Chapter 4

Identification of a Hydroelectric Power Plant

The identification of a hydroelectric power turbine dynamic model is presented in this chapter. The identification has been performed for the Susqueda power plant, which is a hydroelectric plant belonging to the Endesa Group in Spain. Models described in Chapter 3 are taken as an initial step, although some modifications have been carried out in order to adjust the response of the real plant to model behaviour.

Dynamic behaviour of the power plant has been recorded in different conditions and situations, and consists of the gate opening and the measured electric power, which may be considered as the *measured mechanical power* generated by the turbine.

The first identification approach utilises a group of models (Models A, which are based on models WG5, QR52, QR51 and WG4, and a refinement function) with the most complex hydraulic dynamics, since all of them are nonlinear, and consider surge tank effects. The second approach uses other models: nonlinear models with *no* surge tank effects (Models B, which are based on models WG3, QR32, QR31 and WG2, and a refinement function); a linearized model with surge tank (Model C, which is based on model Q_{lin0} and a refinement function) and a linear model with no surge tank (Model D, which is based on model G_{lin0} and a refinement function). This last group of models (B, C, D) is less complex than the first one (Models A).

This chapter is organised as follows: Section 4.1 describes the physical characteristics of the power station. Section 4.2 presents the dynamic equations considering a general nonlinear model with surge tank effects, and proposes some adjustments of the model. Section 4.3 presents the results of the simulation using the models of the groups A, B C and D. Section 4.4 describes a comparative study of the behaviour of the models. Finally, Section 4.5 summarises the contents of this chapter.

4.1 Characteristics of the Hydroelectric Power Station

The Susqueda power plant is situated next to Susqueda’s reservoir in the province of Girona (Spain), which is supplied by the river Ter. The total installed power is 86 MW and it has three groups (2 x 37 MW + 1 x 12 MW). Figure 4.1 presents a diagram of the main characteristics of the Susqueda power plant, and shows the heads and flows that intervene in a hydroelectric plant model that considers surge tank effects.

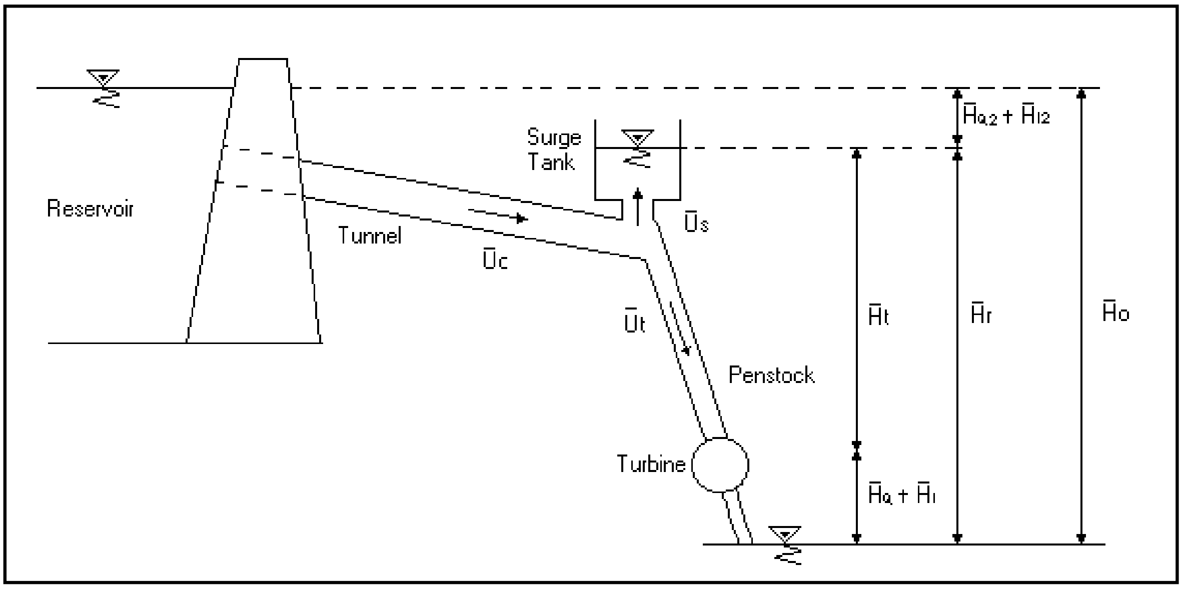


Figure 4.1: Plot of heads and flows distribution in a hydroelectric plant with surge tank.

The hydroelectric power station of Susqueda has the following characteristics:

The Plant	
Total Head (H_0)	174.41 m
Head Losses (H_l+H_{l2})	10.39 m
Maximum Flow (Q_{max})	65 m^3/seg
Installed Power ($2 \cdot P_{elec}$+ P_{elec-1})	2 x 37 MW + 1 x 12 MW = 86 MW
Annual Production	180 GWh

Table 4.1: Plant Characteristics.

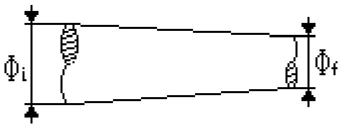
The Conduits	
Surge Tank (Cylindrical)	Height = 100 m Diameter = 9 m
Tunnel (Cylindrical)	Length = 3500 m (L_c) Diameter = 4.3 m Internal Material: concrete.
Penstock  	Length = 250 m (L_p) Initial Diameter: Φ_i = 4.3 m Final Diameter: Φ_f = 3.3 m Internal Material: concrete.

Table 4.2: Conduits Characteristics.

4.2 General Nonlinear Equations

The initial approach is to consider the most general dynamic equations and tune the resulting model to match the reality. The general dynamic equations are taken from models WG5 and WG4 from IEEE working Group (1992), and QR52 and QR51 from Quiroga and Riera (1999). These equations are:

- Equation of continuity

$$\overline{U}_t = \overline{U}_c - \overline{U}_s \qquad (4.\ 1)$$

- Dynamics of the penstock

$$\overline{H}_l = f_{p1} \cdot \overline{U}_t^{\,2} \qquad (4.\ 2)$$

$$\overline{H}_t = \overline{H}_r - \overline{H}_l - z_p \cdot \tanh(T_{ep} \cdot s) \cdot \overline{U}_t = \overline{H}_r - \overline{H}_l - \overline{H}_Q \qquad (4.\ 3)$$

$$\overline{U}_t = \overline{G} \cdot \sqrt{\overline{H}_t} \qquad (4.\ 4)$$

- Dynamics of the tunnel

$$\frac{d\overline{U}_c}{dt} = \frac{1.0 - \overline{H}_r - \overline{H}_{l2}}{T_{WC}} \tag{4. 5}$$

$$\overline{H}_{l2} = f_{p2} \cdot \overline{U}_c \cdot \left|\overline{U}_c\right| \tag{4. 6}$$

