

Viability of the matter bounce scenario in $F(T)$ gravity and Loop Quantum Cosmology for general potentials

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Abstract

We consider the matter bounce scenario in $F(T)$ gravity and Loop Quantum Cosmology (LQC) for phenomenological potentials that at early times provide a nearly matter dominated Universe in the contracting phase, having a reheating mechanism in the expanding or contracting phase, i.e., being able to release the energy of the scalar field creating particles that thermalize in order to match with the hot Friedmann Universe, and finally at late times leading to the current cosmic acceleration. For these potentials, numerically solving the dynamical perturbation equations we have seen that, for the particular $F(T)$ model that we will name *teleparallel version of LQC*, and whose modified Friedmann equation coincides with the corresponding one in holonomy corrected LQC when one deals with the flat Friedmann-Lemaître-Robertson-Walker (FLRW) geometry, the corresponding equations obtained from the well-know perturbed equations in $F(T)$ gravity lead to theoretical results that fit well with current observational data. More precisely, in this *teleparallel version of LQC* there is a set of solutions which leads to theoretical results that match correctly with last BICEP2 data, and there is another set whose theoretical results fit well with *Planck's* experimental data. On the other hand, in the standard holonomy corrected LQC, using the perturbed equations obtained replacing the Ashtekar connection by a suitable sinus function and inserting some counter-terms in order to preserve the algebra of constraints, the theoretical value of the tensor/scalar ratio is smaller than in the teleparallel version, which means that there is always a set of solutions that matches with *Planck's* data, but for some potentials BICEP2 experimental results disfavours holonomy corrected LQC.

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1 Introduction

It's well-known that inflation suffers from several problems (see [1] for a review about these problems), like the initial singularity which normally is not addressed (as an exception, in [2] the problem was addressed concluding that the initial singularity is unavoidable in general inflationary models), the fine-tuning of the degree of flatness required for the potential in order to achieve successful inflation [3], or the following problem related with initial conditions: In inflationary cosmology it is usually assumed that modes well inside the Hubble radius are initially (at the beginning of inflation) in the adiabatic vacuum in order to obtain a nearly scale invariant spectrum. This assumption could be accepted if, as in Linde's early papers about chaotic inflation (see for instance [4] for a review), inflation started at energy densities of the order of Planck's scale, because in that case before inflation it would be impossible to describe classically our Universe. However, from the four-year data set provided by the Cosmic Background Explorer (COBE) satellite or from the seven-year data of Wilkinson Microwave Anisotropy Probe (WMAP), we know that the observed value of the power spectrum for scalar perturbations is constrained to be $P_\zeta(k) \cong 2 \times 10^{-9}$ [5] for modes that exit the Hubble radius 60 e-folds before the end of inflation, which means that in chaotic inflation the slow roll phase started at energy densities of the order $10^{-11} \rho_{pl}$, and consequently, the evolution of the Universe could be described classically before inflation. Then, it is essential to know the evolution of the modes before inflation, because if they re-enter in the Hubble radius, positive and negative frequencies could mix, and thus, those modes would not be in the vacuum state.

In order to avoid these problems, an alternative scenario to the inflationary paradigm, called *matter bounce scenario* [6] (essentially it depicts, at very early times, a matter dominated Universe in the contracting phase that evolves towards the bounce to enter in the expanding phase), has been developed to explain the evolution of our Universe. This model, like inflation, solves the horizon problem that appears in Einstein Cosmology (EC) and improves the flatness problem in EC (where spatial flatness is an unstable fixed point and fine tuning of initial conditions is required), because the contribution of the spatial curvature decreases in the contracting phase at the same rate as it increases in the expanding one (see for instance [7]). However, it suffers from the anisotropy problem that does not exist in other models like Ekpyrotic scenarios [8], for this reason an improved model combining both scenarios could avoid this problem [9], and become a realistic alternative to inflation.

There are essentially two ways to set up a matter bounce scenario in the flat FLRW geometry: within the framework of EC violating the null energy condition at the bouncing point [10], or going beyond EC. In order to violate the null energy condition in EC one needs to incorporate new forms of matter such as phantom [11] or quintom fields [10], Galileons [12] or phantom condensates [13]. Going beyond EC one can introduce higher derivatives in the action [14], braneworld bouncing scenarios [15], Ekpyrotic [16], Pre-Big-Bang [17], loop quantum [18] or

teleparallel cosmologies [19], modified $f(R)$ gravity [20], extended loop quantum cosmology to R^2 gravity [21], etc.

In the present work we only deal with the matter bounce scenario, that in the flat FLRW geometry has the same Friedmann equation as in holonomy corrected Loop Quantum Cosmology (LQC), which leads to the simplest bouncing scenario, and where the numerical calculation can be carried out completely.

We will perform a deep and detailed study of the evolution of cosmological perturbations in this scenario, that improves on those recently made in [22] using standard holonomy corrections and those of [23] where an example of $F(T)$ gravity which leads, in the flat FLRW geometry, to the same modified Friedmann equation as in holonomy corrected LQC, was used to obtain the perturbation equations in the framework of $F(T)$ gravity. In both works, the potential of the used scalar field leads to solutions that at early and late times are in a matter dominated phase, and do not agree with the current acceleration of the Universe. Moreover, since only one analytic solution of the isotropic equations (the unperturbed ones) is known, all the calculations are performed with this analytic solution, which leads to conclusions that do not agree with the current observations. For example, in those works it is claimed that, in order to match theoretical results with observations, the value of the critical density in LQC has to be of the order of $10^{-9}\rho_{pl}$ which contradicts the current value $0.4\rho_{pl}$ [24]. Dealing with tensor perturbations, in holonomy corrected LQC the equation of tensor perturbations has singularities when the energy density is a half of the critical one, meaning that one could consider infinitely many mode solutions because there is not a criterium of continuity at the singular point to decide which mode is the correct one, and thus, there is not a unique way to calculate the power spectrum for tensor perturbations. On the other hand, in the *teleparallel version of LQC*, i.e. using the $F(T)$ -perturbed equation for the model whose isotropic Friedmann equation coincides with the holonomy corrected one of LQC, there is a unique way to calculate this power spectrum, but for the analytical solution of the isotropic equations the ratio of tensor to scalar perturbations is approximately equal to 6, which does not agree with the current observational data.

However, for the other solutions which we have obtained numerically in this work, there is a set of solutions that fit well with the recent BICEP2 data [25], i.e. for the solutions that belong in that set its tensor/scalar ratio satisfy $r = 0.20^{+0.07}_{-0.05}$ [26], and another one whose tensor/scalar ratio is smaller than 0.11, matching correctly with the latest *Planck's* data [27]. Then, our main objective in this work is to generalize this result to more phenomenological potentials (containing a matter domination in the contracting phase at early times, and leading to a reheating process in the expanding phase in order to match with the current Λ CDM model of the Universe, or more generally, with the hot Friedmann Universe plus the current cosmic acceleration). That is, we want to find phenomenological potentials that have a set of solutions that agrees with BICEP2 data, and another one that matches correctly with the latest *Planck's* results. For some of those potentials we present numerical results supporting this match.

