

Appendix A

Nonlinear Models

A.1 Models with Surge Tank Effects

A.1.1 Models K5, K52, K51, K4

$$G(s) = \frac{\bar{H}_s - \bar{H}_0}{\bar{U}_p - \bar{U}_0} = \frac{\Phi_c + s \cdot T_{WC}}{1 + s \cdot C_s \cdot \Phi_c + s^2 \cdot T_{WC} \cdot C_s}$$

- Nonlinear Dynamics:

$$\bar{U}_t = \bar{G} \cdot \sqrt{\bar{H}_t}$$

$$\bar{P}_{\text{mechanical}} = (\bar{U}_t - \bar{U}_{NL}) \cdot \bar{H}_t$$

Transfer Function F(s) (K5):

$$F(s) = \frac{\bar{U}_t - \bar{U}_0}{\bar{H}_t - \bar{H}_0} = - \frac{1 + \frac{G(s)}{z_p} \cdot \tanh(T_{ep} \cdot s)}{\Phi_p + G(s) + z_p \cdot \tanh(T_{ep} \cdot s)}$$

Transfer Function F(s) (K52):

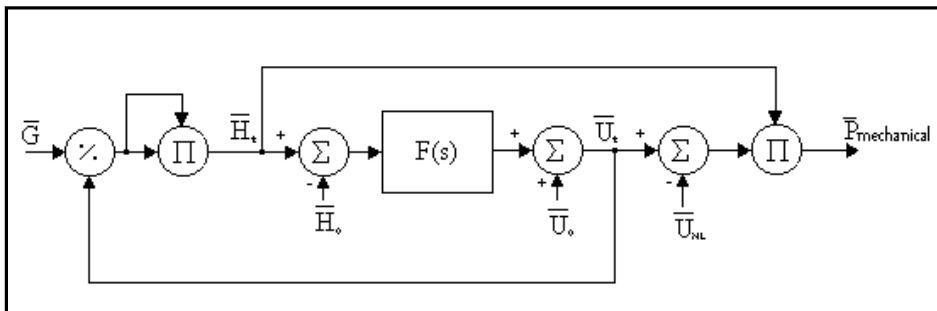
$$F(s) = \frac{\bar{U}_t - \bar{U}_0}{\bar{H}_t - \bar{H}_0} = - \frac{1 + \frac{G(s)}{z_p} \cdot \frac{s \cdot T_{ep} \cdot \left(1 + \left(\frac{s \cdot T_{ep}}{\pi}\right)^2\right) \cdot \left(1 + \left(\frac{s \cdot T_{ep}}{2 \cdot \pi}\right)^2\right)}{\left(1 + \left(\frac{2 \cdot s \cdot T_{ep}}{\pi}\right)^2\right) \cdot \left(1 + \left(\frac{2 \cdot s \cdot T_{ep}}{3 \cdot \pi}\right)^2\right)}}{\Phi_p + G(s) + z_p \cdot \frac{s \cdot T_{ep} \cdot \left(1 + \left(\frac{s \cdot T_{ep}}{\pi}\right)^2\right) \cdot \left(1 + \left(\frac{s \cdot T_{ep}}{2 \cdot \pi}\right)^2\right)}{\left(1 + \left(\frac{2 \cdot s \cdot T_{ep}}{\pi}\right)^2\right) \cdot \left(1 + \left(\frac{2 \cdot s \cdot T_{ep}}{3 \cdot \pi}\right)^2\right)}}$$

Transfer Function F(s) (K51):

$$F(s) = \frac{\bar{U}_t - \bar{U}_0}{\bar{H}_t - \bar{H}_0} = - \frac{1 + \frac{G(s)}{z_p} \cdot \frac{s \cdot T_{ep} \cdot \left(1 + \left(\frac{s \cdot T_{ep}}{\pi}\right)^2\right)}{\left(1 + \left(\frac{2 \cdot s \cdot T_{ep}}{\pi}\right)^2\right)}}{\Phi_p + G(s) + z_p \cdot \frac{s \cdot T_{ep} \cdot \left(1 + \left(\frac{s \cdot T_{ep}}{\pi}\right)^2\right)}{\left(1 + \left(\frac{2 \cdot s \cdot T_{ep}}{\pi}\right)^2\right)}}$$

