RECONFIGURATION OF SPACECRAFT FORMATIONS IN THE VICINITY OF LIBRATION POINTS

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ABSTRACT

Formation flying is an important concept demanding reliable and accurate techniques for the reconfiguration or the deployment of the spacecraft. This paper is a contribution to these set of techniques considering a finite element approach. The formulation allows to compute proximity maneuvers for the spacecraft and, at the same time, to incorporate optimal control techniques which include collision avoidance or many other types of constraints. The computations have been carried out about libration point orbits of the Sun-Earth system and using linearized equations about a nonlinear nominal base orbit. The trajectories obtained for this model have been corrected considering a full nonlinear model (JPL-ephemeris). Also random errors have been added to the nominal reconfiguration maneuvers to simulate small thrust errors. The presented methodology proves to be robust in both the computation of the nominal reconfiguration maneuvers and in the correction of the perturbations.

1 Introduction

In recent years, the idea of constellations or formations of spacecraft have had an important role in technology for some science and astrophysical missions. One of the applications of the formations of spacecraft is the use of formations with baselines of hundreds of meters which form a virtual satellite.

At the same time, missions about the libration points have had interest in the last years, since the ISEE3 was launched in 1978 to study the Sun in a Halo orbit about the Sun-Earth+Moon L_1 point. The special properties of the dynamics around the Lagrangian points L_1 and L_2 give us good places to put spacecraft ([1, 2]). As L_1 is located between the Sun and the Earth, it has been used to missions which study the Sun, such as ISEE3 or SOHO. L_2 keeps the Sun and the Earth at the same direction, and it is a good place to put spacecraft to study deep space, such as ACE, Genesis or the concepts of Darwin or TPF [3, 4, 5].

On the other hand, formation flight for spacecraft demands a lot in terms of technology hardware requirements and also in terms of mission design. Astrodynamic problems like station keeping maintaining relative positions during observation periods, deployment and reconfigurations of formations have to be addressed and solved in an efficient way.

Currently, the existing literature on formation flight trajectory design about the collinear libration points focuses mainly on rough estimates of the mission cost, like in the works of Beichman, Gómez, Lo, Masdemont, Museth and Romans [5, 6], or on the control strategies for the formation. With respect to this, Folta, Hartman, Howell, Marchand [7, 8] consider the formation control of the MAXIM mission about L_2 and more control techniques can be found in Farrar, Thein and Folta [9] and references therein. Also Elosegui, Gómez, Marcote, Masdemont, Mondelo, Perea and Sánchez have studies concerning the transfer of the formation, suitable geometries and control procedures (see [10, 11, 12]).

Also reconfigurations and deployments have been mostly considered for formations about the Earth. Representative techniques of proximity maneuvering have been studied by McInnes [13, 14] by means of

Lyapounov functions and by Hadaegh, Beard, Wang and McLain [15, 16] considering rotations of the formations or using a sequence of simple maneuvers. In general, these techniques have to be adapted someway to the particular reconfiguration problem under consideration. They require some tuning of the parameters or to select an appropriate sequence of motions in a set of permutation possibilities.

In this paper we consider the reconfiguration of a formation in a more or less general way. This is a procedure that presumably will need to be done many times in its lifetime. The deployment of the formation, pointing it to different goals or changing its pattern for different purposes are examples of complex maneuvers that can be studied with the procedure.

The work is carried out computing and obtaining nominal reconfiguration maneuvers in a linearized model about a nonlinear libration point orbit of the restricted three body problem (RTBP) and presenting a methodology developed by the authors in a recent paper. This methodology essentially consists on discretizing the time interval in a set of intervals (mesh) where the nominal maneuvers for the selected reconfiguration are computed in such a way that they minimize a functional related to the fuel expenditure of the spacecraft, and of course, avoiding the collision risk.

Then the paper focus on an example where nonlinearities are added in order to see the magnitude and influence of the deviations with respect to the pure linearized model. For this purpose we consider JPL ephemeris and some random errors in the execution of the maneuvers. The nominal maneuvers demand now of small corrective maneuvers which are compared with them in magnitude and percentage.

2 Methodology

The objective of this paper is to compute the reconfiguration of a set of N spacecraft about a nominal Halo orbit, in a fixed time T. These nominal reconfiguration trajectories will be obtained using a linearized model about a nominal Halo orbit of 120000 km of z-amplitude. The methodology we follow to obtain the reconfiguration trajectories is fully presented in [17, 18] and here we give just a summary.

