CASE STUDIES

Finding the Poynting´s Theorem in a Health Centre in San Pablo (Peru)

Ignacio Echeveste Guzmán and Manuel Lambea Olgado

PHOTO: Mobile telephony supports medical consultation in Cusco, Peru. ONGAWA.

GDEE GLOBAL DIMENSION IN ENGINEERING EDUCATION

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Finding the Poynting´s Theorem in a Health Centre in San Pablo (Peru)

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FINDING THE POYNTING´S THEOREM IN A HEALTH CENTRE IN SAN PABLO (PERU)

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1. Introduction

This case study is based on the Willay Program implemented by ONGAWA in different regions of Peru. Willay, which means “to inform” in Quechua, proposes the use of Information and Communication Technologies (ICTs) in rural areas for democratic governance and citizen participation. The telecommunication infrastructure is based on WiFi for Long-Distance (WILD) technology that offers internet access and Internet Protocol (IP) telephony.

In this case study, we will use the San Pablo I Project to find the electromagnetism Poynting’s theorem [Ramo, p. 137] in different places of the local network in the village of Tumbaden. This local network has been chosen because there is no connection with conventional electric network in this area, and photovoltaic solar energy is needed to feed the infrastructure.

1.1. Disciplines covered

This case study covers subjects based on electromagnetic fields theory (electrodynamics) and on photovoltaic solar energy which is applied to understand the design of the feeding system of the isolated health centre in Tumbaden.

1.2. Learning outcomes

- The student will know how to apply Poynting´s theorem in several practical situations.
- The student will know how to design a photovoltaic solar energy system.
- The student will understand Leontovich approximation [Nikolski, p. 237], and will obtain practical results in different ways to verify that they have the same physical bases.
- The student will realize how important is to take into account the beneficiaries’ opinion in a cooperation project.

1.3. Activities

The class activity: In the first hour, a general brief explanation of the Willay Project in San Pablo will be given. The feeding system of the isolated Tumbaden health centre with solar photovoltaic energy is then presented, including a group discussion to decide the requirements for this centre and to compare it with a typical health centre in the students’ country.

In the second hour, the first Poynting problem is proposed to the students (**Problem 1**: The sunlight incising on the solar panel). They will try to solve it, through group work, and finally, every student will deliver the solution individually to the teacher. Lastly, the teacher solves the proposed problem and briefly presents the homework activity.

The homework activity consists of five problems to be delivered individually by the students, although some group work is allowed:

Problem 2 is to design the solar photovoltaic feeding system of the health centre in Tumbaden (the dimensioning of the solar panels and the battery).

Problem 3 is to apply Poynting´s theorem to a part of the feeding system of the health centre in Tumbaden.

Problem 4 is to apply Poynting´s theorem to the radio communication system between the secondary school and the health centre.

Problem 5 is to apply Poynting´s theorem and the Leontovich approximation to the coaxial cable that connects the antenna with the receptor in the health centre.

Problem 6 is to discover the real physical characteristics of the coaxial cable used by means of the transmission lines theory and to verify the attenuation obtained in Problem 5 using Poynting´s theorem.

2. Description of the context

2.1. Peru at a glance

Peru is the third largest country in Latin America after Brazil and Argentina, with an area of 1,285,216 km2 (2.5 times the area of Spain). It is the fifth most populous country in Latin America after Brazil, Mexico, Colombia and Argentina, with a population density of 23.7 inhabitants / Km ², four times lower than Spain. In Peru socially deep inequalities persist. There is a large contrast in Human Development Index (HDI) scores between the capital and the provinces and between urban and rural areas. Although the country has experienced steady economic growth in recent years, there are still major challenges related to social inclusion and gender equality, for example. Many social conflicts, uprisings and protests from people living in the interior of Peru have taken place as a result of a lack of economic investment in this area, despite the economic boom. There are severe limitations on access to good quality basic services such as education, health, water, housing and electricity; as well as poor promotion of economic opportunity and progress for much of the rural population.

Administratively, Peru is divided into 25 regions, 194 provinces and 1624 districts. The elections of regional and local (provincial and district) authorities are held every five years. The complex and rugged geography and the implementation of population concentration policies have created an unequal and asymmetric occupation of the country. This makes it difficult to overcome the various spatial dimensions of development, promote social cohesion and ensure state presence. In addition, expensive transport and communications infrastructure is required to ensure connectivity.

The country has been experiencing major demographic transition since the mid-1960s. A population explosion has been coupled with increasing migration to the big cities, in particular Lima. It is estimated that the population of Peru in 2014 was 30,814,175 inhabitants, with an annual average growth rate of 1.11%. There is a high concentration of the population in urban areas (73%), especially in Lima, where more than a third of the total population lives. The World Bank report "Peru 2012" stated that 53% of the rural population lives below the national rural poverty line. Peru is characterised by a Human Development Index (HDI) of 0.741 according to data in 2013, which places it in the group of countries with high HDI, ranking 77 of 185, below Cuba, and above Turkey and Brazil. The Adjusted HDI (IDHI), which reflects disparities between the population in income, health and education, is 0.561, 24.3% less than the corresponding HDI.

According to the International Monetary Fund (IMF) in 2013 Peru is considered a middle-income country with a GDP per capita of € 8,132 per inhabitant (compared to € 25,222 per inhabitant in the European Union). Economic reforms during the 1990s were key to an impressive improvement of the Peruvian economy. Important macroeconomic developments and the liberalization of the telecommunications market favoured private investment. During the 1990s the evolution of investment in utility infrastructure, especially telecommunications and energy, mainly benefited households and businesses in urban areas, neglecting investment in rural infrastructure.

2.2. TELECOMMUNICATION SECTOR

Mobile telephony coverage in Peru has had a high annual growth rate, which stood at 82% in 2013, compared with 28.6% in fixed telephony. In comparison, mobile telephony penetration in Europe was 128% in 2013. Internet access of urban households in Peru was 20% in 2013 compared with just 0.9% of households in rural areas. Across Europe in the same period, 73% of households were connected to the Internet. 36% of urban households in Peru had a computer in 2013, compared with 5.8% of households in rural areas in Peru and 77% of households in Europe.

Table 1 summarizes the access to telecommunications services in rural and urban areas.

Table 1: *Population living in areas with telecommunications services coverage*

SERVICE	URBAN	RURAL
Fixed telephony	86%	0%
Mobile + Fixed wireless telephony	92%	53%
Fixed broadband access to the Internet (ADSL)	82%	0%
Mobile access to Internet 2.5G (EDGE)	92%	48%
Mobile broadband access to Internet (UMTS)	56%	3%
Cable TV	67%	0%
Satellite TV	100%	100%
Public telephony	94%	56%

The use of ICT services was measured by the Peruvian National Institute for Statistics and Information (INEI) in the census of poverty levels published on 2012, with the results shown in Figure 1. This shows how both poverty and lack of telecommunications services coincide in rural areas.

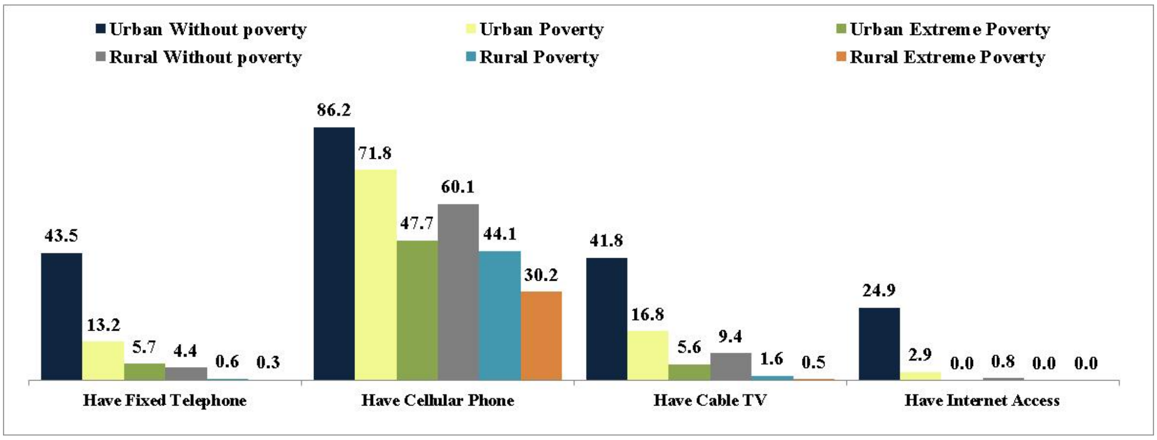


Figure 1: Peruvian *households with ICT access by poverty level and area (Source: INEI 2011)*

2.3. GOVERNANCE IN PERU

In Peru, between 2002 and 2009 the government prioritized the improvement of good governance by putting several laws, regulations and national plans into action. These plans determined and developed the principles of citizen participation, transparency, and accountability of local governments. The state recognized the importance of using ICTs in enhancing organizational management and performance. The National Office of Electronic Government and Information Technology (ONGEI) was established, along with several plans for e-government deployment in central and local public administrations. E-government tools were introduced to and incorporated within the priorities of local public entities.

2.4. HEALTH

The United Nations (UN) recognizes health as one of the key elements of human development, along with education, minimum level of income and the ability to participate in political and social life of the community. The health status of the population is also a factor that affects development.

Poor health reduces work capacity and productivity of people and affects the physical development, schooling and learning of children. There is a link between the improvement in nutrition and health with the increase in productivity and school performance. In relative terms, the economic and education advantages that produce an improvement in health generate greater benefits in the poorest population. This is the reason why health was one of the key issues considered in the Millennium Development Goals (MDGs).

According to the World Health Organization most inequalities in health are due to the conditions in which people are born, live and work, as well as the health system they have access to. That is, access to safe water and adequate sanitation, an adequate supply of safe food, adequate nutrition, adequate housing, healthy working conditions and environment, and adequate social protection. Improving these social determinants of health and reducing inequalities of power, money and resources may help to improve population health.

Often women and men are affected by different social determinates of health, producing gender inequality in access to health. For example, domestic tasks cause women be in contact with contaminated water, fatigue and stress of "double day" of women inside and outside the home, health problems during pregnancy, childbirth and postpartum, etc..

Health is recognized as a Human Right, so governments that have signed international covenants on human rights are obliged to create the conditions that allow all people to live as healthily as possible, including the social determinants of health. The Right to Health is not to be understood as the right to be healthy. Rather, international regulations on the Right to Health require governments to provide access to health care with quality care, non-discrimination and economic conditions that do not prevent access of the poor.

2.5. WILLAY PROGRAM

The Willay program is implemented in two distinct regions; San Pablo in Cajamarca and Acomayo in Cuzco, together having a combined population of 50,000 people. The majority of the population belong to indigenous communities whose main economic activity is farming (84% of the active population).

In Acomayo, 46% of the population does not have access to electricity, 23% do not have access to running water, and 62% do not have access to appropriate sanitation. In terms of HDI, Acomayo is ranked ninth out of the thirteen provinces located in the department of Cuzco, with medium-low HDI similar to that in Sudan. Life expectancy is 63 years, 91% of children between 5 and 18 are in school and the illiteracy rate among women is 42%.

Government implementation of national initiatives related to the use of ICT, which are designed based on a developed urban perspective generated unexpected results in these communities because of the lack of connectivity, capacity for management, and technology at the local level. Since there were neither good connections nor qualified technical staff in rural areas, the rural municipalities opted to establish offices in the respective districts' capitals. These satellite offices added to the municipalities' costs and complicated the human resources management process. There was limited knowledge regarding regulations on adequate use of management tools and deficiencies in using an appropriate language with the population in public entities. Regarding civil society organizations, they had organizational weaknesses; were unaware of their democratic governance rights and experienced limitations in leadership building. Spaces for consensus existed although they were not properly utilized due to a lack of satisfaction on the citizens' side.

The Willay program, meaning “to inform” in Quechua, proposes the use of ICTs in rural areas for democratic governance and citizen participation. The project explores how ICTs could enhance the processes of transparency, citizen participation and the accountability and effectiveness of local governments. This is achieved by building capacities of the stakeholders involved (civil society organizations and public entities like local government, health centres and schools).

In total, 44 local government institutions have been provided with a telecommunication infrastructure shared between them, based on WiFi for Long-Distance (WiLD) technology that offers Internet access and IP telephony. Besides this, it has installed information systems and software and implemented a system of continuous improvement. Public workers and community leaders have also been trained in participatory budgeting, accountability and transparency of institutions public, citizen surveillance, education management and health management.

3. Class activity

The class activity consists of:

In the first hour:

A general brief explanation (15 minutes) of the context outlined before and the presentation of the Willay Project in San Pablo [Araujo]:

This project was developed in Cajamarca department, San Pablo province, in five different districts: San Pablo, San Bernardino, San Luis, Tumbaden and Rodeo Pampa. The project offers internet access and IP telephony in municipalities, health centres and schools in several villages located within the different districts. The telecommunication infrastructure is based on WiFi for Long-Distance (WILD) technology.

The telecommunication network is ordered by trunk and local networks as shown in Figure 2.