The outline of the paper goes as follows: In Section II we review the way to obtain the Mukhanov-Sasaki equations in standard holonomy corrected LQC and in its teleparallel version, i.e. in $F(T)$ gravity for the model that, restricted to the flat FLRW geometry, leads to the same dynamical equations that in holonomy corrected LQC. Section III is devoted to showing that, for a matter bounce scenario, the relation between the Bardeen potential and the curvature fluctuation in co-moving coordinates, in Fourier space, when the Universe is matter dominated at late times, is the same as in inflationary cosmology when the Universe has a phase transition from the quasi de Sitter stage to the matter dominated one, that is, its quotient is equal to $3/5$. As a consequence, since in matter bounce scenario the power spectrum of the curvature fluctuation in co-moving coordinates is scale invariant, the corresponding power spectrum for the Bardeen potential is also scale invariant, which is not trivial to show in this scenario. Section IV is destined to review, with all the details, the calculation of the power spectrum of scalar perturbations in both holonomy corrected LQC and its teleparallel version. In Section V we deal with the problems of the potential currently used to study the matter bounce scenario in holonomy corrected LQC, for example the absence of an explanation for the current cosmic acceleration. In the last Section we suggest some models that include a reheating process and the current acceleration of the Universe, and which lead to solutions whose theoretical results match correctly with current observational data (power spectrum of scalar perturbations, spectral index of scalar perturbations and ratio of tensor to scalar perturbations). Moreover, at the end of this Section we perform a detailed study of the reheating in the matter bounce scenario via gravitational particle production.

The units used in the work are $\hbar = c = 8\pi G = 1$.

2 Mukhanov-Sasaki variables in LQC

Assuming that the dynamics of the Universe is carried on by a scalar field, namely $\varphi = \bar{\varphi} + \delta\varphi$, where $\bar{\varphi}$ is the homogeneous part of the field, in EC, where in the flat Friedmann-Lemaître-Robertson-Walker (FLRW) geometry the Friedmann equation is $H^2 = \frac{\rho}{3}$, the perturbation equations, in the longitudinal gauge, are (see for instance [28])

$$\frac{1}{a^2}\Delta\Phi = \frac{\dot{\bar{\varphi}}^2}{2H} \frac{d}{dt} \left(\frac{H}{\dot{\bar{\varphi}}} \delta\varphi + \Phi \right), \quad \frac{d}{dt} \left(\frac{a\Phi}{H} \right) = \frac{a\dot{\bar{\varphi}}^2}{2H^2} \left(\frac{H}{\dot{\bar{\varphi}}} \delta\varphi + \Phi \right). \quad (1)$$

Introducing the variables

$$v = a(\delta\varphi + \frac{\dot{\bar{\varphi}}}{H}\Phi); \quad z = \frac{a\dot{\bar{\varphi}}}{H}, \quad u = \frac{2\Phi}{\dot{\bar{\varphi}}}; \quad \theta = \frac{1}{z}, \quad (2)$$

one obtains the Mukhanov-Sasaki (M-S) equations

$$c_s \Delta u = z \left(\frac{v}{z} \right)'; \quad \theta \left(\frac{u}{\theta} \right)' = c_s v, \quad (3)$$

where in EC, the velocity of sound, namely c_s , is equal to 1.

On the other hand, in holonomy corrected Loop Quantum Cosmology (LQC), in the flat FLRW geometry the corresponding modified Friedmann equation is (see for a review [29] of LQC)

$$H^2 = \frac{\rho}{3} \left(1 - \frac{\rho}{\rho_c} \right), \quad (4)$$

where ρ_c is the so-called ‘‘critical density’’ (the energy at which the Universe bounces).

Remark 2.1. *It is important to realize that this equation could be obtained for all kind of fluids and scalar fields from the holonomy corrected Hamiltonian in Loop Quantum Gravity (LQG), which for the flat FLRW geometry, and working in the so-called new quantization scheme [30], has the form [31, 32, 33]*

$$\mathcal{H}_{LQC} \equiv -\frac{2}{\gamma^3 \lambda^3} \sum_{i,j,k} \varepsilon^{ijk} \text{Tr}[h_i(\lambda) h_j(\lambda) h_i^{-1}(\lambda) h_j^{-1}(\lambda) h_k(\lambda) \{h_k^{-1}(\lambda), V\}] + \rho V, \quad (5)$$

where $V = a^3$ is the volume, γ is the Barbero-Immirzi parameter (whose value is constrained, but not fixed as earlier believed, by the Bekenstein-Hawking formula for the entropy of a black hole (see [24, 34] for the earlier determination and [35] for updated derivation which shows that the Immirzi parameter is no longer fixed, but only bounded in the LQC setting, by this formula) and $\lambda = \sqrt{\frac{\sqrt{3}}{4}} \gamma$ is the square root of the minimum eigenvalue of the area operator in LQG.

The holonomies are given by

$$h_j(\lambda) \equiv e^{-i \frac{\lambda \beta}{2} \sigma_j} = \cos\left(\frac{\lambda \beta}{2}\right) - i \sigma_j \sin\left(\frac{\lambda \beta}{2}\right), \quad (6)$$

where the Pauli matrices σ_j have been used and β is canonically conjugate to V , with Poisson bracket $\{V, \beta\} = \frac{\gamma}{2}$.

A simple calculation leads to the following holonomy corrected Hamiltonian

$$\mathcal{H}_{LQC} = -3V \frac{\sin^2(\lambda \beta)}{\lambda^2 \gamma^2} + \rho V. \quad (7)$$

Then, from the Hamilton equation $\dot{V} = \{V, \mathcal{H}_{LQC}\}$ one obtains the relation $H = \frac{\sin(2\lambda\beta)}{2\lambda\gamma}$, that together with the Hamiltonian constrain $\mathcal{H}_{LQC} = 0$, lead to the holonomy corrected Friedmann equation in LQC, which depicts, in the plane (H, ρ) , the ellipse given by (4). Note also that in formula (4) the critical density is equal to

$$\rho_c = \frac{3}{\lambda^2 \gamma^2} = \frac{4\sqrt{3}}{\gamma^3}. \quad (8)$$

This method to obtain the holonomy corrected Friedmann equations was obtained independently in [36, 37, 38], and does not depend of the fluid or scalar field used to depict the material composition of the Universe. Another different question is how to obtain equation (4) from the quantum Hamiltonian constrain in LQC. In this situation, using coherent states, it has been proved that equation (4) is obtained for models without potential [39], and what has not been confirmed yet, is whether this equation (4) could be recovered from the quantum Hamiltonian constrain in all kind of fluids and scalar fields.