Let us consider the linearized equations for the motion about the selected Halo orbit of the RTBP. These equations have the form,

$$\dot{\mathbf{X}}(t) = \mathbf{A}(t)\mathbf{X}(t), \qquad (1)$$

where $\mathbf{A}(t)$ is a 6×6 matrix and $\mathbf{X}$ is the state of the satellite. The origin of the reference frame for the $\mathbf{X}$

coordinates is the nominal point in the halo orbit at time t and the orientation of the axis is parallel to the ones of the RTBP.

Since Halo orbits are periodic orbits, $\mathbf{A}(t)$ is also a periodic matrix. The matrix $\mathbf{A}(t)$ has as well some properties related to the characteristics of this kind of orbits: for a fixed value of t, it has six eigenvalues, two of them are real with opposite sign (the ones which give the hyperbolic part to the Halo orbit) and the other 4 ones are pure imaginary numbers and conjugated in pairs (the ones which are related with the rotations about the orbit), as can be seen in [19]. In the case of other libration orbits, this is not exactly in this way, but the hyperbolic and rotation characteristics are maintained.

Since the aim of this work is to perform reconfigurations of a set of spacecraft, the spacecraft must be subjected to a control. Let us consider a control applied to the i-th spacecraft in the formation. Then the equations of motion for this spacecraft are of the form,

$$\dot{\mathbf{X}}_i(t) = \mathbf{A}(t)\mathbf{X}_i(t) + \bar{\mathbf{U}}_i(t),$$

where the control $\bar{\mathbf{U}}_i(t)$ only affects to the acceleration, i.e. it is of the form $\bar{\mathbf{U}}_i(t) = (0, 0, 0, \bar{u}_i^x(t), \bar{u}_i^y(t), \bar{u}_i^z(t))^t$.

Adding the initial and final states of the spacecraft in the reconfiguration problem, we obtain the equations,

$$\begin{cases} \dot{\mathbf{X}}_i(t) = \mathbf{A}(t)\mathbf{X}_i(t) + \bar{\mathbf{U}}_i(t) \\ \mathbf{X}_i(0) = \mathbf{X}_i^0 \\ \mathbf{X}_i(T) = \mathbf{X}_i^T \end{cases} \qquad (2)$$

where $\mathbf{X}_i^0$ and $\mathbf{X}_i^T$ stand for the initial and final state of the i-th spacecraft of the formation. The goal is to find optimal controls, $\bar{\mathbf{U}}_1, \ldots, \bar{\mathbf{U}}_N$, subjected to given constraints, being a fundamental one the collision avoidance.

Using the properties of the Halo orbits inherited by the matrix $\mathbf{A}(t)$ as discussed previously, we can perform a change of variables and to simplify our problem [18] by uncoupling the equations of motion. For each spacecraft (the index i is dropped now for clarity) the equations (2) cast into six equations, each one of the form,

$$\begin{cases} \ddot{x}(t) + \lambda(t)\dot{x}(t) + \tau(t)x(t) = u(t), \\ x(0) = x_0, \qquad x(T) = x_T, \\ \dot{x}(0) = v_0, \qquad \dot{x}(T) = v_T. \end{cases} ,$$

where x refers now to a variable that is a function of the state of the spacecraft. We have to mention that this process of uncoupling variables is not strictly necessary. It is suitable for periodic or quasiperiodic libration point orbits to improve the computational

cost and probably it can be suitable as well when considering formation flight about the Earth, but in general terms, one could skip this step.

Our objective is to find for each spacecraft the set of six controls $u(t)$ which perform the required reconfiguration using a minimum fuel consumption and avoiding collisions. The methodology we implement to obtain these controls is based on the finite element methodology. Our domain is the time interval $[0, T]$ considered for the reconfiguration. We split this time interval in M elements, which are subintervals of the domain. The elements can be of different length, depending on the nature of the reconfiguration problem or on the level of accuracy one wants to attain for the trajectories. Each element joins to the neighbour ones by means of a node, a distinguished point at each end of the subinterval. The nodes are the places where the maneuvers (delta-v) will be performed, and so, the objective of the problem is essentially to determine the maneuvers we need to apply at the nodes.

