The diagonal of the TT transform: computation and meaning

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Abstract: *The TT-transform stands for time-time transform and has been derived as an inverse Fourier transform of the time-frequency S-transform. So far, it has been proposed that the diagonal of the TT-transform can be used for signal characterisation. We show here an alternative and simplified derivation of the TT-transform which enables a better understanding of this transform.*

Keywords: *TT-transform, S-transform, time-frequency localisation, time-time analysis*

1. INTRODUCTION

In disciplines such as music or geophysics, signals are non stationary. The need for processing such signals has led to the appearance of several types of time varying frequency filters, such as the short time Fourier transform [1], wavelets [2], and more recently the S-transform (ST) [3]. These transforms introduce redundancy passing from a 1D time signal to a 2D time-frequency (or time-scale) signal. In 2003, [4] introduced a new transform based on the ST and called it the TT-transform (TT). It includes redundancy in time passing from a 1D time signal to a 2D time-time signal. Until now, this transform has seen little application [5] and in general, interest has mainly been focused on the diagonal part. The aim of this correspondence is to show an simpler way of computing the diagonal of the TT and to give a clear interpretation of it.

In the next section, the *S*- and TT-transforms will be reviewed. Section 3 will demonstrate a simplified way to compute the latter. Section 4 will show examples and the last section will conclude this paper.

2. THE *S*- AND TT-TRANSFORMS

The ST of $u(t)$ is

$$S(\tau, f) = \int_{-\infty}^{\infty} u(t) w(t-\tau, f) \mathrm{e}^{-2i\pi f t} \mathrm{d}t,$$

$w(t,f)$ being a 1-mean window, generally a Gaussian with a variance of $1/f$.

The TT, [4], is the inverse Fourier transform(FT) w.r.t. f of $S(\tau,f)$:

$$TT(\tau, t) = \int_{-\infty}^{\infty} S(\tau, f) \mathrm{e}^{2i\pi f t} \mathrm{d}t.$$

Both transforms are easily invertible.

3. COMPUTING THE DIAGONAL OF THE TT-TRANSFORM

In the applications using the TT, only its diagonal part has been used. We will hence concentrate on this part. For details on the rest of the transform, see [5].

We have shown, [5], that

$$T(t,t) = \mathcal{F}^{-1}\{U(f)G(f)\}$$

where $\mathcal{F}^{-1}$ is the inverse FT, $U(f)$ the FT of $u(t)$ and $G(f) = -k\pi^2|f|$, k being a constant.

This is an important result as, by using this formula, not only can we completely forget about the use of the ST and thus simplify the computation of the TT but we can also much more easily understand its behaviour. Indeed, we see that the diagonal terms of the TT are just a frequency filtered version of the original signal, putting more emphasis on high frequencies.

4. EXAMPLE OF APPLICATION

The example, Fig. 1, top, is a sum of sines. The bottom plot show the diagonal terms of its TT.

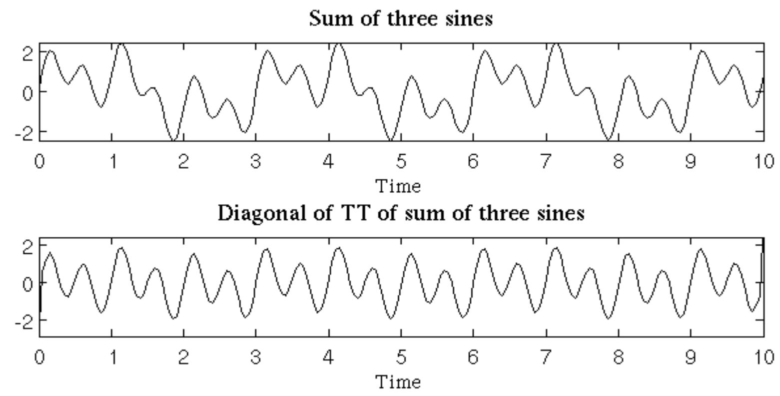

Fig. 1. Original signal and the diagonal of its TT

On its TT, Fig. 2, it is seen how most of the information is centred on the diagonal part.

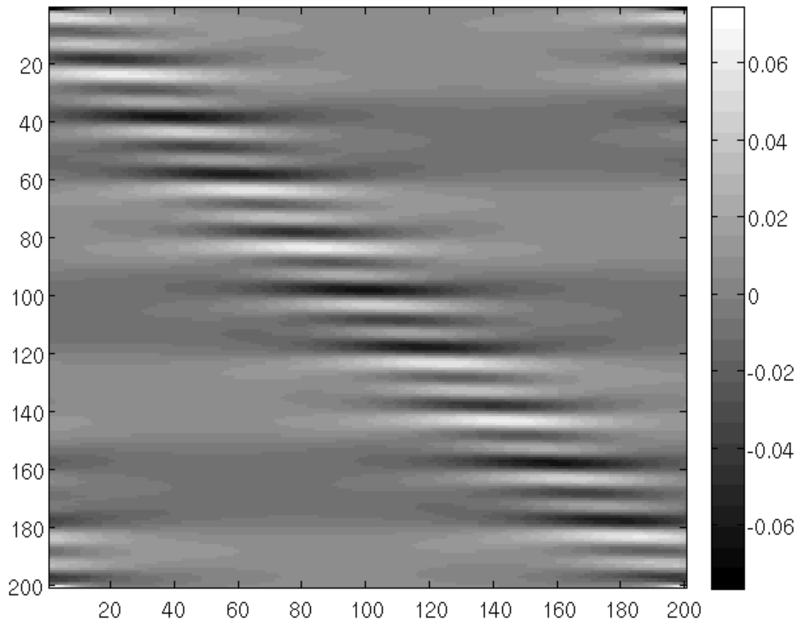

Fig.2. TT of the signal

On Fig. 3, the FFT of the diagonal of the TT as well as the FFT of the original signal is shown. The first one has been normalised to facilitate the comparison. Here, we can clearly see how the TT emphasises high frequencies at the cost of low frequencies.

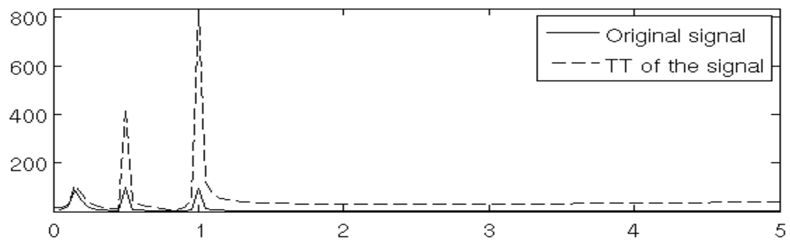

Fig.3. FFT of the signal and of the diagonal of its TT

5. CONCLUSION

In this presentation, we show that computing the diagonal elements of the TT-transform of a signal is equivalent to frequency filtering it; the equivalent filter gives more emphasis to high frequencies with respect to low ones. As well as allowing a clear interpretation of the meaning of the diagonal of the TT-transform, this work thus gives a much simpler and more direct way to compute it.

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