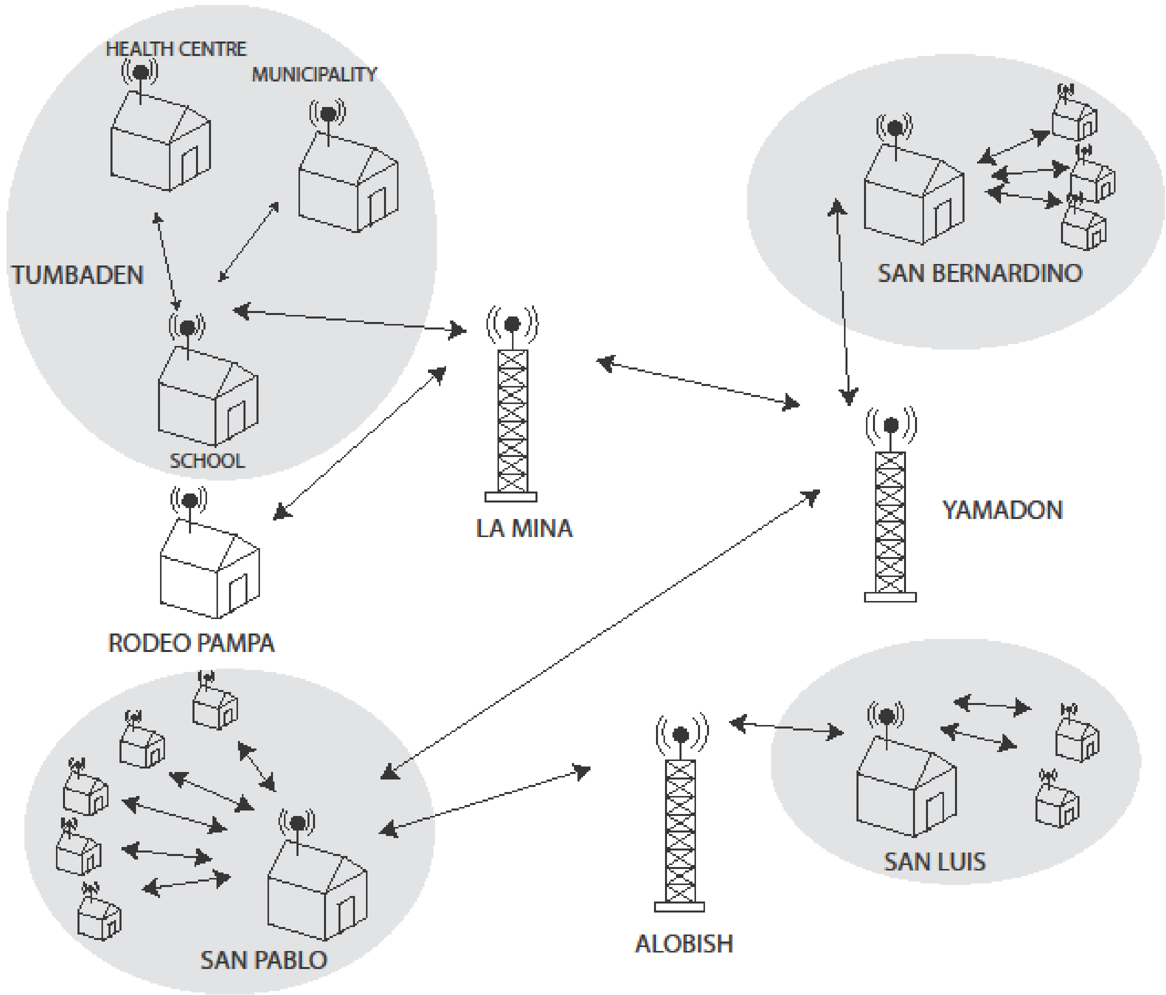


Figure 2: *Willay Network in Cajamarca-San Pablo (Source: [Araujo])*

Trunk networks are composed of point-to-point links between the different local boosters and the trunk boosters (Yamadon, La Mina and Alobish). This allows the villages (San Bernardino, San Luis, Tumbaden and Rodeo Pampa) to have Internet and telephone connection to San Pablo and vice versa.. San Pablo is the only place where conventional telephony and internet access is possible with the company “Telefónica del Perú”.

Local networks are composed of point-multipoint links between the local booster and its clients (usually the municipality, the health centre and the school). The basic operation of the local network is: the local booster receives the trunk network signal (using a directive antenna) and distributes it to its clients (using an omnidirectional antenna) and vice versa. The transmission frequency used is 5.8 GHz.

In this case study, the local network in the village of Tumbaden is used to find the electromagnetism Poynting theorem at different places. This local network connects the health centre and the municipality with the secondary school, where the local booster that connects with the rest of the trunk network is placed. This local network has been chosen because there is no connection via a conventional electric network in this area, and photovoltaic solar energy is needed to feed the infrastructure.

Some services that the Willay-Cajamarca network can provide are:

- Tele-education: on-line courses, tutorials, libraries access …
- Tele-medicine: Tele-assistance, transmission of information as medical images or digitalized electrocardiograms …
- Connection with other institutional offices in different districts, audio conferences, applications as Skype …

There will be a group discussion of 25 minutes to decide the requirements of the health centre in Tumbaden, and compare it with the typical health centre in the students' country. The idea is to think about people's needs, taking into account different genders and the different limitations that the students can imagine.

Similar group work provided the basis for a participative design meeting that took place with municipality, education and health authorities of the district, the rural telecommunication group of "Pontificia Universidad Católica" of Peru and ONGAWA members.

A role playing methodology can be used to help students hold their own participative design meeting, assigning roles to different student groups which have expressed previous views regarding the most important problem to be solved in the region… As an example:

- The municipality members think the most important thing is to have broadband Internet in the Town Hall.
- The education members believe that an antenna in the school will be dangerous for student safety.
- The local health authorities think that it is better to use the money to have a new drainage system.
- The University group shows the advantages of similar projects carried out in other Peruvian regions.
- The ONGAWA members try to give a voice to a local NGO member that was not invited to the meeting…

The lecturer moderates the meeting (no more than 15 minutes) and encourages the students to meet a common agreement. If this is difficult, it will emphasise that making decisions in light of these social problems is not easy, given the different points of view of participants. Furthermore, it highlights that it is essential that range of different people involved participate in the meeting to ensure the project is accepted by the beneficiaries. In the following 10 minutes, the students will compare the situation in their own country with this case.

In the last 20 minutes, the real Tumbaden health centre is presented, focusing on the feeding system with solar photovoltaic energy to enable students to understand how it works:

In Figure 3, the connection scheme of Tumbaden´s health centre is shown.

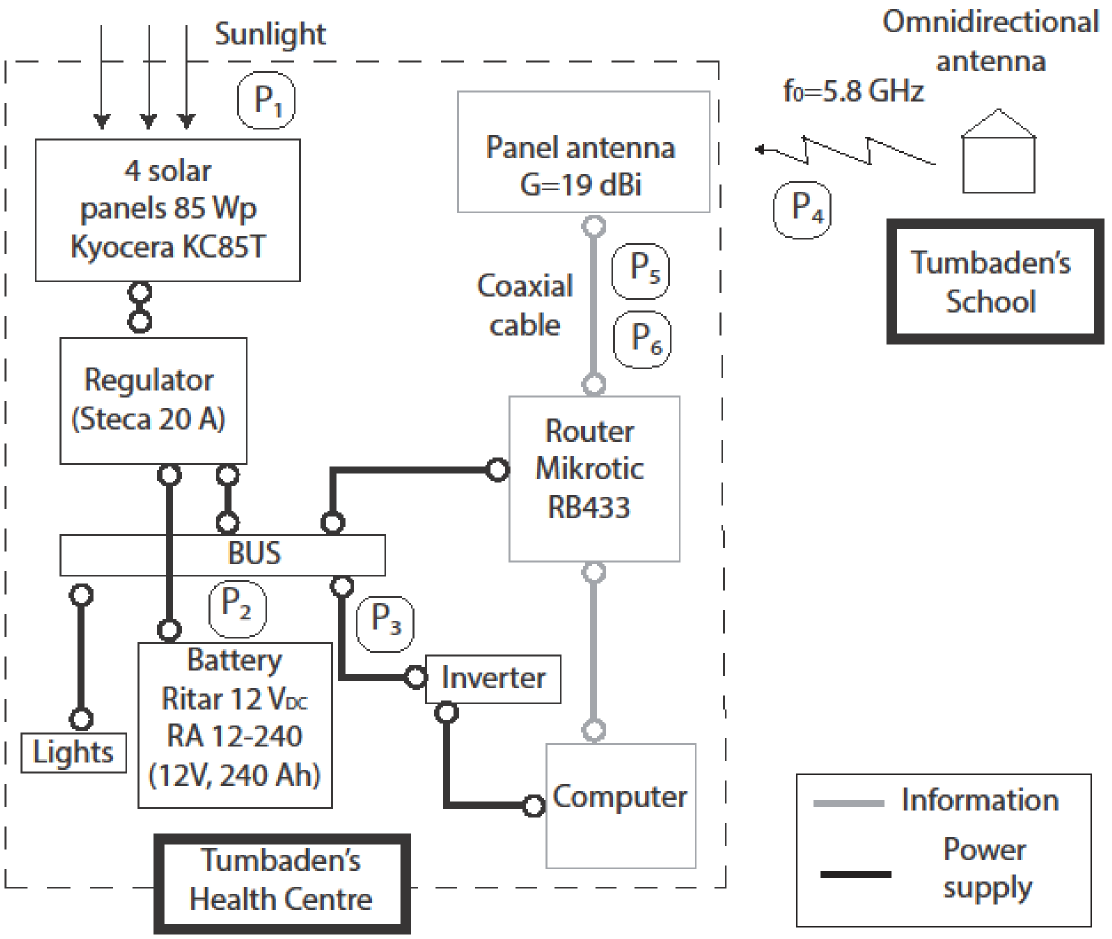


Figure 3: *Tumbaden´s health centre connection scheme (Source: [Araujo]). Pi represents the location of the different problems to solve in this case study.*

This station is a client station of Tumbaden´s local network, where the school is the local booster that connects with the rest of the trunk network.

The photovoltaic system of Tumbaden's Health Centre is designed in order to provide power to one router [Mikrotic] 24 hours a day, one computer 3 hours a day and two lights also 3 hours a day. The system has been designed to have two days of autonomy.

The photovoltaic system has four solar panels of 85 Wp [Kyocera], a 12 V battery of 240 Ah [RITAR] and a 20 Amp regulator [STECA] to feed the router, the computer and the two lights.

So, the sunlight falls on the solar panels and the incoming energy is converted to DC energy that goes to the regulator. The four panels in parallel can be considered as a DC current generator, and in first approximation, the regulator behaves like two switches that are open or closed depending on the work cycle (Figure 4).

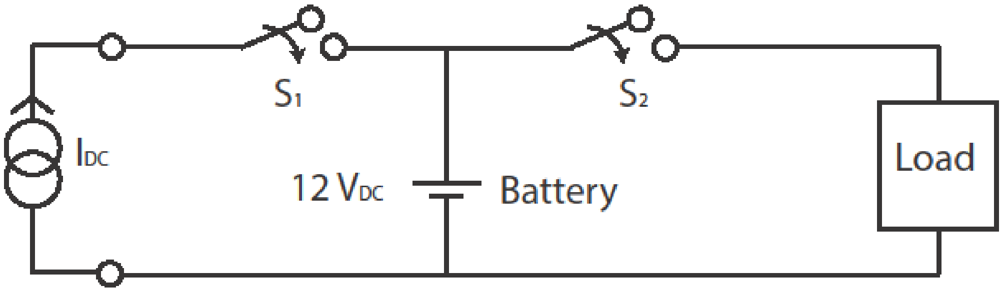


Figure 4: *Electric scheme of the Health Centre feeding system.*

During the day, switch 1 is closed and the battery is charging. During the night, switch 1 is opened because the panels are not working. When a particular load has to be fed, switch 2 will be closed and will be opened if the load stops working. The lights and the router are fed at 12 V_{DC}, but the computer needs AC: 60 Hz and 220 V_{eff}, so an inverter is placed before the computer to convert the DC input to the AC output.

The telecommunication signal is received from the Tumbaden´s school ommidirectional antenna [HyperLink, model HG5812U-PRO] to the panel antenna [HyperLink, model HG5819P] of the health centre, that works in the 5.8 GHz band and has a gain of 19 dBi.

A coaxial cable [CA-400 LMR-400], a low loss cable of characteristic impedance 50 Ω, connects the antenna with the router that controls the communication processes. This router is connected with the computer and also with a telephone that is not included in the schemes to simplify the figures.

The location of the problems to be solved in this case study is indicated in Figure 3:
Problems 1, 2 and 3 are related to the solar photovoltaic feeding system, and
Problems 4, 5 and 6 are related to the telecommunication system.

In the second hour of class activity:

Problem 1 is presented to the students (The sunlight falling on the solar panel). This problem is inspired in a classical problem included in Krauss [p. 421], and is a good first approximation to Poynting´s theorem with mean values, but not in the monochromatic case. For 25 minutes there should be work in groups (of 2 or 3 students) to solve the problem. Afterwards, students have 15 minutes of individual work to write up and outline the solution. In the last 20 minutes, the teacher presents the solution and, finally, briefly presents the homework activity, specifying the lesson containing the theory for every problem to be done.