More precisely, the finite element formulation is carried out for each variable x in an element Ω^k (from time t_k to t_{k+1}) obtaining the equations,

$$
\begin{pmatrix} K_{1,1}^k & K_{1,2}^k \\ K_{2,1}^k & K_{2,2}^k \end{pmatrix} \begin{pmatrix} x_k \\ x_{k+1} \end{pmatrix} = \begin{pmatrix} \Delta u_k^k \\ \Delta u_{k+1}^k \end{pmatrix},
$$

where the values $K_{i,j}^k$ are known (computed from $\lambda(t)$ and $\tau(t)$ evaluated inside Ω^k) and the nodal variables x_k, x_{k+1} of $x(t)$ giving the trajectory, as well as the controls Δu_k^k and Δu_{k+1}^k, are unknown.

Finally, assembling the elements of the mesh by means of the ordinary procedure of the finite element method we end up with a set of $6N$ systems of linear equations of the form (we denote $K_i = K_{2,2}^i + K_{1,1}^{i+1}$),

$$
\begin{pmatrix} K_0 & K_{1,2}^1 & 0 & & \\ K_{2,1}^1 & K_1 & K_{1,2}^2 & & \\ \ddots & \ddots & \ddots & & \\ & K_{2,1}^{M-3} & K_{M-3} & K_{1,2}^{M-2} & \\ & 0 & K_{2,1}^{M-2} & K_{M-2} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{M-2} \\ x_{M-1} \end{pmatrix} +
$$

$$
+ \begin{pmatrix} K_{2,1}^0 x_0 \\ 0 \\ \vdots \\ 0 \\ K_{1,2}^{M-2} x_T \end{pmatrix} = \begin{pmatrix} \Delta u_1 \\ \Delta u_2 \\ \vdots \\ \Delta u_{M-2} \\ \Delta u_{M-1} \end{pmatrix}
$$

which relate the reconfiguration trajectories of the spacecraft, determined by the M nodal x_j values for each satellite and component, with the nodal maneuvers Δu_j applied. In order to simplify the notation, we have used $K^i = K_{2,2}^i + K_{1,1}^{i+1}$.

We then reduce our reconfiguration problem to an optimization problem with constraints. The function to be minimized has to be related to the fuel consumption, and the constraints, in the examples considered, only take into account collision avoidance.

Since the fuel expenditure of spacecraft is directly related to the norm of delta-v, the functional we want to minimize is

$$
J_1 = \sum_{i=1}^{N} \sum_{k=0}^{M_i} \rho_{i,k} ||\Delta \mathbf{v}_{i,k}||, \tag{3}
$$

where $|| * ||$ denote the Euclidean norm and $\rho_{i,k}$ are weight parameters that can be used, for instance, to penalize the fuel consumption of selected spacecraft with the purpose of balancing fuel resources (here for clarity we consider that $\rho_{i,k}$ multiplies the modulus of the delta-v, but in a similar we can impose a weight on each component).

Of course the functional of equation 3 is ill conditioned to compute derivatives when delta-v values are small (and our objective is precisely to find the delta-v as small as possible). In order to avoid this problem, we start minimizing a functional without ill conditioning problems and also somewhat related to fuel consumption,

$$
J_2 = \sum_{i=1}^{N} \sum_{k=0}^{M_i} \rho_{i,k} ||\Delta \mathbf{v}_{i,k}||^2. \tag{4}
$$

From the optimal value of this functional, the mesh is adapted in order to obtain an optimal solution for (3) controlling and avoiding the ill condition (more details in [18] or [20]).

Like the minimization of fuel consumption, collision avoidance is a key problem to be considered in the reconfiguration of spacecraft. Collision avoidance enters in the formulation of our optimization problem as a set of constraints.

One of the advantages of the methodology we use is that we can include easily more constraints in the problem. Due to the nature of the optimization problems, these constraints can be equality constraints, this is $c(\mathbf{x}) = 0$, or inequality constraints, $c(\mathbf{x}) > 0$. Our most important set of constraints is collision avoidance, but we can add some other constraints to the optimization problem, to obtain trajectories with geometrical confinements or with thrust restrictions.

As a final remark, the methodology to avoid collision between spacecraft consists on assuring a mini-

mum distance between them during the reconfiguration time. To accomplish this objective, we surround each spacecraft with an imaginary sphere of radius half the security distance. The constraint we impose is that in all the reconfiguration process the spheres do not intersect, accepting only a tangency point between them.

