# Best Known Solutions for Taillard's Instances Adapted to the Distributed Blocking Flow Shop Scheduling Problem 

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Abstract - The distributed blocking flow shop scheduling problem (DBFSP) allows modeling the scheduling process in companies with more than one factory. It configures production systems as flow shop lines where the blocking constraint must be considered. To the best of our knowledge, this variant of the distributed permutation flow shop scheduling problem has not been studied. In this working paper, we show the best solutions found during our research for the Taillard's instances adapted to this problem.

Keywords-scheduling; heuristics; distributed blocking flow shop; blocking flow shop; distributed permutation flow shop

## Introduction

Distributed manufacturing is a common situation for large enterprises that compete in a globalized market. Because of current globalization trends, production has shifted from single factory production to a multi-factory production network [1]. In this environment, the scheduling problems deal with the allocation of jobs to factories and the scheduling of jobs in each plant. Since the flow shop configuration is the most common processing layout, the flow shop scheduling problem has been studied greatly since the seminal paper of Johnson [2]. However, its extension to a multi-plant environment was first presented by Naderi and Ruiz [3], who referred to it as the Distributed Permutation Flow Shop Scheduling Problem (DPFSP). After the publication of [3], several authors proposed various heuristics to solve this problem ([4]-[13]), but the blocking constraint has been considered in none of them. The blocking flow shop scheduling problem allows many production systems to be modeled when there are no buffers between consecutive machines. In general, it is useful for those systems that have a production line without a drag system that forces a job to be transferred between two consecutive stations at pre-established times. Some industrial examples can be found in the iron and steel industry [14]; in the treatment of industrial waste and the manufacture of metallic parts [15]; or in a robotic cell, where a job may block a machine while waiting for the robot to pick it up and move it to the next stage [16]. The blocking constraint tends to increase the completion time of jobs, because the processed job cannot leave the machine if the next machine is busy. Therefore, the heuristics designed to schedule jobs in this environment have to consider this fact in order to minimize the idle time of machines due to possible blockage. Therefore, the distributed blocking flow shop scheduling problem (DBFSSP) deals with the allocation and scheduling of jobs in a multi-factory production network with the blocking constraint present in the manufacturing system. It is interesting to study this problem in order to design specific procedures, since the adaptation of those designed for the DPFSP probably perform worse than procedures which consider its characteristics.

In this working paper we show the best values found for the Taillard's benchmark [17] applied to the DBFSP.

## Problem definition

The problem is defined as follows: $n$ jobs have to be scheduled in one of the F identical factories. The production configuration of each factory is a flow shop consisting of m machines. Each factory is able to process all jobs. The jobs assigned to a factory have to be processed by all machines in the same order, from machine 1 to machine $m$. Each job $i, i \in\{1,2, \ldots, n\}$ requires a fixed nonnegative processing time $\mathrm{p}_{\mathrm{j}, \mathrm{i}}$ on every machine $j, j \in\{1,2, \ldots, m\}$, which does not change from factory to factory. Setup times are considered to be included in the processing time. The objective is to schedule the jobs to the different factories such that the maximum makespan $\left(\mathrm{C}_{\max }\right)$ among
factories is minimized. Hence, a plant $f$ has a set of $n_{f}$, jobs to be sequenced in order to minimize the $\mathrm{C}_{\max }$ of the plant. We denote $\sigma_{f}$ as the sequence of the $n_{f}$ jobs assigned to plant $f$. Therefore, a solution $\Pi$ is formed by the sequence of jobs in each factory $\left(\Pi=\left(\sigma_{1}, \sigma_{2}, \ldots, \sigma_{f}\right)\right)$.

We denote $\left[k_{f}\right]$ as the job which occupies position $k$ in the sequence of $\sigma_{f}$, and $f_{\max }$ as the factory with the maximum makespan. Let $e_{j, k_{f}, f}$ be the time in which the job $\left[k_{f} f\right]$ starts to be processed in machine $j$, and $c_{j, k, f}$ be the departure time of job $\left[k_{f} f\right]$ in machine $j . \mathrm{C}_{\max }=\mathrm{C}_{\max }(\Pi)$ denotes the global makespan, i.e., the maximum completion time of the last job processed in any of the factory. Therefore, according to this notation the problem can be formalized as follows:
$\operatorname{Min} \max \left\{c_{m, n_{f}, f}\right\} \quad \forall j, k, f$
$e_{j, k, f}=\operatorname{Max}\left\{c_{j, k-1, f} ; c_{j-1, k, f}\right\} \quad \forall j, k, f$
$c_{j, k, f}=\operatorname{Max}\left\{c_{j, k-1, f} ; e_{j, k, f}+p_{j,[k, f]}\right\} \quad \forall j, k, f$
with $c_{j, 0}=0, c_{0, k, f}=0, c_{m+1, k, f}=0 \quad \forall j, k, f$ being the initial conditions.