- Dynamics of the surge tank

$$\overline{H}_r = \frac{1}{C_s} \cdot \int \overline{U}_s \cdot dt - f_0 \cdot \overline{U}_s \cdot \left|\overline{U}_s\right| \tag{4. 7}$$

- Mechanical power

$$\overline{P}_{mechanical} = A_t \cdot \overline{H}_t \cdot \left(\overline{U}_t - \overline{U}_{NL}\right) \tag{4. 8}$$

The parameters of the equations (4. 1) to (4. 8) for the Susqueda power station may be deduced from values of Table 4.1, Table 4.2 and the formulas described in Section 3.1.1. These parameters can be written as

PARAMETERS	VALUES
T_{WP}	0.82 [s]
T_{WC}	9.15 [s]
C_s	140 [s]
T_{ep}	0.208 [s]
f_{p1}	0.0475 [pu]
f_{p2}	0.089 [pu]
f_0	0 [pu]
A_t	1.67 [pu]
$\overline{U}_{NL}$	0.13 [pu]
z_p	3.95
T	225 [s]

Table 4.3: Parameters of Susqueda power station.

Figure 4.2 shows a block diagram for the general nonlinear model with surge tank effects. This diagram includes the nonlinear function $\eta(\overline{G})$.

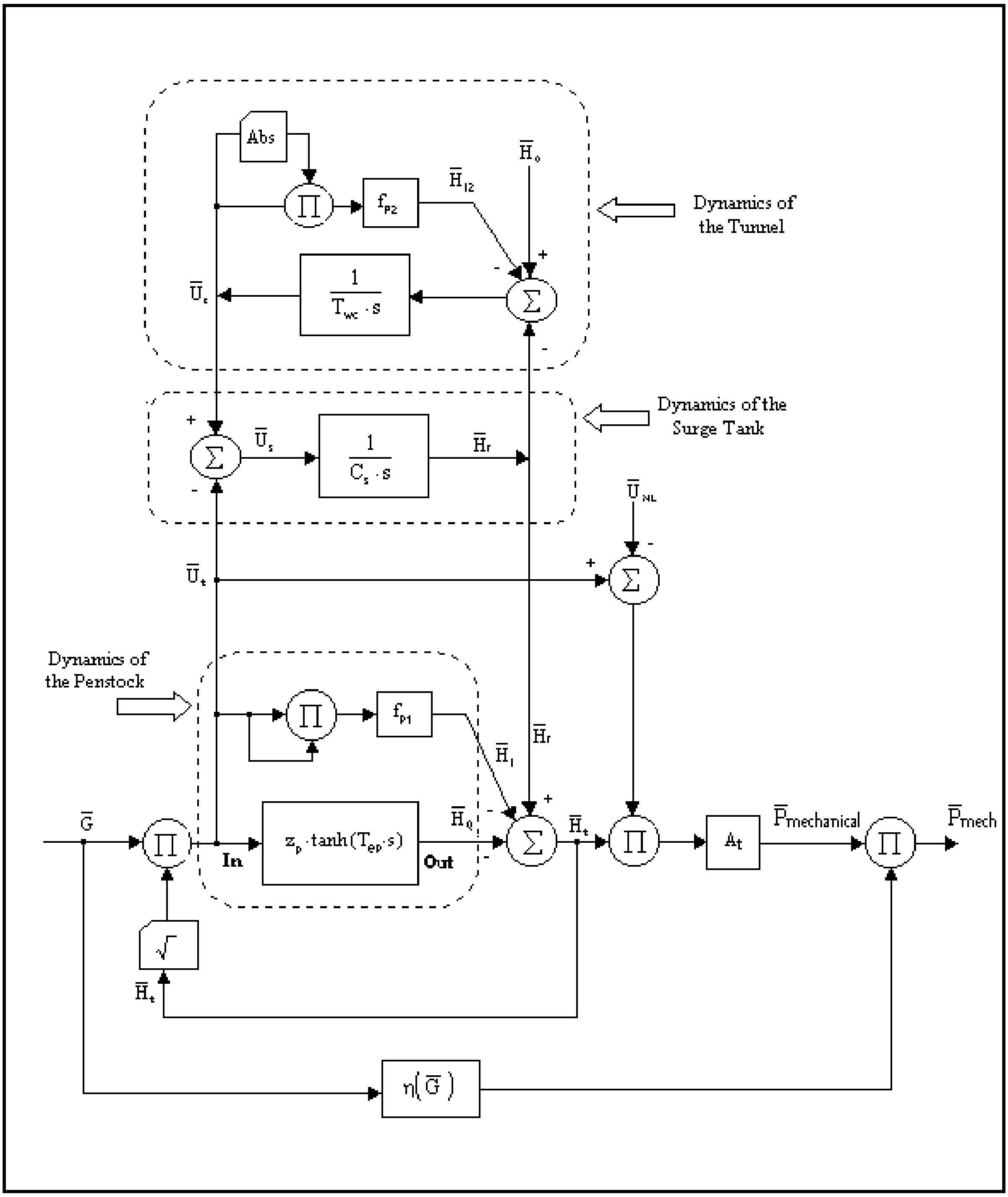


Figure 4.2: Functional diagram for a general nonlinear model with surge tank effects.

Figure 4.3 depicts the block employed to calculate the exact hyperbolic tangent function. This scheme is utilised in the Models A1 and B1.

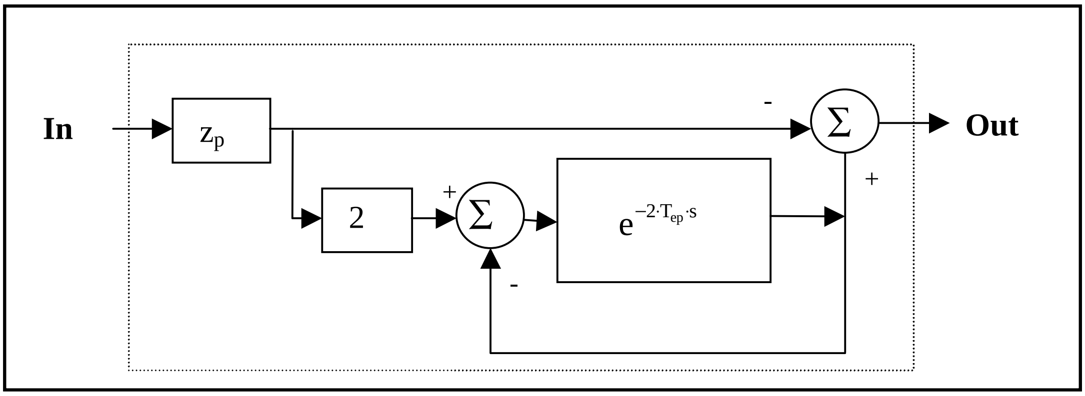


Figure 4.3: Schematic representation of the "In-Out" block used to calculate the exact hyperbolic tangent given by the mathematical expression $z_p \cdot \tanh(T_{ep} \cdot s)$.

