Joint Beamforming and Viterbi Equalizer in Wireless Communications

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Abstract

The presence of a sequence detector in the baseline architecture for base stations for mobile communications has become a standard. Also the use of coding allows the baseband detector to work at very low signal to noise ratios. When spatial diversity is included in the front-end, the joint design of the beamformer, matched filter and the desired impulse response DIR of the Viterbi equalizer VE is mandatory. This work describes the joint design of these stages when the matched DIR response has priority in the maximization of the signal to noise plus interferences ratio at the input of the VE.

Introduction

Taking as the baseline architecture the single channel receiver formed by a matched filter followed by a symbol sampler and the VE, depicted in Figure 1,

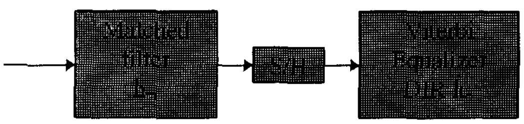

Figure 1. Baseline detector with DIR equal to $\underline{h}_o$ and matched filter response $\underline{h}_m$

the maximum length of the matched filter response hm is four and the DIR of the VE ho is usually between four and six in order to bound the complexity and delay of the VE. When including spatial diversity, several antennas are set together with the corresponding radio-frequency front-ends and baseband beamforming. Again, and in order to bound implementation complexity to reasonable limits, the number of elements is assumed to be four. A scheme of the receiver architecture is shown in Figure 2.

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Note that in the figure the matched filter is considered a separate and single block at the output of the beamformer; this is the case when there are full coherence between the sensors signal. A more general approach is to consider a broadband beamformer where every channel has a different matched filter, i.e. the matched filter is included in a FIR filter on every array sensor. As it is shown this is under the scope of the design procedure describe hereafter. Also in Section IV it is shown how to derive the single matched filter from the broadband beamformer design as well as the parameter to measure the goodness of the approximation.

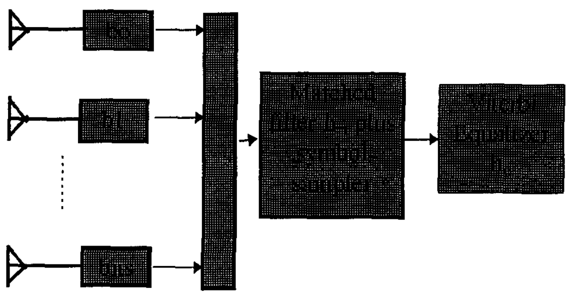

Figure 2. Diversity combiner plus baseline detector

Since the matched filter can be derived from the joint design of the DIR $\underline{h}_o$ and a broadband beamformer, we will assume without loss of generality that the matched filter response corresponds to a rectangular pulse, facing directly the design of the beamformer $\underline{b}$ and the DIR $\underline{h}_o$. Note that the length of the DIR in the VE equalizer determines the maximum delay (interference inter-symbol ISI order) allowed to pass the beamformer. In other words, being the DIR of length equal to four symbol intervals, only late arrivals higher than four symbol intervals will be considered as co-channel interference at the beamformer stage, meanwhile the early arrivals will be managed as desired signal at this stage.

The joint design of the beamformer and the DIR requires to select a suitable objective prioritizing the performance of the VE, which at the end determines the

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performance of the receiver. Before entering the next section on the joint design, it is worthwhile to mention that this joint design problem in terms of beamforming plus DIR represents almost the same problem tools and solutions that the joint design of a Forward equalizer to reduce the target DIR in a sequence detector (see for example [1]). The differences between these two problems is the capacity of the beamformer in nulling out co-channel interferences and that the beamformer preserves the white character of the noise at the input of the VE. These two facts act in favor of the use of spatial diversity, instead of time diversity used by forward equalizers, regardless both have in common the objective of reducing the DIR length of the VE which is the parameter that basically determines the complexity of the metrics computation on it.

DESIGN OBJECTIVE

The primary objective could be the minimization of the square error between the beamformer output, after matched filtering, and the DIR response to the corresponding training sequence vector $\underline{d}_n$ which is the basic metric computation to be performed by the VE when the training sequence is not present. This choice means to design b and ho in order to minimize (1), being Xn the snapshot after matched filtering

$$\rho = E\left(/\underline{b}^h . \underline{X}_n - \underline{h}_o^h . \underline{d}_n /^2 \right) \qquad (1)$$

Without loss of generality we will consider uncorrelated symbols since it does not modify the main ideas around the joint design.

