

## FEEDBACK FORMULATION FOR MULTIPLE SCATTERERS.

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## I. INTRODUCTION.

Numerical methods have been widely used for studying scattering problems. Some of the most frequently used are the Moment Method, which solves an integral equation and calculates the induced currents, the Finite Element Method (F.E.M.) [1], which separately solves the inner and external problems and finally couples them through a surface, and the Boundary Element Method (B.E.M.) [2], which solves an integral equation extended on the boundaries of the objects applying F.E.M. techniques.

The study of the scattering from several objects implies that the system of equations to be solved, usually have very large dimensions, appearing storage and execution time limitations.

In this work, a method allowing to solve these problems in an easy and fast manner is shown. It uses a feedback formulation to model multiple interactions between objects, expressing incident and scattered fields as sums of cylindrical modes, resulting in smaller matrix sizes.

## II. THEORY.

A simple case is considered, as shown in fig.1 where a plane wave is incident on two objects O1 and O2. Applying superposition, this problem can be split in two different ones. Incidence on O1 will be assumed in the first subproblem and incidence on O2 in the second one.

The interactions between both objects are modeled by using the feedback systems shown in fig. 2, where ED stands for scattered field and EI for the incident one.

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The method follows three steps, first the characterization of each object, then the transformation of each emerging mode from one object into incident fields on the others and finally the computation of the scattered fields.

### II.1 OBJECT CHARACTERIZATION.

This is done separately for each object and it will be obtained by calculating the scattered field when there's a field  $J_n(kr) \exp(jn\phi)_{oi}$  incident on it. Subindex  $oi$  stands for modes related to object  $OI$ .

B.E.M. is used to study homogeneous objects. This method starts from the following integral equation for the object's boundary when the incident field is a TM plane wave.

$$\int [G(r, r') H_t(r') - G'(r, r') E_z(r')] dl = E_z(r)$$

where  $G(r, r') = w \mu \cdot 0.25 \cdot H_0^{(3)}(kr)$  and  $G'(r, r')$  is its normal derivative on the boundary.

The scattered field is thus obtained and it can be expressed as a sum of emergent modes  $H_p^{(2)}(kr) \exp(jp\phi)_{oi}$ , as it is shown in fig. 3.a.

Repeating this process for the  $-M \dots +M$  incident modes, it's possible to characterize  $OI$  by a matrix  $DI$  of dimensions  $(2M+1)(2N+1)$ , whose elements  $d_{mn}$  will be the amplitude of the emerging modes  $H_n^{(2)}(kr) \exp(jn\phi)_{oi}$  when the incident field is  $J_m(kr) \exp(jm\phi)_{oi}$ .

### II.2 TRANSFORMATION OF FIELDS.

The field scattered by  $OI$  becomes an incident field on  $OJ$ , as it is shown in fig. 3.b. It is necessary to represent it as a sum of modes  $J_n(kr) \exp(jn\phi)_{oj}$ . This is easily done by expressing the emerging modes as

$$H_p^{(2)}(kr) \exp(jp\phi)_{oi} = \sum_q t_{pq} J_q(kr) \exp(jq\phi)_{oj}.$$

This results in a  $(2N+1)(2M+1)$  matrix  $TJI$ , completely defining the graphs.

### II.3 SCATTERED FIELD COMPUTATIONS.

By following the usual rules of graph analysis, each field EDIJ scattered by OI when only OJ is illuminated, can be easily calculated. The total scattered field will be derived from these:

$$\begin{aligned} ED11 &= (I-D1*T21*D2*T12)^{-1} * D1*EI1. \\ ED21 &= (I-D2*T12*D1*T21)^{-1} * D2*EI1. \\ ED12 &= (I-D1*T21*D2*T12)^{-1} * D1*T21*D2*EI2 \\ ED21 &= (I-D2*T12*D1*T21)^{-1} * D2*EI2. \end{aligned}$$

### III. RESULTS.

The geometry shown in fig.1, when the incident field is a TM plane wave, has been studied by the B.E.M. and the feedback method. As it can be seen there is a good agreement between both. B.E.M.'s formulation requires solving a system of dimension  $(208) \times (208)$ , whereas in the feedback one, we have used  $N=9$  (scattered field) and  $M=12$  (incident field), the largest matrix dimension being  $(25) \times (25)$ .

### REFERENCES

- [1]. Chang and Mei. "Application of the Unimoment Method to Electromagnetic Scattering of Dielectric Cylinders". IEEE T.A.P., vol. AP-24, no. 1, pp. 35 - 42, January 1976.
- [2]. Yashiro and Ohkawa. "Boundary Element Method for Electromagnetic Scattering from Cylinders". IEEE T.A.P., vol. AP-33, no. 4, April 1983.

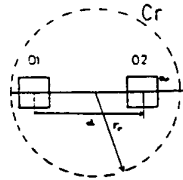


Fig.1. Two dielectric square cylinders

$$\begin{aligned} a &= 1. \\ ka &= 7.6. \\ \epsilon_r &= 2.89. \\ \epsilon_r &= 1.75. \\ d &= 2*a. \end{aligned}$$

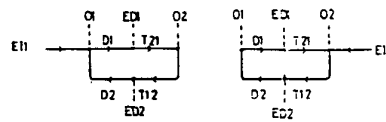


Fig.2. Feedback systems modeling problem of fig.1.

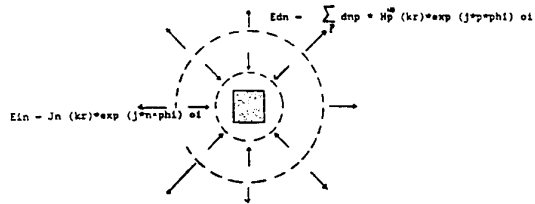


Fig.3.a Characterization : an incident mode and the scattered field.

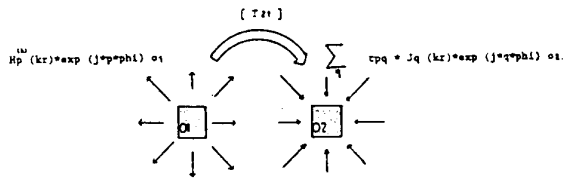


Fig.3.b Transformation : an emergent mode from O1 and the incident field on O2.

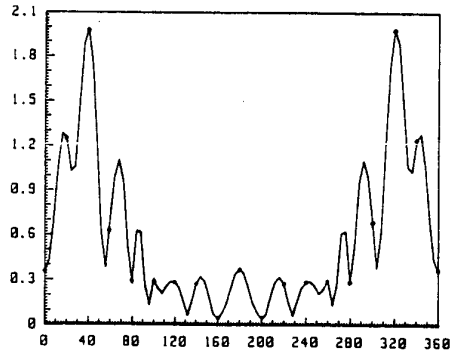


Fig.4. Amplitude of scattered field on Cr for objects of fig.1.

\_\_\_\_\_ B.E.M.

. . . feedback formulation.