In holonomy corrected LQC the perturbation equations were obtained for the first time in [40] using holonomy corrections, i.e., replacing the Ashtekar connection by a suitable sinus function [41], and adding some counter-terms to the perturbed Hamiltonian in order to preserve the algebra of constrains. These equations are

$$\frac{1}{a^2}\Delta\Phi = \frac{\dot{\dot{\varphi}}^2}{2H} \frac{d}{dt} \left(\frac{H}{\dot{\varphi}} \delta\varphi + \Phi \right), \quad \frac{d}{dt} \left(\frac{a\Phi}{H} \right) = \frac{a\dot{\dot{\varphi}}^2 c_s^2}{2H^2} \left(\frac{H}{\dot{\varphi}} \delta\varphi + \Phi \right), \quad (9)$$

and they differ from the classical ones in the square of the velocity of sound that appears in the right hand side of the second equation, and whose value is

$$c_s^2 \equiv \Omega = 1 - \frac{2\rho}{\rho_c}. \quad (10)$$

Introducing the variables

$$v = a(\delta\varphi + \frac{\dot{\varphi}}{H}\Phi); \quad z = \frac{a\dot{\varphi}}{H}, \quad u = \frac{2\Phi}{\dot{\varphi}c_s}; \quad \theta = \frac{1}{c_s z}, \quad (11)$$

one obtains the corresponding M-S equations (3) in holonomy corrected LQC.

Note that in the super-inflationary phase, i.e. when $\rho > \rho_c/2$, the velocity of the sound becomes imaginary which could lead, during this stage, to a Jeans instability for ultra-violet modes satisfying $k^2 |c_s^2| \gg \left| \frac{z''}{z} \right|$, and as a consequence, these growing modes could condensate and produce undesirable cosmological consequences. This is a problem that needs to be addressed, because the validity of the linear perturbation equations during this regime is not clear.

This is one of the reasons why a *Teleparallel version of LQC* has recently been introduced in [19]. This theory is based in the fact that, in the flat FLRW geometry, the holonomy corrected Friedmann equation introduced above could be obtained as a particular case of a teleparallel $F(T)$ theory. In [19] this example has been found to be

$$F_{\pm}(T) = \pm \sqrt{-\frac{T\rho_c}{2}} \arcsin \left(\sqrt{-\frac{2T}{\rho_c}} \right) + \frac{\rho_c}{2} \left(1 \pm \sqrt{1 + \frac{2T}{\rho_c}} \right), \quad (12)$$

where + corresponds to the super-inflationary phase, i.e. to $\rho > \rho_c/2$, and – to the deflationary one, i.e. to $\rho < \rho_c/2$.

Remark 2.2. The function (12) is easily obtained, isolating ρ as a function of T in equation (4) and inserting it in the general Friedmann equation for $F(T)$ gravity $\rho = -2\frac{dF(T)}{dT} + F(T)$ [19]. Note that, in this case, in order to obtain (12) it is not necessary that the critical density be given by the expression (8). It can be understood just as a parameter in the theory.

The perturbation equations were recently obtained in [23] using the well-known perturbed equations in $F(T)$ gravity [42] applied to this particular $F(T)$ model. The result is

$$\frac{c_s^2}{a^2}\Delta\Phi = \frac{\dot{\varphi}^2\Omega}{2H} \frac{d}{dt} \left(\frac{H}{\dot{\varphi}} \delta\varphi + \Phi \right), \quad \frac{d}{dt} \left(\frac{a\Phi}{H} \right) = \frac{a\dot{\varphi}^2\Omega}{2H^2} \left(\frac{H}{\dot{\varphi}} \delta\varphi + \Phi \right), \quad (13)$$

where, in this teleparallel version of LQC, the square of the velocity of sound is

$$c_s^2 \equiv |\Omega| \frac{\arcsin\left(2\sqrt{\frac{3}{\rho_c}}H\right)}{2\sqrt{\frac{3}{\rho_c}}H}, \quad (14)$$

which is always positive.

Performing the change of variables

$$v = a \frac{\sqrt{|\Omega|}}{c_s} \left(\delta\varphi + \frac{\dot{\varphi}}{H} \Phi \right); \quad z = \frac{a\sqrt{|\Omega|\dot{\varphi}}}{c_s H}, \quad u = \frac{2\Phi}{\dot{\varphi}\sqrt{|\Omega|}}; \quad \theta = \frac{1}{c_s z}, \quad (15)$$

one obtains the following M-S equations in teleparallel LQC, that differ a little bit from (3)

$$\frac{|\Omega|}{\Omega} c_s \Delta u = z \left(\frac{v}{z} \right)'; \quad \frac{|\Omega|}{\Omega} \theta \left(\frac{u}{\theta} \right)' = c_s v. \quad (16)$$

Note that, since in this version of LQC the velocity of sound is always positive, modes that satisfy $k^2 |c_s^2| \gg \left| \frac{z''}{z} \right|$ are sound waves which never condensate, and thus, they will not produce any cosmological consequence.

Remark 2.3. Here, it is important to realize that holonomy corrected LQC and what we call teleparallel LQC only coincide in the homogeneous and isotropic case, i.e., in the flat FLRW geometry. When one deals with cosmological perturbation the theories are completely different, and lead to different perturbed dynamical equations, because these equations are obtained using different approaches: In holonomy corrected LQC the perturbation equations are obtained working in the Hamiltonian framework, where the isotropic part of the Ashtekar connection, which does not have a quantum version in the Hilbert space of the almost periodic functions, has to be replaced by a suitable sinus. After this replacement, the anomalies that will appear in the algebra of constraints must be removed introducing some counter-terms. On the other hand, the perturbed equations in the teleparallel version are obtained in the Lagrangian framework using the well-known equations in $F(T)$ gravity.

3 Calculation of the Bardeen potential

In this Section, we will show that the formula that relates the Bardeen potential with the curvature fluctuation in co-moving coordinates, in the matter bounce scenario, is the same as the one obtained in inflationary cosmology when the Universe enters in the matter dominated phase. In fact, we will see that, the formula only depends of the fact that, at late times, the Universe is in a matter dominated phase.