It is evident that the direct minimization of (1) produces a trivial solution and an additional constrain have to be set or a more realistic objective has to be selected. In this sense, and being our priority the performance of the VE, it seems that an adequate choice will be to maximize the input signal to noise ratio of the VE defined as (2),

$$\mathrm{SNR} = / \underline{b}^h . \underline{\mathbf{G}} /^2 \ / \rho \qquad (2)$$

where matrix $\underline{\mathbf{G}}$ is the signature of vector dn on the measured snapshot.

$$\underline{\mathbf{G}} = E\left(\underline{X}_n . \underline{d}_n^h \right) \qquad (3)$$

A better choice is to select what is named as the effective signal to noise ratio SNRe to be maximu,which consists on the error sequence $\underline{\varepsilon}$, over all possible candidates d_n, which produces minimum distance / $\underline{h}_o^h . \underline{\varepsilon}$ / divided by ρ [2].

$$\mathrm{SNRe} = \min_{\underline{\varepsilon}} . / \underline{h}_o^h . \underline{\varepsilon} /^2 \ / \ \rho \qquad (4)$$

This last measure of optimality, directly related with a bound for the bit error rate, is rather more complicated than (2) since low complexity is always a demand on the baseband receiver of a base-station. Also the short time duration on every frame of the training sequence and the fact that at low signal to noise ratios exist a close correspondence between (2) and (4) dictates the use of the first. It is interesting to mention that an alternative is to maximize what could be consider an upper bound of (4). This objective, formed by the ratio between the norm of the DIR and the mean square error (5)

$$\Psi = / \underline{h}_o /^2 / \ \rho \qquad (5)$$

This objective does not maximize the signal to noise ratio and for low signal to noise ratio of the desired signal presents significative losses with respect to the maximization of (2). Nevertheless, this is an existing alternative and, as it was for the forward equalizer design, can be used in spatial diversity [3].

THE MATCHED DIR SOLUTION

The maximization of (2) is performed by the minimization of the MSE with norm one for the numerator of the SNR. Going ahead with this constrained minimization, the Lagrangian and its derivatives are:

$$\Lambda = \rho - \lambda . \underline{b}^h . \underline{\mathbf{G}} . \underline{\mathbf{G}}^h . \underline{b} \qquad (6.a)$$
$$\delta \Lambda / \delta \underline{b}^h = \underline{\mathbf{R}} . \underline{b} - \underline{\mathbf{G}} . \underline{h}_o = \lambda . \underline{\mathbf{G}} . \underline{\mathbf{G}}^h . \underline{b} \qquad (6.b)$$
$$\delta \Lambda / \delta \underline{h}_o^h = \underline{\mathbf{G}}^h . \underline{b} - \underline{h}_o = 0 \qquad (6.c)$$

where $\underline{\mathbf{R}}$ is the autocorrelation matrix of the received snapshots and λ is the Lagrange multiplier of the constrained minimization problem.

Equation (6.b) gives the name to the procedure, as it was in the forward equalizer design, since it reveals that the DIR $\underline{h}_o$ is the matched filter to the response of the beamformer to the transmitted sequence dn. Thus, this procedure can be refered as the Matched DIR (M-DIR) method. Also (6.c) can be viewed as the spatial constraint since it forces that each useful arrival (every row of $\underline{\mathbf{G}}^h$) is constrained to have a gain equal to the corresponding component of the DIR. Note also that the energy of the DIR is one as it will be in the maximization of (5).

