DISPERSION CHARACTERISTICS AND FIELD STRUCTURE OF AN AXIALLY MAGNETIZED FERRITE LOADED RECTANGULAR WAVEGUIDE

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Abstract

A basic limitation in an earlier work on the axially magnetized, ferrite loaded rectangular guide has led to a detailed re-examination of this problem.[1] Both a series solution and a perturbational technique are used to find dispersion curves and field patterns.

Introduction

By assuming an $e^{j(\omega t-\beta z)}$ dependence we are led to the familiar coupled equations[2,3]

$$\left.\begin{aligned}(\nabla_t^2 + \alpha_1)e_z &= -j\beta k_f \frac{\kappa}{1+\chi} h_z \\ (\nabla_t^2 + \alpha_2)h_z &= j\beta k_f \frac{\kappa}{1+\chi} e_z\end{aligned}\right\} \qquad (1)$$

Application of a technique described elsewhere leads to a solution of the form[3]

$$e_z = u_1 + u_2, \quad h_z = q_1u_1 + q_2u_2,$$

where

$$(\nabla_t^2 + \sigma_{1,2}^{\;2})u_{1,2} = 0 \quad .$$

This yields a simple solution for the circular guide[3,4]

For the rectangular guide we postulate a solution of the form

$$u_{1,2} = \sum_n (A_n^{1,2}\cos\frac{n\pi x}{a} + B_n^{1,2}\sin\frac{n\pi x}{a})(C_n^{1,2}\cos k_n^{1,2}y + D_u^{1,2}\sin k_n^{1,2}y)$$

where each term satisfies (1), and the eight boundary conditions are imposed on the series. There are no relations of orthogonality between terms, but four sets of coefficients can be expressed as functions of the remaining four. This leads to an infinite dimensional homogeneous system, and, by truncation, the problem is reduced to finding the zeros of a complex determinant.

Alternatively, it is seen that equations (1), together with the boundary conditions, decouple for $\beta=0$, giving pure TE or TM modes. In view of this fact we write the fields in the guide as

$$e_z = \sum_{n=0}^{\infty} f_n(x,y)\beta^n \;, \quad h_z = \sum_{n=0}^{\infty} g_n(x,y)\beta^n$$

and $f = \sum_0^\infty a_n^f\beta^n$. Similar expressions are written for the remaining frequency dependent parameters.

The following equations are thus obtained:

$$(\nabla_t^2+a_o^{\alpha_1})f_n + \sum_{i=1}^{n} a_i^{\alpha_1} f_{n-i} = -j\sum_{i=0}^{n-1} a_i^\sigma g_{n-i-1}$$

$$(\nabla_t^2+a_o^{\alpha_2})g_n + \sum_{i=1}^{n} a_i^{\alpha_2} g_{n-i} = j\sum_{i=0}^{n-1} a_i^\sigma f_{n-i-1} \quad ,$$

plus the conditions at the boundaries

$$f_n = 0 \quad ,$$

$$-j\sum_{i=0}^{n-2} a_i^{\kappa'} \frac{\partial g_{n-i-2}}{\partial \tau} + j\sum_{i=0}^{n-1} a_i^\sigma \frac{\partial f_{n-i-1}}{\partial n} + \sum_{i=0}^{n} a_i^{\alpha_1} \frac{\partial g_{n-i}}{\partial n} = 0.$$

Use of Green's functions of electric and magnetic type lead to general expressions for f_i and g_i ,

$$f_i = \sum_{nm}\sum F_{mn}^i \sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}$$

$$g_i = \sum_{nm}\sum G_{mn}^i \cos\frac{n\pi x}{a}\cos\frac{m\pi y}{b} \quad .$$

The a_n^f's are obtained by applying Green's identity to (f_n,f_{n+2}) or (g_n,g_{n+2}), depending on the type of mode.[5]

It can be shown that only powers of β^2 enter in the expression for f, as expected, and that h_z contains odd powers of β when e_z contains even ones (quasi-TM modes) and vice versa (quasi-TE modes). It is obvious that the method is good only for quasi-TE, -TM modes.