Figure 4.4 is a representation of the "In-Out" block used to calculate the approximations n=0, 1, 2 of the hyperbolic tangent function.

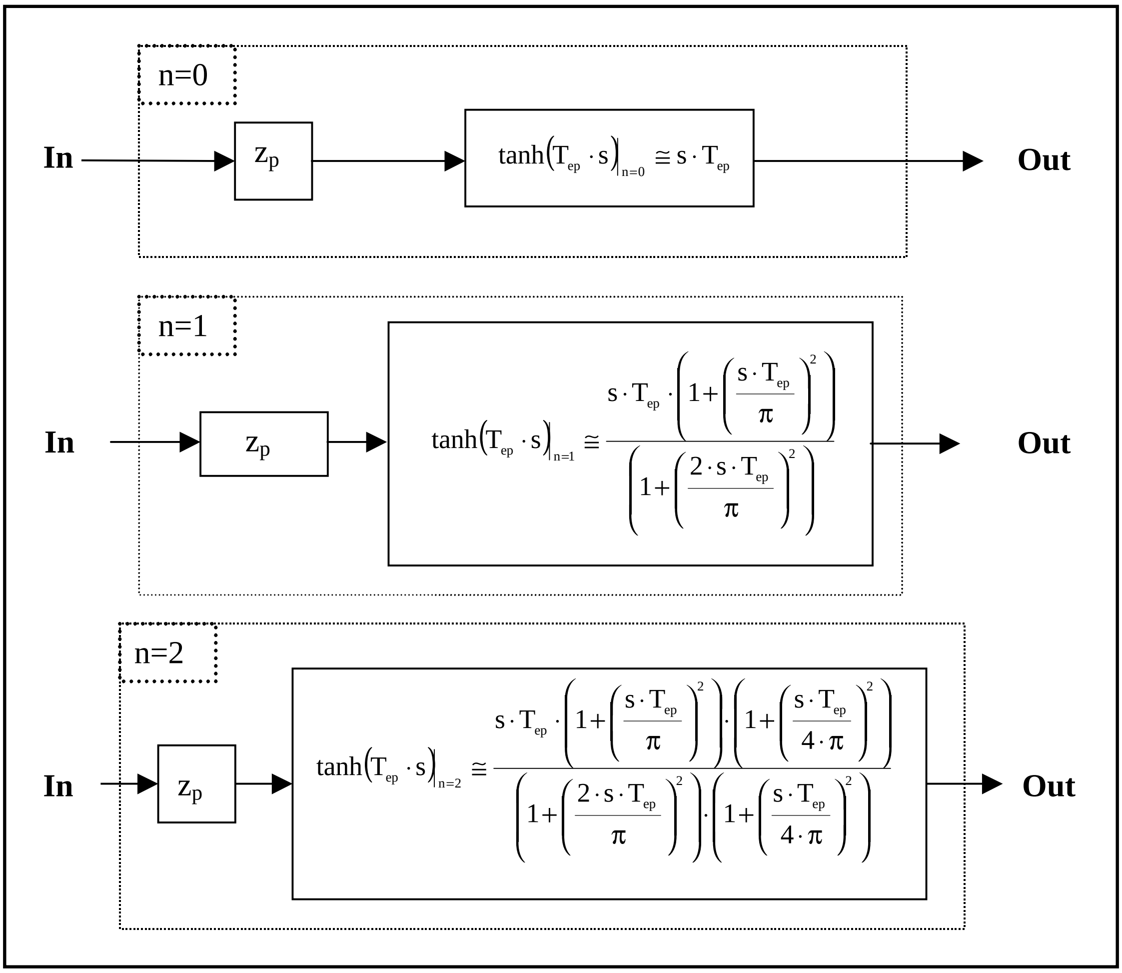


Figure 4.4: Representation of the block used to calculate the approximations n=0, n=1 and n=2 of the hyperbolic tangent function.

4.2.1 Adjustment of the Equations

In order to match model behaviour to real plant response, two kinds of adjustments must be performed. The first one is needed to adjust the static gain of the model to the Susqueda power plant. The second one is related to the pressure oscillations caused by the surge tank.

4.2.2 Output Power Adjustment

To adjust the output power ($\overline{P}_{mechanical}$) it is necessary to multiply the value given by equation (4. 8), by a nonlinear function $\eta(\overline{G})$ that represents the efficiency of the turbine. This function depends on the gate opening and its shape is similar to the efficiency curve of a Francis hydraulic turbine (Zipparro and Hasen, 1993).

$$\overline{P}_{mech} = \eta(\overline{G}) \cdot \overline{P}_{mechanical} = \eta(\overline{G}) \cdot A_t \cdot \overline{H}_t \cdot (\overline{U}_t - \overline{U}_{NL}) \qquad (4.\ 9)$$

Table 4.4 gives the values of $\eta(\overline{G})$ deduced from steady state values of experimental registers.

$\overline{G}$ (pu)	P_{elec} (MW)	$\eta(\overline{G})$ (pu)
0.13	13.1	0.8922
0.18	13.6	0.9012
0.25	14.2	0.9107
0.36	14.5	0.9111
0.411	15.3	0.9113
0.603	27.3	0.9180
0.6635	30.15	0.9048
0.752	30.2	0.8410
0.8	30.75	0.8174
0.85	31.3	0.7874
0.896	31.8	0.7610

Table 4.4: Values of the nonlinear function $\eta(\overline{G})$ for different gate positions deduced from experimental tests.

Moreover, the nonlinear function is calculated by means of the method of the least squares. The 1st, 2nd, 3rd and 5th order polynomials can be easily found using for example the function *polyfit* from the MATLAB mathematical package. This function finds the coefficients of a polynomial of degree ‘n’ that fits the values given by the first and third column of Table 4.4 in the least square sense.