Transfer Function F(s) (K4):

$$F(s) = \frac{\bar{U}_t - \bar{U}_0}{\bar{H}_t - \bar{H}_0} = - \frac{1 + \frac{G(s)}{z_p} \cdot T_{ep} \cdot s}{\Phi_p + G(s) + z_p \cdot T_{ep} \cdot s}$$



A.1.2 Models WG5, QR52, QR51, WG4

- Nonlinear Dynamics:

$$\bar{U}_t = \bar{U}_c - \bar{U}_s$$

$$\bar{P}_{\text{mechanical}} = A_t \cdot \bar{H}_t \cdot (\bar{U}_t - \bar{U}_{\text{NL}}) - D_1 \cdot \bar{G} \cdot \Delta \bar{\omega}$$

- Dynamics of the Tunnel:

$$\bar{H}_r = 1.0 - \bar{H}_{12} - \bar{H}_{Q2}$$

$$\bar{H}_{12} = f_{p2} \cdot \bar{U}_c \cdot |\bar{U}_c|$$

$$\bar{H}_{Q2} = T_{\text{WC}} \cdot \frac{d\bar{U}_c}{dt}$$

- Dynamics of the Surge Tank:

$$\bar{H}_r = \frac{1}{C_s} \cdot \int \bar{U}_s \cdot dt - f_0 \cdot \bar{U}_s \cdot |\bar{U}_s|$$

- Dynamics of the Penstock (WG5):

$$\bar{H}_t = \bar{H}_r - \bar{H}_1 - \bar{H}_Q$$

$$\bar{H}_1 = f_{p1} \cdot \bar{U}_t^2$$

$$\bar{H}_Q = z_p \cdot \tanh(T_{\text{ep}} \cdot s) \cdot \bar{U}_t$$

- Dynamics of the Penstock (QR52):

$$\bar{H}_t = \bar{H}_r - \bar{H}_1 - \bar{H}_Q$$

$$\bar{H}_1 = f_{p1} \cdot \bar{U}_t^2$$

$$\bar{H}_Q = z_p \cdot \frac{s \cdot T_{\text{ep}} \cdot \left(1 + \left(\frac{s \cdot T_{\text{ep}}}{\pi}\right)^2\right) \cdot \left(1 + \left(\frac{s \cdot T_{\text{ep}}}{2 \cdot \pi}\right)^2\right)}{\left(1 + \left(\frac{2 \cdot s \cdot T_{\text{ep}}}{\pi}\right)^2\right) \cdot \left(1 + \left(\frac{2 \cdot s \cdot T_{\text{ep}}}{3 \cdot \pi}\right)^2\right)} \cdot \bar{U}_t$$

- Dynamics of the Penstock (QR51):

