Capacity results on frequency-selective Rayleigh MIMO channels*

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Abstract: *Recently, MIMO channels arising from the use of multi-element antenna (MEA) systems that use spatial diversity at both the transmitter and the receiver have drawn considerable attention. In this paper we will focus on the information-theoretic channel capacity for frequency-selective channels. Since the capacity is, indeed, a function of the random channel realization, it is treated as a random quantity. The capacity of a channel is highly dependent on factors such as the correlation of the fades, the power allocation strategy used in transmission and the system configuration (number of transmit-receive antennas). Monte-Carlo simulations were performed in order to compare different system configurations and channel statistics.*

Introduction

Recently, MIMO channels arising from the use of multi-element antenna (MEA) systems that use spatial diversity at both the transmitter and the receiver have drawn considerable attention [1][2]. In this paper we will focus on the information-theoretic channel capacity, which is a measure that represents the maximum achievable bit rate free of errors. Since the capacity is, indeed, a function of the random channel realization, it is treated as a random quantity.

It will be shown that an n_T-input, n_R-output multiple antenna channel consists of $n=min(n_T,n_R)$ parallel subchannels or eigenmodes. Therefore, the channel capacity of the MEA can be computed as the sum of the individual subchannel capacities [3]. Depending on the knowledge that the transmitter has about the channel, different power allocation strategies can be used, leading to different capacities [4]. Multiple high bit rate algorithms have been published recently concerning the realistic implementation of techniques to achieve bit rates close to the theoretical capacity, such as [5][6] for the case of unknown channel in transmission and [7][8] for the known channel case.

It will also be illustrated that, when the fades are correlated, the channel capacity can be significantly smaller than when the fades are i.i.d. [9]. The capacity as a function of the frequency-selectivity of the channel is also analyzed.

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Capacity

The capacity of a channel depends completely on the channel realization, noise, and transmitted signal power (along with its distribution). In this section, the expression of the capacity is reviewed for SISO/MIMO frequency-nonselective/selective channels.

Capacity of a SISO channel

For a single-input single-output (SISO) channel, the received signal model is given by

$$y(t) = \alpha \cdot x(t) + n(t) \qquad (1)$$

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where $x(t)$, y(t) and n(t) are the transmitted, received and noise signals respectively, and α is the channel fading. The capacity for such a model, as was derived by Shannon in 1948, is[1]

$$C = \log_2\left(1 + \frac{P}{\sigma_n^2}|\alpha|^2\right). \tag{2}$$

where P is the transmitted power and σ_n^2 is the noise power within the channel band.

In case the channel is frequency-selective, the received signal can be expressed as

$$y(t) = \sum_{l=0}^{L} \alpha_l x(t-\tau_l) + n(t) \tag{3}$$

or, equivalently, after subdividing the signal into flat-fading frequency bands as

$$Y(f) = \alpha(f) \cdot X(f) + N(f) \tag{4}$$

where now $Y(f)$, $\alpha(f)$, $X(f)$ and $N(f)$ stand for the Fourier-transformed $y(t)$, α_l, $x(t)$ and $n(t)$ respectively. The capacity expression is then given by

$$C = \frac{1}{W}\int_W \log_2\left(1 + \frac{P(f)}{\Phi_{nn}(f)}|\alpha(f)|^2\right)df \tag{5}$$

with $\Phi_{nn}(f)$ the noise power spectral density, W the transmitting bandwidth and $P(f) = E\left\{|X(f)|^2\right\}$. The transmitted power is constrained by $\int_W P(f)df \le P_{av}$.

Capacity of a MIMO channel

The received signal model for the case of a flat MIMO channel is

$$\mathbf{y}(t) = \mathbf{H}\mathbf{x}(t) + \mathbf{n}(t), \tag{6}$$

where $\mathbf{x}(t)$, $\mathbf{y}(t)$ and $\mathbf{n}(t)$ are the transmitted, received and noise vectors respectively, and $\mathbf{H}$ is the channel matrix that contains the fading from each transmission antenna to each receiving one.