Degree (n)	POLYNOMIALS	Norm [pu]
1st	$\eta(\overline{G}) = -0.155147 \cdot \overline{G} + 0.95188$	0.1158
2nd	$\eta(\overline{G}) = -0.654517 \cdot \overline{G}^2 + 0.5181708 \cdot \overline{G} + 0.8258124$	0.0402
3rd	$\eta(\overline{G}) = -0.7932113 \cdot \overline{G}^3 + 0.56973402 \cdot \overline{G}^2 - 0.026849177 \cdot \overline{G} + 0.8895742$	0.0287
5th	$\eta(\overline{G}) = 15.8112 \overline{G}^5 - 39.05104 \overline{G}^4 + 34.7891776 \overline{G}^3 - 14.1045755 \overline{G}^2 + 2.6587286 \overline{G} + 0.7179617$	0.0128

Table 4.5: Polynomials obtained from the first and third columns of Table 4.4 using the method of the least squares.

Figure 4.5 shows the quadratic and cubic polynomials that represent $\eta(\overline{G})$ versus the steady state values taken from the third column of Table 4.4.

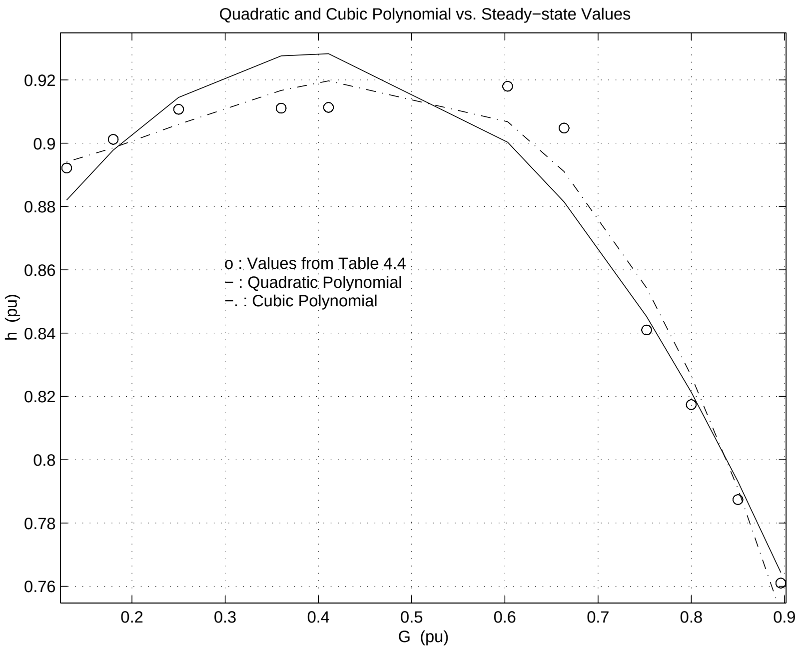


Figure 4.5: Plot of the nonlinear function $\eta(\overline{G})$ calculated by the quadratic and cubic polynomials.

4.2.3 Surge Tank Natural Period

The period (T) of the pressure waves due to the surge tank is a function of three physical parameters of the hydroelectric plant: the cross section area of the surge tank (A_s), the cross section area of the tunnel (A_c) and the length of the tunnel (L_c), see e.g. Section 3.1.

$$T = 2 \cdot \pi \cdot \sqrt{\frac{L_c \cdot A_s}{g \cdot A_c}} \tag{4. 10}$$

The surge tank is cylindrical, so its diameter ϕ_s is used to adjust the surge tank natural period (T). Since the surge tank in Susqueda is very large, there exists the possibility that the diameter of the surge tank could vary a small percentage along its 100m of height. The modified value of the diameter is five per cent less than the original diameter. Apart from this, the surge tank has not got an orifice; for this reason the value of the loss coefficient f_0 is equal to zero. The physical parameters ϕ_c and L_c may be used to adjust T also; however, the modified values for the diameter or length of the tunnel are greater than the five per cent obtained after adjusting ϕ_s.

4.3 Simulation Results

Several different simulation programs can be implemented by starting from the general model equations. The first approach is obtained by calculating the exact hyperbolic tangent function as depicted in Figure 4.3. This scheme is used for the models A1 and B1. Other approaches are obtained by approximating the hyperbolic tangent function as shown in Figure 4.4. This figure is formed by three schemes that are used as follows: n=0 for the models A4 and B4; n=1 for the models A3 and B3 and n=2 for the models A2 and B2.

4.3.1 Identification Using the Models A

Table 4.6 presents a group of four nonlinear models for a power plant with a surge tank. All these models may be used in the identification process of Susqueda hydroelectric plant.

DERIVED MODELS	REFERENCED MODELS
Models A Given by equations: (4. 1) to (4. 7) and (4. 9).	Nonlinear models *with* surge tank effects: A1. Elastic water column in the penstock and non-elastic water column in the tunnel. Hyperbolic tangent calculated by means of the “In-Out” block of Figure 4.3. A2. Elastic water column in the penstock and non-elastic water column in the tunnel. Hyperbolic tangent calculated by means of the “In-Out” block of Figure 4.4, approximation n=2 A3. Elastic water column in the penstock and non-elastic water column in the tunnel. Hyperbolic tangent calculated by means of the “In-Out” block of Figure 4.4, approximation n=1. A4. Non-elastic water columns. Hyperbolic tangent calculated by means of the “In-Out” block of Figure 4.4, approximation n=0.

Table 4.6: Description of the Models A and their references.

The nonlinear function $\eta(\bar{G})$ is not considered in the general models given by IEEE Working Group (1992) and Quiroga and Riera (1999).

Registers of the gate opening are used as input for the Models A. Figure 4.6 shows the measured electric power (equivalent to mechanical power) labelled as P_{mech}(register), and the mechanical power calculated by means of the Models A. For all figures (4.6 to 4.14) the responses of the mechanical power calculated by means of the Models A1, A2, A3 and A4 are almost the same (details are explained in Section 4.4). This is due to the fact that the gate opening in the real plant has not a real step function shape (as the step function used in Section 3.6.1 for exciting the gate opening), and for the worst case the gate opening has the shape of a ramp. This can be observed in Figures 4.7, 4.9, 4.11 and 4.13. That is the reason why the measured electric power is labelled as P_{mech}(Models A)

Moreover, these last mentioned figures represent on the one hand the gate opening versus the measured electric power and labelled as P_{mech}(register). On the other hand, they show the gate opening versus the mechanical power calculated by means of the Models A, and labelled in figures as P_{mech}(Models A). All figures in this section show different operating situations of the power plant.