3 Dealing with nonlinearities

The methodology of the previous section gives us reconfiguration trajectories computed by means of linearized equations about a nonlinear libration point orbit. Our objective now is to check up to which extent the model is good enough to obtain nominal trajectories when considering further nonlinearities. Of course, in principle this is something directly related to the size of the formation. For small diameter formations the effects of the nonlinearities of the model will be almost negligible while they will be much more apparent when the diameter is big.

In this section more than to evaluate and quantify this fact in general situations, and to decide in what conditions the methodology is good enough and in which ones is not, we will present a technique that could be used to deal with nonlinearities. In particular we will focus on the results for an ordinary example. The main idea is to consider the trajectory of the previous methodology as the nominal one for the spacecraft, and to add some corrective maneuvers between the nodes to force the spacecraft to attain the nominal nodal states.

Since we work with small formations in size when compared to the Halo orbit, the linearized equations give us a good approximation for the nonlinear model. Our objective is to give a way to study how the truncated nonlinear terms, as well as other perturbations, affect to the nominal reconfiguration trajectory, and the corrections that must be done to the nominal maneuvers (*corrective maneuvers*) in order to reach the same goal. The study of the influence of these new nonlinearities is done in two steps: the first one, taking into account the full RTBP equations, and the second one using JPL-ephemeris.

The corrective maneuvers will be computed using a strategy similar to [6]. Our nominal trajectory, the one that the spacecraft must follow, is the trajectory obtained with the finite element methodology and consists of some nodal states that the spacecraft must follow, plus the nominal maneuvers to be applied at the nodes in order to follow the states. When we consider a nominal trajectory in the full RTBP or the JPL-ephemeris model (for instance by means of
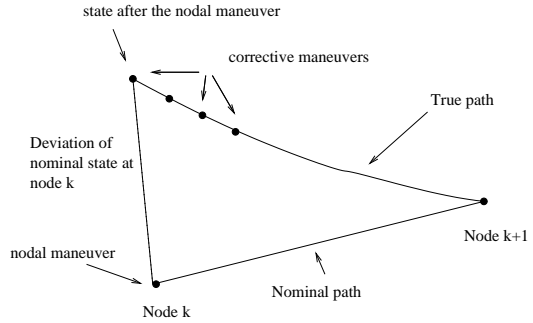

Figure 1: Corrective maneuvers inside the element Ω^k (from node k to node $k+1$).

giving the initial condition plus the set of maneuvers at the nodal times), or when we include an execution error in the maneuvers, we obtain a new trajectory differing from the nominal one. Let us call it the true trajectory. The idea is that in each element, the difference between the nominal and the true trajectories will be corrected by the addition of some small corrective maneuvers (see figure 1).

The correction of the trajectory that we implement uses a fixed number of corrective maneuvers inside the element Ω^k, $\Delta\hat{v}_k^0, \Delta\hat{v}_k^1, \ldots, \Delta\hat{v}_k^n$, which will be applied at some given times $\hat{t}_0, \hat{t}_1, \ldots, \hat{t}_n$. Eventually, these maneuvers should be applied as soon as possible in order to avoid the inherent exponential grow of errors with respect to time, and will be computed in order to satisfy that the state of spacecraft at node $k+1$ is the corresponding one of the nominal path.

The maneuvers $\Delta\hat{\mathbf{v}}_k^j$ are obtained solving an equation, which in the case of two corrective maneuvers is given by,

$$\phi_{(1-\alpha)\Delta t}\left[\phi_{\alpha\Delta t}\left(\mathbf{x}_k+\begin{pmatrix}0\\\Delta\hat{\mathbf{v}}_k^1\end{pmatrix}\right)+\begin{pmatrix}0\\\Delta\hat{\mathbf{v}}_k^2\end{pmatrix}\right]=$$
$$=\mathbf{x}_{k+1},$$

where $\mathbf{x}_k$ is the initial state, $\mathbf{x}_{k+1}$ the final state and ϕ_t is the time-t flow of the equations of motion. In case of considering more corrective maneuvers, the controller is constructed in a similar way.

The delta-v are also chosen to minimize the functional:

$$\sum_{j=0}^{n_i} 2^{-j}||\Delta\hat{v}_i^j||^2,$$

where the weights 2^{-j} grant someway that the corrective delta-v decay at each step approximately by a factor of two.

