A Hybrid Modal-Boundary Element Method for Electromagnetic Scattering from Arbitrary Conducting Wedges

J.M. Alvarez, J.C. Cruellas, *M.Ferrando
Departamento de Teoría de la Señal y Comunicaciones
E.T.S.E.T., UPC, Apdo. 30.002
08080 Barcelona - Spain

Abstract

A hybrid technique is applied to find the electromagnetic scattering contribution of arbitrary conducting wedges in electrically large objects. The boundary integral equation is discretized in a ficticious contour, and the field outside is expanded in modal series. From the knowledge of fields at the conducting surface, the method could be used jointly with a Physical Optics (PO) high frequency approximation to obtain the Radar Cross Section (RCS) of electrically large objects.

Introduction

Low frequency methods are widely used to obtain the scattered field by objects of resonant size. When the objects are electrically large, the resulting matrix equations are too large for actual computers, and high frequency approximations are sought. If the arbitrary wedges in a electrically large object are dealt with a low frequency method, the equivalent currents obtained could be added to the model of the object, therefore simplifying the forthcoming analysis.

In this paper, we present a hybrid method aimed at solving the scattering of arbitrary conducting wedges, where high frequency approximations fail or are difficult to use.

The arbitrary wedge is closed by a ficticious contour as shown in Fig. 1. The solution inside the contour is obtained imposing known boundary conditions on the contour and applying the Boundary Element Method (BEM) [1].

The solution in the unbounded region is expressed as modal series in a similar way to the Unimoment Method [2], where the solution outside the mathematical circle enclosing an arbitrary cylinder was expanded in Hankel functions. Continuity conditions in the electric and magnetic field are imposed on the boundary of both regions to adjust the solutions.

General Formulation

The solution inside a contour can be obtained by means of the boundary integral equation

$$E_z(\vec{r}) = \int_c \left(G(\vec{r},\vec{r}\,')H_t(\vec{r}\,') - \frac{\partial G(\vec{r},\vec{r}\,')}{\partial n} E_z(\vec{r}\,') \right) dc \tag{1}$$

if the electric and tangential magnetic field are known in the closed contour.

The BEM formulation [1] expressed in a matrix form reduces to,

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$$
\begin{aligned}
Ae &= Bh \\
e &= (E_1, E_2, \ldots, E_j, \ldots, E_k) \\
h &= (H_1, H_2, \ldots, H_j, \ldots, H_k)
\end{aligned} \tag{2}
$$

where E_j and H_j are the values of electric and magnetic field at boundary element j on the boundary sorrounding the homogeneous region R_1 described in Fig. 1.

The matrices A and B in eq. (2) depend only on the shape of the wedge end and the frequency of the incident wave, but not on boundary conditions.

We suppose boundary conditions

$$
\begin{aligned}
E_z &= \sin \upsilon(\phi - \alpha) \qquad \text{on } C_1 \qquad \upsilon = \frac{n\pi}{2(\pi - \alpha)} \\
E_z &= 0 \qquad \text{on } C_2 \qquad n = 1..N
\end{aligned} \tag{3}
$$

where α is the equivalent semiangle of the wedge and N is the number of significant modes used in the unbounded region R_2.

By means of eq. (2) we obtain H_t on C_1 and C_2. If we express the resulting magnetic field on C_1 as a sine series

$$
H_t^n = \sum_{m=1}^{N} A_{nm} \sin l(\phi - \alpha) \qquad l = \frac{m\pi}{2(\pi - \alpha)} \tag{4}
$$

we can find the electric and magnetic field in R_1 and R_2 by adjusting the solution on C_1 to a modal series.

The resulting equations for electric and magnetic field on C_1, are

$$
\begin{aligned}
E_z|_{\rho=a} &= \sum_{n=1}^{N} (a_n J_\upsilon(Ka) + b_n H_\upsilon^{(2)}(Ka)) \sin \upsilon(\phi - \alpha) = \sum_{n=1}^{N} c_n \sin \upsilon(\phi - \alpha) \\
H_t|_{\rho=a} &= \frac{-jK}{\omega\mu} \sum_{n=1}^{N} (a_n J'_\upsilon(Ka) + b_n H_\upsilon^{(2)\prime}(Ka)) \sin \upsilon(\phi - \alpha) = \sum_{n=1}^{N} A_n c_n \sin \upsilon(\phi - \alpha)
\end{aligned} \tag{5}
$$

where A_n is known and a_n are those of an infinite conducting wedge [3].