Problem 1: *Sunlight is composed by a wide range of frequencies, as infrared, visible or ultraviolet. Not taking into account* Mercury, *Venus and the Moon, free space can be considered between the Sun and the Earth.*

The average power density of solar radiation arriving at the top of the Earth's atmosphere (solar irradiance) is 1367 W/m^2. The distance between the Earth and the Sun is approximately of 150 million kilometers.

a) *Applying Poynting's theorem (time-average and integral form) obtain the total radiated power of the Sun assuming that it radiates uniformly in all directions. Draw the coordinate system that has been selected and express the mean value of the Poynting vector in such a coordinate system.*
b) *The energy received by the Earth's surface is 360 Joules (ifor every cm^2 per hour and considering optimal conditions: summer, noon and cloudless day). What is the mean value of the Poynting vector at the Earth's surface in W/m^2? Use the same coordinate system as in the previous question. Note: The difference between the density power in the top of the atmosphere and iat the Earth's surface, which is due to reflection, absorption and scattering of the solar radiation in the atmosphere by different physical phenomena.*
c) *The power density obtained in b) is used in photovoltaic designs when standard conditions are assumed. Supposing the same conditions, a solar panel (of dimensions 1 m x 0.652 m) is placed in the health centre of Tumbaden, such that sunlight radiation falls with normal direction. From the datasheet of the panel, it is known that the panel provides a voltage of 17.4 V and a current of 5.02 A in standard conditions. What is the panel efficiency? (Efficiency is defined as the ratio of the output and input power.)*
d) *If there is an error of 30° between the panel and the sunlight radiation, what would the output power of the panel be?*

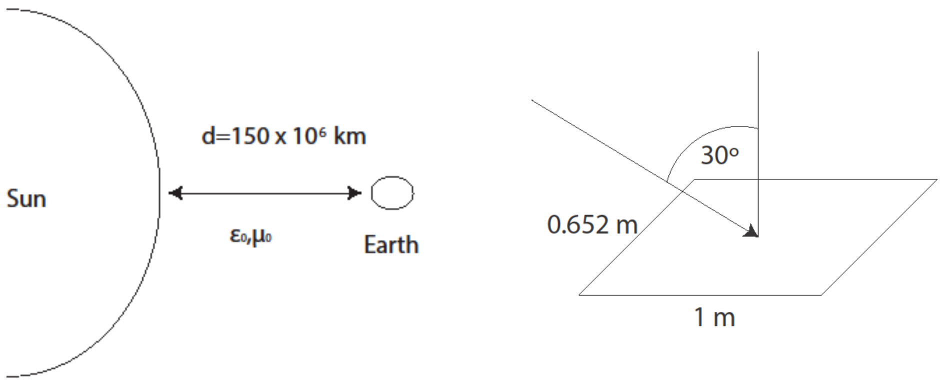


Figure 5: *Left: Scheme of the Sun – Earth distance and placement. Right: Dimensions and representation of the panel and the solar radiation for d).*

3.1. SOLUTION AND EVALUATION CRITERIA

The evaluation criteria (total of10 points) could be:
a) Giving one point to the students that have been active in the group discussion.
b) The evaluation of Problem 1 (9 points).

The detailed solution of Problem 1 is in Annex 1.

Solution of Problem 1:

a) [3 p.]: $Radiated\ power = 3.86 \times 10^{26}\ W$ with the origin of coordinates in the Sun.

$$< \bar{S} >|_{\substack{out.\\atm.}} = 1367\,\hat{r}\ \ (W/m^2)$$

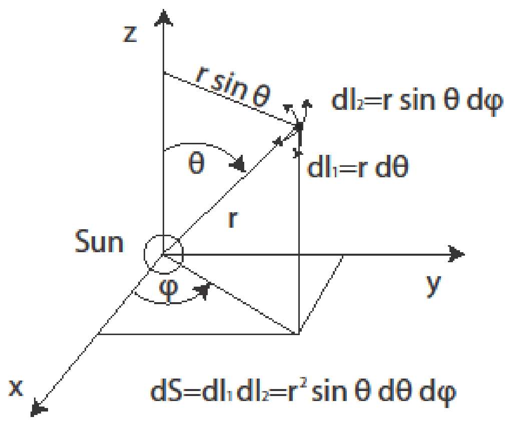


Figure 6: *Spherical coordinate system with the origin of coordinates in the Sun.*

b) [2 p.]: $< \bar{S} >= |< \bar{S} >|\,\hat{r} = 1000\,\hat{r}\ ({}^{W}/_{m^2})$

c) [2 p.]: $\eta = 13{,}4\%$

d) [2 p.]: $P_{Out} = 75.66\,W$

4. HOMEWORK ACTIVITY

The homework activity consists of five problems to be delivered individually by the students, although some group work is allowed:

Problem 2 is to design the solar photovoltaic feeding system of the health centre in Tumbaden (the dimensioning of the solar panels and the battery). If the students have not studied this issue, all the information needed to solve the problem is included in the problem statement [Victoria].

The rest of the problems to be solved (3, 4, 5, 6) cover subjects based on electromagnetic fields theory (electrodynamics). Different topics are included in these problems, so, the idea is that the students will do every problem at the time when they have studied the corresponding theme.

The order of the problems presented in this case study corresponds with the implementation sequence of the subject “Fields and waves in Telecommunication”, which is taught in the second course of “Degree in Engineering Technology and Telecommunication Services” at ETSIT-UPM: The first lesson consists of an introduction to electrodynamics, where Poynting´s theorem in time and frequency domains and the active and reactive power balances in the monochromatic case are studied. Lessons 2 and 3 introduce the homogeneous plane wave and the normal incidence on stratified media. In lesson 4, fields in conducting media are studied and lesson 5 introduces transmission lines.

Problem 3 is to apply Poynting´s theorem to a part of the feeding system of the health centre in Tumbaden and to complete the understanding of the feeding system of the health centre, covered by problems 1 (class activity), 2 and 3. Poynting´s theorem is applied in a very simple case, where there is no radiated power. As a result, it can be seen as the application of the conservation of energy theorem in a simple low frequency circuit

Problem 4 is a typical Poynting problem in the monochromatic case, applied to the communication between the local booster in the school and the health centre in Tumbaden

Problem 5 is to apply Poynting´s theorem and the Leontovich approximation to the coaxial cable that connects the antenna with the receptor in the health centre. It is also a typical Poynting problem: the study of the propagation of an incident field in a coaxial cable, but includes Leontovich approximation that is studied in lesson 4.As a result, this type of Poynting problem cannot be done at the beginning of the semester, and the teacher should propose it at the appropriate moment in the course.

Problem 6 is to discover the real physical characteristics of the used coaxial cable by means of the transmission lines theory and to verify the attenuation obtained in Problem 5 using Poynting´s theorem. It is a typical problem of lesson 5, where transmission line concepts have to be applied to the coaxial cable. To find the attenuation of the coaxial used in Tumbaden´s health centre, the high frequency resistance per unit length of the coaxial has to be used [Ramo, p. 154 or Collin, p. 87]. The attenuation constant obtained in this case can be compared with that calculated in Problem 5, applying Leontovich approximation and Poynting´s theorem. Both expressions agree because in the high frequency resistance method Leontovich approximation is also applied.

Some of these problems are inspired by exam problems of the subjects “Electromagnetic Fields 1” (Study Plan 94) and “Fields and waves in Telecommunication” (Study Plan 2010), some of them are compiled in the problems compilation “Problemas de Campos Electromagnéticos” by Jaime Esteban, Miguel Á. González, Manuel M. Lambea and Jesús M. Rebollar [Departamento de Electromagnetismo y Teoría de Circuitos].

Problem 2: *Design the photovoltaic* system *of Tumbaden’s health centre (the peak power of the panel and the capacity of the batteries) in order to provide power to:*

a) One router (maximum power of 25 W, 24 hours per day).
b) One computer (power consumption of 90 W, 3 hours per day).
c) Two lights (power consumption of 22 W each, 3 hours per day).

with an autonomy of two days.

Two equations are used for dimensioning the panels and batteries of the health centre, the first one obtains the peak power of the panel (P_P) and the second one obtains the nominal capacity of the batteries (C_n):

The peak power of the panel (P_P measured in W) can be obtained as:

$$\mathrm{P_P} = \frac{\mathrm{L}}{\eta \cdot \frac{\mathrm{G_{wm}}}{\mathrm{G}}}$$

where L is the daily power consumption in $\frac{Wh}{day}$, η is the efficiency of the system (usually 0.7) and G_{wm} is the radiation incident over the Earth’s surface in the worst month in $\frac{Wh}{m^2\, day}$, G is the radiation incident over the Earth’s surface in standard conditions (STC):

- *Sunlight radiation of G=1.000 W/m^2*
- *Temperature of 25° C*
- *Average atmosphere AM=1.5*

The batteries’ nominal capacity (C_n measured in Ah) is obtained as:

$$C_n = \frac{L \cdot D}{V \cdot \mu \cdot P_D}$$

where D is the number of days of autonomy desired, V is the voltage of the batteries (usually 12 or 24 V, 12 V in this case because of the selected router: Mikrotic RB433), μ is the efficiency of the regulator (usually 0.8-0.9, in this case 0.9 will be choosen) and $P_D$is the maximum depth of discharge of the batteries (usually 0.8).

On NASA’s website (http://eosweb.larc.nasa.gov/sse), detailed information is provided regarding the radiation incident over the Earth’s surface depending on coordinates, month, tilt of the panel, etc… The tilt is chosen as Latitude + 15 for various advantageous reasons (to avoid the accumulation of dust in the panels for example).

The Tumbaden's health centre is located at the following coordinates:

- *Latitude: South 07° 01' 30.3''*
- *Longitude: West 78° 44' 22.9''*

Select the least favorable conditions to ensure that the system can work in every possible atmospheric situation.

Look for batteries and panels on the internet that satisfy the obtained characteristics. In Tumbaden's health centre, Kyocera Solar Panels and Ritar batteries are used.

Problem 3: *A simplified electric scheme of the photovoltaic feeding system in Tumbaden health centre is shown in Figure 7. In this scheme, the current sources are the panels and the loads are every electronic device that has to be fed.*

Four panels are connected so that they can be considered as generating a current of 5.02 A each.

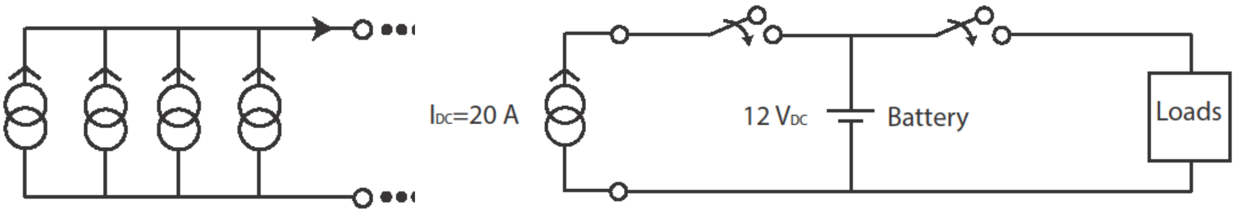


Figure 7: *Simplified electric scheme of the photovoltaic feeding system in Tumbaden health centre.*

The total current generated by the panels is $I = 4 \times 5.02 \cong 20\ A$ *(considering standard conditions: 1000 W/*m^2 *falling on the panels).*

There are several scenarios: During the day, the panels transform the sunlight into electric current and recharge the batteries. In this configuration, the first switch is closed, while the second one is opened if the loads are not being used or closed if they are being used. During the night, the first switch is opened because the panels are not working.

Now, the case where the first switch is opened and the second one is closed is going to be considered. The previous scheme is detailed as follows:

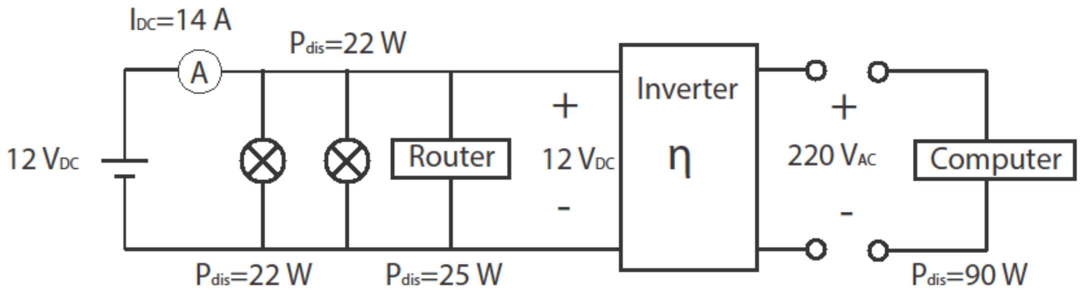


Figure 8: *Complete electric scheme of the photovoltaic system.*

The battery provides direct current that is used to feed the lights and the router. In order to feed the computer, the direct current has to be transformed to alternating current by the inverter of efficiency η.

With an ammeter, the output current of the battery is measured, obtaining 14 A.

a) Apply Poynting's theorem to deduce how much power is dissipated in the inverter
b) What is the efficiency of the inverter? (the efficiency is the ratio between the output and the input power).