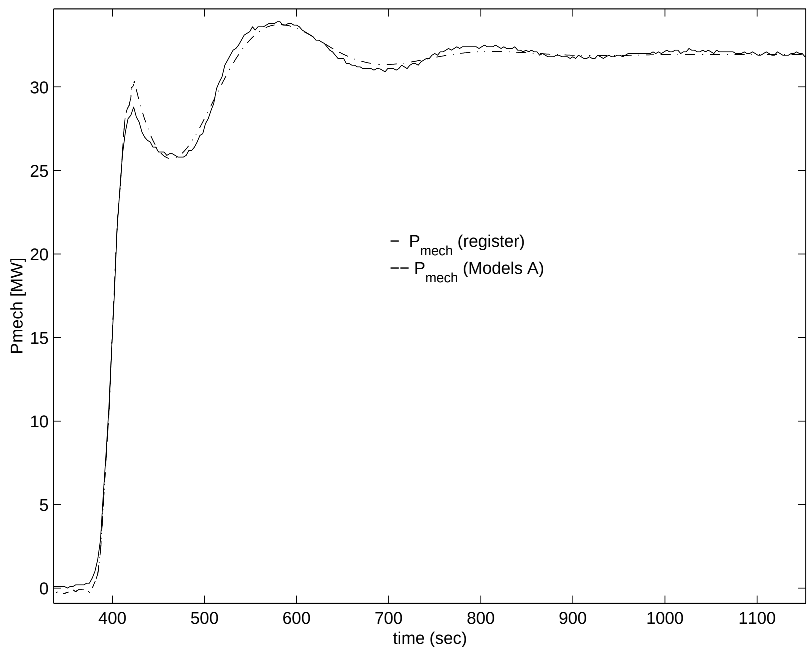


Figure 4.6: Identification of Susqueda using the Models A.

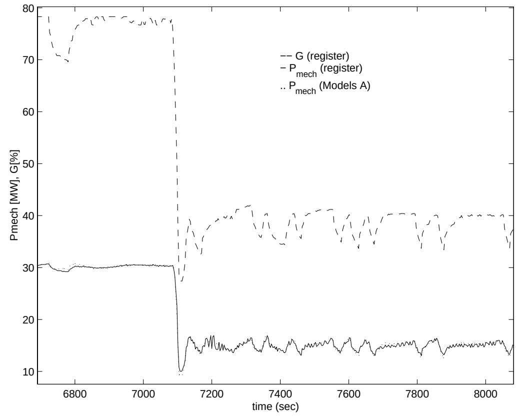


Figure 4.7: Identification of Susqueda using the Models A.

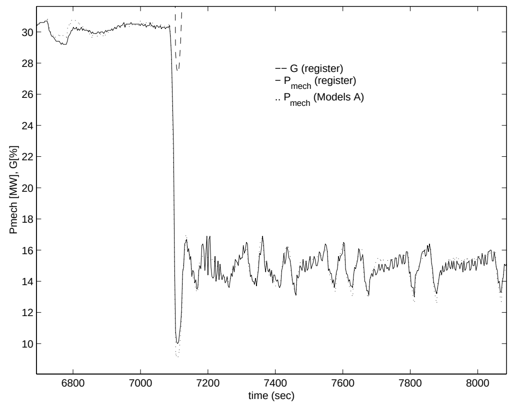


Figure 4.8: Detail of Figure 4.7.

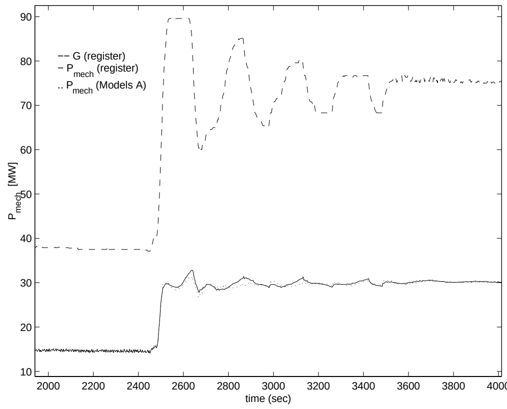


Figure 4.9: Identification of Susqueda using the Models A.

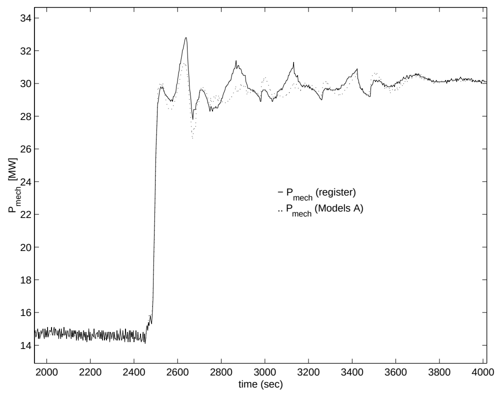


Figure 4.10: Detail of Figure 4.9.

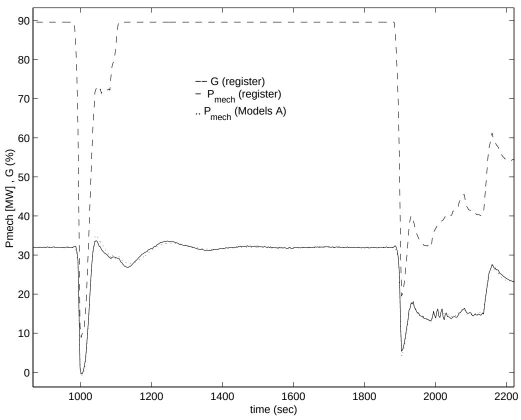


Figure 4.11: Identification of Susqueda using the Models A.

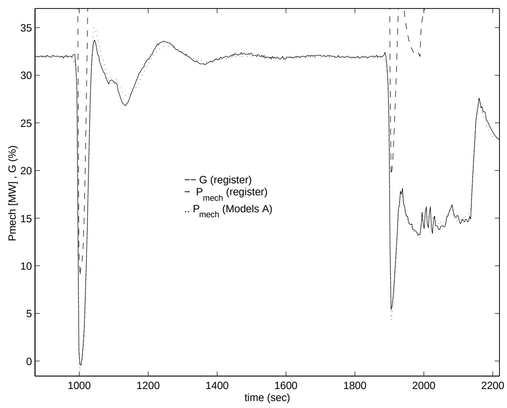


Figure 4.12: Detail of Figure 4.11.