To prove this, first of all we perform the Laplacian in the second equation of (3) and use the first one, to get the M-S equation

$$v'' - c_s^2 \Delta v - \frac{z''}{z} v = 0. \quad (17)$$

Now, inserting the second equation of (3) in the first one, one gets

$$u'' - c_s^2 \Delta u - \frac{\theta''}{\theta} u = 0. \quad (18)$$

Remark 3.1. *The same happens with equations (16), that is, in the teleparallel version of LQC the M-S equations (17) and (18) also remain valid. Effectively, performing the Laplacian in the first equation and using the second one, one gets*

$$\frac{|\Omega|}{\Omega} \theta \left(\frac{\Omega}{|\Omega|} z^2 \left(\frac{v}{z} \right)' \right)' = c_s \Delta v. \quad (19)$$

Now, using that

$$\frac{|\Omega|}{\Omega} \left(\frac{\Omega}{|\Omega|} z^2 \left(\frac{v}{z} \right)' \right)' = \left(z^2 \left(\frac{v}{z} \right)' \right)' = v'' z - z'' v, \quad (20)$$

one finally obtains

$$\theta(v'' z - z'' v) = c_s \Delta v \iff v'' - c_s^2 \Delta v - \frac{z''}{z} v = 0. \quad (21)$$

In the same way, inserting the second equation of (16) in the first one, one gets

$$c_s \Delta u = z \frac{\Omega}{|\Omega|} \left(\frac{|\Omega|}{\Omega} \theta^2 \left(\frac{u}{\theta} \right)' \right)' \iff c_s \Delta u = z \left(\theta^2 \left(\frac{u}{\theta} \right)' \right)', \quad (22)$$

which is equivalent to

$$c_s \Delta u = z(u'' \theta - \theta'' u) \iff u'' - c_s^2 \Delta u - \frac{\theta''}{\theta} u = 0. \quad (23)$$

Coming back to equation (17), we will obtain an equivalent integral equation. In Fourier space, we write (17) as follows:

$$v_k'' - \frac{z''}{z} v_k = -k^2 c_s^2 v_k, \quad (24)$$

and find the solution for the ‘‘associate equation’’, namely $v_k'' - \frac{z''}{z}v_k = 0$. Finally, we use the method of variation of constants to find a particular solution. The solution of the ‘‘associate equation’’ is

$$v_k(\eta) = A_1(k)z(\eta) + A_2(k)z(\eta) \int^\eta \frac{d\bar{\eta}}{z^2(\bar{\eta})}, \quad (25)$$

and the method of variation of constants, after some algebra, gives the following solution of equation (24) as an integral equation

$$v_k(\eta) = A_1(k)z(\eta) + A_2(k)z(\eta) \int^\eta \frac{d\bar{\eta}}{z^2(\bar{\eta})} - k^2 z(\eta) \int^\eta \frac{d\bar{\eta}}{z^2(\bar{\eta})} \int^{\bar{\eta}} z(\tilde{\eta})c_s^2(\tilde{\eta})v_k(\tilde{\eta})d\tilde{\eta}. \quad (26)$$

To obtain the corresponding integral equation for u_k we use the first equation of (3), $-c_s k^2 u_k = z \left(\frac{v_k}{z}\right)'$. Inserting in it the expression (26) one gets

$$u_k(\eta) = -\frac{A_2(k)}{k^2}\theta(\eta) + \theta(\eta) \int^\eta \frac{c_s(\bar{\eta})}{\theta(\bar{\eta})}v_k(\bar{\eta})d\bar{\eta}. \quad (27)$$

From this formula we can calculate the power spectrum for the Bardeen potential at late times when the Universe is matter dominated. To do that, we consider modes well outside of the Hubble radius, i.e., modes that satisfy $c_s^2 k^2 \ll 1/\eta^2 \sim a^2 H^2 \sim z''/z$. In that case formula (25) becomes

$$v_k(\eta) \cong A_1(k)z(\eta), \quad (28)$$

that is, the curvature fluctuation in co-moving coordinates, defined as

$$\zeta_k(\eta) \equiv \frac{v_k(\eta)}{z(\eta)}, \quad (29)$$

is constant.

Then, since at late times $\theta(\eta) \propto \frac{1}{\eta^2}$, this means that $\theta(\eta)$ is the decaying mode, and thus, inserting (28) into (27) one obtains

$$u_k(\eta) = \zeta_k \theta(\eta) \int^\eta \frac{d\bar{\eta}}{\theta^2(\bar{\eta})}. \quad (30)$$

Now, taking into account that, for a matter dominated Universe, in EC one has $z(\eta) = \sqrt{3}a(\eta)$, and using the following classical relations

$$\Phi_k = \frac{u_k}{2}\sqrt{\rho}; \quad \rho(t) = \frac{4}{3t^2}; \quad t \propto \eta^3, \quad (31)$$

one finally obtains the relation between the Bardeen potential and the curvature fluctuation in co-moving coordinates

$$\Phi_k(\eta) = \frac{3}{5}\zeta_k(\eta). \quad (32)$$

Moreover, in EC one of the perturbation equations is

$$-\frac{k^2}{a^2}\Phi_k - 3H\dot{\Phi}_k - 3H^2\Phi_k = \frac{1}{2}\delta\rho_k, \quad (33)$$

and since for modes well outside the Hubble radius ($k^2 \ll a^2 H^2$) Φ_k is constant, the density contrast $\delta_k \equiv \frac{\delta\rho_k}{\rho}$ is related with the curvature in co-moving coordinates as follows

$$\delta_k = -2\Phi_k = -\frac{6}{5}\zeta_k(\eta). \quad (34)$$

Note the remarkable fact that relations (32) only depend of the fact that, at late times, the Universe obeys EC and is matter dominated. That happens in inflation when one considers a transition from the de Sitter phase to the matter domination (see for instance [43]) and in the matter bounce scenario. Then, our result complements the duality pointed out in [44] where was showed that in EC the de Sitter phase, where $a(\eta) \propto -\frac{1}{\eta}$, and the matter-domination $a(\eta) \propto \eta^2$ lead to the same equation (17)

$$v'' - \Delta v - \frac{2}{\eta^2}v = 0, \quad (35)$$

meaning that de Sitter inflation and the matter bounce scenario give rise to a scale invariant spectrum for the curvature fluctuation in co-moving coordinates. In fact, in next section we will calculate the value of the arbitrary function $A_1(k)$ that appears in (28), and we will show that is of the order $k^{-3/2}$ (see formulas (45) and (47)).

Now, from the relation (32) and the fact that in the matter bounce scenario the power spectrum of the curvature fluctuation in co-moving coordinates is scale invariant (see next Section) one can conclude that, in the matter bounce scenario, the Bardeen potential is also scale invariant.

4 Calculation of the power spectrum of scalar perturbations in LQC

In this section we perform an study of the way to calculate analytically the power spectrum of the curvature fluctuation in co-moving coordinates, and from (34) the density contrast, when one considers the matter bounce scenario in holonomy corrected LQC and in its teleparallel version, which provides the easiest model to calculate analytically it.