Problem 4: *The local network of the project is composed by links port-multiport between a local repeater and its clients, using WILD (Wi-Fi for Large Distances). The local network of Tumbaden is composed of a local repeater (Tumbaden school) and two clients (health centre and town hall).*

In Tumbaden School an omnidirectional antenna provides coverage to the health centre and the town hall. Let us suppose that the antenna of the Tumbdan School generates a monochromatic electromagnetic field of frequency $f_0 = 5.8\, GHz$ and that the expressions of the generated fields, in spherical coordinates and valid for $z > 0$, are:

$$E_\theta = A\, w\, \mu_0\, sin\,\theta \left\{ \frac{1}{\beta_0 r} + \frac{1}{j\beta_0^2 r^2} - \frac{1}{j\beta_0^3 r^3} \right\} e^{-j\beta_0 r} \; \left({}^{V}\!/_{m} \right)$$

$$H_\varphi = A\, \beta_0\, sin\,\theta \left\{ \frac{1}{\beta_0 r} + \frac{1}{j\beta_0^2 r^2} \right\} e^{-j\beta_0 r} \; \left({}^{A}\!/_{m} \right)$$

$$E_r = -2A\, w\, \mu_0\, cos\,\theta \left\{ \frac{1}{\beta_0^2 r^2} + \frac{1}{j\beta_0^3 r^3} \right\} e^{-j\beta_0 r} \; \left({}^{V}\!/_{m} \right)$$

$$H_r = H_\theta = E_\varphi = 0$$

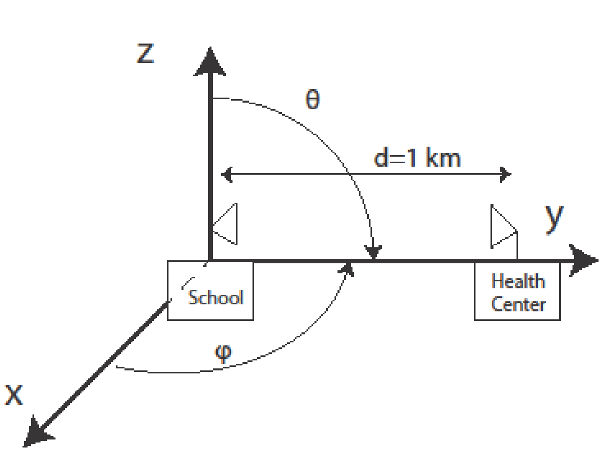
defined for $0 \le \theta \le \pi/2$, $0 \le \varphi \le 2\pi$, where $\beta_0 = w_0\sqrt{\mu_0 \varepsilon_0}$ and being $A \in \mathbb{C}$.

The distance between the school and the Health Centre is $1\, km$ in the direction of $\theta = \pi/2$ and $\varphi = \pi/2$, and their antennas are placed in the same plane ($z = 0$), as represented in Figure 9.

It is known that the lowest power density that has to arrive to the health centre's antenna to be able to establish a communication is $1.62 \times 10^{-8}\; \left({}^{W}\!/_{m^2} \right)$ (*),considering that there are not any losses regarding the medium. Establish:

a) *The lowest radiated power from the antenna in Tumbaden School needed in order to establish communication with the health centre. Specify also the magnitude and the units of A in this case. What can you say about the phase of A?*
b) *The direction when minimum power density occurs.*
c) *Write the expression of the magnetic field (time form).*
d) *In order to study the power received by the students, the head of a Tumbaden School student is simulated as a dielectric sphere of radius $a = 10\, cm$, relative permittivity $\varepsilon_r = \varepsilon_r' - j\varepsilon_r'' = 40 - j10$ and relative permeability $\mu_r = 1$. The head of the*

student is placed at a distance of $r = 20\ m$ in the direction of $\theta = \pi/2$, $\varphi = \pi/2$ and $z > 0$. What is the power intercepted by the student's head if the radiated power by the antenna is $100\ mW$? (Make any approximations needed). Supposing that the entire intercepted power is absorbed by the student's head and that the field inside the head is uniform, obtain the intensity of the electric field inside the head.



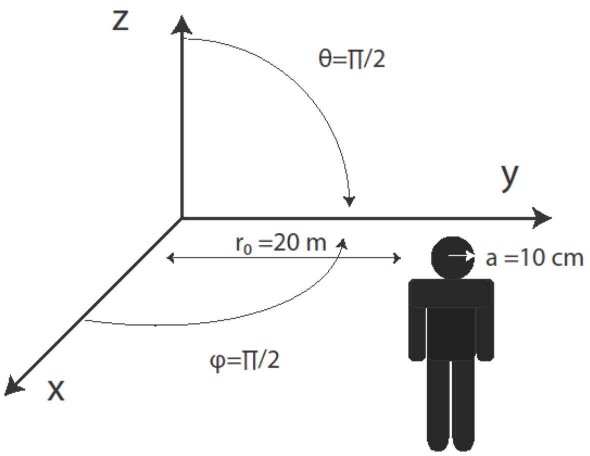


Figure 9: *Left: Distances and placement of the school and the health centre. Right: Position of the student with the school antenna in the origin of coordinates.*

(*) To estimate the minimum power density that the antenna of the Health Centre should receive to establish a communication, the following data have been used:

- *The gain of the antenna is $G_0 = 19\ dBi$.*
- *The sensitivity of the receptor is $S_r = -97 dBm$.*
- *The losses in the cable and connectors are approximated as $L = 1.4\ dB$.*
- *The effective area of the antenna is:*

$$A_{eff} = \frac{\lambda^2}{4\pi} G_0 = 0.017\ m^2$$

Where $\lambda = \frac{c_0}{f_0} = 0.052\ m$ is the wave length at the frequency work, c_0 is the speed of light in the vacuum, and $G_0 = 10^{1.9}$.

The power at the output of the antenna should be:

$$P = S_r + L = -97 + 1.4 = -95.6\ dBm$$

It is known that the power at the output of the antenna is the multiplication of the received power and the effective area of the antenna:

$$P = |<\bar{S}>| \cdot A_{eff}$$

Therefore, the minimum received power can be obtained:

$$|<\bar{S}>|_{min} = \frac{P}{A_{eff}} = \frac{10^{-9.56}}{0.017} = 1.62\ \times 10^{-8}\ (W/_{m^2})$$

Problem 5: A coaxial cable is used to connect the antenna of the health centre with the rest of the system. The antenna receives a monochromatic signal of frequency $f_0 = 5.8\ GHz$ from Tumbaden school.

If the load of the coaxial cable is matched there will be only incident wave and the electromagnetic fields in the dielectric can be written, in an intermediate segment of length L, with these approximated expressions:

$$E_r = \frac{A}{r}e^{-\gamma z}, \qquad E_\varphi = 0, \qquad E_z \neq 0$$

$$H_\varphi = \frac{A}{\eta r}e^{-\gamma z}, \qquad H_z = 0, \qquad H_r = 0$$

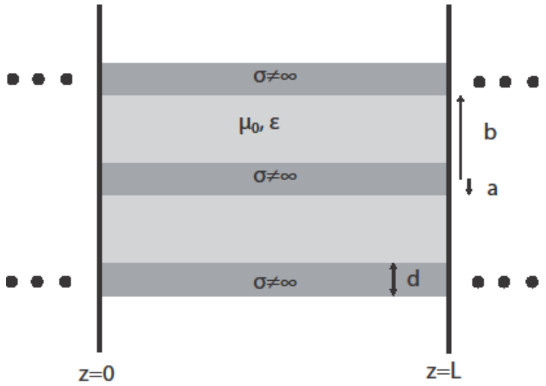


Figure 10: *Representation of a segment of the coaxial cable.*

defined in the region $(a \leq r \leq b, 0 \leq \varphi \leq 2\pi \text{ and } 0 \leq z \leq L)$ with $A \in \mathbb{C}$, $\gamma = \alpha + j\beta$ (α and β are real and positive values) and η is the intrinsic impedance of the dielectric material $\eta = \sqrt{\frac{\mu_0}{\varepsilon}}$.

The monochromatic source is outside the region $0 \leq z \leq L$, because the antenna is supposed to be in $z \ll 0$. Losses are not considered in the dielectric of the coaxial cable but they are in the conductors, as $\sigma \neq \infty$.

a) Obtain the parallel electric fields in the surfaces of the conductors, $r = a \text{ and } r = b$, using the Leontovich approximation and specifying the necessary conditions in order to use this approximation.
b) Draw the mean value of the Poynting vector in the dielectric and in the conductors of the segment $0 \leq z \leq L$.
c) Obtain the expression of the main value of the transmitted power through the dielectric in the planes $z = 0$ and $z = L$.
d) Obtain the expression of the mean value of the power that penetrates the surfaces $r = a$ and $r = b$ in the segment $0 \leq z \leq L$.
e) Apply Poynting's theorem to deduce the expression of α in terms of the rest of the parameters. Which physical phenomenon is responsible for this value?
f) Obtain the expression of the mean value of the dissipated power in the coaxial cable in the segment $0 \leq z \leq L$ by two different methods, justifying why both values agree.

g) Obtain the expression of the difference between the mean values of the stored energies by the magnetic and electric fields in the dielectric in the segment $0 \le z \le L$.
h) Obtain an approximate expression of the magnetic field inside the conductors. Using these approximations, obtain the expression of the mean value of the stored magnetic energy in the conductors (without computing the integral).

Problem 6: *In order to connect the antenna with the rest of the equipment, a low loss coaxial cable of characteristic impedance $Z_0 = 50\,\Omega$ is used. In the datasheet of the coaxial cable [C400, LMR400] some information about the characteristics of the coaxial cable is provided, the outer diameter (OD) of the inner conductor (d_a) is $2.74\,mm$, the OD of the outer conductor (d_{ext}) is $8.08\,mm$ and the capacitance per unit length of the coaxial cable is $78\,pF/m$.*

a) Obtain the permittivity and the coaxial dimensions a and b supposing an ideal dielectric.
b) The attenuation at the operating frequency($f_0 = 5.8$ GHz) is $357.6\,dB/km$. It is known that at high frequencies the losses are mainly due to the losses at the conductor. Supposing both conductors have the same conductivity, obtain the value of the conductivity σ (S/m), using the high frequency resistance per unit length of the coaxial to solve the problem. Which conductor could be used for this coaxial cable?
c) Check that the expression of the attenuation α obtained in Problem 5 question e), is the same as the one obtained in this problem using the approximations of high frequency. Prove that the Leontovich boundary conditions are accomplished.
d) Obtain the total attenuation constant in dB/km of the coaxial cable, supposing losses in the dielectric as well with $\tan\delta = 3 \times 10^{-4}$.

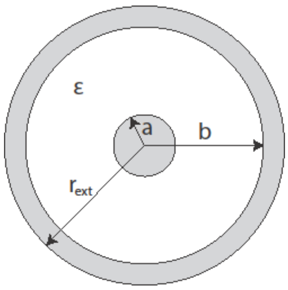


Figure 11: *Representation of the coaxial cable.*

4.1. SOLUTION AND EVALUATION CRITERIA

The evaluation of the delivered problems will be done considering their relative level of difficulty:

Problem 2 can be evaluated as 15 % of the total.
Problem 3 can be evaluated as 10 % of the total.
Problem 4 can be evaluated as 25 % of the total.
Problem 5 can be evaluated as 30 % of the total.
Problem 6 can be evaluated as 20 % of the total.

The detailed solution of Problems 2, 3, 4, 5 and 6 is in Annex 2.

Solution Problem 2:

a) [5 p.]

$$P_P = 338.4\ W_{peak}$$

In order to obtain enough power, four panels of 85 W each are selected (Kyocera KC85T), the total peak power is 85×4=340 W_{peak}.

b) [5 p.] $C_n = 231.94\ Ah$

A battery of 240 Ah (Ritar RA12-240) of 12 V is selected.

Solution Problem 3:

a) [5 p.] $P_{conv} = 9\ W$

b) [5 p.] $\eta = 0.91$

Solution Problem 4:

a) [4 p.] $|A| \geq 9.27 \times 10^{-3}\ (A)$

There is no information about the phase of A, so a generic value φ is considered $A = |A|e^{j\varphi}$.

$$W_{source} \geq 68\ mW$$

b) [1 p.] The minimum power density occurs at $\theta = 0$.

c) [2 p.] $\bar{H}_\varphi(\bar{r}, t) = |A\,|\beta_0 \sin\theta \left[\frac{\cos(w_0 t - \beta_0 r + \varphi)}{\beta_0 r} + \frac{1}{\beta_0^2 r^2}\cos\left(w_0 t - \beta_0 r - \frac{\pi}{2} + \varphi\right)\right]\ \left({}^{A}\!/_{m}\right)$

d) [3 p.] $W_{int} = 1.87 \times 10^{-6}\ (W)$

$|\bar{E}| = 17\ \left({}^{mV}\!/_{m}\right)$

Solution Problem 5:

a) [1p.]

$$\bar{E}_{||}\Big|_{r=a} = \frac{1+j}{\sigma\delta}\frac{A}{\eta a}e^{-\gamma z}\,\hat{z}$$

where $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$ with $a \gg \delta$.

$$\bar{E}_{||}\Big|_{r=b} = \frac{1+j}{\sigma\delta}\frac{A}{\eta b}e^{-\gamma z}(-\hat{z})$$

with $b \gg \delta$ and $d \gg \delta$.

b) [1 p.]

