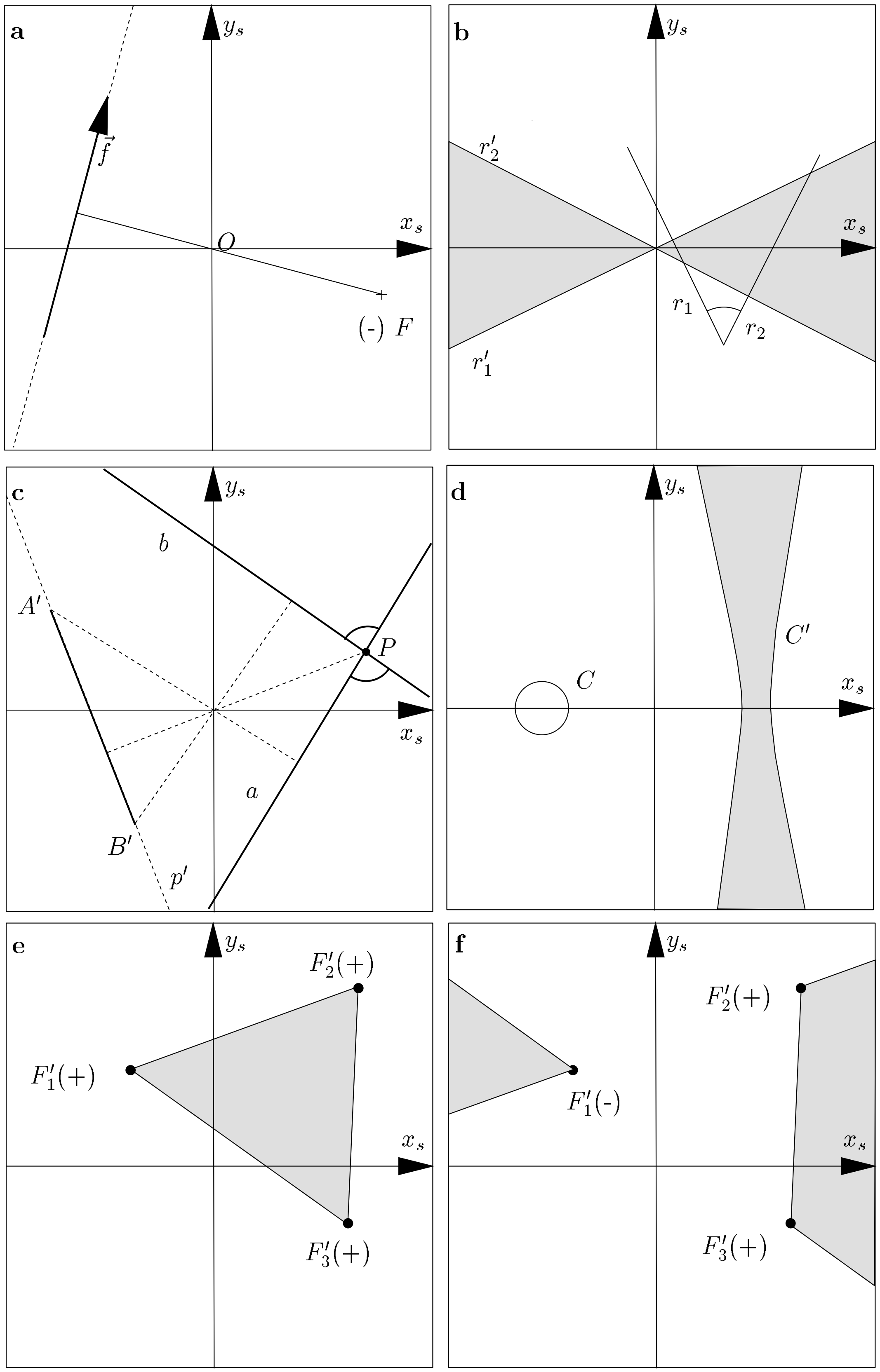

Figure B.2: *Properties of the dual representation of forces.*