Solving the holonomy corrected Friedmann equation in the flat FLRW space-time and the conservation equation for a matter dominated Universe

$$H^2 = \frac{\rho}{3} \left(1 - \frac{\rho}{\rho_c}\right); \quad \dot{\rho} = -3H\rho, \quad (36)$$

one obtains the following quantities [47]

$$a(t) = \left(\frac{3}{4} \rho_c t^2 + 1 \right)^{1/3}, \quad H(t) = \frac{\frac{1}{2} \rho_c t}{\frac{3}{4} \rho_c t^2 + 1} \quad \text{and} \quad \rho(t) = \frac{\rho_c}{\frac{3}{4} \rho_c t^2 + 1}. \quad (37)$$

For small values of the energy density ($\rho \ll \rho_c$), EC is recovered and equation (17) becomes the usual M-S equation that for a matter-dominated Universe, working in Fourier space, is given by

$$v_k'' + \left(k^2 - \frac{a''}{a} \right) v_k = 0 \iff v_k'' + \left(k^2 - \frac{2}{\eta^2} \right) v_k = 0. \quad (38)$$

Assuming that at early times the Universe is in the Bunch-Davies (adiabatic) vacuum, one must take for $\eta \rightarrow -\infty$ the following mode function

$$v_k(\eta) = \frac{e^{-ik\eta}}{\sqrt{2k}} \left(1 - \frac{i}{k\eta} \right). \quad (39)$$

At early times all the modes are inside the Hubble radius, and when time moves forward the modes leave this radius. Then, for a matter-dominated Universe in EC, the modes well outside the Hubble radius are characterized by the long wavelength condition

$$k^2 \eta^2 \ll 1 \iff k^2 \ll \left| \frac{a''}{a} \right| \iff k^2 \ll \left| \frac{1}{c_s^2} \frac{z''}{z} \right|, \quad (40)$$

because for small values of ρ one recovers EC where $z = \sqrt{3}a$ and $c_s = 1$.

And, when holonomy effects are not important, for modes well outside the Hubble radius the M-S equation becomes

$$v_k'' - \frac{z''}{z} v_k = 0, \quad (41)$$

which can be solved using the method of reduction of the order, giving as a result the following long wavelength approximation

$$v_k(\eta) = B_1(k)z(\eta) + B_2(k)z(\eta) \int_{-\infty}^{\eta} \frac{d\bar{\eta}}{z^2(\bar{\eta})}, \quad (42)$$

where for convenience we have taken a definite integral. The reason why we have made this choice is that, in teleparallel LQC, it is impossible to calculate explicitly the primitive of $1/z^2(\bar{\eta})$. However, for $\eta \rightarrow -\infty$, if we make the approximation $z \cong \sqrt{3}a$ we will obtain $\int_{-\infty}^{\eta} \frac{d\bar{\eta}}{z^2(\bar{\eta})} \cong \int_{-\infty}^{\eta} \frac{d\bar{\eta}}{3a^2(\bar{\eta})}$, and this last integral can be analytically calculated.

Note that, at early times in the contracting phase, for modes well outside the Hubble radius, the expressions (39) and (42) give the same solution. The solution

given by (39) could be expanded in terms of $k\eta \ll 1$, and retaining the leading terms in the real and imaginary parts of v_k , one gets

$$v_k(\eta) \cong -\frac{k^{3/2}\eta^2}{3\sqrt{2}} - \frac{i}{\sqrt{2}k^{3/2}\eta}. \quad (43)$$

On the other hand, the explicit solution of (42), as we have already explained, is obtained using the approximation $z \cong \sqrt{3}a = \frac{1}{4\sqrt{3}}\rho_c\eta^2$, which gives as a result

$$v_k(\eta) \cong \frac{B_1(k)}{4\sqrt{3}}\rho_c\eta^2 - \frac{4B_2(k)}{\sqrt{3}\rho_c} \frac{1}{\eta}. \quad (44)$$

Matching both solutions one obtains

$$B_1(k) = -\sqrt{\frac{8}{3}} \frac{k^{3/2}}{\rho_c} \quad \text{and} \quad B_2(k) = i\sqrt{\frac{3}{8}} \frac{\rho_c}{2k^{3/2}}. \quad (45)$$

Once we have calculated the coefficients $B_1(k)$ and $B_2(k)$ we use equation (42) to calculate v_k at late times. More precisely, we calculate v_k in the classical regime of the expanding phase for modes that are still well outside of the Hubble radius. Note that we are considering modes that in the contracting phase leave the Hubble radius and then evolve satisfying $k^2 \ll \left| \frac{1}{c_s^2} \frac{z''}{z} \right|$. Then, we can use the long wavelength approximation

$$v_k(\eta) = (B_1(k) + B_2(k)R)z(\eta), \quad (46)$$

where $R \cong \int_{-\infty}^{\infty} \frac{d\bar{\eta}}{z^2(\bar{\eta})}$, because η is large enough.

From (46) one has

$$\zeta_k(\eta) = \frac{v_k(\eta)}{z(\eta)} = B_1(k) + B_2(k)R \cong B_2(k)R, \quad (47)$$

and thus, the scalar power spectrum is given by

$$\mathcal{P}_\zeta(k) \equiv \frac{k^3}{2\pi^2} |\zeta_k(\eta)|^2 = \frac{3\rho_c^2}{64\pi^2} R^2 = \frac{3\rho_c^2}{\rho_{pl}} R^2, \quad (48)$$

because since in our units $8\pi G = 1$, one has $\rho_{pl} = 64\pi^2$.

In the case of holonomy corrected LQC one has $z(t) = \frac{2a^{5/2}(t)}{\sqrt{\rho_c t}}$ (see [22]), which leads to a simple calculation of R^2 giving as a result $\frac{\pi^2}{27\rho_c}$. Consequently, in holonomy corrected LQC one has

$$\mathcal{P}_\zeta(k) = \frac{\rho_c}{576} = \frac{\pi^2}{9} \frac{\rho_c}{\rho_{pl}}. \quad (49)$$

On the other hand, in teleparallel LQC one has

$$z(t) = 2 \left(\frac{3}{\rho_c} \right)^{1/4} \frac{a(t)|t|^{1/2}}{t \sqrt{\arcsin \left(\frac{\sqrt{3}\rho_c |t|}{a^3(t)} \right)}}, \quad (50)$$

giving as a power spectrum

$$\mathcal{P}_\zeta(k) = \frac{\rho_c}{144\pi^2} \left(\int_0^{\pi/2} \frac{x}{\sin x} dx \right)^2 = \frac{\rho_c}{36\pi^2} \mathcal{C}^2 = \frac{16}{9} \frac{\rho_c}{\rho_{pl}} \mathcal{C}^2, \quad (51)$$

where $\mathcal{C} = 1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \dots = 0.915965\dots$ is Catalan's constant.

The key point to obtain the scale invariant power spectrum (49) (resp. (51)) is that one only considers modes that after leaving and before re-entering the Hubble radius, when holonomy effects could be disregarded, satisfy the long wavelength condition $k^2 |c_s^2| \ll \left| \frac{z''}{z} \right|$, that is, the term $c_s^2 \Delta v$ in the M-S equation is disregarded between the exit and the reentry of the modes in the Hubble radius.

In holonomy corrected LQC and in its teleparallel version, since the symmetric function z''/z is increasing for $-\infty < t < 0$ and decreasing for $0 < t < \infty$, and $|c_s^2|$ satisfy $|c_s^2| \leq 1$ and $\lim_{t \rightarrow \pm\infty} |c_s^2| = 1$, all the modes that leave the Hubble radius at a early time $-|T|$ satisfy the relation $k^2 |c_s^2| \ll \left| \frac{z''}{z} \right|$ up to late time $|T|$. Then, we can conclude that formulae (49) and (51) are correct for all modes that leave the Hubble radius at early times (when the Universe is in the classical regime).