The capacity of a flat MIMO channel is given by the expression [2][1]:

$$C = \log_2\left[\det\left(\mathbf{I} + \frac{1}{\sigma_n^2}\mathbf{H}\mathbf{Q}\mathbf{H}^H\right)\right], \tag{7}$$

where $\mathbf{Q} = \mathrm{E}\left\{\mathbf{x}(t)\mathbf{x}^H(t)\right\}$. Note that the transmission power constraint can be expressed as $trace(\mathbf{Q}) \le P_{av}$.

For the case of frequency-selective MIMO channels, the capacity is computed by integrating over the utilized bandwidth

$$C = \frac{1}{W}\int_W \log_2\left[\det\left(\mathbf{I} + \frac{1}{\Phi_{nn}(f)}\mathbf{H}(f)\mathbf{Q}(f)\mathbf{H}(f)^H\right)\right]df \tag{8}$$

with the power constraint as $\int_W trace(\mathbf{Q}(f))\,df \le P_{av}$.

Capacity as a random variable

We presume that the communication is carried out using bursts. The burst duration is assumed to be short enough so that the channel can be regarded as essentially fixed during a burst, but long enough that the standard information-theoretic assumption of infinitely long code block lengths is a useful idealization. In this quasi-static scenario, it is meaningful to associate a channel capacity with a given realization of the channel matrix $\mathbf{H}$.

Since the channel capacity is a function of the random channel matrix, it can be regarded as a random quantity whose distribution is determined by the distribution of $\mathbf{H}$. In such cases, an important

[1] The capacity expressions given throughout the paper are normalized with respect to the bandwidth, i.e., they are given in terms of bits/sec/Hz.

measure for the channel capacity is the channel capacity at a given outage probability q, denoted by C_{out}. It simply means that the channel capacity is less than C_{out} with probability q or, in other words, it is greater than C_{out} with probability (1-q). In the following, capacity results for different system configurations will be given by means of cumulative distribution functions (CDF) of the capacity, expressing it as the probability (1-q) that the capacity is greater than C_{out}.

Power Allocation Strategies

Unlike in the case of a flat SISO channel, where there is only one available channel, for the case of frequency-selective and/or MIMO channels, the available transmission power can be distributed over the antennas and/or frequency bands according to different strategies. The power allocation techniques will depend on the knowledge of the channel [4].

Uniform distribution

The uniform distribution of the available transmission power has to be used when the channel is unknown for the transmitter. Note that the channel is always assumed known by the receiver. The capacity expression for the flat MIMO channel is then

$$C = \log_2\left[\det\left(\mathbf{I} + \frac{P/n_T}{\sigma_n^2}\mathbf{H}\mathbf{H}^H\right)\right] \tag{9}$$

and for a frequency-selective MIMO channel

$$C = \frac{1}{W}\int_W \log_2\left[\det\left(\mathbf{I} + \frac{P/(n_T W)}{\mathbf{\Phi}_{nn}(f)}\mathbf{H}(f)\mathbf{H}(f)^H\right)\right]df\ . \tag{10}$$

Water-filling

In some situations, such as cases in which channel reciprocity between uplink and downlink can be applied (TDD mode of UTRA) or explicit feedback information is used, the channel is known at the transmitter and, therefore, optimum distribution of power over the antennas and frequency bands can be performed. The maximization of the capacity gives a power distribution technique commonly referred to as "water-filling" or "water-pouring" because it resembles the act of filling a bowl [3].

The "water-filling" technique can be easily derived after performing the singular value decomposition of the channel matrix $\mathbf{H} = \mathbf{U}\mathbf{D}\mathbf{V}^H$ and expressing the flat MIMO channel as a set of $L=min(n_T, n_R)$ parallel channels,

$$\tilde{\mathbf{y}}(t) = \mathbf{D}\tilde{\mathbf{x}}(t) + \tilde{\mathbf{n}}(t) \tag{11}$$

or, equivalently, as

$$\tilde{y}_k(t) = \lambda_k^{1/2}\tilde{x}_k(t) + \tilde{n}_k(t) \qquad 1 \le k \le L \tag{12}$$

where λ_k is the gain of channel k. The optimum power allocation for the set of parallel channels is given by:

$$p_k = K - \frac{\sigma_n^2}{\lambda_k} \tag{13}$$

where K is a constant to meet power constraints. The global capacity is the sum of the capacity of each subchannel [2]. Note that for frequency-selective MIMO channels, the optimum power allocation has to be done simultaneously over the set of parallel channels resulting from the spatial channel parallelization and the orthogonal frequency bands.