Solution

For the first method it was found that for $n > 3$ (i.e., e_z,h_z approximated by more than 24 terms) the values of the resulting determinant are very unstable as a result of imaginary K_n's leading to hyperbolic functions. For n=2 (8×8 determinant) the zeros are well defined, but the accuracy is poor and cannot be improved.

In the perturbational method one can write general expressions for any arbitrary term of order n in terms of the previous ones. On the other hand, convergence for only a limited range of β is expected, since even for the dielectric guide

$$\omega^2\mu_o\varepsilon_o = k_c^2 + \beta^2 \quad , \quad f \sim [1 + (\frac{\beta}{k_c})^2]^{1/2}$$

and f is given as a series of powers of β^2 for $\beta^2 < k_c^2$. However this range can be extended by analytic continuation.

Note that this difficulty does not exist in the dielectric guide if we express $f^2 = f^2(\beta^2)$, but in our problem both f^2 and f appear.

Results

Dispersion curves for the lowest modes of the rectangular guide as shown on Fig. (1) have been computed from both methods. In the series solution, the quasi-TE/TM modes are easily identified for β small. Other modes are under investigation.

Figure 1 shows the dispersion curves for the quasi-TE_{10} modes, as obtained from the perturbational equations, with

$$f = \sum_{n=0}^{16} a_n^f\beta^n \quad .$$

The series turns out to be alternating, thus providing error bounds.

The region of convergence is $0 < \beta < \sim 2.6$ rad/cm for $H_{dc} = 0$, decreases as we approach resonance, and becomes fairly large ($\beta \sim 5$ rad/cm) above resonance. Within this region convergence is fast; for example, for $\beta = 2$ rad/cm ($H_{dc} = 0$), $f = 7.2542 \pm 0.0005$ Ghz, and even for $\beta = 2.4$ rad/cm the error is 0.01 Ghz. This first region of convergence increases for higher order modes.

The parabolic approximation for $\alpha_2 = \omega^2\mu_o\epsilon_f - \beta^2/(1+\chi)$ (dotted line) was found to provide a value of f accurate to better than a 1% through the whole region of convergence (This expression is equivalent to $\omega^2\mu_o\epsilon - \beta^2 = (\pi/a)^2$ for the dielectric guide).

Figure 2 shows the rotating nature of the transverse H field even for a situation close to cutoff. The transverse E field is very similar to that of the pure TE_{10} mode.

The fields at the walls of the guide are not plotted since the trigonometric series giving them do not converse there to the real values (Gibbs' phenomenon).

Discussion

The perturbational method is capable of being extended to wider ranges of β and to other geometries in a direct manner. On the other hand, for modes other than quasi-TE/TM we must return to a consideration of the first method outlined above.

Both can be extended to include ferrite losses by a straightforward modification of the ferrite parameters.

