

# 1D-SPACE DYNAMIC MATHEMATICAL MODEL OF THE COMPOSTING PROCESS

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## INTRODUCTION

Mathematical modelling is a helpful tool to analyse complex systems, but its development and application to the composting process has been rather scarce up to date. The use of mathematical models is common in complex biological processes such as the activated sludge wastewater treatment and the anaerobic digestion process, both systems operating on a single isothermal phase. Composting occurs in a porous media where the three phases, solid, liquid and gaseous, are present and interact between them in a non-isothermal state. A global methodology for the presentation of the biological and physical phenomena involved is not yet fully developed. A first step in that direction was the model of a completely mixed composting reactor (Solé-Mauri *et al.*, 2007).

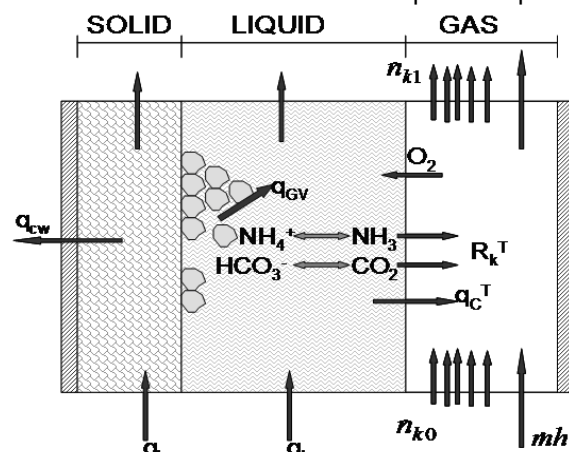
When trying to expand the model to space all state variables become space-distributed and all physical transport phenomena originated by mass and temperature gradients as well as bulk gas flow in the porous media and mass settlement must be considered. Although there are some published models of the composting process, few consider the mass and energy transport phenomena in detail, not allowing an explanation of many of the observed characteristics of the composting process operation.

The present work presents the developed methodology to build up a deterministic mathematical model of the composting process, based on the basic laws of physical and biological elementary processes, with the objectives to simulate and to explain the behaviour of one-dimension (1D) column composting reactor, and to have a framework for further mathematical modelling developments.

## MODEL DEVELOPMENT

A composting media is considered, in a general approach, as the superposition of three vector fields: mass settlement, leachate movement and gas flow. In a first step, only the gas flow field was considered, although the settlement field was characterized (Illa *et al.*, 2012), and could be included in a further development step.

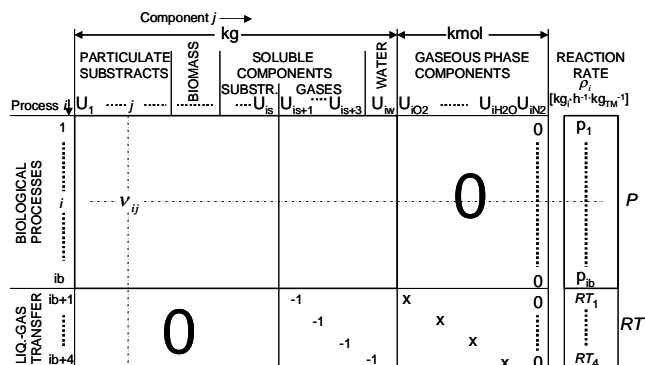
Gas is considered an ideal mixture of O<sub>2</sub>, CO<sub>2</sub>, NH<sub>3</sub>, H<sub>2</sub>O and N<sub>2</sub>, which is flowing through the inter particle porous space. Liquid-gas mass transfer of O<sub>2</sub>, CO<sub>2</sub>, NH<sub>3</sub> and water evaporation is caused by the difference of partial pressure and saturation pressure. Generated heat by microbial activity is transferred to the surroundings and to the gas phase as sensible and latent heat (Fig. 1). As shown in Fig. 1, the mass transfer processes are located at the liquid-gas interphase ( $R_k^T$ ) and at the gas phase ( $n_{k0}$ ,  $n_{k1}$ ). The terms of energy transfer are: generation by biochemical reactions ( $q_{GV}$ ), dissipation at the reactor bounds ( $q_{cw}$ ), conductive dissipation ( $q_k$ ), convective dissipation ( $q_c^T$ ) and convective transfer ( $mh$ ).