$$\bar{H}_t = \bar{H}_r - \bar{H}_1 - \bar{H}_Q$$

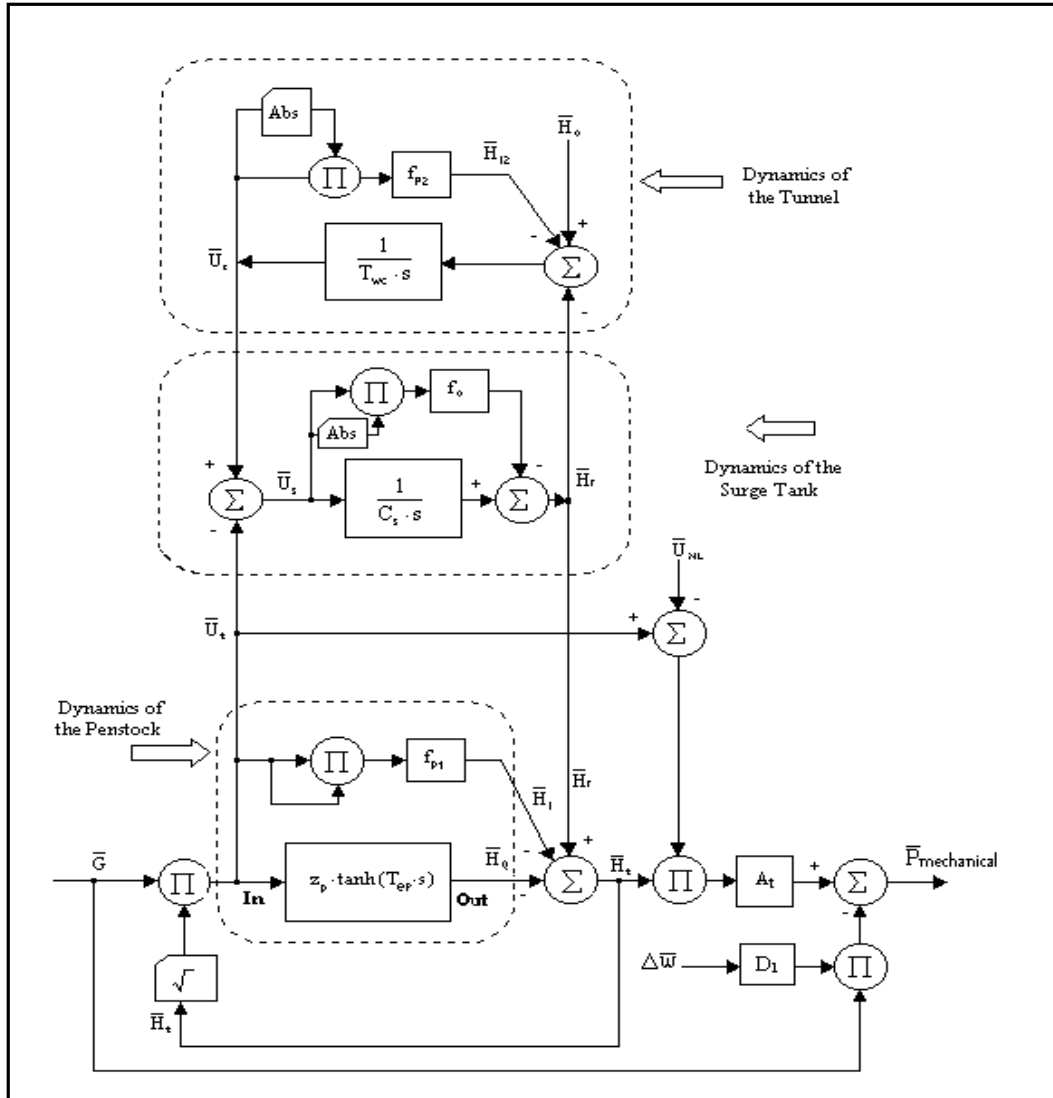
$$\bar{H}_1 = f_{p1} \cdot \bar{U}_t^2$$

$$\bar{H}_Q = z_p \cdot \frac{s \cdot T_{\text{ep}} \cdot \left(1 + \left(\frac{s \cdot T_{\text{ep}}}{\pi}\right)^2\right)}{\left(1 + \left(\frac{2 \cdot s \cdot T_{\text{ep}}}{\pi}\right)^2\right)} \cdot \bar{U}_t$$