The optimum beamformer results from the solution of (7) for the eigenvector associated with the minimum eigenvalue; this eigenvalue is the inverse of the signal to noise ratio achieved at the VE input, as defined in (2)

$$(\underline{\mathbf{R}} - \underline{\mathbf{G}} \cdot \underline{\mathbf{G}}^h) \cdot \underline{b} = \lambda \cdot \underline{\mathbf{G}} \cdot \underline{\mathbf{G}}^h \cdot \underline{b} \qquad (7)$$

A refinement could be added to the above procedure and it concerns the influence of small levels of interferences and late arrivals in the beamformer output. It is well known how sensitive the VE uses to be in the presence of these interference residuals. In fact, the likelihood metric managed by the VE, in strict sense, has to be modified taking into account, not only the interference level but also its internal structure. In fact, the adequate scheme has to manage the interference as in a multi-user receiver in order to compute the exact likelihood among candidate sequences both for the interference and for the desired. These effects of residual interference may show up when the interference impinges the aperture with a structure similar to that assumed here for the desired, i.e. with multiple arrivals. A more practical solution to this problem, at the expense of a degradation in the resulting signal to noise ratio (2), is to select a beamforming with maximum nulling to such interferences. This is achieved by selecting in (7) a few eigenvectors associated to the minimum eigenvalues. The beamformer is formed from a linear combination of these eigenvectors and the optimum choice for the linear combination arise to an equation similar to (7). At the end, it can be shown that the new beamformer results for the same equation (7), where the matrix on the left side is reduced to its noise subspace. Nevertheless, the reduced number of sensors in the base-station use to cause the reduction of the noise subspace of the mentioned matrix to a single eigenvector, with no differences between both techniques.

The Broadband beamformer

The M-DIR design is also valid for the case of broadband beamforming including a four taps delay line per symbol interval in every channel. As announced, highly dispersive channels do not allow the use of a single matched filter as it is depicted in Figure 2 for the baseband receiver.

Yet preserving the symbol sampler at the output of the beamformer, the beamforming as the dot product of the extended snapshot for the beamformer,

$$y(n) = \underline{b}_e^h \cdot \underline{X}_{ne} \qquad (8)$$

can be written as:

$$y(n) = \text{trace } \underline{\mathbf{B}}^h \cdot \underline{\mathbf{X}}_n \qquad (9)$$

where

$$\underline{\mathbf{B}} = \begin{bmatrix} b_{11}^* & b_{12}^* & \cdots & b_{1,p}^* \\ b_{21}^* & b_{12}^* & \cdots & b_{2,p}^* \\ \vdots & \vdots & & \vdots \\ b_{ns,1}^* & b_{ns,2}^* & \cdots & b_{ns,p}^* \end{bmatrix} \qquad (10.a)$$

$$\underline{\mathbf{X}}_{ne} = \begin{bmatrix} x_1(n) & x_1(n-1) & \cdots & x_1(n-p) \\ x_2(n) & x_2(n-1) & \cdots & x_2(n-p) \\ \vdots & \vdots & & \vdots \\ x_{ns}(n) & x_{ns}(n-1) & \cdots & x_{ns}(n-p) \end{bmatrix} \qquad (10.b)$$

where * indicates complex conjugate and the labels ranging from one to ns is the sensor label. p indicates the number of taps per sensor.

From (9) is clear that the narrowband beamformer plus single matched filter can be derived from a rank one approximation of the matrix $\underline{\mathbf{B}}$. The left and right eigenvectors associated with the maximum eigenvalue provide the conjugate of the narrowband beamformer and the matched filter response respectively

$$\underline{\mathbf{B}}_1 = \lambda_{max.} \cdot \underline{b}^* \cdot \underline{h}_m^h \qquad (11)$$

The goodness of the narrowband approximation can be derived from the quotient of the maximum eigenvalue and the trace of the original matrix. This parameter measured for low dispersive channels range between 70 and 90%. Nevertheless, the major appeal of the approximation is the complexity reduction of the front-end.

This procedure is also adequate to design the forward equalizer, which is the case when the wideband beamformer include more than a symbol interval in the tap delay line of every sensor. In fact, with the same procedure applied to the fractional forward equalizer resulting from the MDIR design, the alternative of separate matched filter and symbol by symbol FE can be derived. The major drawback, as announced before, is that when the tap delay line overpasses the pulse length the symbol noise cannot be considered uncorrelated from symbol to symbol. As a consequence of this temporal correlation the differences observed between the SNR and the BER bound provided by the SNRe are, most of the cases, unacceptable. In this case the SNR is no longer a valid figure of merit for the front-end. In fact, even the SNRe has to be changed in its denominator since the VE noise depends on the direction of the error sequence.

