

# OPTIMAL ARRAY COMBINER AND SEQUENCE DETECTOR IN MOBILE RADIO CHANNELS

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## ABSTRACT\*

The use of spatial diversity at the receiver front-end together with a sequence detector implies a joint design problem of the spatial combiner and the sequence detector impulse response. This joint design is usually faced under the constraint that the impulse response of the sequence detector is matched to the channel and combiner response. This procedure maximizes the signal to noise ratio at the input of the detector but, as it is well known, this does not guarantee a minimum probability of error, which is more related to the so-called effective signal to noise ratio. This work presents a procedure that, starting from a simple structure for the space-time receiver aims directly at the maximization of the effective signal to noise ratio, yet preserving all the features of the spatial processor in terms of co-channel and high order intersymbol interference rejection.

## I. INTRODUCTION

The increasing traffic demands in mobile communications move the manufacturers to seek for the potential of spatial diversity techniques to alleviate congestion problems. On the other hand, base-stations are expensive, sophisticated systems, both in technology and operation, that impose constraints on the use of spatial diversity being crucial that the spatial processor is designed jointly with the rest of the subsystems forming the baseband processor. In this respect, optimal multichannel/multiuser receivers are often unaffordable structures. Suo-optimal solutions like multichannel receivers which deal with interference as if it was noise give performances far from the desired, and hence more practical solutions have to be found.

To specify the relationship between the spatial combiner and the temporal processor, namely the matched filter and the sequence detector, it is necessary to assign the appropriate role to each part. The sequence detector, normally a Viterbi Equalizer (VE) is the optimum procedure to combat intersymbol interference (ISI); but it is quite sensitive to co-channel interferers or temporal correlated noise [1]. At the same time, the VE is based

in a metric computation measuring the distance of the received sample with the product of a candidate sequence by the impulse response of the communication channel, which is denoted by the Desired Impulse Response (DIR). The major impact of the DIR length is both in complexity and decoder delay, since the number of candidate sequences increases with the DIR length. In summary, any pre-processing aims at the reduction of the DIR's length and to remove co-channel interferers. The solution, still in use [2], is to set a pre-equalizer, named as Forward Equalizer, whose major objective is to reduce the length of the DIR. The drawbacks of the FE are twofold; first, it does not help in reducing interference; second, it introduces temporal correlation in the noise. Both effects have a negative impact in the performance of the ML sequence detector [3].

The spatial processor removes the two difficulties of the FE, namely it does not introduce temporal correlation in the noise and it is able to remove co-channel interferers being quite effective in this role. Furthermore, under a training sequence it is capable of removing late arrivals thus reducing the length of the DIR. With respect to the matched filter, we can consider as well the case of a broadband combiner which is equivalent to include a matched filter for every diversity channel. The broadband combiner is mandatory for the cases where the received signal has low coherence among sensors, mainly because either the size of the aperture is too large or the channel spread is high enough. When this is not the case, a single unique matched filter for all sensors is the optimum receiver and the combiner reduces to a narrowband one. Since our presentation is valid for the two cases, we will preserve the broadband architecture. In [4] it is shown a procedure to reduce a broadband design to a narrowband design based in a rank-one approximation of the first. The joint design, under the premises described before, was reported in [4][5], where the departing design criteria were different but the solutions finally obtained turned out to be the same. Also the basic concepts involved in these works can be found in [6][3][2] as a FE design problem both for sequence detectors and for Decision Directed equalizers.

This work will present a refinement of the so-called Matched DIR (MDIR) procedure [4], which consist in the direct maximization of the Effective Signal to Noise Ratio ( $SNR_e$ ) of the sequence detector. Since MDIR is based on the SNR at the

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input of the sequence detector it does not ensure that it is working under optimum conditions. Since the sequence detector is controlled by the ratio between minimum distance between candidate sequences and the gaussian noise power, the procedure to be reported hereafter focus directly on the  $SNR_e$  and, as a consequence, produces always better performance than the MDIR design. The reason is that only the  $SNR_e$  acts in controlling the Bit Error Rate (BER) bound. The resulting algorithm improves the  $SNR_e$ , yet preserving the advantages and features of the MDIR design. The complexity associated with the algorithm is very low. The procedure will be named Minimum Distance (MDIS) and will be described in section III, right after and introduction in section II of the MDIR method.