References

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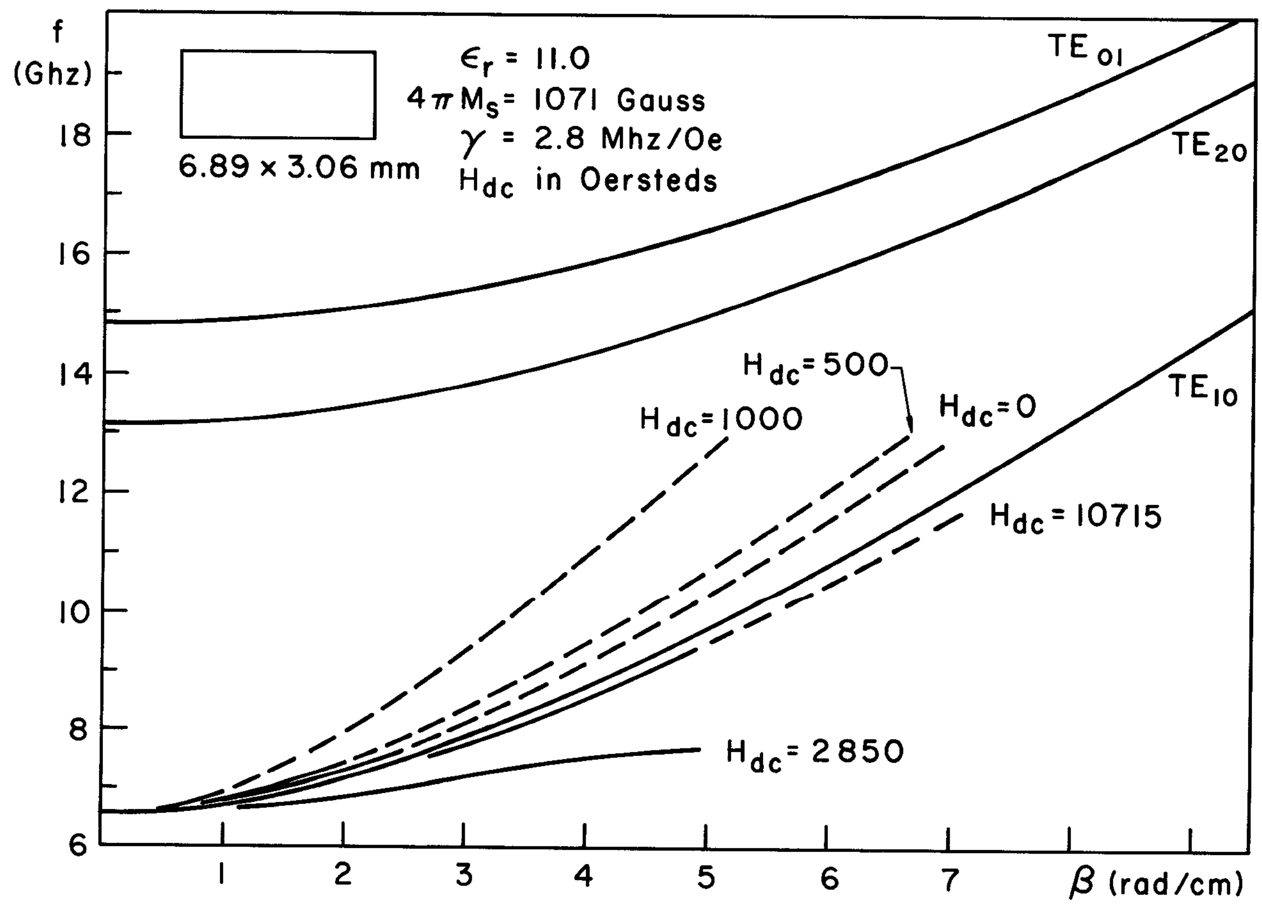


FIG. 1

$H_{dc} = 0$, $4\pi M_s = 1071$ Gauss , $F = 7.25$ Ghz

$\frac{|E_z|_{max}}{|E_t|_{max}} = 0.02$ $\frac{|H_z|_{max}}{|H_t|_{max}} = 1.78$ $\frac{|H_t|_{max}}{|E_t|_{max}} \times Z_{TE} = 1.25$