Simulation examples

The capacity results for the different system configurations are given as curves of CDF or outage probability. They are computed using Monte-Carlo simulations based on 10,000 random channel realizations. Different configurations will depend on many factors such as the number of transmit and receive antennas (n_T,n_R), whether the channel is known or not in transmission (it is always assumed known by the receiver), whether the fading is fully correlated or completely uncorrelated, the transmission power, and the frequency-selectivity of the channel (or its power delay profile). Note that, unless stated otherwise, the transmitted power is kept constant for fair comparisons (SNR=21dB at each receiving antenna independently of the number of transmit antennas).

Capacity of flat SIMO/MISO vs. MIMO channels

As illustrated in Figure 1, where some capacity curves for different configurations of uncorrelated flat SIMO and MISO Rayleigh channels are plotted, the capacity increases with the number of either transmitting or receiving antennas. Nevertheless, when arrays of antennas are utilized both in transmission and reception simultaneously (see Figure 2), i.e. when MIMO channels are used, the capacity boosts (note that for the case of (4,4) C=21 bits/sec/Hz at P_{out}=0.1). This significant increase of capacity is due to the existence of parallel channels, which do not exist in SIMO/MISO channel configurations.

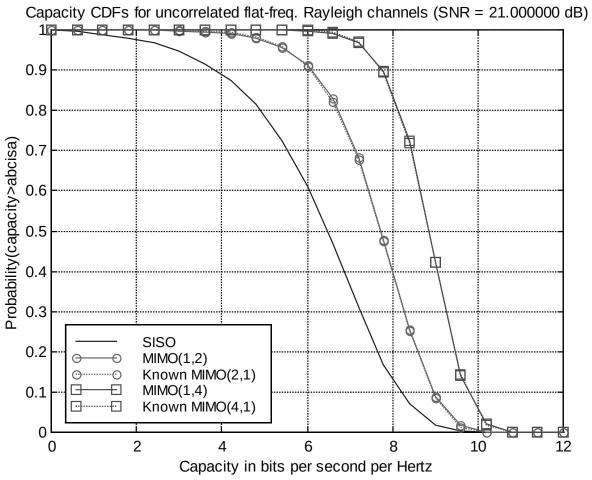

Figure 1 Flat uncorrelated SIMO/MISO channels.

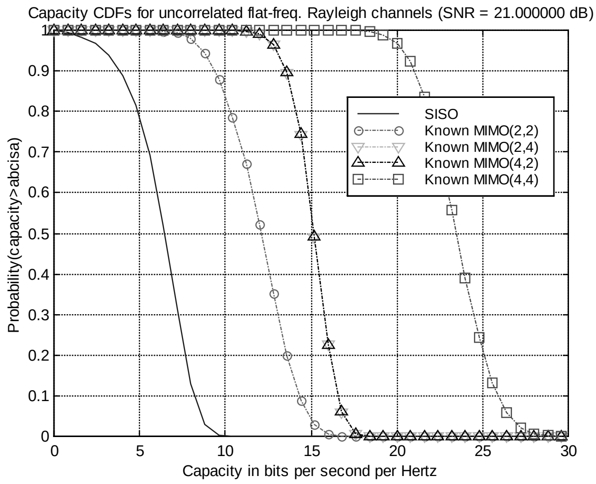

Figure 2 Flat uncorrelated MIMO channels.

Capacity as a function of the fading correlation

The effect of the fading correlation can be seen comparing Figure 2 with Figure 3 (for the (4,4) case, the capacity decreases from 21 to 8 bits/sec/Hz at P_{out}=0.1).