Remark 4.1. *Dealing with holonomy corrected LQC, where the square of the velocity of sound is $c_s^2 = \Omega = 1 - 2\frac{\rho}{\rho_c}$, one has to be cautious because in the super-inflationary phase the speed of sound becomes negative. In this analysis we are only taking into account modes satisfying the long wavelength conditions, and thus, for those modes the speed of sound does not have any importance in their evolution. However, and this is a problem that has not been addressed yet, the modes with a shorter wavelength (modes remaining inside the Hubble radius) will suffer Jeans instability, leading to some undesirable cosmological consequences. On the other hand, this never happens in the teleparallel version of LQC where the speed of sound is always a real number.*

In contrast, the quantity $\left| \frac{\theta''}{\theta} \right|$ vanishes at the bouncing time. Then, in bouncing scenarios it is impossible to calculate u_k (and the Bardeen potential Φ_k) using the long wavelength approximation

$$u_k(\eta) = C_1(k)\theta(\eta) + C_2(k)\theta(\eta) \int^\eta \frac{d\bar{\eta}}{\theta^2(\bar{\eta})}, \quad (52)$$

because the relation $k^2 |c_s^2| \ll \left| \frac{\theta''}{\theta} \right|$ after leaving and before re-entering the Hubble radius, doesn't hold for any mode. Instead of (52), in order to calculate u_k , one has to use the exact expression given by formula (27).

However, in inflationary EC formula (52) leads to the correct power spectrum for the Bardeen potential. Effectively, the vacuum state is given by the modes (39), which together with the first equation of (3) allow us to calculate the modes u_k at

early times. Then, for modes well outside the Hubble radius ($|k\eta| \ll 1$) one has

$$u_k(\eta) \cong \frac{1}{\sqrt{2k}}\eta - \frac{i}{\sqrt{2}k^{3/2}}. \quad (53)$$

This expression has to be matched with (52) during the quasi de Sitter phase. Since in EC one has $z = \frac{a\sqrt{-2\dot{H}}}{H}$, during the quasi de Sitter epoch we can approximate z by $-\frac{\sqrt{\epsilon}}{H\eta}$ where H and $\epsilon \equiv -\frac{2\dot{H}}{H^2}$ could be considered constants. Then, a simple calculation gives rise to

$$C_1(k) = -\sqrt{\frac{\epsilon}{2k}}\frac{1}{H}, \quad C_2(k) = \frac{-iH}{\sqrt{2\epsilon}k^{3/2}}. \quad (54)$$

Now, to calculate the Bardeen potential at late times, we use the classical relations $\Phi_k = \frac{\sqrt{-2\dot{H}}}{2a}u_k$ and $\theta = \frac{1}{z}$ to obtain

$$\Phi_k(t) = \frac{C_1(k)H(t)}{2a(t)} - \frac{C_2(k)H(t)}{a(t)} \int^t \frac{a(\bar{t})\dot{H}(\bar{t})}{H^2(\bar{t})} d\bar{t}. \quad (55)$$

The first term is decaying and can be disregarded. The second one, after integration by parts, leads to

$$\Phi_k(t) = C_2(k) \frac{d}{dt} \left(\frac{1}{a(t)} \int^t a(\bar{t}) d\bar{t} \right). \quad (56)$$

When the Universe is matter dominated, i.e., when $a(t) \propto t^{2/3}$ one obtains

$$\Phi_k(t) = \frac{3}{5}C_2(k). \quad (57)$$

Finally, calculating (25) in the quasi de Sitter phase and matching the result with (43) one easily obtains $A_1(k) = C_2(k)$, and taking into account that the mode $z(t) \int^t \frac{d\bar{t}}{z^2(\bar{t})}$ is decaying in the matter dominated stage one concludes that $\zeta_k(t) = C_2(k)$, and thus, we obtain in inflationary cosmology the relation (32).

5 The current model

To calculate the power spectrum provided by LQC in the matter bounce scenario, first of all one has to look for a potential of the scalar field such that its non-perturbed solutions (the background solutions) lead to a matter dominated Universe, i.e., they depict, at very early times, a matter dominated Universe in the contracting phase that evolves towards a bounce to enter in the expanding phase.

The simplest way to find one such potential is to impose that the pressure vanishes, i.e., $\frac{\dot{\varphi}^2}{2} - V(\varphi) = 0$, which leads to the equation

$$\dot{\varphi}^2(t) = \rho(t) \iff \dot{\varphi}^2(t) = \frac{\rho_c}{\frac{3}{4}\rho_c t^2 + 1}, \quad (58)$$

where we have used the third equation of (37).

This equation has the particular solution

$$\bar{\varphi}(t) = \frac{2}{\sqrt{3}} \ln \left(\sqrt{\frac{3}{4}\rho_c t} + \sqrt{\frac{3}{4}\rho_c t^2 + 1} \right), \quad (59)$$

which, after reconstruction, i.e., isolating $\frac{3}{4}\rho_c t^2 + 1$ as a function of $\bar{\varphi}$ and using the relation $\frac{\rho_c}{\frac{3}{4}\rho_c t^2 + 1} = 2V(\bar{\varphi})$, leads to the potential

$$V(\varphi) = 2\rho_c \frac{e^{-\sqrt{3}\varphi}}{\left(1 + e^{-\sqrt{3}\varphi}\right)^2}. \quad (60)$$

It is important to realize that the solution (59) is special in the sense that it satisfies for all time $\dot{\bar{\varphi}}^2(t)/2 = V(\bar{\varphi}(t))$, that is, if the Universe is described by this solution it will be matter dominated all the time. However, the other solutions, that is, the solutions of the non-perturbed conservation equation

$$\ddot{\bar{\varphi}} + 3H_{\pm}\dot{\bar{\varphi}} + V_{\varphi}(\bar{\varphi}) = 0, \quad (61)$$

where $H_- = -\sqrt{\frac{\rho}{3}(1 - \frac{\rho}{\rho_c})}$ in the contracting phase and $H_+ = \sqrt{\frac{\rho}{3}(1 - \frac{\rho}{\rho_c})}$ in the expanding phase, do not lead to a matter-dominated Universe all the time. Only at early and late times the Universe is matter dominated because the solution (59) is a global repeller at early times and a global attractor at late times. But, what is important to realize is that all these solutions depict a matter bounce scenario: matter domination at early times in the contracting phase, evolution towards a bounce and finally entrance in the expanding phase.