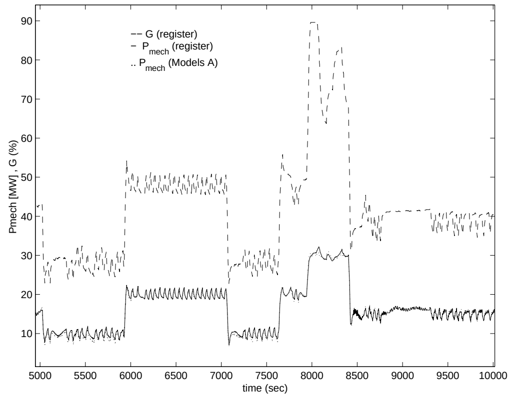


Figure 4.13: Identification of Susqueda using the Models A.

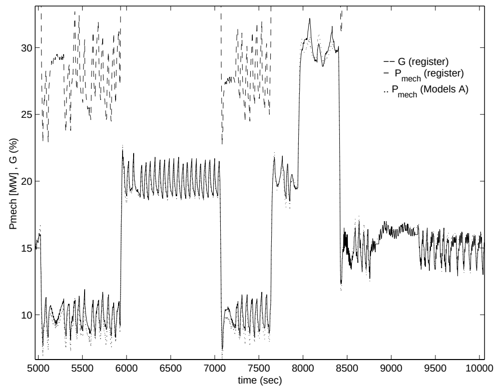


Figure 4.14: Detail of Figure 4.12.

Figures 4.6 to 4.14 show that the models A give a very good identification of the hydraulic plant of Susqueda. For these models the static gain of the mechanical power and the surge tank time period are both adjusted.

4.3.2 Identification Using the Models B

In this section the group of the models B are presented. Table 4.7 shows in detail the models B1, B2, B3 and B4. Similar to the case of the models A, the models B differ only in the manner of calculating the hyperbolic tangent function.

DERIVED MODELS	REFERENCED MODELS
Models B Given by equations: (4. 1) to (4. 4) and (4. 9).	Nonlinear models *with no* surge tank effects: B1. Elastic water column in the penstock. Hyperbolic tangent calculated by means of the “In-Out” block of Figure 4.3. B2. Elastic water column in the penstock. Hyperbolic tangent calculated by means of the “In-Out” block of Figure 4.4, approximation n=2 B3. Elastic water column in the penstock. Hyperbolic tangent calculated by means of the “In-Out” block of Figure 4.4, approximation n=1. B4. Non-elastic water column in the penstock. Hyperbolic tangent calculated by means of the “In-Out” block of Figure 4.4, approximation n=0.

Table 4.7: Description of the Models B and their references.

In Figure 4.15 are shown the measured electrical power, labelled as P_{mech}(register), and the mechanical power calculated by means of the Models B. The responses of the mechanical power calculated by means of the Models B1, B2, B3 and B4 are almost the same, and that is the reason that the measured electric power is labelled as P_{mech}(Models B).

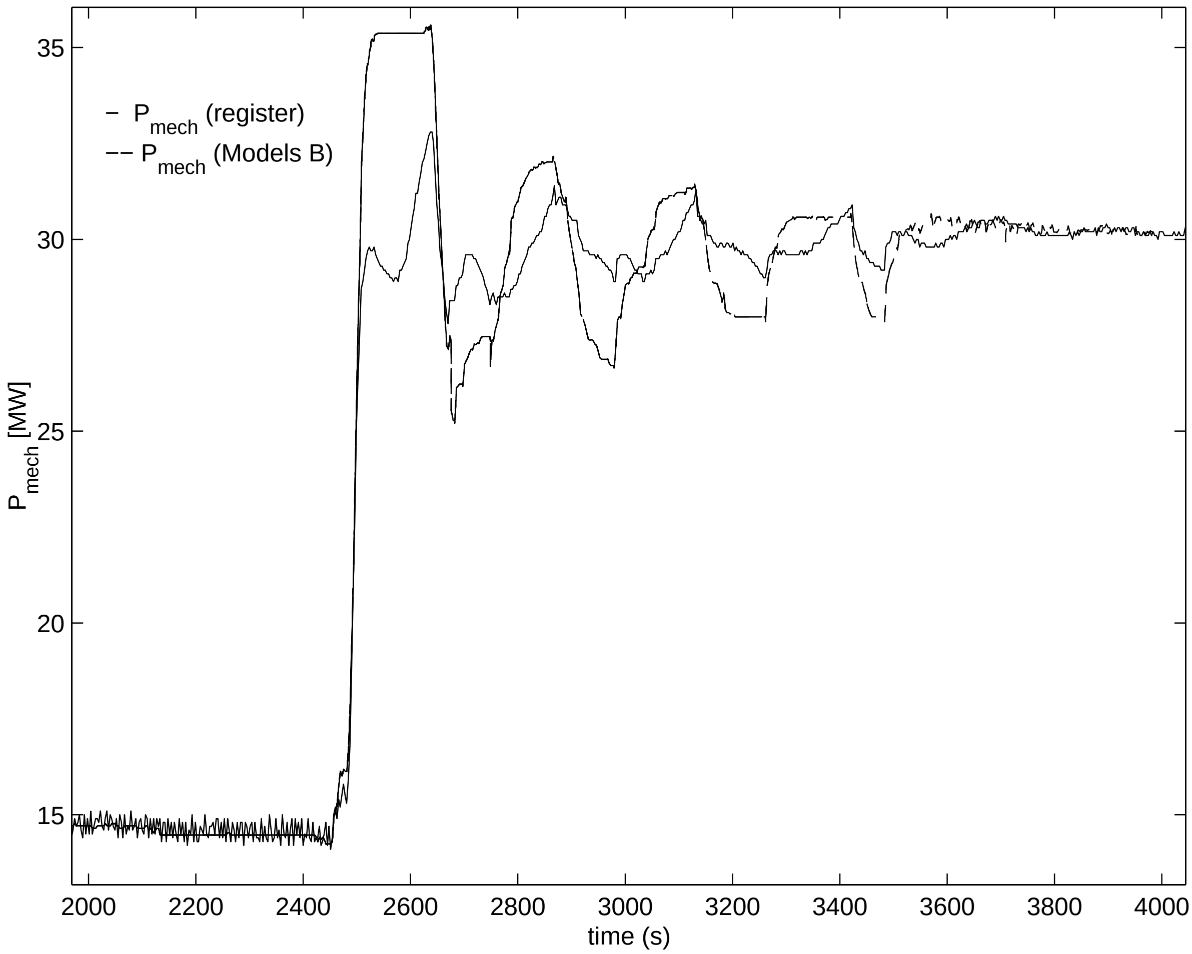


Figure 4.15: Identification of Susqueda using the Models B.