4

4 Case example using the procedure

In this section we present a case example to see how the techniques apply. The problem consists in switching the position of two spacecraft. In a first step we consider the direct application of the methodology to obtain low thrust trajectories for this reconfiguration example. In a second step we see how we can study the way that the nonlinearities of the model affect the nominal solution. Finally, we consider also random execution errors in the nominal maneuvers. We see how these errors are corrected by the methodology and the cost associated with such corrections.

Switching positions of two spacecraft

Let us consider two spacecraft about a Halo orbit of z-amplitude 120000 km about L_1 in the Sun-Earth system. One of the spacecraft is located 100 meters far from the orbit, in the positive direction of the X coordinate of the local frame which is parallel to the one of the RTBP. The second spacecraft is located symmetrically to the first one with respect to the orbit, i.e., in the point $(-100, 0, 0)$ in local coordinates centered in the Halo orbit nominal position.

The reconfiguration we consider is a switch in position between these two spacecraft in 8 hours time, with a security distance of 20 meters. For each spacecraft, to attain the new desired position, the optimal trajectory is essentially a bang-bang control. But if both spacecraft follow a bang-bang trajectory at the same time, the result is a collision after 4 hours of the reconfiguration process.

We have computed the nominal reconfiguration using different number of initial nodes in the mesh. First of all, we start with 10 elements and then we use the solution with 10 elements as a initial seed to obtain the solution using 20 elements and so on. The delta-v obtained using 10, 20, 50 and 100 elements are shown in figure 2. We see that when the number of elements is increasing, the delta-v of the reconfiguration tend to a low thrust profile.

Nonlinearities of the model

We now take as a parameter of the reconfiguration the baseline between the spacecraft. In the previous computation, we consider the switch between two spacecraft that are initially at a distance of 200 meters. Now we consider the same reconfiguration process, but changing the distance between them and also the security distance. Just for illustrative purposes of the methodology, for each reconfiguration the

Dist. (m)	Δv_L	$\Delta \hat{v}_{max}$	$\Delta \hat{v}$	% $\Delta \hat{v}$
200	3.78	5.3×10^{-4}	0.023	0.61
350	6.64	6.4×10^{-4}	0.042	0.63
500	9.50	7.1×10^{-4}	0.071	0.74
750	14.32	2.2×10^{-3}	0.145	1.01
1000	19.05	3.0×10^{-3}	0.234	1.23
2000	39.13	8.9×10^{-3}	0.587	1.50
5000	94.37	2.5×10^{-2}	1.719	1.82
7500	142.18	8.2×10^{-2}	2.890	2.03
10000	188.70	2.1×10^{-1}	3.911	2.07

Table 1: Corrective maneuvers (n=3) for switching two spacecraft as in the case example depending on their initial base distance. Model equations are given by JPL ephemeris and all the delta-v are given in cm/s.

Dist. (m)	Δv_L	$\Delta \hat{v}_{max}$	$\Delta \hat{v}$	% $\Delta \hat{v}$
200	3.78	3.5×10^{-4}	0.021	0.55
350	6.64	4.2×10^{-4}	0.039	0.59
500	9.50	5.7×10^{-4}	0.063	0.66
750	14.32	1.2×10^{-3}	0.133	0.93
1000	19.05	2.1×10^{-3}	0.202	1.03
2000	39.13	6.5×10^{-3}	0.557	1.42
5000	94.37	1.9×10^{-2}	1.673	1.77
7500	142.18	6.8×10^{-2}	2.820	1.98
10000	188.70	9.9×10^{-2}	3.829	2.03

Table 2: Corrective maneuvers (n=4) for switching two spacecraft as in the case example depending on their initial base distance. Model equations are given by JPL ephemeris and all the delta-v are given in cm/s.

security distance has been taken 10% of the initial distance between spacecraft. For each of the reconfigurations, considering nonlinear model equations given by JPL ephemeris, we compute the nominal delta-v required as a result of using the linearized equations about the nonlinear halo orbit of the RTBP (Δv_L), and some magnitudes measuring the corrective maneuvers like: the maximum of the corrective maneuvers ($\Delta \hat{v}_{max}$), the total amount of corrective maneuvers ($\Delta \hat{v}$) and the percentage of them with respect to Δv_L. On each of the elements, we put 3, 4 and 5 corrective maneuvers. Numerical results are shown in tables 1, 2 and 3, while figure 3 represents the percentage of corrective maneuvers depending on the initial spacecraft distance, using 4 corrective maneuvers on each element.