- Dynamics of the Penstock (WG4):

$$\bar{H}_l = f_{p1} \cdot \bar{U}_t^2$$

$$\frac{d\bar{U}_t}{dt} = \frac{\bar{H}_r - \bar{H}_t - \bar{H}_l}{T_{WP}}$$



A.2 Models with no Surge Tank Effects

A.2.1 Models K3, K32, K31, K2

- Nonlinear Dynamics:

$$\bar{U}_t = \bar{G} \cdot \sqrt{\bar{H}_t}$$

$$\bar{P}_{\text{mechanical}} = (\bar{U}_t - \bar{U}_{NL}) \cdot \bar{H}_t$$

Transfer Function F(s) (K3)

$$F(s) = \frac{\bar{U}_t - \bar{U}_0}{\bar{H}_t - \bar{H}_0} = - \frac{1}{\Phi_p + z_p \cdot \tanh(T_{ep} \cdot s)}$$

Transfer Function F(s) (K33)

$$F(s) = \frac{\bar{U}_t - \bar{U}_0}{\bar{H}_t - \bar{H}_0} = - \frac{1}{\Phi_p + z_p \cdot \frac{s \cdot T_{ep} \cdot \left(1 + \left(\frac{s \cdot T_{ep}}{\pi}\right)^2\right) \cdot \left(1 + \left(\frac{s \cdot T_{ep}}{2 \cdot \pi}\right)^2\right) \cdot \left(1 + \left(\frac{5 \cdot s \cdot T_{ep}}{3 \cdot \pi}\right)^2\right)}{\left(1 + \left(\frac{2 \cdot s \cdot T_{ep}}{\pi}\right)^2\right) \cdot \left(1 + \left(\frac{2 \cdot s \cdot T_{ep}}{3 \cdot \pi}\right)^2\right) \cdot \left(1 + \left(\frac{2 \cdot s \cdot T_{ep}}{5 \cdot \pi}\right)^2\right)}$$

Transfer Function F(s) (K32)

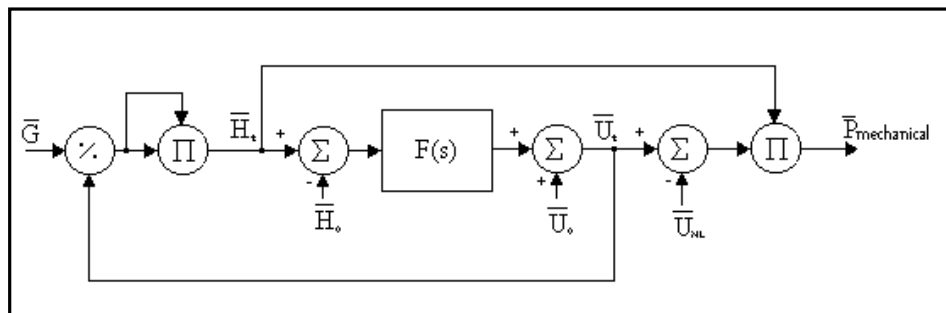
$$F(s) = \frac{\bar{U}_t - \bar{U}_0}{\bar{H}_t - \bar{H}_0} = - \frac{1}{\Phi_p + z_p \cdot \frac{s \cdot T_{ep} \cdot \left(1 + \left(\frac{s \cdot T_{ep}}{\pi}\right)^2\right) \cdot \left(1 + \left(\frac{s \cdot T_{ep}}{2 \cdot \pi}\right)^2\right)}{\left(1 + \left(\frac{2 \cdot s \cdot T_{ep}}{\pi}\right)^2\right) \cdot \left(1 + \left(\frac{2 \cdot s \cdot T_{ep}}{3 \cdot \pi}\right)^2\right)}$$

Transfer Function F(s) (K31)

$$F(s) = \frac{\bar{U}_t - \bar{U}_0}{\bar{H}_t - \bar{H}_0} = - \frac{1}{\Phi_p + z_p \cdot \frac{s \cdot T_{ep} \cdot \left(1 + \left(\frac{s \cdot T_{ep}}{\pi}\right)^2\right)}{\left(1 + \left(\frac{2 \cdot s \cdot T_{ep}}{\pi}\right)^2\right)}}$$