**Fig. 1. The three phases in an elemental volume unit. Arrows indicate main fluxes direction.**

The state variables considered in an elemental volume unit represent either a mass quantity of a biological or chemical component or a phase temperature. A different temperature is assumed for the gas and the solid-liquid phases, which allows establish the energy transfer equations between them. The governing set of ordinary differential equations is obtained from the mass balance for every component of the system and from the energy balances in the solid-liquid and gas phases. The subset of differential equations representing the components mass balance can be expressed in matrix form.

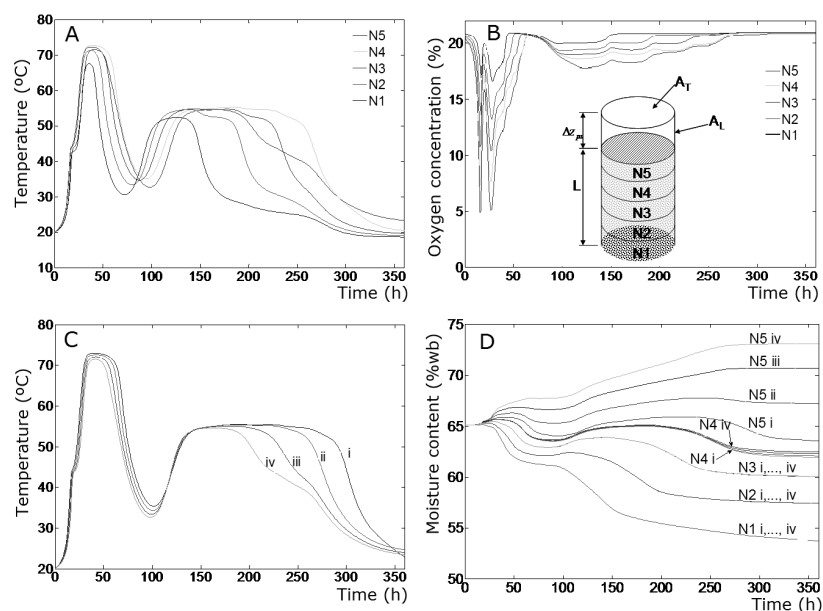
The time derivatives vector of the mass-related state variables is the sum of the net mass flow rates vector and the reaction terms vector, which can be obtained as the product of the Petersen matrix and the reaction rates vector (Fig. 2). A modified version of the biological model of Solé-Mauri *et al.* (2007) was adopted, including 22 state variables and 13 biological processes in each elemental volume. Given an initial state, this set of differential equations governs the time evolution of the system.



**Fig. 2.** Structure of the Petersen matrix, the vector of biological reaction rates (P) and the vector of liquid-gas transfer rates (RT), coupled into the reaction rates vector  $p$ .

## SOME RESULTS OF THE NUMERICAL SIMULATION

Fig. 3 shows the results of the numerical simulation of a column composting reactor, discretized into 5 layers, with the physical, biochemical and operating parameters described by Illa (2012).



**Fig. 3.** A) Temperature evolution at 5 different layers (N1, ..., N5); B) Evolution of the oxygen concentration in the gas phase at each layer. Effect of different top plenum surfaces (i<ii<iii<iv) on: C) The temperature of the upper layer (N5); D) Moisture content of the upper layer (N5).

Considering different top plenum surfaces ( $A_T$  and  $A_L$ ) at ambient air temperature, and not insulated (surfaces area  $i < ii < iii < iv$ ), the different vapour condensation levels affect the temperature and moisture content of the upper layer (N5), as shown in Fig. 3C and D, reproducing the phenomenon found in industrial plants. Other phenomena that have been satisfactorily simulated by the 1D model are the temperature decrease due to water evaporation during the composting process, the evolution of the overpressure at the bottom plenum when operating at constant inflow rate and the evolution of the process when operating at natural convection air flow.

## CONCLUSIONS

The developed model simulates satisfactorily the main trends in the evolution of the observed process variables, integrates the different phenomena present in the composting process and allows the quantification of its relative importance. The open structure of the model facilitates the inclusion of new phenomena and biological models from other authors.

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