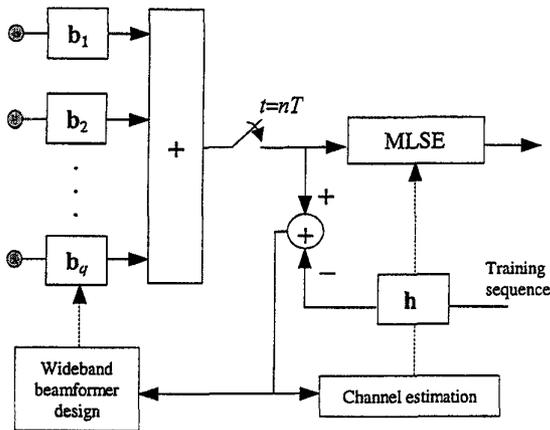


Figure 1. The broadband spatial combiner, including a matched filter for every channel.

## II. THE MDIR METHOD

Assume we deal with the narrowband combiner, that is, with escalar gains in each sensor path. The joint design of the spatial combiner  $\mathbf{b}$  and the DIR  $\mathbf{h}$  of the sequence detector is based on the minimization of the mean square error (MSE)  $\eta$  :

$$E\{|e(n)|^2\} = E\{|\mathbf{b}^H \mathbf{x}(n) - \mathbf{h}^H \mathbf{d}(n)|^2\} = \eta \quad (1)$$

where  $\mathbf{x}(n)$  is the snapshot from the spatial aperture and  $\mathbf{d}(n)$  is a vector, with length equal to that of the DIR, containing successive symbols of a given training sequence. Since the presence of channel coding does not modify our results, we will assume that the symbols of the training sequence are uncorrelated, i.e.  $E\{\mathbf{d}(n)\mathbf{d}(n)^H\}$  is equal to the identity matrix, with no loss of generality. A scheme of the temporal processor is depicted in Figure 1. Perfect synchronism is assumed at the symbol sampler in our presentation.

When selecting a constraint for the joint minimization problem, in order to avoid the trivial solution, there are several choices [3]. Nevertheless, the most efficient is based in the control of the energy at the output of the spatial combiner associated to the symbols included in the training sequence; note that these

symbols will not cause any detector problem, because they are optimally combined by the DIR. To formulate the constraint, let us assume that the received snapshot is formed by two terms; the first term is the multichannel response to the symbols that can be accommodated in the sequence detector and a second term which includes late multipath arrivals, interferences and noise:

$$\mathbf{x}(n) = \mathbf{G}\mathbf{d}(n) + \mathbf{v}(n) \quad (2)$$

The signal part of the combiner output will be provided only by the first term and its energy will be constrained to one:

$$\mathbf{b}^H \mathbf{G} \mathbf{G}^H \mathbf{b} = 1 \quad (3)$$

When this constraint is used for the minimization of the MSE defined in (1), the resulting  $\mathbf{b}$  and  $\mathbf{h}$  maximize the signal to noise ratio at the input of the sequence detector defined as (4); where  $\mathbf{R}_v$  is the covariance matrix of  $\mathbf{v}(n)$ .

$$SNR = \frac{\mathbf{b}^H \mathbf{G} \mathbf{G}^H \mathbf{b}}{\mathbf{b}^H \mathbf{R}_v \mathbf{b}} \quad (4)$$

The estimation of the channel matrix  $\mathbf{G}$  and the covariance matrix  $\mathbf{R}_v$ , which are the basis of a vector MLSE formulation are defined as:

$$\mathbf{G} = E\{\mathbf{x}(n)\mathbf{d}(n)^H\} \quad (5.a)$$

$$\mathbf{R} = E\{\mathbf{x}(n)\mathbf{x}(n)^H\} \quad (5.b)$$

and can be estimated during the presence of the training frame. Note that with those definitions, the noise matrix can be formulated as:

$$\mathbf{R}_v = \mathbf{R} - \mathbf{G} \mathbf{G}^H \quad (5.c)$$

Going back to the constrained minimization problem, it is easy to find that the optimum combiner is given by the eigenvector associated to the minimum eigenvalue of:

$$\mathbf{R} \mathbf{b} = \lambda \mathbf{G} \mathbf{G}^H \mathbf{b} \quad (6.a)$$

and the optimum DIR is given by:

$$\mathbf{h} = \mathbf{G}^H \mathbf{b} \quad (6.b)$$

The optimum SNR is given by

$$SNR_{opt} = \frac{1}{1 - \lambda_{min}} \quad (6.c)$$

Equation (6.b) is responsible for the MDIR name to the procedure, since the DIR  $\mathbf{h}$  is matched to the response of the communication channel plus the spatial combiner when signal vector.

The design is also valid for a broadband beamformer with symbol-rate sampling at the output of the combiner. In this case, the definition of the criterion is the same:

$$E\{|e(n)|^2\} = E\{|\text{trace}[\mathbf{B}^H \mathbf{X}(n)] - \mathbf{h}^H \mathbf{d}(n)|^2\} = \eta$$

with the following definitions (assuming sampling at  $K$  samples per symbol,  $q$  sensors, and  $r$  samples per matched filter):

$$\mathbf{B} = \begin{bmatrix} b_{1,1} & b_{1,2} & \cdots & b_{1,r} \\ b_{2,1} & b_{2,2} & \cdots & b_{2,r} \\ \vdots & \vdots & \ddots & \vdots \\ b_{q,1} & b_{q,2} & \cdots & b_{q,r} \end{bmatrix} = [\mathbf{b}_1 \quad \mathbf{b}_2 \quad \cdots \quad \mathbf{b}_q]$$

$$\mathbf{X}(n) = \begin{bmatrix} x_1(nT) & x_1(nT - T/K) & \cdots & x_1(nT - rT/K) \\ x_2(nT) & x_2(nT - T/K) & \cdots & x_2(nT - rT/K) \\ \vdots & \vdots & \ddots & \vdots \\ x_q(nT) & x_q(nT - T/K) & \cdots & x_q(nT - rT/K) \end{bmatrix}$$

### III. MAXIMUM EFFECTIVE SIGNAL-TO-NOISE RATIO

The signal to noise ratio defined for the MDIR design is just a measure of how good is the signal-plus-ISI-to-noise ratio at the input of the sequence detector. In other words, this SNR does not tell us how difficult it could be for the sequence detector to combat the residual ISI. When a signal time slot is under processing, the DIR is matched as (6.b) and, ideally, no interferers or late arrivals are present at the combiner output; the input of the sequence detector is gaussian distributed with power equal to the MSE  $\eta$ , and mean equal to  $\mathbf{h}^H \mathbf{d}(n)$ . Thus the metric to be computed by the sequence detector is the square of the difference between the received signal and the mentioned mean. An error deciding the optimum sequence will be produced, with the highest probability, to the closest vector  $\mathbf{d}(n)$ ; in consequence a bound of the BER will be given by the minimum distance between two valid sequences in the detection space. In summary, the so-called effective SNR<sub>e</sub> can be defined as:

$$SNR_e = \min_i \left( \frac{|\mathbf{h}^H \mathbf{e}_i|^2}{4\eta} \right) \quad (7)$$

where the min(.) operator stands for the search of the most dangerous error pattern  $\mathbf{e}_i$  (i.e. the difference of two valid  $\mathbf{d}(n)$ ) which minimizes the numerator. The BER is then bounded by:

$$BER \leq K_0 Q(SNR_e^{1/2}) \quad (8)$$

where  $K_0$  depends on the probability of the error sequence. As an example, for a BPSK signal and  $p=4$ , 40 significant different sequences  $\mathbf{e}_i$  are obtained.

Being our objective to increase the SNR<sub>e</sub>, the algorithm introduces a small perturbation to the DIR. Since this perturbation has to increase the numerator in (7), the perturbation is chosen in the direction of the worst error sequence:

$$\mathbf{h}_j = \mathbf{h}_{j-1} + \mu \mathbf{e}_i \text{sign}(\text{Re}(\mathbf{h}^H \mathbf{e}_i)) \quad (9)$$

where  $\text{sign}(\cdot)$  stands for the sign function and  $\text{Re}$  for the real part. This perturbation avoids the use of the dot product because it is assumed that the error sequence has zero quadrature component as corresponds with a BPSK modulation. In other words, the DIR is only in-phase perturbed leaving constant their quadrature components. Alternatives to this algorithm can be found in [8] in the framework of linear discriminants for pattern recognition purposes.