Transfer Function F(s) (K2)

$$F(s) = \frac{\bar{U}_t - \bar{U}_0}{\bar{H}_t - \bar{H}_0} = - \frac{1}{T_{WP} \cdot s}$$



A.2.2 Models WG3, QR33, QR32, QR31, WG2

- Nonlinear Dynamics:

$$\bar{U}_t = \bar{G} \cdot \sqrt{\bar{H}_t}$$

$$\bar{P}_{\text{mechanical}} = A_t \cdot \bar{H}_t \cdot (\bar{U}_t - \bar{U}_{\text{NL}}) - D_1 \cdot \bar{G} \cdot \Delta \bar{\omega}$$

- Dynamics of the Penstock (WG3):

$$\bar{H}_t = \bar{H}_r - \bar{H}_1 - \bar{H}_Q$$

$$\bar{H}_1 = f_{p1} \cdot \bar{U}_t^2$$

$$\bar{H}_Q = z_p \cdot \tanh(T_{ep} \cdot s) \cdot \bar{U}_t$$

- Dynamics of the Penstock (QR33):

$$\bar{H}_t = \bar{H}_r - \bar{H}_1 - \bar{H}_Q$$

$$\bar{H}_1 = f_{p1} \cdot \bar{U}_t^2$$

$$\bar{H}_Q = z_p \cdot \frac{s \cdot T_{ep} \cdot \left(1 + \left(\frac{s \cdot T_{ep}}{\pi}\right)^2\right) \cdot \left(1 + \left(\frac{s \cdot T_{ep}}{2 \cdot \pi}\right)^2\right) \cdot \left(1 + \left(\frac{5 \cdot s \cdot T_{ep}}{3 \cdot \pi}\right)^2\right)}{\left(1 + \left(\frac{2 \cdot s \cdot T_{ep}}{\pi}\right)^2\right) \cdot \left(1 + \left(\frac{2 \cdot s \cdot T_{ep}}{3 \cdot \pi}\right)^2\right) \cdot \left(1 + \left(\frac{2 \cdot s \cdot T_{ep}}{5 \cdot \pi}\right)^2\right)} \cdot \bar{U}_t$$

- Dynamics of the Penstock (QR32):

$$\bar{H}_t = \bar{H}_r - \bar{H}_1 - \bar{H}_Q$$

$$\bar{H}_1 = f_{p1} \cdot \bar{U}_t^2$$

$$\bar{H}_Q = z_p \cdot \frac{s \cdot T_{ep} \cdot \left(1 + \left(\frac{s \cdot T_{ep}}{\pi}\right)^2\right) \cdot \left(1 + \left(\frac{s \cdot T_{ep}}{2 \cdot \pi}\right)^2\right)}{\left(1 + \left(\frac{2 \cdot s \cdot T_{ep}}{\pi}\right)^2\right) \cdot \left(1 + \left(\frac{2 \cdot s \cdot T_{ep}}{3 \cdot \pi}\right)^2\right)} \cdot \bar{U}_t$$

- Dynamics of the Penstock (QR31):

$$\bar{H}_t = \bar{H}_r - \bar{H}_1 - \bar{H}_Q$$

$$\bar{H}_1 = f_{p1} \cdot \bar{U}_t^2$$

$$\bar{H}_Q = z_p \cdot \frac{s \cdot T_{ep} \cdot \left(1 + \left(\frac{s \cdot T_{ep}}{\pi} \right)^2 \right)}{\left(1 + \left(\frac{2 \cdot s \cdot T_{ep}}{\pi} \right)^2 \right)} \cdot \bar{U}_t$$

- Dynamics of the Penstock (WG2):

$$\frac{d\bar{U}_t}{dt} = \frac{1.0 - \bar{H}_t - \bar{H}_1}{T_{WP}}$$

$$\bar{H}_1 = f_{p1} \cdot \bar{U}_t^2$$