Corrections of thrust errors

Let us consider now the case where the nominal maneuvers are executed with some random error. At
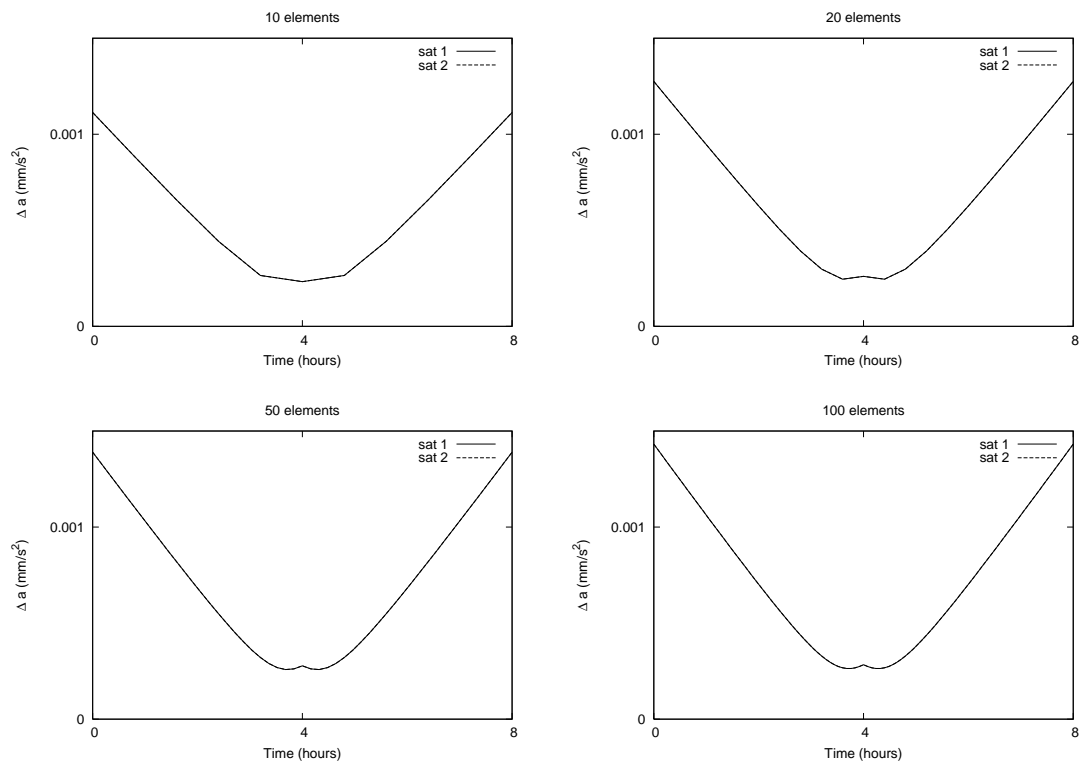
Figure 2: Delta-v divided by the element length for the case example of switching two spacecraft, using 10, 20, 50 and 100 elements. The results tend to a low-thrust profile.

Dist. (m)	Δv_L	$\Delta \hat{v}_{max}$	$\Delta \hat{v}$	% $\Delta \hat{v}$
200	3.78	3.1×10^{-4}	0.020	0.53
350	6.64	3.9×10^{-4}	0.037	0.56
500	9.50	5.3×10^{-4}	0.060	0.63
750	14.32	9.8×10^{-4}	0.131	0.91
1000	19.05	1.3×10^{-3}	0.189	0.99
2000	39.13	5.6×10^{-3}	0.549	1.40
5000	94.37	1.2×10^{-2}	1.667	1.77
7500	142.18	5.7×10^{-2}	2.813	1.98
10000	188.70	8.0×10^{-2}	3.820	2.02

Table 3: Corrective maneuvers (n=5) for switching two spacecraft as in the case example depending on their initial base distance. Model equations are given by JPL ephemeris and all the delta-v are given in cm/s.
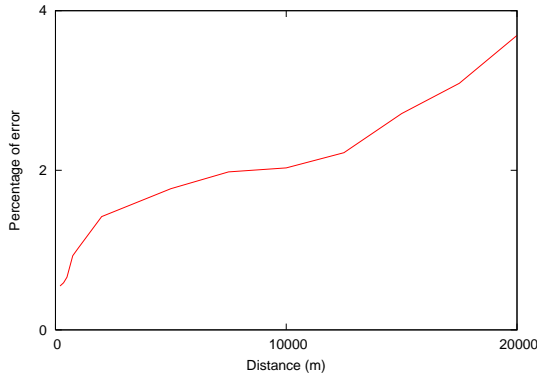

Figure 3: Percentage of corrective maneuvers with respect to the nominal amount of maneuvers for the case example of switching two spacecraft depending on their initial base distance and considering as the new model JPL ephemeris.