## Best solutions for Taillard's instances

The Taillard's benchmark was generated to test algorithms for the permutation flow shop problem with makespan criterion, although they have also been used under other criteria and conditions. In particular, these instances were adapted to the DPFSP in [3] and used later in [18] and [9] to test their algorithms for the same problem. The benchmark is composed of 12 sets of 10 instances, ranging from 20 jobs and 5 machines to 500 jobs and 20 machines, where $n \in\{20,50,100,200$, $500\}$ and $m \in\{5,10,20\}$, although not all combinations of $n$ and $m$ are available. In particular, sets $200 \times 5,500 \times 5$ and $500 \times 10$ are missing. These 120 instances were augmented with six values of $F \in\{2,3,4,5,6,7\}$.

The next table shows the best makespan value known for each instance.

| num | $n$ | m | F |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 2 | 3 | 4 | 5 | 6 | 7 |
| 1 | 20 | 5 | 771 | 583 | 493 | 440 | 409 | 384 |
| 2 | 20 | 5 | 793 | 589 | 496 | 438 | 404 | 381 |
| 3 | 20 | 5 | 704 | 522 | 446 | 398 | 373 | 360 |
| 4 | 20 | 5 | 805 | 616 | 521 | 469 | 432 | 413 |
| 5 | 20 | 5 | 758 | 573 | 489 | 438 | 406 | 385 |
| 6 | 20 | 5 | 745 | 565 | 483 | 436 | 404 | 383 |
| 7 | 20 | 5 | 757 | 568 | 478 | 439 | 430 | 430 |
| 8 | 20 | 5 | 758 | 568 | 489 | 440 | 412 | 386 |
| 9 | 20 | 5 | 767 | 577 | 478 | 427 | 396 | 376 |
| 10 | 20 | 5 | 699 | 513 | 434 | 392 | 365 | 347 |
| 11 | 20 | 10 | 1085 | 888 | 789 | 732 | 693 | 670 |
| 12 | 20 | 10 | 1155 | 941 | 840 | 781 | 735 | 707 |
| 13 | 20 | 10 | 1044 | 845 | 754 | 700 | 673 | 650 |
| 14 | 20 | 10 | 969 | 778 | 689 | 635 | 605 | 586 |
| 15 | 20 | 10 | 1005 | 817 | 727 | 671 | 650 | 628 |
| 16 | 20 | 10 | 978 | 786 | 695 | 648 | 613 | 591 |
| 17 | 20 | 10 | 1019 | 832 | 743 | 696 | 671 | 671 |
| 18 | 20 | 10 | 1075 | 875 | 773 | 720 | 692 | 692 |
| 19 | 20 | 10 | 1070 | 860 | 769 | 713 | 702 | 702 |
| 20 | 20 | 10 | 1127 | 915 | 808 | 760 | 723 | 707 |
| 21 | 20 | 20 | 1715 | 1482 | 1363 | 1305 | 1257 | 1237 |
| 22 | 20 | 20 | 1602 | 1391 | 1297 | 1235 | 1191 | 1191 |
| 23 | 20 | 20 | 1772 | 1528 | 1405 | 1347 | 1320 | 1320 |
| 24 | 20 | 20 | 1665 | 1443 | 1354 | 1297 | 1259 | 1239 |
| 25 | 20 | 20 | 1726 | 1490 | 1377 | 1309 | 1280 | 1253 |
| 26 | 20 | 20 | 1687 | 1468 | 1356 | 1286 | 1256 | 1256 |
| 27 | 20 | 20 | 1701 | 1474 | 1362 | 1297 | 1251 | 1232 |
| 28 | 20 | 20 | 1648 | 1422 | 1321 | 1258 | 1232 | 1227 |
| 29 | 20 | 20 | 1699 | 1483 | 1369 | 1302 | 1257 | 1240 |
| 30 | 20 | 20 | 1638 | 1416 | 1302 | 1240 | 1201 | 1166 |
| 31 | 50 | 5 | 1563 | 1076 | 828 | 688 | 603 | 536 |
| 32 | 50 | 5 | 1657 | 1143 | 887 | 738 | 648 | 582 |
| 33 | 50 | 5 | 1565 | 1072 | 837 | 699 | 609 | 543 |
| 34 | 50 | 5 | 1633 | 1135 | 882 | 736 | 642 | 578 |
| 35 | 50 | 5 | 1648 | 1129 | 882 | 736 | 636 | 572 |
| 36 | 50 | 5 | 1649 | 1145 | 897 | 749 | 657 | 591 |
| 37 | 50 | 5 | 1589 | 1109 | 872 | 730 | 640 | 575 |
| 38 | 50 | 5 | 1582 | 1092 | 856 | 715 | 622 | 560 |
| 39 | 50 | 5 | 1511 | 1052 | 825 | 688 | 602 | 540 |
| 40 | 50 | 5 | 1616 | 1116 | 873 | 727 | 631 | 565 |
| 41 | 50 | 10 | 1985 | 1452 | 1190 | 1041 | 940 | 870 |
| 42 | 50 | 10 | 1910 | 1394 | 1143 | 1003 | 904 | 844 |
| 43 | 50 | 10 | 1919 | 1399 | 1146 | 1009 | 921 | 856 |
| 44 | 50 | 10 | 1989 | 1450 | 1188 | 1042 | 948 | 882 |
| 45 | 50 | 10 | 1976 | 1437 | 1190 | 1042 | 947 | 879 |
| 46 | 50 | 10 | 1978 | 1432 | 1181 | 1036 | 935 | 869 |
| 47 | 50 | 10 | 2038 | 1488 | 1227 | 1069 | 973 | 901 |
| 48 | 50 | 10 | 1981 | 1443 | 1195 | 1045 | 948 | 875 |
| 49 | 50 | 10 | 1931 | 1404 | 1148 | 1008 | 911 | 844 |
| 50 | 50 | 10 | 2005 | 1474 | 1220 | 1067 | 971 | 902 |