Every perturbation introduced in the DIR causes an increase in the MSE. Since the metric is valid whenever the DIR remains matched to the combiner response, the following constraint has to be set for the new combiner:

$$\mathbf{G}^H \mathbf{b} = \mathbf{h} \quad (10)$$

with respect the new MSE, since its remains with the same formulation  $\mathbf{b}^H \mathbf{R}_v \mathbf{b}$ , it turns out that the optimum combiner, after perturbation is given by:

$$\mathbf{b} = \mathbf{R}_v^{-1} \mathbf{G} (\mathbf{G}^H \mathbf{R}_v^{-1} \mathbf{G})^{-1} \mathbf{h} \quad (11)$$

and the corresponding MSE:

$$\eta = \mathbf{h}^H (\mathbf{G}^H \mathbf{R}_v^{-1} \mathbf{G})^{-1} \mathbf{h} \quad (12)$$

The perturbations are iterated while the SNR<sub>e</sub> increases. During the procedure the target error sequence  $\mathbf{e}_i$  which minimizes the SNR<sub>e</sub> changes, and hence the procedure cannot be carried over in a single iteration. It is important to note that the major problem of the procedure is the presence of strong interferers because, in such a case, the MSE will show a significative increase, even for low perturbations. There are two problems associated with the strong interferences; first, small perturbations do not increase the SNR<sub>e</sub>; and, the increase of interferers power at the combiner output promotes that the metric used in the sequence detector is not longer in correspondence with the exact likelihood of the signal.

### IV. PERFORMANCE EVALUATION

*Simulation 1.* In order to evaluate the performance of the procedure, the following scenario was prepared: The array was an eight sensor ULA array and the length of the estimated channel was set to four; the desired source was located at the array broadside with variable CNR ranging from -4 up to 14 dB; the ISI was set of order 3 (actual DIR of four taps) with DOAs equal to 10,-7 and 40 degrees; the coefficients of these rays were set to 0.7,-0.6 and 0.3 with respect the direct path at the broadside. Figure 2 shows the gain of SNR<sub>e</sub> in dB of the MDIR versus the corresponding MDIR (top) and the loss observed in the SNR, also in dB. Regardless the target SNR is the effective one, it is evident that a gain of 4 dB is maintained in all the range of desired C/N (carrier to noise ratio).

It is important to note that, although the SNR is not the crucial parameter, the amount of SNR<sub>e</sub> gained by MDIR is achieved with a small loss in the input SNR. The second scenario is the

same as in Figure 2, but adding a late arrival (five symbol intervals delay), impinging the aperture from  $-30^\circ$  and with a coefficient, relative to the desired, equal to  $-0.9$ . Moreover, one interference, CIR equal to  $-10$  dB and DOA equal to  $20^\circ$  is included. Again the improvement obtained from MDIS, versus MDIR, is quite evident. Figure 3 shows the gain and loss respectively for this new scenario.

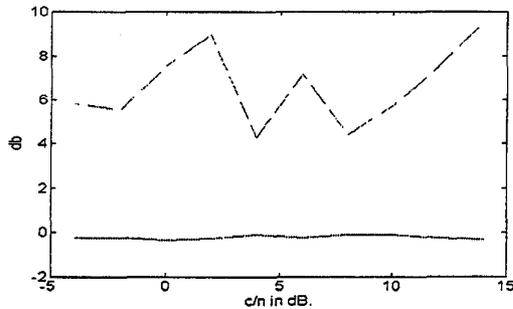


Figure 2.  $SNR_e$  improvement for MDIS versus MDIR for different CNR of the desired (Top). Loss in the input SNR to the sequence detector of MDIS versus MDIR (Bottom).