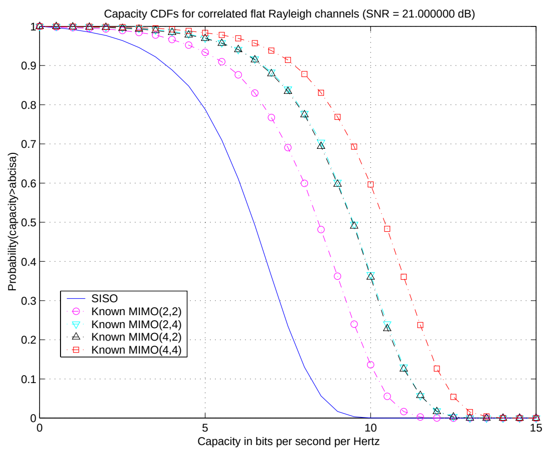

Figure 3 Flat correlated MIMO channels.

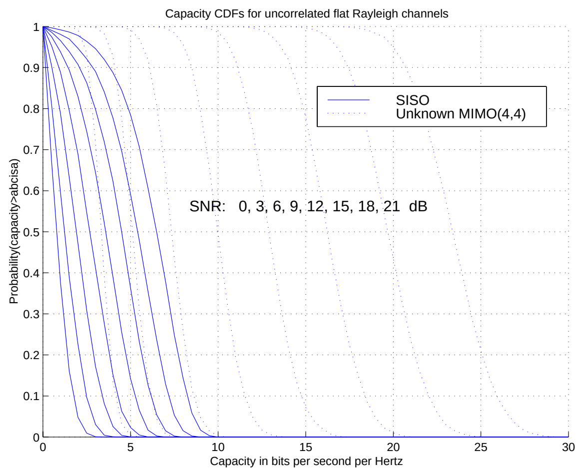

Figure 4 Capacity for different SNRs.

It can be seen how the huge potential capacity of the uncorrelated channels vanishes when the channel becomes fully correlated. The explanation for that substantial difference is the fact that, when the channel gets correlated, the number of parallel channels decreases to the point of having just one single channel, which corresponds to the fully correlated case. In such cases, the capacity gain is obtained only by beamforming.

Capacity as a function of the transmitted power

In Figure 4, capacity CDF curves are plotted for the uncorrelated flat MIMO (4,4) case as a function of the transmitted power (or equivalently the received average SNR at each antenna element). Note that for high SNR values, whereas the capacity of the SISO channel increases 1 bit per 3dB increase of SNR, the capacity of a uncorrelated (n,n) channel increases n bits per 3 dB increase of SNR (see eq.(9)).

Capacity as a function of the number of antenna elements

In Figure 5, the capacity CDF curves of flat MIMO (n,n) channels are plotted. As predicted by eq. (9), the capacity grows without limit as n increases for the case of uncorrelated channel. (Actually, for large n, it increases at least linearly with increasing n [1].)

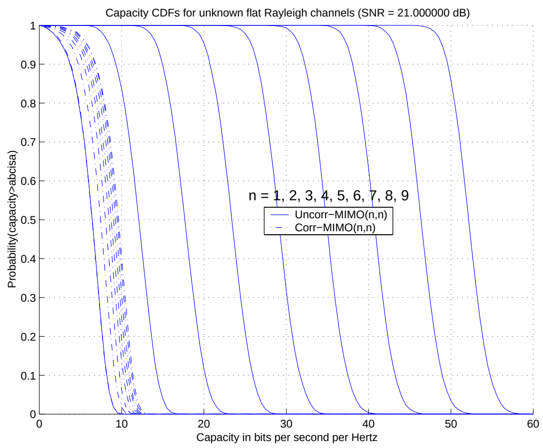

Figure 5 Capacity for different number of antennas.