| num | n | m | F |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 2 | 3 | 4 | 5 | 6 | 7 |
| 51 | 50 | 20 | 2736 | 2147 | 1864 | 1692 | 1582 | 1502 |
| 52 | 50 | 20 | 2597 | 2043 | 1761 | 1610 | 1501 | 1423 |
| 53 | 50 | 20 | 2598 | 2045 | 1765 | 1603 | 1494 | 1419 |
| 54 | 50 | 20 | 2641 | 2073 | 1797 | 1638 | 1531 | 1452 |
| 55 | 50 | 20 | 2589 | 2042 | 1765 | 1609 | 1504 | 1428 |
| 56 | 50 | 20 | 2586 | 2043 | 1769 | 1602 | 1495 | 1426 |
| 57 | 50 | 20 | 2608 | 2047 | 1766 | 1603 | 1497 | 1417 |
| 58 | 50 | 20 | 2613 | 2063 | 1791 | 1624 | 1515 | 1436 |
| 59 | 50 | 20 | 2653 | 2094 | 1820 | 1652 | 1539 | 1468 |
| 60 | 50 | 20 | 2667 | 2097 | 1808 | 1644 | 1531 | 1453 |
| 61 | 100 | 5 | 3138 | 2135 | 1628 | 1331 | 1127 | 982 |
| 62 | 100 | 5 | 3088 | 2091 | 1594 | 1302 | 1101 | 963 |
| 63 | 100 | 5 | 3018 | 2057 | 1568 | 1273 | 1082 | 942 |
| 64 | 100 | 5 | 2941 | 1993 | 1519 | 1236 | 1049 | 912 |
| 65 | 100 | 5 | 3054 | 2067 | 1579 | 1278 | 1093 | 954 |
| 66 | 100 | 5 | 2969 | 2021 | 1533 | 1248 | 1059 | 924 |
| 67 | 100 | 5 | 3070 | 2074 | 1586 | 1287 | 1095 | 957 |
| 68 | 100 | 5 | 2985 | 2025 | 1541 | 1250 | 1056 | 922 |
| 69 | 100 | 5 | 3119 | 2125 | 1621 | 1328 | 1125 | 985 |
| 70 | 100 | 5 | 3122 | 2113 | 1611 | 1309 | 1109 | 967 |
| 71 | 100 | 10 | 3677 | 2549 | 2004 | 1687 | 1469 | 1323 |
| 72 | 100 | 10 | 3558 | 2449 | 1911 | 1595 | 1384 | 1239 |
| 73 | 100 | 10 | 3614 | 2496 | 1952 | 1638 | 1422 | 1277 |
| 74 | 100 | 10 | 3775 | 2616 | 2051 | 1718 | 1502 | 1353 |
| 75 | 100 | 10 | 3584 | 2481 | 1953 | 1632 | 1427 | 1280 |
| 76 | 100 | 10 | 3502 | 2420 | 1877 | 1568 | 1364 | 1222 |
| 77 | 100 | 10 | 3581 | 2476 | 1934 | 1608 | 1407 | 1263 |
| 78 | 100 | 10 | 3594 | 2514 | 1967 | 1645 | 1437 | 1292 |
| 79 | 100 | 10 | 3712 | 2579 | 2022 | 1692 | 1475 | 1319 |
| 80 | 100 | 10 | 3656 | 2543 | 1991 | 1666 | 1461 | 1314 |
| 81 | 100 | 20 | 4350 | 3183 | 2606 | 2278 | 2043 | 1891 |
| 82 | 100 | 20 | 4351 | 3167 | 2589 | 2250 | 2037 | 1881 |
| 83 | 100 | 20 | 4342 | 3163 | 2608 | 2272 | 2051 | 1897 |
| 84 | 100 | 20 | 4351 | 3173 | 2605 | 2261 | 2043 | 1891 |
| 85 | 100 | 20 | 4355 | 3175 | 2605 | 2269 | 2051 | 1900 |
| 86 | 100 | 20 | 4407 | 3236 | 2654 | 2319 | 2094 | 1934 |
| 87 | 100 | 20 | 4410 | 3197 | 2636 | 2294 | 2070 | 1908 |
| 88 | 100 | 20 | 4465 | 3264 | 2695 | 2353 | 2125 | 1963 |
| 89 | 100 | 20 | 4398 | 3230 | 2650 | 2304 | 2084 | 1931 |
| 90 | 100 | 20 | 4421 | 3243 | 2661 | 2317 | 2097 | 1934 |
| 91 | 200 | 10 | 6902 | 4719 | 3605 | 2956 | 2499 | 2197 |
| 92 | 200 | 10 | 6831 | 4639 | 3576 | 2907 | 2483 | 2170 |
| 93 | 200 | 10 | 6848 | 4674 | 3593 | 2929 | 2496 | 2185 |
| 94 | 200 | 10 | 6830 | 4675 | 3559 | 2915 | 2480 | 2170 |
| 95 | 200 | 10 | 6830 | 4674 | 3585 | 2921 | 2481 | 2169 |
| 96 | 200 | 10 | 6739 | 4585 | 3514 | 2863 | 2444 | 2137 |
| 97 | 200 | 10 | 6961 | 4759 | 3644 | 2987 | 2534 | 2220 |
| 98 | 200 | 10 | 6909 | 4698 | 3603 | 2946 | 2504 | 2190 |
| 99 | 200 | 10 | 6842 | 4657 | 3560 | 2915 | 2479 | 2169 |
| 100 | 200 | 10 | 6867 | 4676 | 3586 | 2917 | 2492 | 2175 |