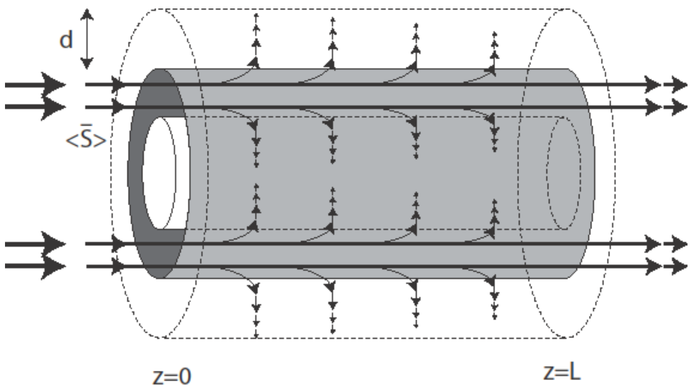


Figure 12: *Representation of the mean value of the Poynting vector in the dielectric and in the conductors.*

c) [1 p.]

$$W_T\Big|_{z=0} = \frac{|A|^2}{\eta}\pi \ln\left(\frac{b}{a}\right)$$

$$W_T\Big|_{z=L} = \frac{|A|^2}{\eta}\pi \ln\left(\frac{b}{a}\right) e^{-2\alpha L}$$

d) [1p.]

$$W_T|_{r=a} = \frac{|A|^2\pi\,(1-e^{-2\alpha L})}{2\sigma\delta\eta^2 a\alpha}$$

$$W_T|_{r=b} = \frac{|A|^2\pi\,(1-e^{-2\alpha L})}{2\sigma\delta\eta^2 b\alpha}$$

e) [2p.]

$$\frac{|A|^2\pi}{\eta}\ln\left(\frac{b}{a}\right)(1-e^{-2\alpha L}) = \frac{|A|^2\pi\,(1-e^{-2\alpha L})}{2\sigma\delta\eta^2\alpha}\left(\frac{1}{a}+\frac{1}{b}\right) \quad [1]$$

$$\alpha = \frac{1}{2\sigma\delta\eta\ln\left(\frac{b}{a}\right)}\left(\frac{1}{a}+\frac{1}{b}\right)\ \left({}^{Nep}/_{m}\right) \quad [2]$$

This attenuation constant is because the conductors are not ideal, so $\alpha = \alpha_c$.

f) [1p.]

$$W_{dis} = W_T\Big|_{z=0} - W_T\Big|_{z=L} = \frac{|A|^2}{\eta}\pi\ln\left(\frac{b}{a}\right)(1-e^{-2\alpha L})$$

or

$$W_{dis} = W_T\Big|_{r=a} + W_T\Big|_{r=b} = \frac{|A|^2\pi\,(1-e^{-2\alpha L})}{2\sigma\delta\eta^2\alpha}\left(\frac{1}{a}+\frac{1}{b}\right)$$

It is easy to justify that both values agree because they are the two terms of Poynting's theorem applied in e), equation [1].

g) [2p.]

$$\langle W_H\rangle - \langle W_E\rangle = \frac{-1}{2w_0}\left[\frac{|A|^2\pi\,(1-e^{-2\alpha L})}{2\sigma\delta\eta^2\alpha}\left(\frac{1}{a}+\frac{1}{b}\right)\right]\ (J)$$

h) [1p.]

$$\overline{\mathrm{H}}_{(\mathrm{r}\leq\mathrm{a})} = \frac{A}{\eta a}e^{-(\alpha+j\beta)z}e^{\frac{1+j}{\delta}(r-a)}\hat{\varphi}$$

$$\overline{\mathrm{H}}_{(\mathrm{b}\leq\mathrm{r}\leq\mathrm{b+d})} = \frac{A}{\eta b}e^{-(\alpha+j\beta)z}e^{-\frac{1+j}{\delta}(r-b)}\ \hat{\varphi}$$

$$\langle W_H\rangle_{r\leq a} = \frac{\mu_0}{4}\int_{r=0}^{a}\int_{\varphi=0}^{2\pi}\int_{z=0}^{L}\frac{AA^*}{\eta^2a^2}e^{-2\alpha z}e^{\frac{2}{\delta}(r-a)}r\,d\varphi drdz$$

$$\langle W_H\rangle_{b\leq r\leq b+\mathrm{d}} = \frac{\mu_0}{4}\int_{r=b}^{b+d}\int_{\varphi=0}^{2\pi}\int_{z=0}^{L}\frac{AA^*}{\eta^2b^2}e^{-2\alpha z}e^{\frac{-2}{\delta}(r-b)}r\,d\varphi drdz$$

$$\langle W_H\rangle_{cond} = \langle W_H\rangle_{r\leq a} + \langle W_H\rangle_{b\leq r\leq b+d}$$

<u>Solution Problem 6:</u>

a) [3p.]

$$\varepsilon_r' = 1.365$$

$$a = 1.37\ mm, \qquad b = 3.62\ mm$$

b) [3p.]

$$\alpha_c({}^{\mathrm{Nep}}/_{m}) = \frac{R_{cond}({}^{\Omega}/_{m})}{2\mathcal{R}_e[Z_0]} = \frac{\frac{1}{a}+\frac{1}{b}}{2\sigma\,\delta\ \mathcal{R}_e[\eta]\ln\frac{b}{a}} \qquad [3]$$

$$\sigma = \left(\frac{\left(\frac{1}{a}+\frac{1}{b}\right)\sqrt{\pi f\mu_0}}{2\alpha_c\ \mathcal{R}_e[\eta]\ln\frac{b}{a}}\right)^2 = 3.47\times10^7\ \frac{S}{m}$$

The aluminum conductivity is approximately $\sigma_{Al} = 3.5\times10^7\ S/m$.

c) [2p.]

It can be observed that equation [2] in question e) of Problem 5 is the same expression as the one obtained in equation [3] . This is because the Leontovich approximation is used to obtain the resistance per unit length of the conductors in high frequency.

The thickness is obtained as:

$$t = r_{ext} - b = 4.04 - 3.62 = 0.42\ mm$$

$$\delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma}} = 1.12\ \mu m$$

As it can be observed $t \gg \delta,\ a \gg \delta\ and\ b \gg \delta,$ therefore the Leontovich approximation can be applied.

d) [2p.]

$$\alpha = 542.9 \frac{dB}{km}$$

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ANNEX 1 DETAILED SOLUTION OF CLASS ACTIVITY (PROBLEM 1)

a) It is known that the average power of the sources in volume V, defined in the Figure 13, has to be equal to the flux of the mean value of the Poynting vector going out through the surface S_{ur} plus the losses in V.

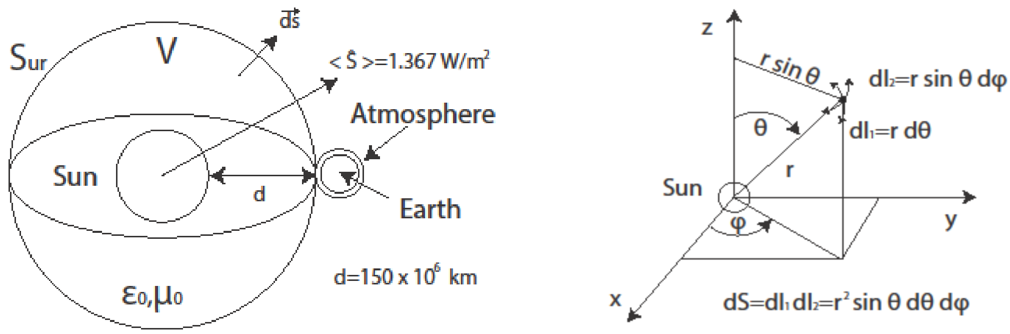

Figure 13: *Left: Scheme of the Poynting vector applied to the volume V. Right: Spherical coordinate system with the origin of coordinates in the Sun.*

As it is considered to be a vacuum, there are no losses. A spherical coordinate system has been selected to solve the problem where the Sun has been placed in the origin of the coordinates, as it is shown in the second picture, where

$$< \bar{S} >|_{\substack{out.\\ atm.}} = 1367\,\hat{r}\ \ (W/m^2)$$

$$\begin{matrix} Average\ power\ of \\ the\ sources\ in\ V \end{matrix} = \begin{matrix} Flux\ of\ the\ mean\ value\ of\ P. \\ vector\ passing\ through\ \ S_{ur} \end{matrix} + Losses\ in\ V$$

The total radiated power by the Sun can be obtained considering there are no losses:

$$Radiated\ power\ by\ the\ Sun = \oint_{S_{ur}} < \bar{S} > \cdot\, d\bar{s} + 0$$

Substituting the values provided in the problem (considering r the distance between the Sun and the Earth $\mathrm{r} = 150 \times 10^6\ km$):

$$Radiated\ power = \int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi} 1367\,\hat{r}\cdot\hat{r}\,r^2 \sin\theta\ \ d\theta\, d\varphi = 1367(150\times10^9)^2[-\cos\theta]_0^{\pi}\ 2\pi$$

Finally, the total radiated power by the Sun is obtained:

$$Radiated\ power = 3.86 \times 10^{26}\ W$$

b) Using the same coordinate system, the value of the Poynting vector in the Earth's surface can be obtained as:

$$|< \bar{S} >| = \frac{360\ Joules}{60\times 60\ seconds\ (10^{-2}meters)^2} = \frac{360}{3600}\times 10^4 = 1000\ ({}^{W}/_{m^2})$$

$$< \bar{S} >= |< \bar{S} >|\,\hat{r} = 1000\,\hat{r}\ ({}^{W}/_{m^2})$$

c) In order to obtain the efficiency, the input and output power must be previously obtained.

$$P_{Out_{(DC)}} = V \times I = 17.4 \times 5.02 = 87.35\ W$$

$$P_{In} = density\ power \times area = 1000 \times 0.652 = 652\ W$$

$$\eta = \frac{P_{Out}}{P_{In}} = \frac{87.35}{652} = 13{,}4\%$$

d) In case of an error of 30°, the input and output power in that scenario would be:

$$P_{In} = \oint_{\substack{S_{ur} \\ panel}} < \bar{S} > \cdot\, d\bar{s} = \oint_{\substack{S_{ur} \\ panel}} |< \bar{S} >|\ \cos\psi\, ds = 1000\ \times \frac{\sqrt{3}}{2} \times 0.652$$

$$P_{In} = 326\sqrt{3}\ W$$

$$P_{Out} = \eta \cdot P_{In} = 0.134\ \times 326\sqrt{3} = 75.66\ W$$

Annex 2 Detailed solution of Homework Activity (Problems 2, 3, 4, 5, and 6)

Solution Problem 2:

Two days autonomy is desired. The daily power consumption can be obtained:

$$L_T = L_{router} + L_{comp} + L_{lights} = 25\ \mathrm{W} \times 24\ \frac{\mathrm{h}}{\mathrm{day}} + 90\ \mathrm{W} \times 3\ \frac{\mathrm{h}}{\mathrm{day}} + 2 \times 22\ \mathrm{W} \times 3\ \frac{\mathrm{h}}{\mathrm{day}}$$

$$L_T = 1002\ \frac{\mathrm{Wh}}{\mathrm{day}}$$

In order to obtain the radiation incident over the Earth's surface in Tumbaden, the longitude and latitude (expressed as degrees/minutes/seconds) should be included:

- Latitude: South 07° 01' 30.3''
- Longitude: West 78° 44' 22.9''

It can also be expressed in decimal degrees (mandatory in the NASA website *http://eosweb.larc.nasa.gov/sse*) as:

- Latitude: -7.025 (Negative because is South)
- Longitude: -78.74 (Negative because is West)

The photovoltaic system is going to be designed taking into account the worst scenario in order to ensure it can work in every possible situation.

Minimum Radiation Incident On An Equator-pointed Tilted Surface (kWh/m²/day)

Lat -7.025 Lon -78.74	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Annual Average
Tilt 0	4.66	4.32	4.53	4.75	4.70	4.08	4.92	5.23	5.49	5.12	5.49	5.15	4.87
Tilt 7	4.72	4.34	4.52	4.85	4.90	4.27	5.17	5.40	5.53	5.13	5.56	5.24	4.97
Tilt 22	4.69	4.23	4.37	4.88	5.14	4.54	5.52	5.55	5.43	4.96	5.53	5.26	5.01
Tilt 90	2.30	1.89	1.76	2.46	3.16	3.03	3.59	3.02	2.18	1.91	2.50	2.61	2.54

Figure 14: *Table of the minimum radiation obtained from the NASA website.*

Looking at the table, the minimum radiation for the worst month using a tilt of 22 degrees (Latitude + 15) is 4.23 kWh/m^2/day, corresponding to February.

Once the data has been obtained, the peak power can be obtained as:

$$P_P = \frac{L}{\eta \cdot \frac{G_{wm}}{G}} = \frac{1002}{0.7 \cdot \frac{4230}{1000}} = 338.4\ W_{peak}$$

In order to obtain enough power, four panels of 85 W each are selected (Kyocera KC85T), the total peak power is 85×4=340 W_{peak}.

The nominal capacity of the batteries can be obtained as:

$$C_n = \frac{L \cdot D}{V \cdot \mu \cdot P_D} = \frac{1002 \cdot 2}{12\ V_{DC} \cdot 0.9 \cdot 0.8} = 231.94\ Ah$$

A battery of 240 Ah (Ritar RA12-240) of 12 V is selected.

