Circuital Analysis of a Two-port Angular Sector

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ABSTRACT

The objective of this communication is to introduce an element that allows us to analyze two-dimensional diffraction problems in a circuital way. The circuital analysis of electromagnetic problems, together to the segmentation technique, allow to analyse big problems, in an electromagnetic sense, with the additional advantage that a change in a material is readily computed. This makes this method very appropriate for its use in CAD tools.

In this communication the problem of a sector considered as a cuadripole is only presented, in such a way that we can see the advantages of the method. It is easy to see how we can enlarge the problem to a four-port net. Finally, we must say that only the TM case is presented here.

I. INTRODUCTION

The method of Generalised Admittance Matrix (GAM) is being one of those most broadly method used in scattering problems. Initially this method was used in closed problems [1]. The scattering matrix (S matrix) was reserved for open problems.

However, nowdays it has been seeing the possibility of using the GAM for open problems [2]. It is starting from the caracterization of simple elements , as the one presented in this communication, and through its admittance matrix, how we can build bigger systems and calculate its scattering. For this last setp you can use the scattering matrix calculate from the admittance matrix.

So far, the MoM and the contour elements method [3-4] have been used to do it. Their problems, in front the proposed circuital analysis, are their little versatility to variations of some of the scatterer elements. With the GAM method, together to the segmentation one, these problems are eliminated. It is important to see that with this last technique -the segmentation one- we can create any structure with cannonical problems. In this case our cannonical problem is the circular sector in cylindrical coordinates.

The objective of this communication will be to outline the problem of the two-port circular sector and its circuital solution as a cuadripole. That is to say, we do not still outline the case of scattering, but it is clear their generalization starting from a four-port circular sector and not two.

II. POSITIONING THE PROBLEM

In figure 1 the geometry of our problem is drawn. We have a dielectric circular sector delimited between radii a and b and angles φ_1 and φ_2. The numeration of the ports is also indicated.

In order to have the full caraterization of the element we should calculate its admittance matrix. This is the matrix that relates the electric and magnetic fields They are given by their modal form and the admittance matrix will relate the coeficients of their serial representation.

For the calculation of each Y_{i1} element we use a shortcircuit in port 2. And, in a

reciprocity way, we can calculate the Y_{i2} elements placing a short circuit in port 1.
Let us see the first case. We know that the potential inside is given by:

$$\Psi = \sum_{\nu} f_{\nu}(r)\cdot a_{\nu} \sin(\nu\cdot(\varphi - \varphi_1)) \tag{1}$$

where $f_\nu(r)$ is a linear combination of Bessel functions of order ν that verifies the boundary conditions in the electric walls placed in r=a and in r=b. These orders can be obtained following an eigenfunction and eigenvalue method similar to the one proposed in [5], but solving in the order ν and not in the wave number.
The a_ν coefficients are obtained from the contour condition placed in port 1, that is the excitation port and where we have the incident field. This incident field is (TM one):

$$E_z = \sum_{m=0}^{\infty} \alpha_m \cdot \sin\left(2\cdot\pi\cdot m\cdot\frac{r-a}{b-a}\right) + \beta_m\cdot\cos\left(2\cdot\pi\cdot m\cdot\frac{r-a}{b-a}\right) \tag{2}$$

where α_m and β_m are the coefficients of the serie. So, thanks to the ortogonality of the Bessel functions, we have:

$$a_\nu = \frac{1}{-j\omega\mu}\cdot\frac{1}{\int_{r=a}^{b} f_\nu^2(r)\frac{dr}{r}}\cdot\frac{1}{\sin(\nu\cdot(\varphi_2-\varphi_1))}\cdot$$

$$\cdot\sum_{m=1}^{\infty}\left[\begin{array}{l}\alpha_m\cdot\int_{r=a}^{b} f_\nu(r)\cdot\sin\left(2\cdot\pi\cdot m\cdot\frac{r-a}{b-a}\right)\frac{dr}{r}+\\ +\beta_m\cdot\int_{r=a}^{b} f_\nu(r)\cdot\cos\left(2\cdot\pi\cdot m\cdot\frac{r-a}{b-a}\right)\frac{dr}{r}\end{array}\right] \tag{3}$$

and knowing that the magnetic field in port 1 is:

$$H_r = \frac{1}{r}\cdot\sum_{\nu} f_\nu(r)\cdot a_\nu\cdot\nu\cdot\cos(\nu\cdot(\varphi_2-\varphi_1)) =$$

$$= \sum_{p=0}^{\infty} C_p\cdot\sin\left(2\cdot\pi\cdot p\cdot\frac{r-a}{b-a}\right) + D_p\cdot\cos\left(2\cdot\pi\cdot p\cdot\frac{r-a}{b-a}\right) \tag{4}$$

where C_p y D_p are the coefficients of the magnetic field, we have that the Y_{11} element of the admittance matrix is:

$$\begin{pmatrix}\overline{C}\\ \overline{D}\end{pmatrix} = Y_{11}\cdot\begin{pmatrix}\overline{\alpha}\\ \overline{\beta}\end{pmatrix} = \begin{pmatrix}\overline{\overline{Y}}_{11}^{(ss)} & \overline{\overline{Y}}_{11}^{(cs)}\\ \overline{\overline{Y}}_{11}^{(sc)} & \overline{\overline{Y}}_{11}^{(cc)}\end{pmatrix}\cdot\begin{pmatrix}\overline{\alpha}\\ \overline{\beta}\end{pmatrix} \tag{5}$$