This method can also be applied for TE incidence with slight modifications.

Results

Fig. 2 shows the total field in a infinite conducting wedge under TM plane wave illumination. This result was obtained computing the eigenfunction solution. From this solution, we can easily find the currents at the surface of the wedge. Substracting the PO current, we isolate the diffraction current caused by the edge . Fig. 3 shows the diffraction current for TE plane wave incidence, that agrees with the results obtained by numerically integrating an expression for the diffraction currents [4].

Fig. 4 shows the total field obtained in a wedge ended with a cylinder under TE plane wave illumination, and agrees with the analytical solution. Other results have been obtained in wedges ended with shapes defined by splines but they can not be compared to analytical solutions.

Conclusions

A new method to characterize arbitrary wedges is devised, obtaining the total field near an arbitrary conducting wedge, and as a result, the equivalent currents on the surface.

The edge currents in Fig. 3, could be extended to arbitrary wedges by direct application of the presented method, to obtain the scattered field as a surface integral of uniform and nonuniform currents.

The Physical Theory of Diffraction (PTD) and Method of Equivalent Currents (MEC) extend the PO solution by treating edge diffraction and adding the diffraction currents to the solution. The method presented could complement the PTD and MEC solutions since it includes the phenomenon of diffraction by curved surfaces. The diffraction currents can be obtained on the conducting surface or on the ficticious contour C_1 by direct application of the Equivalence Theorem.

A closed-form approximation of surface currents on an infinite conducting wedge under plane wave illumination was obtained in [4], and a more accurate closed-form expression has been found recently [5], but are not extended to arbitrary wedges. Although we do not obtain the surface currents in a closed-form, but numerically under certain conditions, the posibilities of the method to applying electrically large objects should be considered. The application of the method is currently under investigation and the results would be communicated in due course.

References

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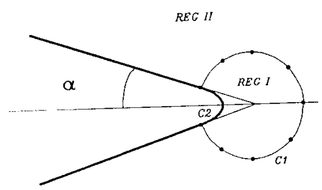

Fig. 1. Analysis regions in an arbitrary conducting wedge.

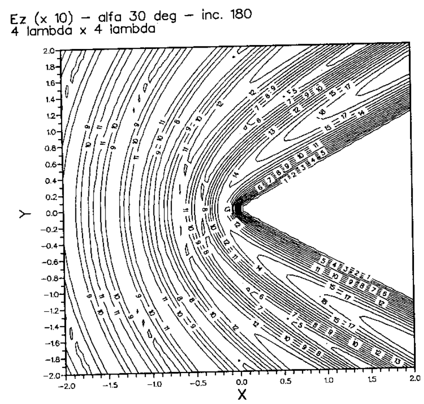

Fig. 2. Total field on an infinite conducting wedge under TM plane wave illumination.

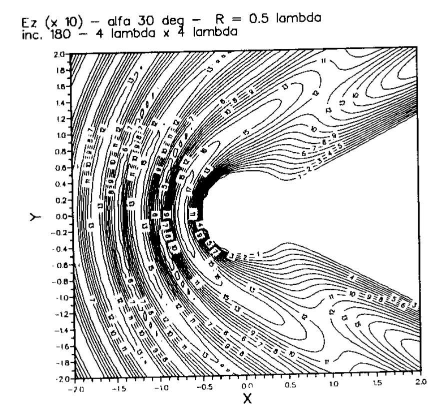

Fig. 4. Total field on an infinite conducting wedge ended with a cylinder under TM plane wave illumination.

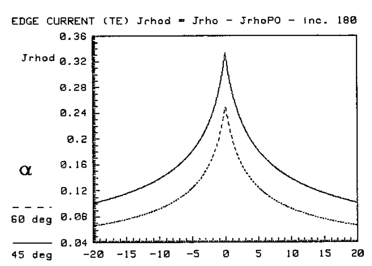

Fig. 3. Edge current on an infinite wedge under TE plane wave illumination.