Solution Problem 3:

a) The battery power is obtained as:

$$P_b = 12 \times 14 = 168\ W$$

Using the Poynting's theorem it is known that the power generated by the battery has to be equal to the radiated power plus the losses in the system. As there is no radiated power, the power of the battery is distributed between every load:

$$P_{batt} = P_{lights} + P_{router} + P_{inv} + P_{computer} = 2 \times 22 + 25 + P_{inv} + 90 = 168\ W$$

The dissipated power in the inverter is:

$$P_{inv} = 9\ W$$

b) The efficiency is the ratio between the output and the input power.

$$\eta = \frac{P_{out}}{P_{in}}$$

The dissipated power in the inverter is 9 W, so:

$$P_{dis} = P_{in} - P_{out} = 9\ W$$

$$P_{dis} = \frac{P_{out}}{\eta} - P_{out} = P_{out}\left[\frac{1-\eta}{\eta}\right]$$

The output power is dissipated in the computer (90 W), so the efficiency is obtained:

$$\eta = \frac{P_{out}}{P_{out} + P_{dis}} = 0.91$$

Solution Problem 4:

a) Applying Poynting's theorem and assuming that there are not any losses:

$$\begin{matrix} Power\ of \\ sources\ in\ V \end{matrix} = \begin{matrix} Flux < \bar{S} > \\ through\ S_{ur} \end{matrix} + \begin{matrix} Losses \\ in\ V \end{matrix}$$

for $0 \le \theta \le \pi/2,\ 0 \le \varphi \le 2\pi.$ The power radiated from the source is obtained as:

$$W_{source} = \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \frac{1}{2} \mathbb{R}e[\bar{E} \times \bar{H}^*] \cdot \overline{ds}$$

where $\overline{ds} = r^2 \sin\theta\, d\theta\, d\varphi\, \hat{r}$. We are only interested in the mean value of the Poynting vector in the $\hat{r}$ direction. The Poynting vector is:

$$< \bar{S}_r(\bar{r},t) >= \frac{1}{2}\mathbb{R}e\left[\bar{E}_\theta \times \bar{H}_\varphi^*\right]$$

where $\bar{E}_\theta$ and $\bar{H}_\varphi$ are the components of the electric and magnetic field provided.

$$\begin{aligned}< \bar{S}_r(\bar{r},t) >&= \frac{1}{2}\mathbb{R}e\left[Aw\mu_0 \sin^2\theta \left\{\frac{1}{\beta_0 r} + \frac{1}{j\beta_0^2 r^2} - \frac{1}{j\beta_0^3 r^3}\right\} e^{-j\beta_0 r}\ A^*\ \beta_0 \left\{\frac{1}{\beta_0 r} + \frac{1}{-j\beta_0^2 r^2}\right\} e^{+j\beta_0 r}\right]\hat{r} \\ &= \frac{|A|^2}{2} w\mu_0 \sin^2\theta\, \beta_0\, \mathbb{R}e\left[\frac{1}{\beta_0^2 r^2} + \frac{1}{\beta_0^4 r^4} - \frac{1}{\beta_0^4 r^4} + j\left(\frac{1}{\beta_0^3 r^3} - \frac{1}{\beta_0^3 r^3} - \frac{1}{\beta_0^5 r^5}\right)\right]\hat{r} \\ &= \frac{|A|^2}{2} w\mu_0 \sin^2\theta\, \beta_0\, \frac{1}{\beta_0^2 r^2}\hat{r} = \frac{|A|^2 w\mu_0 \sin^2\theta}{2\beta_0 r^2}\hat{r}\end{aligned}$$

Once the Poynting vector has been obtained, we are going to integrate it to obtain the power of the sources:

$$\begin{aligned}W_{source} &= \int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi/2} \frac{|A|^2 w\mu_0 \sin^2\theta}{2\beta_0 r^2} r^2 \sin\theta\, d\theta\, d\varphi\, \hat{r}\cdot\hat{r} = \frac{|A|^2}{2} w\mu_0 \frac{1}{\beta_0}\int_{\varphi=0}^{2\pi} d\varphi \int_{\theta=0}^{\pi/2} \sin^3\theta\, d\theta \\ &= \frac{|A|^2}{2} w\mu_0 \frac{1}{\beta_0} 2\pi \frac{2}{3} = |A|^2 80\pi^2\end{aligned}$$

The lowest power density has to be greater than the threshold provided:

$$\begin{aligned}|< \bar{S}(\bar{r},t) >|_{\substack{\theta=\pi/2 \\ r=1km}} &= \left.\frac{|A|^2 w\mu_0 \sin^2\theta}{2\beta_0 r^2}\right|_{\substack{\theta=\pi/2 \\ r=1km}} = \frac{|A|^2 w\mu_0 \sin^2\pi/2}{2\beta_0 10^6} = \frac{|A|^2 6\pi}{10^5} \geq \\ &\geq 1.62\ \times 10^{-8}\ \left(W/_{m^2}\right)\end{aligned}$$

If a communication is going to be established, $|A|$ should be greater than:

$$|A|^2 \geq \frac{1.62\ \times 10^{-3}}{6\pi} = 8.59 \times 10^{-5}$$

$$|A|\ \geq 9.27 \times 10^{-3}\ (A)$$

There is no information about the phase of A, so a generic value φ is considered $A = |A|e^{j\varphi}$. In that way, the power will be:

$$W_{source} \geq |A|^2 80\pi^2 = 68\ mW$$

b) The minimum power density occurs at $\theta = 0$ as it depends of $\sin^2\theta$.

c) The expression of the magnetic field in time form is:

$$\begin{aligned}\bar{H}_\varphi(\bar{r},t) &= \mathbb{R}e\left[A\,\beta_0 \sin\theta \left\{\frac{1}{\beta_0 r} + \frac{1}{j\beta_0^2 r^2}\right\} e^{-j\beta_0 r} e^{jw_0 t}\right] \\ &= |A\,|\beta_0 \sin\theta \left[\frac{\cos(w_0 t - \beta_0 r + \varphi)}{\beta_0 r}\right. \\ &\quad \left. + \frac{1}{\beta_0^2 r^2}\cos\left(w_0 t - \beta_0 r - \frac{\pi}{2} + \varphi\right)\right]\ \left(A/_{m}\right)\end{aligned}$$

d) A student is placed at a distance of 20 meters as it is shown in Figure 15. The head of the student is approximated as a sphere of ten centimeters in radius. As the values of the power density are almost the same in every point of the sphere, the intercepted power by the student's head will be obtained as a multiplication of the power density at the maximum value $(\theta = \pi/2,\ \varphi = \pi/2)$ and the interception surface of the head of the student:

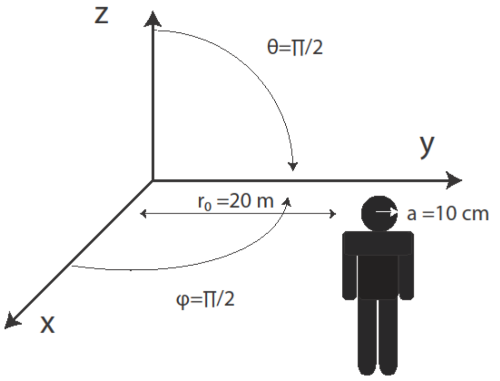

Figure 15: *Position of the student with the school antenna in the origin of coordinates.*

$$W_{int} = |<\bar{S}(\bar{r},t)>| \cdot Intercetion\ surface$$

Now the $W_{source} = 100\ mW$, $|A'|^2$ is obtained as:

$$|A'|^2 = \frac{W_{source}}{80\pi^2} = 1.27 \times 10^{-4}$$

$$|<\bar{S}(\bar{r},t)>|_{\substack{\theta=\pi/2 \\ \varphi=\pi/2 \\ r=20\,m}} = \frac{|A'|^2 w \mu_0 \sin^2\theta}{2\beta_0 r^2} = 6 \times 10^{-5}\ \left({}^{W}\!/_{m^2}\right)$$

When the distance d is similar to a, the interception surface is smaller than the surface of an equivalent circle of radius a. However, when $d \gg a$, as it is in this case, the intercepted surface of the sphere can be approximated as the surface of a circle of radius a.

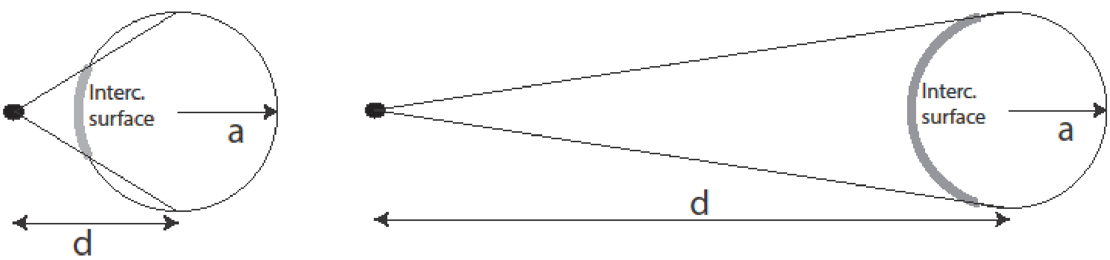

Figure 16: *Left: Intercepted surface for short distances. Right: Intercepted surface for long distances.*

As $20\ m \gg 10\ cm$, the intercepted surface can be approximated as πa^2. The intercepted power by the sphere is obtained as:

$$W_{int} = 6 \times 10^{-5}\, \pi a^2\Big|_{a=10\ cm} = 1.87 \times 10^{-6}\ \ (W)$$

Supposing that the entire intercepted power is dissipated in the head:

$$W_{int} = \frac{w\varepsilon''}{2} \int_V |\bar{E}|^2\, dV$$

Assuming that the electric field is uniform inside the head:

$$W_{int} = \frac{w\varepsilon''}{2} |\bar{E}|^2 \int_V\ dV = \frac{w\varepsilon''}{2} |\bar{E}|^2 \frac{4}{3} \pi a^3$$

Finally, the electric field can be obtained as:

$$|\bar{E}|\ = \sqrt{\frac{3W_{int}}{2w\varepsilon''\pi a^3}} = 17\ \ (^{mV}/_{m})$$

With $\varepsilon'' = 10\varepsilon_0$, $\varepsilon_0 = 8.854 \times 10^{-12}\ {}^{F}/_{m}$, $a = 0.1\, m$ *and* $w = 2\pi \times 5.8 \times 10^9\ {}^{rad}/_{s}$.

Solution Problem 5:

a) The magnetic field parallel to the conductor in $r = a$ is:

$$\bar{H}_\varphi\Big|_{r=a} = \frac{A}{\eta a} e^{-\gamma z} \hat{\varphi}$$

As $\sigma \neq \infty$, the Poynting vector penetrates the conductor generating losses. With the Leontovich approximation in $r = a$ there is a parallel electric field $\bar{E}_{||}\Big|_{r=a}$ and a plane wave propagates inside the conductor $r \leq a$:

$$\bar{E}_{||}\Big|_{r=a} = \eta_\sigma \left[\bar{H}_{||}\Big|_{r=a} \times \hat{n}\right] = \frac{1+j}{\sigma\delta}\left[\frac{A}{\eta a} e^{-\gamma z}\ \hat{\varphi} \times (-\hat{r})\right] = \frac{1+j}{\sigma\delta} \frac{A}{\eta a} e^{-\gamma z}\ \hat{z} = E_z\Big|_{r=a}\ \hat{z}$$

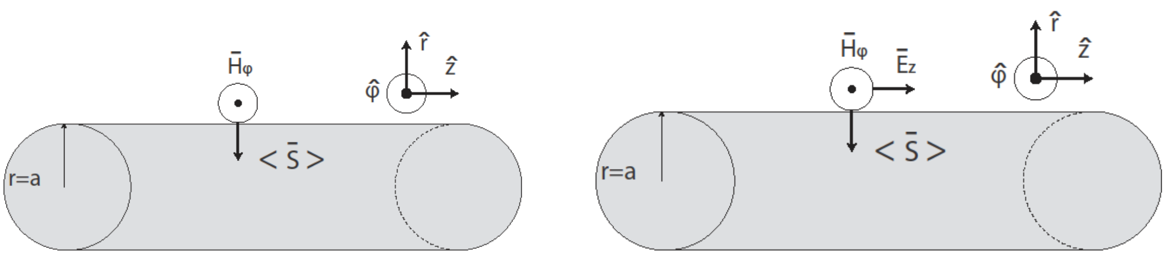

Figure 17: *Left: Poynting vector penetrating the conductor. Right: Poynting vector penetrating the conductor and the representation of the parallel electric field.*

where $\eta_\sigma = \frac{1+j}{\sigma\delta}$ is the intrinsic impedance in a conductor σ and $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$ is the penetration depth in the conductor.

In order to accomplish the Leontovich approximation the thickness and the radius of curvature of the conductor have to be much greater than δ, so $a \gg \delta$.
For the exterior conductor:

$$\bar{E}_{||}\Big|_{r=b} = \eta_\sigma \left[\bar{H}_{||}\Big|_{r=b} \times \hat{n}\right] = \frac{1+j}{\sigma\delta}\left[\frac{A}{\eta b} e^{-\gamma z}\,\hat{\varphi} \times (+\hat{r})\right] = \frac{1+j}{\sigma\delta}\frac{A}{\eta b} e^{-\gamma z}(-\,\hat{z})$$
$$= E_z\Big|_{r=b}(-\,\hat{z})$$

As in the previous case, in order to accomplish the Leontovich approximation the thickness and the radius of curvature have to be much greater than δ, so $b \gg \delta$ and $d \gg \delta$.

b) Therefore there are mean values of the Poynting vector in $\hat{z}$ and $\mp\hat{r}$ directions. The power density that penetrates at $z = 0$ is divided between the surfaces $r = a, r = b$ and $z = L$ (the dielectric medium is lossless).