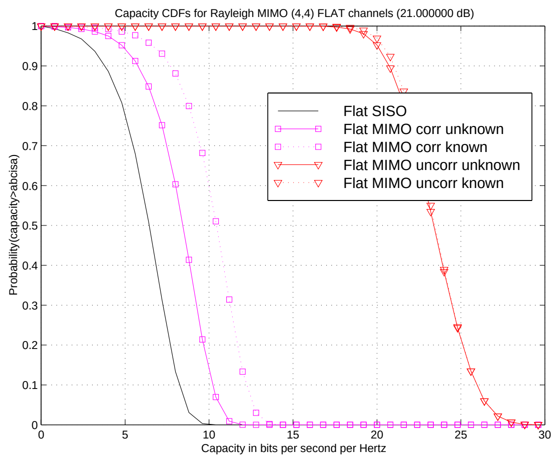

Figure 6 Capacity of a Rayleigh flat MIMO channel.

Capacity as a function of the frequency-selectivity of the channel

In Figures 6-8, capacity CDF curves for a MIMO(4,4) configuration over a flat channel and two frequency-selective channels (with delay profiles PED-A and VEH-A according to [11]) are plotted. It can be observed how the increase of frequency diversity increases the slope of the capacity curves, but does not shift it (as for the increase of n in the (n,n) case), improving the capacity at low probabilities of outage.

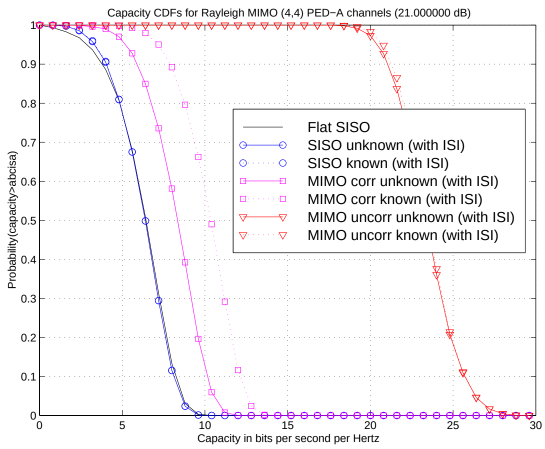

Figure 7 Capacity of a Rayleigh MIMO PED channel.

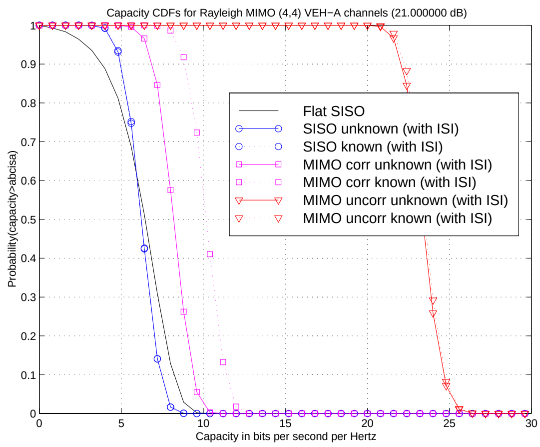

Figure 8 Capacity of a Rayleigh MIMO VEH channel.

The present capacity analysis has been performed over two extreme channel situations: completely uncorrelated and fully correlated fading. In a real situation (see [9]), the channel correlation can be analyzed by means of the eigenvalues associated with the MIMO channel (gains of the parallel channels of (12)). For a low angle spread, the channel becomes fully correlated, whereas for high angle spread, the channel decorrelates. An interesting possibility to further decorrelate the fades is the use of dual polarization antennas.

Conclusions

The present analysis of the capacity of Rayleigh MIMO channels can be summarized as follows:

- The capacity of a MEA system generally decreases as the fades of the MIMO channel become more correlated or, in other words, as the angular spread decreases.
- For the case of uncorrelated fading, there is a large amount of capacity available that increases linearly with n in (n,n) systems for large n and with the transmitted power (n bits per 3dB increase in high SNR regime).
- The performance of a wireless communication system that has multiple transmit-receive antennas depends on how the transmitted power is distributed among the parallel channels. The difference between the capacities achieved by uniform and optimum power allocation is small when the fades associated with transmit-receive antenna pairs are independent, but can become very large when the fades are highly correlated. Therefore, the additional complexity of optimum power allocation over uniform power allocation is justified only in the correlated channel case.
- The frequency-selectivity of MIMO channels increases the slope of the capacity CDF curves.

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