An Example

The following signal scenario has been selected to check the behavior of the Matched DIR design. The desired signal is BPSK modulated and is assumed to arrive to the aperture with four order ISI of coefficients equal to [0.5 -0.7 -0.6 0.9] normaliced with respect the direct path level. The aperture was formed by an ULA array of four antennas half wavelength of separation. The DOA of the desired signal was the array broadside and the DOAs of the four multipaths were [10 -7 -40 -30] degrees respectively. An interference 10 dB above the direct path of the desired signal was located a 20 degrees. The training signal was formed as a periodic extension of the TSC0 code [4] of the GSM standard [5]. The interference has the same structure but with code TSC4. Note that with a DIR of length four, the last arrival represents a co-channel interference for the VE.

Figure 3 shows the beamformer response and Figure 5 the evolution of the SNR and the effective SNRe for different signal to noise ratios of the direct path of the desired signal. The processing was wideband with four taps per chanel.

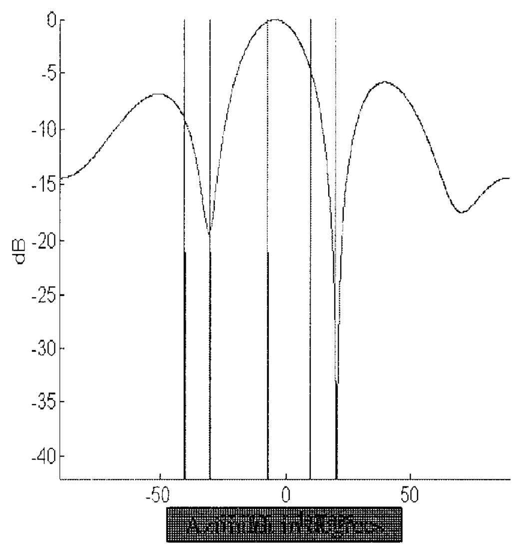

Figure 3. Beamformer response for the signal scenario described in the text

The beamformer response shown in Figure 3, shows how the spatial filtering is performed, passing by the arrivals to be used by the VE, and attenuating the late arrival at -30 degrees of DOA. At the same time the beamformer rejects the co-channel interference, 10 dB above the direct path with a nulling close to 40 dB. This plot has been obtained for -2.4 dB of signal to noise ratio for the direct path. The DOAs of non-coherent multipaths and interferences are indicated by the vertical lines, excluding the direct path at the broadside.

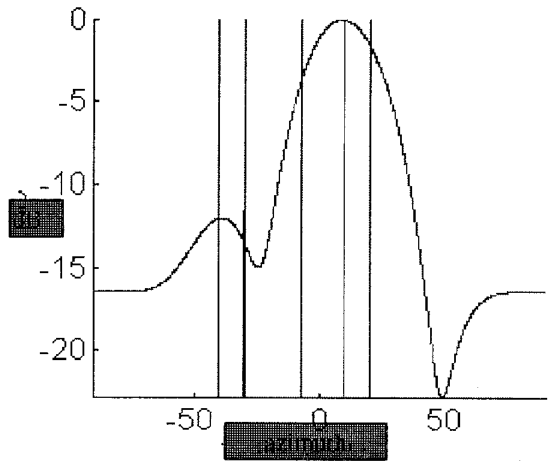

Figure 4. Quiescent response

How this beamforming differs from the quiescent, can be seen in Figure 4. As in the GSLC formulation, this quiescent corresponds to the beamformer that verifying the matched DIR constrain has minimum norm. The BER for this beamformer was observed always above 0.2. In a practical implementation, when the number of arrivals is high or the number of interferences is high, the quiescent is interesting because it may avoid catastrophic performance by resetting to its response for a few frames. This situation may shown up in urban scenarios with a low number of sensors available.

In Figure 5 it can be seen the correspondence between the measured signal to noise ratio SNR and the effective SNRe for the VE for different signal to noise ratio for the direct path. The graphic depicts the evolution for both signal to noise ratios for values of the direct path, ranging from -11.4 dB up to 2.4 dB. Note that in the margin of -6 dB and above both SNRs evolve with a similar tendency being almost coincident for high signal to noise ratio for the desired source. This close correspondence between the two SNRs in the margin mentioned before proves the utility of (2) as the design objective; only at very low SNR for the desired the effective SNR shows a threshold effect. The BER observed above -6 dB was always above 10^{-2}.