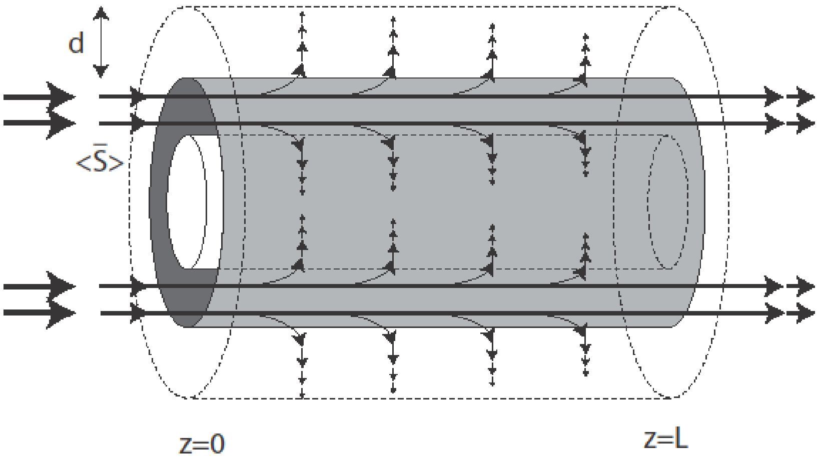

Figure 18: *Representation of the mean value of the Poynting vector in the dielectric and in the conductors.*

c) $W_T\Big|_{z=0}$ and $W_T\Big|_{z=L}$ are the flux of the mean value of the Poynting vector through the surfaces $z = 0$ and $z = L$, therefore only $< \bar{S}_z >$ has to be calculated.

Figure 19: *Representation of the flux of the mean value of the Poynting vector through the surfaces $z = 0$ and $z = L$.*

$$W_T(z) = \int_{sur} < \bar{S}_z > \cdot\, ds\ \hat{z} = \int_{r=a}^{b} \int_{\varphi=0}^{2\pi} \frac{1}{2}\, \mathbb{R}e\left[\bar{E}_r \times \bar{H}_\varphi^*\right] \cdot\, r\, d\varphi dr\ \hat{z}$$

$$= \int_{r=a}^{b} \int_{\varphi=0}^{2\pi} \frac{1}{2}\, \mathbb{R}e\left[\frac{AA^*}{r^2\eta} e^{-(\gamma+\gamma^*)z}\right] r\, d\varphi dr\, (\hat{r} \times \hat{\varphi}) \cdot \hat{z} = \frac{|A|^2}{2\eta} 2\pi \ln\left(\frac{b}{a}\right) e^{-2\alpha z}$$

Particularized for $z = 0$ and $z = L$

$$z = 0 \rightarrow W_T\Big|_{z=0} = \frac{|A|^2}{\eta} \pi \ln\left(\frac{b}{a}\right)$$

$$z = L \rightarrow W_T\Big|_{z=L} = \frac{|A|^2}{\eta} \pi \ln\left(\frac{b}{a}\right) e^{-2\alpha L}$$

d) The main value of the transmitted power through the dielectric at $r = a$ and $r = b$ is obtained as:

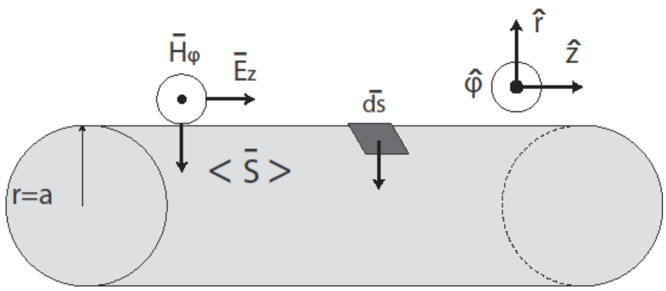

Figure 20: *Poynting's vector going inside the inner conductor.*

$$< \bar{S}_r >\Big|_{r=a} = \frac{1}{2}\, \mathbb{R}e\left[\bar{E}_z \times \bar{H}_\varphi^*\right]\Big|_{r=a} = \frac{1}{2}\, \mathbb{R}e\left[\frac{1+j}{\sigma\delta} \frac{A}{\eta a} e^{-\gamma z}\, \hat{z} \times \hat{\varphi}\, \frac{A^*}{\eta a} e^{-\gamma^* z}\right]$$

$$= \frac{1}{2} \frac{1}{\sigma\delta} \frac{|A|^2}{\eta^2 a^2} e^{-2\alpha z}\, (-\hat{r})$$

$$W_T|_{r=a} = \int_{z=0}^{z=L} \int_{\varphi=0}^{2\pi} < \bar{S}_r >\Big|_{r=a} \cdot ds\, (-\hat{r}) = \int_{z=0}^{z=L} \int_{\varphi=0}^{2\pi} \frac{1}{2} \frac{1}{\sigma\delta} \frac{|A|^2}{\eta^2 a^2} e^{-2\alpha z}\, (-\hat{r}) \cdot (-\hat{r})\, a\, d\varphi dz$$

$$= \frac{|A|^2 2\pi}{2\sigma\delta\eta^2 a} \frac{e^{-2\alpha z}}{-2\alpha}\Bigg|_{z=0}^{L} = \frac{|A|^2 \pi\, (1 - e^{-2\alpha L})}{2\sigma\delta\eta^2 a\alpha}$$

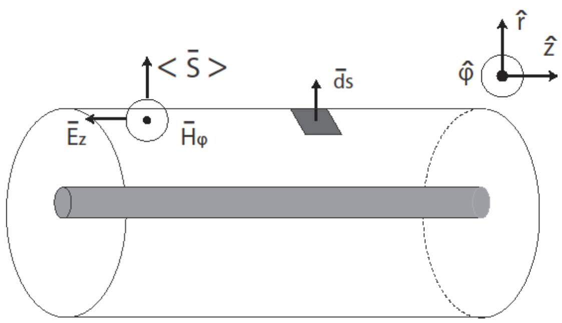

Figure 21: *Poynting's vector going inside the outer conductor.*

$$< \bar{S}_r >\Big|_{r=b} = \frac{1}{2}\,\mathbb{R}e\left[\bar{E}_z \times \bar{H}_\varphi^*\right]\Big|_{r=b} = \frac{1}{2}\,\mathbb{R}e\left[\frac{1+j}{\sigma\delta}\frac{A}{\eta b}e^{-\gamma z}\,(-\hat{z}) \times \hat{\varphi}\,\frac{A^*}{\eta b}e^{-\gamma^* z}\right]$$

$$= \frac{1}{2}\frac{1}{\sigma\delta}\frac{|A|^2}{\eta^2 b^2}e^{-2\alpha z}\,(\hat{r})$$

$$W_T|_{r=b} = \int_{z=0}^{z=L}\int_{\varphi=0}^{2\pi} < \bar{S}_r >\Big|_{r=b} \cdot ds\;\hat{r} = \int_{z=0}^{z=L}\int_{\varphi=0}^{2\pi}\frac{1}{2}\frac{1}{\sigma\delta}\frac{|A|^2}{\eta^2 b^2}e^{-2\alpha z}\;\hat{r}\cdot\hat{r}\;b\;d\varphi dz$$

$$= \frac{|A|^2 2\pi}{2\sigma\delta\eta^2 b}\frac{e^{-2\alpha z}}{-2\alpha}\Big|_{z=0}^{L} = \frac{|A|^2\pi\,(1-e^{-2\alpha L})}{2\sigma\delta\eta^2 b\alpha}$$

e) Applying the Poynting's theorem in the dielectric volume delimited by $0 \le z \le L$ and $a \le r \le b$:

$$\begin{matrix}\textit{Power of} \\ \textit{sources in } V\end{matrix} = \begin{matrix}\textit{Flux} <\bar{S}> \\ \textit{through } S_{ur}\end{matrix} + \begin{matrix}\textit{Losses} \\ \textit{in } V\end{matrix}$$

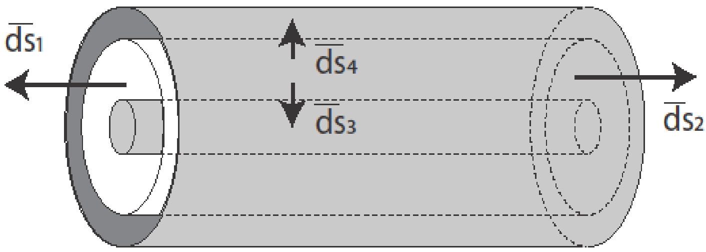

Figure 22: *Representation of the different surfaces in V.*

As there are neither sources nor losses in the volume V, the flux of the Poynting vector is the only parameter that should be calculated. The surface S_{ur} is composed of four different surfaces as it is depicted in the figure and each Poynting vector has been previously obtained in the different questions.

$$0 = -\frac{|A|^2}{\eta}\pi\ln\left(\frac{b}{a}\right) + \frac{|A|^2}{\eta}\pi\ln\left(\frac{b}{a}\right)e^{-2\alpha L} + \frac{|A|^2\pi\,(1-e^{-2\alpha L})}{2\sigma\delta\eta^2 a\alpha} + \frac{|A|^2\pi\,(1-e^{-2\alpha L})}{2\sigma\delta\eta^2 b\alpha}$$

$$\frac{|A|^2\pi}{\eta}\ln\left(\frac{b}{a}\right)\,(1-e^{-2\alpha L}) = \frac{|A|^2\pi\,(1-e^{-2\alpha L})}{2\sigma\delta\eta^2\alpha}\left(\frac{1}{a}+\frac{1}{b}\right) \qquad [1]$$

$$\frac{(1-e^{-2\alpha z})}{2\sigma\delta\eta\;\alpha}\left(\frac{1}{a}+\frac{1}{b}\right) = \ln\left(\frac{b}{a}\right)(1-e^{-2\alpha L})$$

$$\frac{1}{2\sigma\delta\eta\;\alpha}\left(\frac{1}{a}+\frac{1}{b}\right) = \ln\left(\frac{b}{a}\right)$$

$$\alpha = \frac{1}{2\sigma\delta\eta\ln\left(\frac{b}{a}\right)}\left(\frac{1}{a}+\frac{1}{b}\right)\;\left({}^{Nep}/_{m}\right)$$

This attenuation constant is because the conductors are not ideal, so $\alpha = \alpha_c$.

f) The dissipated power in the coaxial cable can be obtained in two different ways. In both cases it is assumed that the power through $r = a$ and $r = b$ are completely dissipated in the conductor because $a \gg \delta$ and $d \gg \delta$. The first method involves obtaining the difference between the power in $z = 0$ and the power in $z = L$. The second method involves calculating the power that goes through $r = a$ and $r = b$ because there are only losses in the conductors. Using the first method:

$$W_{dis} = W_T\Big|_{z=0} - W_T\Big|_{z=L} = \frac{|A|^2}{\eta}\pi \ln\left(\frac{b}{a}\right) - \frac{|A|^2}{\eta}\pi \ln\left(\frac{b}{a}\right) e^{-2\alpha L} = \frac{|A|^2}{\eta}\pi \ln\left(\frac{b}{a}\right)\left(1 - e^{-2\alpha L}\right)$$

Using the second method:

$$W_{dis} = W_T\Big|_{r=a} + W_T\Big|_{r=b} = \frac{|A|^2\pi\left(1 - e^{-2\alpha L}\right)}{2\sigma\delta\eta^2 a\alpha} + \frac{|A|^2\pi\left(1 - e^{-2\alpha L}\right)}{2\sigma\delta\eta^2 b\alpha} = \frac{|A|^2\pi\left(1 - e^{-2\alpha L}\right)}{2\sigma\delta\eta^2\alpha}\left(\frac{1}{a} + \frac{1}{b}\right)$$

It is easy to justify that both values agree because they are the two terms of Poynting's theorem applied in e) (Equation [1]).

g) Applying the reactive power balance in the dielectric volume, the following relationship is obtained:

$$\begin{matrix} Reactive\ power \\ of\ sources\ in\ V \end{matrix} = \begin{matrix} Flux\ \frac{1}{2}\Im m\left[\bar{E} \times \bar{H}^*\right] \\ through\ S_{ur} \end{matrix} + 2w_0\left[\langle W_H\rangle - \langle W_E\rangle\right]$$

where $\langle W_H\rangle$ and $\langle W_E\rangle$ are the mean value of the magnetic and electric energies stored in V respectively.