The method to prove the asymptotical behavior of all the solutions is similar to the one used in [48], and goes as follows:

At early and late times one can disregard holonomy corrections, and since we are considering the early and late time dynamics of the system (what happens for large values of $|\varphi|$), our potential (60) reduces to $\bar{V}(\varphi) = V_0 e^{-\sqrt{3}\varphi}$. Then, performing the change of variable $\bar{\varphi} = \frac{2}{\sqrt{3}} \ln \psi$ the corresponding non-perturbed Klein-Gordon equation (61) (or equivalently, the conservation equation) reads

$$\frac{d\dot{\psi}}{d\bar{\varphi}} = F_{\pm}(\psi), \quad (62)$$

with

$$F_{\pm}(\psi) = \frac{3\sqrt{3}}{4\dot{\psi}} \left(\frac{2}{3}\dot{\psi}^2 + V_0 \right) \mp \frac{3}{2} \sqrt{\frac{2}{3}\dot{\psi}^2 + V_0}, \quad (63)$$

where in F_{\pm} , the sign + (resp. -) means that the Universe is in the expanding (resp. contracting) phase.

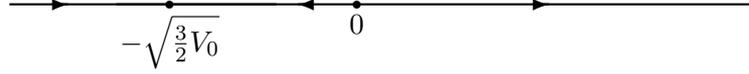


Figure 1: Phase portrait, in the contracting phase, using $\bar{\varphi}$ as a time. With this time, 0 is a repeller and $-\sqrt{\frac{3}{2}}V_0$ an attractor.

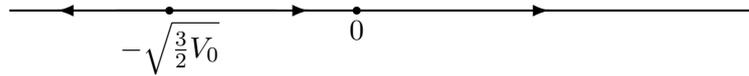


Figure 2: Phase portrait, in the contracting phase, using the cosmological time t . Now $-\sqrt{\frac{3}{2}}V_0$ is a global repeller.

The equation (62) describes two (one for the contracting and other one for the expanding phase) one dimensional first order autonomous dynamical systems, that are completely understood calculating its critical points and evaluating the function F_{\pm} at the right and left hand sides of each critical point in order to know whether these critical points are attractors or repellers. In our case, these critical points are $\dot{\psi}_+ = \sqrt{\frac{3}{2}}V_0$ for the expanding phase and $\dot{\psi}_- = -\sqrt{\frac{3}{2}}V_0$ for the contracting one. Taking into account the sign of F_{\pm} in the right and left hand sides of each critical point one will deduce the dynamics of the system in time $\bar{\varphi}$. For example, in figure 1 we show the phase portrait (in the straight line $\dot{\psi}$) when the Universe is in the contracting phase, and from the phase portrait in time $\bar{\varphi}$, we will deduce the corresponding one in cosmic time, changing the direction of the arrows for negative values of $\dot{\psi}$ (see figure 2), because $\dot{\psi} < 0$ which is the same that $\dot{\bar{\varphi}} < 0$, means that $\bar{\varphi}$ is decreasing in cosmic time, then in order to obtain the dynamics in cosmic time (figure 2), the direction of the arrows must be changed in the phase portrait 1.

The conclusion is that in the expanding (resp. contracting) phase $\dot{\psi}_+$ (resp. $\dot{\psi}_-$) is a global attractor (resp. repeller). These points correspond to the solutions $\bar{\varphi}_{\pm} = \frac{2}{\sqrt{3}} \ln(\pm\sqrt{\frac{3}{2}}V_0 t)$, that of course, satisfy $\rho = \dot{\bar{\varphi}}^2 = \frac{4}{3t^2}$, i.e., all the solutions of the conservation equation depict a matter-dominated Universe at early (in the contracting phase) and late times (in the expanding one).

Then, once we have proved the behavior of the solutions at early and late times,

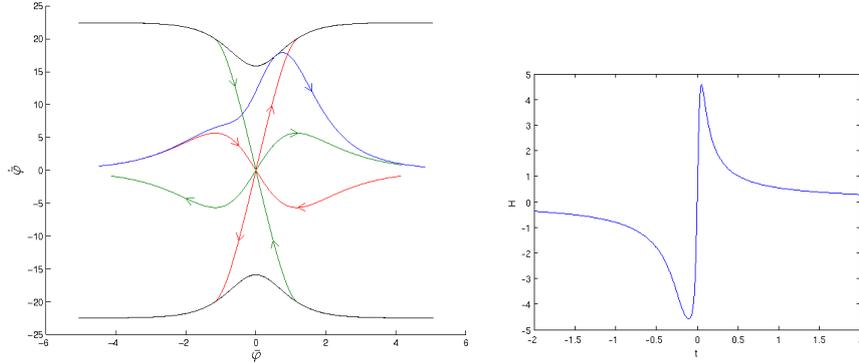


Figure 3: In the first picture we have the phase portrait: black curves defined by $\rho = \rho_c$ depict the points where the Universe bounces. The point $(0, 0)$ is a saddle point, red (resp. green) curves are the invariant curves in the contracting (resp. expanding) phase. The blue curve corresponds to an orbit different from the analytically computed one (59). Note that, before (resp. after) the bounce the blue curve does not cut the red (resp. green) curves. Finally, it is important to realize that the allowed orbits are those that touch the black curve in the region delimited by an unstable red curve and a stable green curve, because for orbits that do not satisfy this condition, $\dot{\varphi}$ vanishes at some time, meaning that its corresponding power spectrum diverges. In the second picture we have drawn the Hubble parameter for the blue curve of the first picture.

to calculate analytically the power spectrum we need the evolution of $a(t)$, $\bar{\varphi}(t)$ and $H(t)$ during all the time, and we only have, analytically, this evolution for the particular solution (59). For the other solutions, numerical calculations are needed (In figure 3 we show the phase portrait in the plane $(\bar{\varphi}, \dot{\bar{\varphi}})$ for the dynamical system given by equation (61)). In fact, using this analytical solution, to obtain a theoretical value of the power spectrum that matches correctly with observations, one can see that the value of the critical energy density has to be of the order $10^{-9}\rho_{pl}$ (see [22]). This does not mean, contrary to the claim of [23, 22], that the correct value of the critical density has to be of this order. What it really means is that the value of the critical density has to be of this order if the Universe is described by orbits near this analytical one, but there could be other orbits such that for the current value of the critical density, approximately $0.4\rho_{pl}$, the numerical results obtained from the model might match correctly with current observations (we will show at the end of the Section that this is not the case, but it could be possible in principle). One has to imagine ρ_c as a parameter, whose value depends on the orbit we have chosen, and is determined by current observations. This is exactly what happens in chaotic inflation. More precisely, if one considers, for the sake of simplicity, the quadratic model $V(\varphi) = \frac{1}{2}m^2\varphi^2$ the number of e-folds

before the end of inflation is approximately

$$N = \int_{t_N}^{t_{end}} H(t) dt \cong \int_{\bar{\varphi}_{end}}^{\bar{\varphi}_N} \frac{V(\bar{\varphi})}{V_\varphi(\bar{\varphi})} d\bar{\varphi} \cong \frac{\bar{\varphi}_N^2}{4}. \quad (64)$$

On the other hand, the power spectrum of scalar perturbations, in slow-roll inflation, is given by

$$\mathcal{P}_\zeta(k) = \frac{V^3(\bar{\varphi}_N)}{12\pi^2 V_\varphi^2(\bar{\varphi}_N)} = \frac{m^2 \bar{\varphi}_N^4}{96\pi^2} = \frac{32N^2 m^2}{3\rho_{pl}}. \quad (65)$$

Then, to choose the number of e-folds is equivalent to choosing the orbit, because with the value of N one obtains $\bar{\varphi}_N$, and with the slow-roll equation $V(\bar{\varphi}_N)\dot{\bar{\varphi}}_N + V_\varphi(\bar{\varphi}_N) = 0$ one obtains $\dot{\bar{\varphi}}_N$. Finally, for a given value of N , using the current constrain $\mathcal{P}_\zeta(k) \simeq 2 \times 10^{-9}$ and the formula (65) one determines de value of the parameter m , to match correctly theoretical results with observational data.