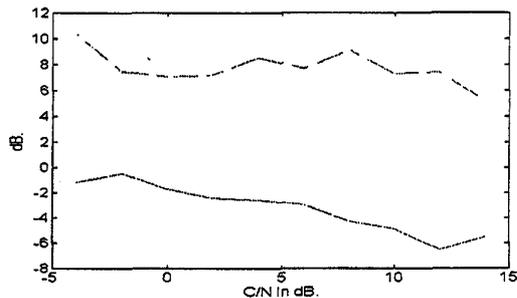


Figure 3. The same as in figure 2 when the scenario includes a late arrival and a co-channel interference.

**Simulation 2.** In order to evaluate the method in a realistic mobile scenario, we have carried out simulations based on a Gaussian stationary uncorrelated hypothesis for the channel, assuming independence between angular and Doppler spread, as it has been experienced from measures taken in downtown Stockholm [10]. There, it is empirically shown that azimuth spectrum follows a Laplacian law, along with Gaussian distribution for the directions of arrival for each user. The angular spread (that is the standard deviation of the Gaussian) is taken  $8^\circ$ . The number of rays impinging the array is found as a Poisson random variable of mean 25. An exponential law is found in [10] for the power delay spread. The delay associated to each impinging ray is also an exponential random variable of mean  $0.85 \mu\text{s}$ , thus allowing for non-flat fading channel when using the GSM/DCS-1900 air interface. The mobile speed is set to 50 km/h, and Doppler spread is assumed to follow the typical spectrum, thus assuming multiple reflections close around the mobile. The array is sectored to  $120^\circ$ , with eight sensors ( $q=8$ ) linearly and uniformly spaced at  $d/\lambda=0.5$ . The length of the estimated channel is  $p=4$ .

One interferer is included, for different mean CIR between  $-15$  and  $10$  dB, thus obtaining in each time slot different instantaneous C/I ratios. Mean signal power to noise power is taken to 20 dB in all cases. The training sequences are formed, as in the GSM standard [7], by 26 symbols located in the middle of 116 information symbols frame (midamble). Sequences TSC4 and TSC7 [9] are used for the desired and interferent users.

Plots showing the BER are displayed in figure 4 versus the instantaneous CIR in the time slot, both for the perturbed MDIR and the VMLSE (without estimation of the covariance matrix of noise-plus-interference). Narrowband beamformer has been used and perfect knowledge of the sampling time has been assumed. MDIS exhibits a behavior which is pretty much steady with respect to CIR, and suitable for DCS requirements, while VMLSE is much more dependent on the strength of the interferer.

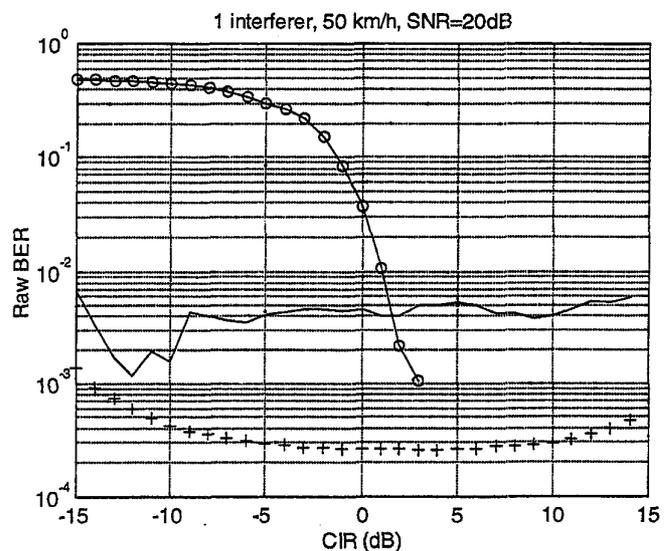


Figure 4. Probability of error of VMLSE (circle-solid line) and MDIS (solid line) versus instantaneous CIR. Cross line indicates the margin of confidence in the estimation.

## V. CONCLUSIONS

A simple and efficient algorithm for combining temporal and spatial diversity has been presented. Good performance of the receiver relies on the fact that enough sensors and taps of the DIR are available to null interferers and late arrivals of the desired user. Simulations have been carried on with accurate models of real mobile channels for the GSM signal structure, showing that the method outperforms the VMLSE receiver in a wide range of CIR at a much lower complexity.

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