As in question e), the surface S_{ur} is composed of four different surfaces concerning the two conductors and the limits of the segment, delimited by $z = 0$ and and $z = L$. In $z = 0$ and $z = L$ the term $\frac{1}{2}\Im\mathrm{m}\left[\bar{E}_r \times \bar{H}_\varphi^*\right]$ is null because the product $\left[\bar{E}_r \times \bar{H}_\varphi^*\right]$ is real. At the conductor surfaces, at $r = a$ and $r = b$, $\frac{1}{2}\Im\mathrm{m}\left[\bar{E}_z \times \bar{H}_\varphi^*\right] = \frac{1}{2}\ \mathbb{R}e\left[\bar{E}_z \times \bar{H}_\varphi^*\right]$ so, the fluxes have the same results as calculated in question d). Obtaining:

$$\langle W_H\rangle - \langle W_E\rangle = \frac{-1}{2w_0}\left[\frac{|A|^2\pi\left(1 - e^{-2\alpha L}\right)}{2\sigma\delta\eta^2\alpha}\left(\frac{1}{a} + \frac{1}{b}\right)\right]\ (J)$$

h) Leontovich approximation: Inside the conductor, the electromagnetic field is an homogeneus plane wave that becomes vanishing as it goes inside the conductor, with a $\gamma_{cond} = \gamma_\sigma = \frac{1+j}{\delta}$

$$\bar{H}\Big|_{r=a} = \frac{A}{\eta a} e^{-\gamma z}\ \hat{\varphi}$$

$$\bar{H}\Big|_{r=b} = \frac{A}{\eta b} e^{-\gamma z}\ \hat{\varphi}$$

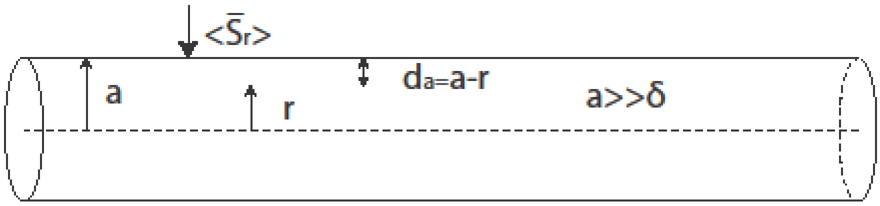

Figure 23: *Poynting's vector going inside the inner conductor.*

$$\underset{(r\leq a)}{\overline{\mathrm{H}}_{\mathrm{inside}}} = \overline{H}\Big|_{r=a} e^{-\gamma_\sigma d_a} = \frac{A}{\eta a} e^{-\gamma z} e^{-\frac{1+j}{\delta}(a-r)}\hat{\varphi} = \frac{A}{\eta a} e^{-(\alpha+j\beta)z} e^{\frac{1+j}{\delta}(r-a)}\hat{\varphi}$$

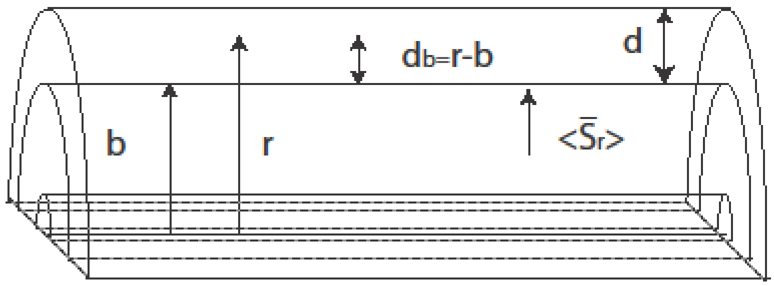

Figure 24: *Poynting's vector going inside the outer conductor.*

$$\underset{(b\leq r\leq b+d)}{\overline{\mathrm{H}}_{\mathrm{inside}}} = \overline{H}\Big|_{r=b} e^{-\gamma_\sigma d_b} = \frac{A}{\eta b} e^{-\gamma z} e^{-\frac{1+j}{\delta}(r-b)}\hat{\varphi} = \frac{A}{\eta b} e^{-(\alpha+j\beta)z} e^{-\frac{1+j}{\delta}(r-b)}\hat{\varphi}$$

where d_a and d_b are the depth that the wave goes inside in each conductor: $d_a = a - r$ and $d_b = r - b$, as it is depicted in the figures.

$$\langle W_H\rangle_{r\leq a} = \frac{\mu'}{4}\int_V |\overline{H}|^2\, dV = \frac{\mu_0}{4}\int_{r=0}^{a}\int_{\varphi=0}^{2\pi}\int_{z=0}^{L} \frac{AA^*}{\eta^2 a^2} e^{-2\alpha z} e^{\frac{2}{\delta}(r-a)} r\, d\varphi dr dz$$

$$\langle W_H\rangle_{b\leq r\leq b+d} = \frac{\mu'}{4}\int_V |\overline{H}|^2\, dV = \frac{\mu_0}{4}\int_{r=b}^{b+d}\int_{\varphi=0}^{2\pi}\int_{z=0}^{L} \frac{AA^*}{\eta^2 b^2} e^{-2\alpha z} e^{\frac{-2}{\delta}(r-b)} r\, d\varphi dr dz$$

$$\langle W_H\rangle_{cond} = \langle W_H\rangle_{r\leq a} + \langle W_H\rangle_{b\leq r\leq b+d}$$

Solution Problem 6:

a) In order to obtain the permittivity of the dielectric material placed between conductors, the expression of the capacitance is used:

$$C = \frac{\varepsilon'}{k_g} = 78 \times 10^{-12} \ {}^{F}/_{m}$$

where ε' is the real part of the permittivity and the term k_g in a coaxial can be expressed as:

$$k_g = \frac{1}{2\pi} \ln \frac{b}{a}$$

It is known that, in the case of ideal dielectric, the permittivity is $\varepsilon' = \varepsilon_0 \varepsilon_r'$, where ε_0 is the vacuum permittivity ($\varepsilon_0 = 8.85 \times 10^{-12} \ {}^{F}/_{m}$) and ε_r' is the relative permittivity.

Although the ratio b/a is unknown, it is also known that the characteristic impedance Z_0 can be stated as:

$$Z_0 = \eta \cdot k_g = \frac{\eta_0}{\sqrt{\varepsilon_r'}} \cdot \frac{1}{2\pi} \ln \frac{b}{a} = 50 \ \Omega$$

So the $\ln \frac{b}{a}$ can be obtained as:

$$\ln \frac{b}{a} = Z_0 \frac{2\pi \sqrt{\varepsilon_r'}}{\eta_0}$$

The expression of the capacitance that was expressed at the beginning can be rewritten now as:

$$C = \frac{\varepsilon'}{k_g} = \frac{\varepsilon_0 \varepsilon_r'}{\frac{1}{2\pi} \ln \frac{b}{a}} = \frac{\varepsilon_0 \varepsilon_r'}{\frac{1}{2\pi} Z_0 \frac{2\pi \sqrt{\varepsilon_r'}}{\eta_0}} = \frac{\varepsilon_0 \sqrt{\varepsilon_r'}\, \eta_0}{Z_0} = \frac{\varepsilon_0 \sqrt{\varepsilon_r'} \sqrt{\frac{\mu_0}{\varepsilon_0}}}{Z_0} = \frac{\sqrt{\varepsilon_r'} \sqrt{\varepsilon_0 \mu_0}}{Z_0}$$

where μ_0 is the vacuum permeability ($\mu_0 = 4\pi \times 10^{-7} \ {}^{H}/_{m}$). Finally, the relative permittivity is obtained:

$$\varepsilon_r' = \frac{C^2 Z_0^2}{\varepsilon_0 \mu_0} = 1.365$$

Now that the relative permittivity is known, the ratio b/a can be obtained from:

$$\ln \frac{b}{a} = Z_0 \frac{2\pi \sqrt{\varepsilon_r'}}{\eta_0} = 50 \frac{2\pi \sqrt{1.368}}{120\pi} = 0.973$$

Lastly, a and b are obtained as:

$$a = \frac{2.74}{2} = 1.37 \ mm, \qquad b = 3.62 \ mm$$

The values obtained agree with the data provided in the datasheet.

b) Using the data of the attenuation, so that the losses of the conductor are related to the attenuation of the coaxial cable:

$$At(dB) = 10\log\frac{W_{T_1}}{W_{T_2}} = 10\log\frac{W_{T_1}}{W_{T_1}e^{-2\alpha_c L}} = 20\,\alpha_c L\log e$$

α_c is the attenuation constant in the conductor in $({}^{Nep}/_{m})$ and L is the length ($L = 1\ km$) , so:

$$\alpha_c = \frac{At(dB)}{L20\log e} = \frac{357.6}{8.686\times 10^3} = 0.0412\ \frac{Nep}{m}$$

In high frequency, the impedance of the conductor (Z_{cond}) can be approximated as the multiplication of the conductor's internal impedance per unit length and unit width (Z_S) , and the term k_j [Collin, p.87]:

$$Z_S = \frac{1+j}{\sigma\,\delta}$$

$$k_j = \frac{\oint_{C_\sigma}|j_s|^2\,dl}{\left|\oint_{C_\sigma} j_s\,dl\right|^2}$$

where C_σ is the conductor outline and $\bar{J}_s$ is the superficial density current in the conductor ($\sigma\ \to\infty$). When $\bar{J}_s$ is uniform, as in the coaxial case, $k_j = 1/perimeter$, so :

$$Z_{cond} = Z_S\ \cdot k_j = \begin{cases}\dfrac{1+j}{\sigma_1\delta_1}\,\dfrac{1}{2\pi a}\\[2ex] \dfrac{1+j}{\sigma_2\delta_2}\,\dfrac{1}{2\pi b}\end{cases}$$

where k_j has been particularized for the radius of both conductors [Ramo, p.154]. The resistance per unit length in high frequency is:

$$R_{cond}({}^{\Omega}/_{m}) = R_{C_1} + R_{C_2} = \frac{1}{\sigma_1\delta_1}\,\frac{1}{2\pi a} + \frac{1}{\sigma_2\delta_2}\,\frac{1}{2\pi b} = 2\alpha_c\mathcal{R}_e[Z_0] = 2\alpha_c\,k_g\mathcal{R}_e[\eta]$$

From this and assuming the same conductivity in both conductors, the attenuation constant due to non-ideal conductors is:

$$\alpha_c({}^{\text{Nep}}/_{m}) = \frac{R_{cond}}{2k_g\mathcal{R}_e[\eta]} = \frac{\dfrac{1}{a}+\dfrac{1}{b}}{2\sigma\,\delta\ \mathcal{R}_e[\eta]\ln\dfrac{b}{a}} \qquad [3]$$

$$\alpha_c({}^{\text{Nep}}/_{m}) = \frac{\left(\dfrac{1}{a}+\dfrac{1}{b}\right)\sqrt{\pi f\mu_0}}{2\sqrt{\sigma}\ \mathcal{R}_e[\eta]\ln\dfrac{b}{a}}$$

As it has been pointed out, both conductors are supposed to have the same characteristics so:

$$if\ \sigma_1 = \sigma_2 \Rightarrow\ \delta_1 = \delta_2 = \delta = \frac{1}{\sqrt{\pi f\mu_0\sigma}}$$

Lastly, the conductivity of the conductor can be obtained as:

$$\sigma = \left(\frac{\left(\frac{1}{a} + \frac{1}{b}\right) \sqrt{\pi f \mu_0}}{2\alpha_c \, \mathcal{R}_e[\eta] \ln \frac{b}{a}} \right)^2 = 3.47 \times 10^7 \, \frac{S}{m}$$

That is approximately the aluminum conductivity $\sigma_{Al} = 3.5 \times 10^7 \, S/m$.

c) It can be observed that equation [2] in question e) of Problem 5 is the same expression as the one obtained in equation [3]. This is because the Leontovich approximation is used to obtain the resistance per unit length of the conductors in high frequency.

In order to see if the Leontovich boundary condition can be applied, the thickness t of the outer conductor is going to be obtained. Since the outer and inner radiuses of the conductor are known, the thickness is obtained as:

$$t = r_{ext} - b = 4.04 - 3.62 = 0.42 \, mm$$

$$\delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma}} = 1.12 \, \mu m$$

As can be observed, $t \gg \delta$, $a \gg \delta$ *and* $b \gg \delta$, therefore the Leontovich approximation can be applied.

d) The total attenuation constant is the summation of the attenuation constants in the conductor and in the dielectric:

$$\alpha = \alpha_c + \alpha_d$$

The propagation constant for ideal conductors ($\sigma \rightarrow \infty$) is:

$$\gamma = jw\sqrt{\mu_0 \varepsilon_0 \varepsilon_r'(1 - j \tan \delta)} = jw\sqrt{\mu_0 \varepsilon_0 \varepsilon_r'}\sqrt{1 - j3 \times 10^{-4}}$$

In the case of low losses, the mathematical approximation of the square root for small values of x can be used:

$$if \ x \rightarrow 0 \Rightarrow \sqrt{1 + x} \approx 1 + \frac{x}{2}$$

Therefore, the propagation constant can be approximated as:

$$\gamma \approx jw\sqrt{\mu_0 \varepsilon_0 \varepsilon_r'}(1 - \frac{j3 \times 10^{-4}}{2})$$

The attenuation constant in the dielectric is characterized as:

$$\alpha_d = \frac{2\pi \, 5.8 \times 10^9}{3 \times 10^8} \sqrt{1.365} \frac{3 \times 10^{-4}}{2} = 0.0213 \, \frac{Nep}{m}$$

The attenuation constant of the coaxial cable is:

$$\alpha = \alpha_c + \alpha_d = 0.0412 + 0.0213 = 0.0625 \, \frac{Nep}{m}$$

Expressed in dB/km:

$$\alpha = 542.9\frac{dB}{km}$$

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