| num | $\boldsymbol{n}$ | $\boldsymbol{m}$ | $\mathbf{F}$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ |  |
|  |  |  |  |  |  |  |  |  |  |
| 101 | 200 | 20 | 7820 | 5487 | 4314 | 3608 | 3151 | 2824 |  |
| 102 | 200 | 20 | 7929 | 5533 | 4336 | 3644 | 3183 | 2855 |  |
| 103 | 200 | 20 | 8000 | 5610 | 4402 | 3682 | 3212 | 2871 |  |
| 104 | 200 | 20 | 7948 | 5556 | 4373 | 3645 | 3183 | 2854 |  |
| 105 | 200 | 20 | 7859 | 5484 | 4297 | 3599 | 3139 | 2809 |  |
| 106 | 200 | 20 | 7940 | 5519 | 4321 | 3611 | 3136 | 2819 |  |
| 107 | 200 | 20 | 7944 | 5554 | 4370 | 3659 | 3203 | 2869 |  |
| 108 | 200 | 20 | 7965 | 5574 | 4377 | 3660 | 3202 | 2864 |  |
| 109 | 200 | 20 | 7908 | 5540 | 4349 | 3635 | 3172 | 2845 |  |
| 110 | 200 | 20 | 7951 | 5538 | 4367 | 3651 | 3202 | 2863 |  |
|  |  |  |  |  |  |  |  |  |  |
| 111 | 500 | 20 | 18399 | 12525 | 9619 | 7869 | 6704 | 5849 |  |
| 112 | 500 | 20 | 18541 | 12670 | 9724 | 7959 | 6766 | 5919 |  |
| 113 | 500 | 20 | 18344 | 12517 | 9609 | 7863 | 6701 | 5860 |  |
| 114 | 500 | 20 | 18469 | 12610 | 9688 | 7908 | 6742 | 5888 |  |
| 115 | 500 | 20 | 18374 | 12490 | 9575 | 7856 | 6667 | 5831 |  |
| 116 | 500 | 20 | 18494 | 12618 | 9655 | 7893 | 6699 | 5889 |  |
| 117 | 500 | 20 | 18348 | 12510 | 9595 | 7845 | 6665 | 5806 |  |
| 118 | 500 | 20 | 18399 | 12595 | 9643 | 7883 | 6700 | 5881 |  |
| 119 | 500 | 20 | 18313 | 12480 | 9559 | 7857 | 6647 | 5798 |  |
| 120 | 500 | 20 | 18496 | 12597 | 9680 | 7911 | 6713 | 5872 |  |

Best solutions for the DBFSP

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