Figure 4.15 shows that using the models B a not adequate identification of Susqueda hydropower plant is obtained. The reason is that only the static gain of the mechanical power is adjustable, since these four models have not got a surge tank so the natural period T is not adjustable.

4.3.3 Identification Using the Model C

This point describes the identification of the hydro plant of Susqueda using the model C. This model is described in Table 4.8. Figures 4.16 and 4.17 show a not good identification of the hydro power plant of Susqueda: the surge tank natural period (T) is adjusted, but the static gain is not adjusted.

<table>
<tr>
<td>Model C</td>
<td>C. <u>Linearized</u> model with surge tank effects, <u>elastic water column in the penstock</u> and <u>non-elastic water column in the tunnel</u>

This model is based on the equation (3.41), which is:

$$\frac{\Delta\overline{P}_m}{\Delta\overline{G}} = \frac{1-\Phi_p - z_p \cdot T_{ep} \cdot s + \frac{G(s)}{z_p} \cdot T_{ep} \cdot s - G(s)}{1+0.5\cdot\Phi_p + 0.5\cdot z_p \cdot T_{ep} \cdot s + 0.5\cdot G(s) + \frac{G(s)}{z_p}\cdot T_{ep}\cdot s}$$

This transfer function must be adjusted by multiplying by the nonlinear function $\eta(\overline{G})$; hence,

$$\overline{P}_{mech} = \frac{1-\Phi_p - z_p \cdot T_{ep} \cdot s + \frac{G(s)}{z_p} \cdot T_{ep} \cdot s - G(s)}{1+0.5\cdot\Phi_p + 0.5\cdot z_p \cdot T_{ep} \cdot s + 0.5\cdot G(s) + \frac{G(s)}{z_p}\cdot T_{ep}\cdot s} \cdot \eta(\overline{G}) \cdot \Delta\overline{G}$$</td>
</tr>
</table>

Table 4.8: Description of the Model C and its references.

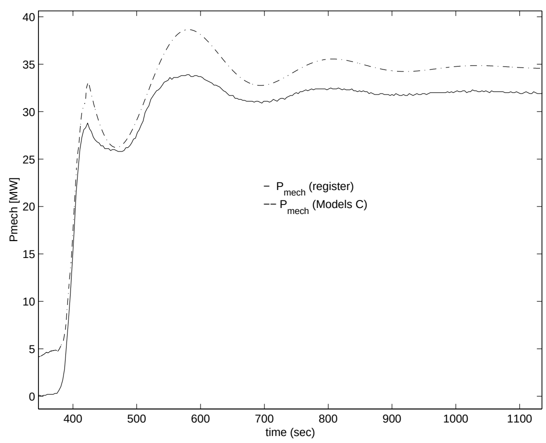


Figure 4.16: Identification of Susqueda using the Model C.

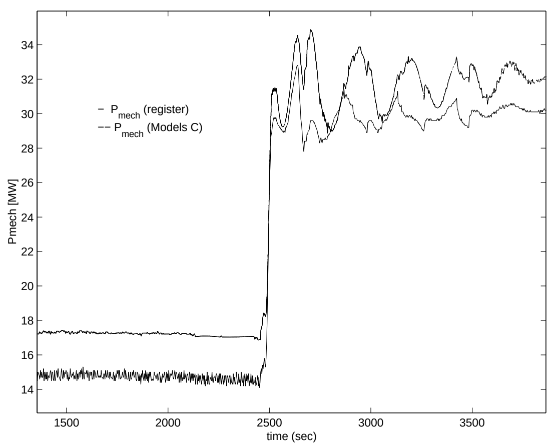


Figure 4.17: Identification of Susqueda using the Model C.

4.3.4 Identification Using the Model D

This is the last model proposed to identify the hydropower plant of Susqueda. Table 4.9 describes the characteristics of the model D.

Model D	D. Linear model *with no* surge tank effects and non-elastic water column. This model is based on the equation (3.43), which is: $$\frac{\Delta\overline{P}_m}{\Delta\overline{G}} = \frac{1 - z_p \cdot T_{ep} \cdot s}{1 + 0.5 \cdot z_p \cdot T_{ep} \cdot s} = \frac{1 - T_{Wp} \cdot s}{1 + 0.5 \cdot T_{Wp} \cdot s}$$ This transfer function must be adjusted by multiplying by the nonlinear function $\eta(\overline{G})$; hence, $$\frac{\Delta\overline{P}_m}{\Delta\overline{G}} = \frac{1 - T_{Wp} \cdot s}{1 + 0.5 \cdot T_{Wp} \cdot s} \cdot \eta(\overline{G}) \cdot \Delta\overline{G}$$

Table 4.9: Description of the Model D and its references.