With the ortogonality of the trigonometric functions is easy to find out each $Y_{11}^{(ab)}$ element. Here we put just one of them:

$$Y_{11_{mp}}^{(ss)} = \frac{2}{b-a}\cdot\sum_{\nu}\nu\cdot\frac{\cos(\nu\cdot(\varphi_2-\varphi_1))}{\sin(\nu\cdot(\varphi_2-\varphi_1))}\cdot\frac{-1}{j\omega\mu}\cdot\frac{1}{\int_{r=a}^{b} f_\nu^2(r)\frac{dr}{r}}\cdot$$

$$\cdot\left[\int_{r=a}^{b} f_\nu(r)\cdot\sin\left(2\cdot\pi\cdot m\cdot\frac{r-a}{b-a}\right)\cdot\frac{dr}{r}\right]\cdot\left[\int_{r=a}^{b} f_\nu(r)\cdot\sin\left(2\cdot\pi\cdot p\cdot\frac{r-a}{b-a}\right)\cdot\frac{dr}{r}\right] \tag{6}$$

Following the same method we can calculate the Y_{21} element from the magnetic field in port 2.

By symmetry considerations, we can state that $Y_{12}=-Y_{21}$ and that $Y_{22}=-Y_{11}$

III. CONNECTING NETS

One of the biggest advantages of this proposed method is their flexibility in front some dielectric variations. We only need to recalculate the admittance matrix for the element that changes. After that we need to connect all the elements again in order to have all the system well characterized. For example, let us see figure 2. We can have the two sectors characterized by their admitance matrix and we need the global one.

We can state that, when in a N-port net, ports k and l are united we can put the new admittance matrix and each element as:

$$\overline{\overline{Y}}'_{pq}=\overline{\overline{Y}}_{pq}+\left(\overline{\overline{Y}}_{pk}+\overline{\overline{Y}}_{pl}\right)\cdot\left(\overline{\overline{Y}}_{lk}+\overline{\overline{Y}}_{ll}-\overline{\overline{Y}}_{kl}-\overline{\overline{Y}}_{kk}\right)^{-1}\cdot\left(\overline{\overline{Y}}_{kq}+\overline{\overline{Y}}_{lq}\right) \qquad (7)$$

IV.RESULTS

To check this method the field inside the structure of figure 3 has been calculated. The radii interior and exterior are a=0.1989 m and b=2·a. The angles are $\varphi_1=60°$ and $\varphi_2=30°$ and the work frequency is f=2.4 GHz.

The structure has been excited by port 1with a sinus function: $E_z=\sin\left(\pi\cdot\frac{r-a}{b-a}\right)$.

In the following table we have the ν values that we have in each dielectric media:

	ν					
$\epsilon_{r1}=4$	9.4380	17.3458	21.6997	25.1965	29.0004	33.6787
$\epsilon_{r2}=1$	4.7586	11.1084	15.0001	j·10.8531	j·17.3456	j·22.9305

The field distribution (E_z component) is shown in figure 4. The results has been compared with the FEM ones and they are best than 1% in all the points.

The method is also useful for lossy media. In figure 5 we have the results when the first media has losses (module of E_z component). The following table shows the value of orders ν that are complex because of the losses:

	ν					
$\epsilon_{r1}=4$-j	12.4431-j·7.9757	6.9207-j·14.6648	18.1924-j·5.2770	22.0772-j·4.3669	4.6472-j·22.1619	25.4447-j·4.3407
$\epsilon_{r2}=1$	4.7586	11.1084	15.0001	j·10.8531	j·17.3456	j·22.9305

V.CONCLUSIONS

With this method one can think of calculating the inhomogeneous scattered field by an cilindrycal structure using the circuital method and segmentation. This way we are able to characterize elements separately and then uniting them in order to have the full characterization of the inhomogeneous structure.

VI.REFERENCES

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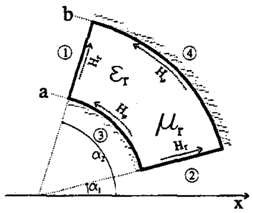

Figure 1.-Two port circular sector

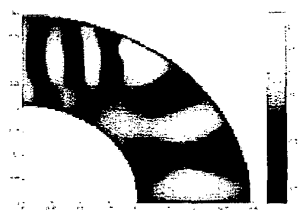

Figure 4.-Lossless circular sector with two media

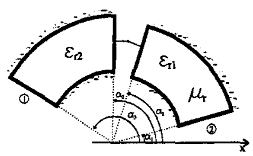

Figure 2.-Union of two cuadripoles

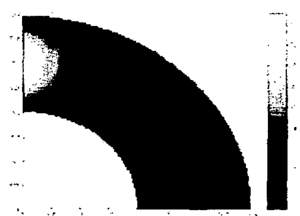

Figure 5.-Two media circular sector. One lossless and other with losses

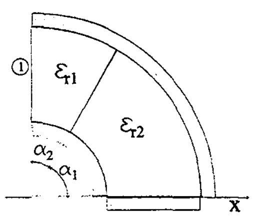

Figure 3.-Two dielectric problem