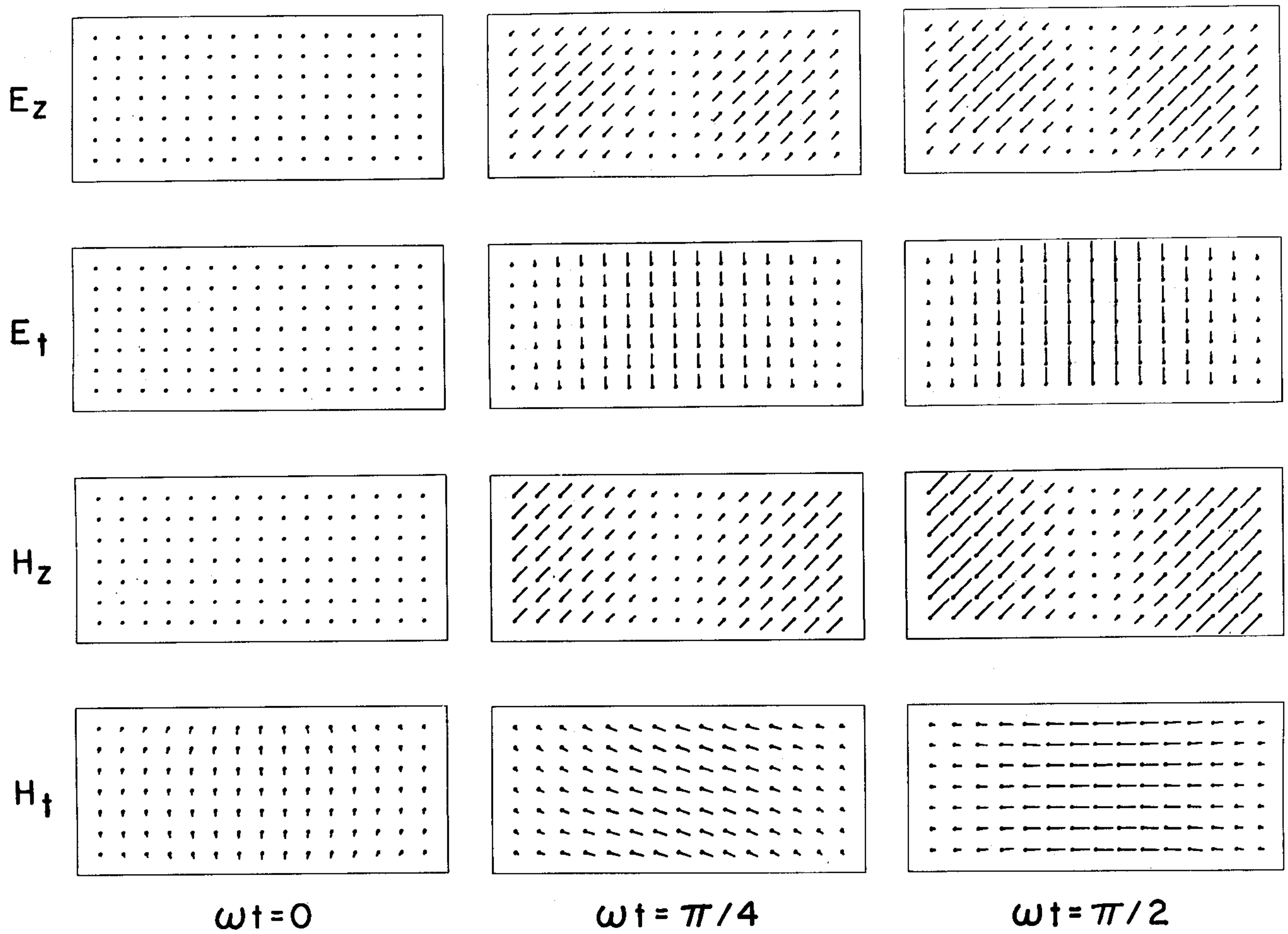
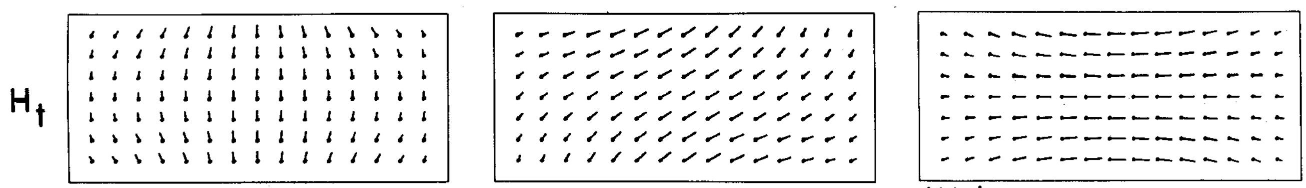


FIG.2



$H_{dc} = 2850$ Oe $4\pi M_s = 1071$ Gauss $\frac{|H_t|_{max}}{|E_t|_{max}} \times Z_{TE} = 0.50$

FIG.3