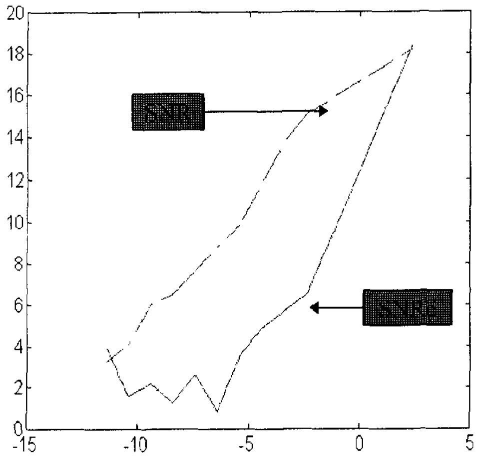

Figure 5. SNR and SNRe as a function of the desired signal to noise ratio (C/N of the broadside path) for the scenario describe in the text.

In order to check the performance of the system, the MDIR technique was evaluated for the narrowband case, being the single matched filter an integrator of four samples per symbol. The signal to noise ratio for the desired path was -2.4 dB and the rest of the signal scenario was the same used in the previous figures of this section. The framing of the desired signal was formed by 26 symbol training sequences followed by 128 symbols of information. Again the training sequence for the desired was selected the TSC0 code. It is important to remark that these sequences are formed by 15 symbols with periodic extrapolation both sides. Also, from the 26 symbols, the length of the training vector, equal to the length of the DIR, reduces to 18 the number of vectors of the training sequence available for the estimation of matrix **G**, and in consequence also for the estimation of **R**.

The estimation of $\underline{\mathbf{R}}$ and $\underline{\mathbf{G}}$ is done in block for the available training sequence in every GSM frame. In addition a forgetting factor β is used to update R and G at the end of every frame, before the beamformer and the VE start to process the information symbols

$$\underline{\mathbf{R}}_k = \beta\, \underline{\mathbf{R}}_{k-1} + (1-\beta)\, \underline{\mathbf{X}}_k \cdot \underline{\mathbf{X}}_k^{h} \qquad (11.a)$$
$$\underline{\mathbf{G}}_k = \beta\, \underline{\mathbf{G}}_{k-1} + (1-\beta)\, \underline{\mathbf{X}}_k \cdot \underline{\mathbf{d}}_k^{h} \qquad (11.b)$$

Being k the frame index and the second term in the right hand side the corresponding matrix arrangements for snapshots and training vectors.

Figure 6 shows the evolution of the SNR as a function of the number of frames processed. Note that, even in the case of fast updating the SNR never goes below 7 dB which is adequate to maintain the BER above 10^{-2}. Nevertheless, a period of five frames to adapt to the spatial scenario use to be adequate, being in this case 0.5 the adequate choice for the forgetting factor enhancing more stability on the received SNR, with fluctuations below 2 dB. along successive frames for stationary scenarios.

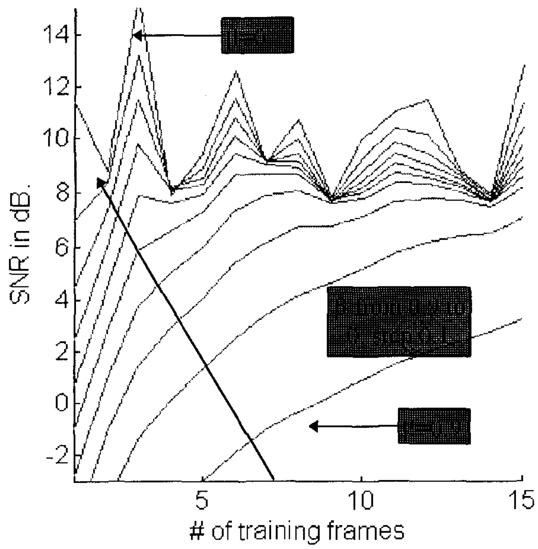

Figure 6. Signal to noise ratio for different forgetting factor as a function of the number of training frames (1-15) of 26 symbols each.

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