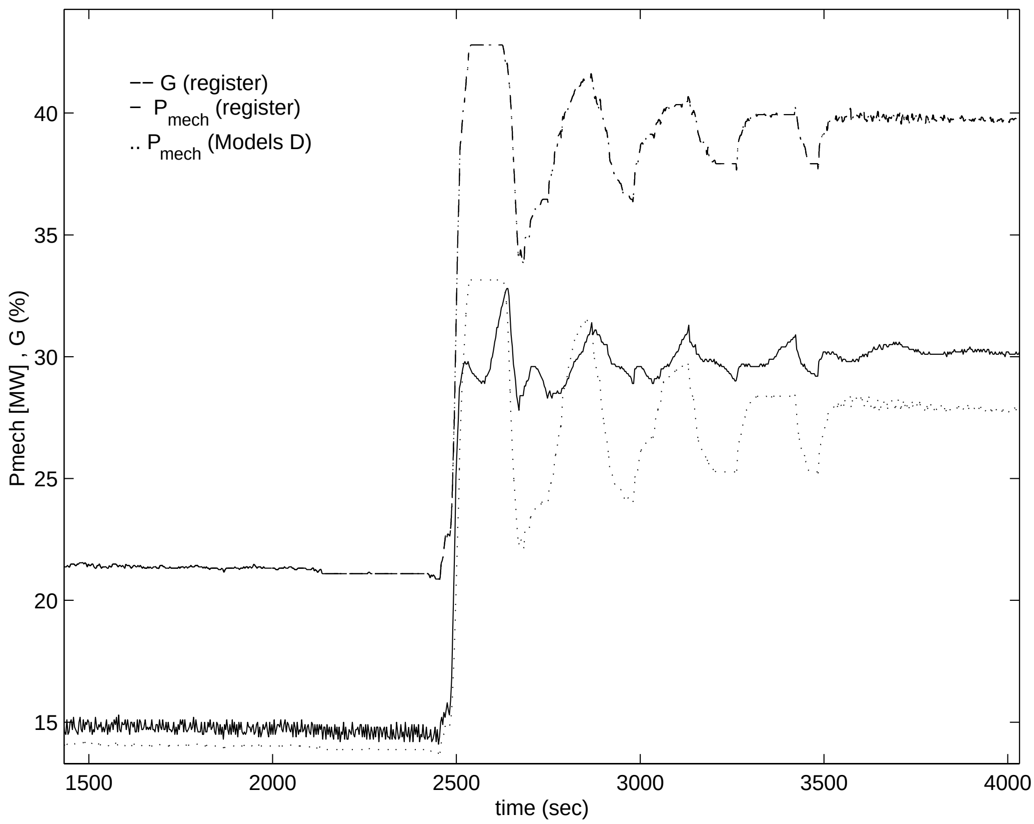


Figure 4.18: Identification of Susqueda using the Model D.

Figure 4.18 depicts a poor identification for the plant of Susqueda. This case does not give an appropriate adjustment of the static gain of the mechanical power nor of the surge tank natural period (T).

4.4 Comparative Study of the Quadratic Error

This section proposes a comparative study of the responses of the models A, which have the best identification of the Susqueda hydropower plant. A quadratic error index is defined by means of the following expression

$$\text{quadratic error} = \int_0^{\infty} \left(\overline{P}_{mech(Register)} - \overline{P}_{mech(ModelAx)}\right)^2 \cdot dt$$

where Ax represents the models A1, A2, A3 or A4.

The registers chosen to calculate the quadratic error are depicted in Figure 4.19 and 4.20, where the hydraulic system reaches the steady state value after a variation in the gate opening. Table 4.10 describes the quadratic error of the models A for polynomials with degree 2 and 10, which represent the nonlinear function $\eta(\overline{G})$.

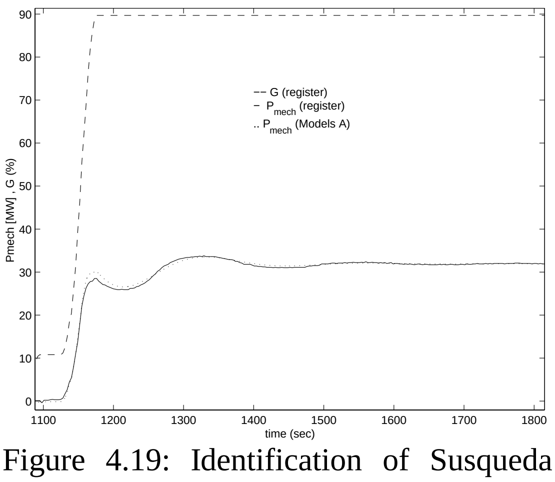


Figure 4.19: Identification of Susqueda using the Models A.

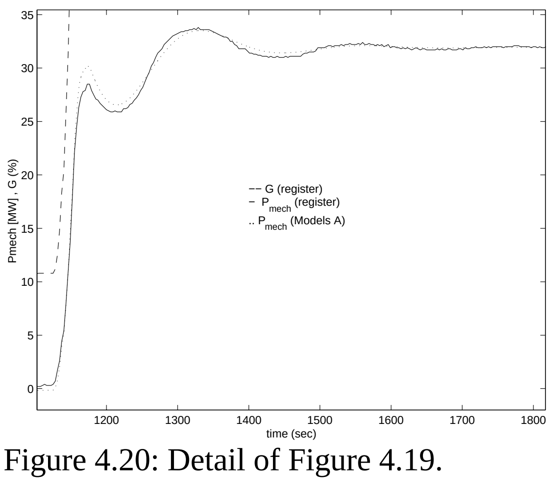


Figure 4.20: Detail of Figure 4.19.

MODELS	QUADRATIC ERROR: for the polynomial of $\eta(\overline{G})$ (n = 5)	QUADRATIC ERROR: for the polynomial of $\eta(\overline{G})$ (n = 2)
A1	0.1530	0.1554
A2	0.3830	0.3878
A3	0.3832	0.3885
A4	0.3835	0.3891

Table 4.10: Quadratic error found using the models A, where the nonlinear function $\eta(\overline{G})$ is approximated by two polynomials with degree 2 and 5.

4.5 Summary and Conclusions

This chapter has presented an identification process of the hydroelectric power station of Susqueda using four groups of hydraulic models.

The main identification process takes into account models that correspond to the group of the models A, which are based on models WG5, QR52, QR51 and WG4. In order to identify the hydropower plant of Susqueda, the models A have been adjusted.

The identification follows two adjustment procedures. The first one tunes the static gain of the Susqueda power plant by taking into account a nonlinear function $\eta(\overline{G})$ in the calculation of the mechanical power. The shape of this function is similar to the efficiency curve of a Francis turbine.

The second one adjusts the oscillation time period (T), caused by the surge tank, by a five per cent modification of the measured value of the surge tank diameter.

The comparative study between the real records and the response of the model shows that the difference in the value of the quadratic error is between 0.15 to 0.39 for the Models A, giving quite a good representation of the real plant. Models A2 (based on QR52), A3 (based on QR51) and A4 (based on WG4) have a quadratic error value that is a sixty per cent greater than the value of the quadratic error of the model A1 (based on WG5), while the differences among the models A2, A3 and A4 are around 0.1 per cent.

Apart from this, models B (based on models WG3, QR32, QR31 and WG2), C (based on Q_{lin0}) and D (based on G_{lin0}) give a bad identification for the Susqueda power plant. This is due to the fact that they are not able to reproduce important phenomena such as the pressure waves in the surge tank or the static gain of the power plant when a nonlinear function $\eta(\overline{G})$ is considered in the calculation of the mechanical power.

Model A1 has the least quadratic error between 0.1530 (for n=5) and 0.1554 (for n=2) and thus the best approximation to the real power plant, although the models A2, A3 and A4 are also good approximations.

While, on one hand the models A1, A2 and A3 need more complex expressions to calculate the hyperbolic tangent. On the other hand, the model A4 is the easiest to simulate since it considers non-elastic water columns and the hyperbolic tangent function is represented by means of a simple derivative function. Furthermore, the model A4 may be expressed as a nonlinear system in the state space and thus may be used in the nonlinear controller design.