Δv_L	% p	n	$\Delta \hat{v}_{LRmax}$	$\Delta \hat{v}_E$	% $\Delta \hat{v}_E$
0.63	0	4	8.7×10^{-5}	0.007	1.06
0.63	2	4	2.5×10^{-4}	0.011	1.80
0.63	4	4	3.2×10^{-4}	0.019	3.08
0.63	6	4	6.5×10^{-4}	0.029	4.62
0.63	8	4	9.4×10^{-4}	0.034	5.41
0.63	10	4	1.3×10^{-3}	0.042	6.64
0.63	12	4	2.0×10^{-3}	0.056	8.84
0.63	14	4	2.5×10^{-3}	0.064	10.16
0.63	16	4	2.8×10^{-3}	0.070	11.13
0.63	18	4	3.1×10^{-3}	0.075	11.92
0.63	20	4	3.6×10^{-5}	0.085	13.42

Table 4: Corrective maneuvers for the execution error in the model equations given by JPL ephemeris.

each node we consider the state given by the initial computations considering the linear equations about the nonlinear halo orbit and the maneuver to be applied. We consider that the thruster has an error on the magnitude of the delta-v. To simulate such fact we scale each maneuver by a factor which depends on the percentage of the error (a parameter p we can choose) and a random variable η, which follows a normal distribution $N(0,1)$ (i.e., at each node we apply a scaled delta-v: $\Delta \bar{v} = \Delta v(1 + \eta p)$, where Δv is the nominal maneuver obtained by the finite element methodology described).

Considering the equations motion given by the JPL ephemeris, we perform corrective maneuvers to account for two errors at the same time: the truncation error due to the nonlinear equations and the engine error. On each element, we insert n small delta-v to attain the nodal position and velocity given by the finite element methodology solution at the end of the element.

In table 4, we present some results for a low thrust example. In this case, we maintain $n = 4$ and change p, the percentage of error of the thrust. For each value of the parameters (p and n), we compute the mean for $\Delta \hat{v}_{LRmax}$ (the maximum delta-v of corrections), $\Delta \hat{v}_E$ (the total delta-v of corrections) and % in $\Delta \hat{v}_E$ (the percentage of the corrective delta-v with respect to the nominal delta-v obtained in the initial computations) with 500 simulations. We have not continued after 20% of execution error since the corrections would be considered very big and the accuracy of the thruster too poor. However, we have performed computations up to this big execution error to see the robustness of the methodology.

In figure 4 we show the result of a simulation using 25 different examples of reconfigurations and making
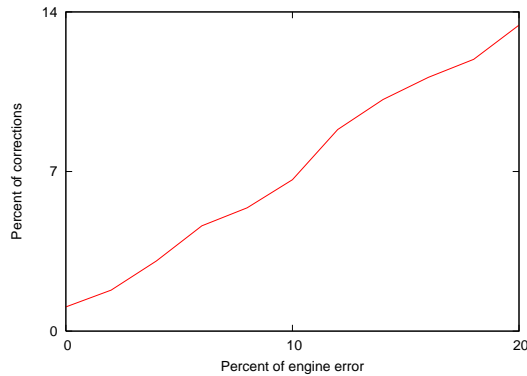
Figure 4: Percentage of the corrective maneuvers with respect to the total amount of nominal delta-v of the reconfiguration, as a function of p. The plot is obtained using a test bench of 25 reconfigurations, with $n = 4$, and 500 simulations in each scenario.

500 simulations for each one of the examples. Again we fix $n = 4$, and we study how the percentage of error grows depending on the parameter p. With the hypothesis taken into account, essentially we see a linear behavior with respect to this variable.

5 Conclusions

In this paper we present a systematic methodology to compute reconfigurations of spacecraft formations. The methodology is flexible enough to be tuned for different constraints and also some techniques to deal with nonlinear effects or perturbations can be introduced when needed. Although the computations and example of this paper is carried out in a libration point regime, the methodology is general enough to be applied in other scenarios like for missions about the Earth.

Acknowledgements

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