Coming back to LQC, in the matter bounce scenario the power spectrum is given by equation (48)

$$\mathcal{P}_\zeta(k) = \frac{3\rho_c^2}{\rho_{pl}} R^2, \quad (66)$$

with $R \cong \int_{-\infty}^{\infty} \frac{d\eta}{z^2(\eta)} = \int_{-\infty}^{\infty} \frac{dt}{a(t)z^2(t)}$, where in holonomy corrected LQC $z = \frac{a\dot{\varphi}}{H}$, and in teleparallel LQC z is given by formula (15). It is important to realize that, when we have done the matching between equations (43) and (44) we have used, at early times, the following scale factor $a(t) \cong \left(\frac{3}{4}\rho_c t^2\right)^{1/3}$. Then, to perform numerical calculations with formula (66), one has to use, as a scale factor, the solution of $\frac{\dot{a}}{a} = H$ that, at early times, satisfies $a(t) \cong \left(\frac{3}{4}\rho_c t^2\right)^{1/3}$.

Finally, performing the change of variable $\tilde{t} = \sqrt{\rho_c} t$, we can see that the conservation equation for the homogeneous part of the field (61) becomes

$$\frac{d^2\bar{\varphi}}{d\tilde{t}^2} + 3\tilde{H}_\pm \frac{d\bar{\varphi}}{d\tilde{t}} + \tilde{V}_\varphi(\bar{\varphi}) = 0, \quad (67)$$

with $\tilde{V} = \frac{V}{\rho_c}$ and $\tilde{H}_\pm = \frac{H_\pm}{\sqrt{\rho_c}} = \pm \sqrt{\frac{\tilde{\rho}}{3}(1 - \tilde{\rho})}$, being $\tilde{\rho} = \frac{\rho}{\rho_c} = \frac{1}{2} \left(\frac{d\bar{\varphi}}{d\tilde{t}}\right)^2 + \tilde{V}$.

This means that $\frac{\dot{\bar{\varphi}}(t)}{H(t)} = \frac{1}{H(\tilde{t})} \frac{d\bar{\varphi}(\tilde{t})}{d\tilde{t}}$ does not depend on ρ_c . In the same way, $a(\tilde{t})$ does not depend on ρ_c because it satisfies the equation $\frac{1}{a(\tilde{t})} \frac{da(\tilde{t})}{d\tilde{t}} = \tilde{H}(\tilde{t})$, and from the definition of the velocity of sound we see that $c_s(\tilde{t})$ is independent on the critical density. From this, we can conclude that $z(\tilde{t})$ does not depend on the value of the critical density, and then

$$\mathcal{P}_\zeta(k) = \frac{3\rho_c}{\rho_{pl}} \left(\int_{-\infty}^{\infty} \frac{d\tilde{t}}{a(\tilde{t})z^2(\tilde{t})} \right)^2, \quad (68)$$

which is of the order ρ_c . Finally, depending on the chosen orbit, one will numerically obtain different values of $\left(\int_{-\infty}^{\infty} \frac{d\tilde{t}}{a(\tilde{t})z^2(\tilde{t})}\right)^2$, which determine the corresponding value of the critical density using the constraint $\mathcal{P}_\zeta(k) \cong 2 \times 10^{-9}$.

5.1 Some comments about the current model

An important difficulty of the model given by the potential (60) is that it cannot explain the current acceleration of the Universe. Moreover, in previous studies any mechanism to explain the reheating of the Universe creating light particles like an oscillatory behavior of the field [49], an instant preheating [50] or a phase transition to a quasi de Sitter stage to a radiation dominated Universe [54], has not taken into account this acceleration.

Another problem appears when one deals with tensor perturbations. In the case of holonomy corrected LQC this equation is

$$v_T'' - c_s^2 \Delta v_T - \frac{z_T''}{z_T} v_T = 0, \quad (69)$$

where $c_s^2 = \Omega$ and $z_T \equiv a\Omega^{-1/2}$. Note that z_T becomes imaginary in the super-inflationary phase $\rho \in (\rho_c/2, \rho_c]$. Moreover, this equation is singular when $\rho = \rho_c/2$, then there are infinitely many ways to match solutions at this value, and consequently infinitely many modes could be used to calculate the power spectrum of tensor perturbations giving completely different results (see [23]). On the other hand, in teleparallel LQC the corresponding M-S equation for tensor perturbations does not contain singularities but, when one uses the analytic solution (59), the ratio of tensor to scalar perturbations which is given by

$$r \cong \frac{1}{3} \left(\frac{\int_{-\infty}^{\infty} \frac{1}{z_T^2} d\eta}{\int_{-\infty}^{\infty} \frac{1}{z^2} d\eta} \right)^2 = \frac{1}{3} \left(\frac{\int_{-\infty}^{\infty} \frac{1}{az_T^2} dt}{\int_{-\infty}^{\infty} \frac{1}{az^2} dt} \right)^2, \quad (70)$$

where now $z_T \equiv \frac{ac_s}{\sqrt{|\Omega|}}$, is $r = 3 \left(\frac{Si(\pi/2)}{c} \right)^2 \cong 6.7187$, as $Si(x) \equiv \int_0^x \frac{\sin y}{y} dy$ is the Sine integral function [26], which is in contradiction with the current observational bound. Recall that BICEP2 data constrain this ratio to $r = 0.20_{-0.05}^{+0.07}$ with $r = 0$ disfavored at 7.0σ [25], and *Planck's* data bounds this ratio to be $r < 0.11$ (95 % CL) [27].

Remark 5.1. *Note that our correct definition of z_T differs from the one of [23] by the factor $\sqrt{2}$. In fact, for small values of the energy density one has $z_T \cong a$ which coincides with the classical definition of z_T , and which does not happen if one uses the definition given in [23]. On the other hand, to obtain the formula (70), one has to follow the same steps as in Section 4. Obtaining the formula*

$$v_{T,k}(\eta) = B_{T,1}(k)z_T(\eta) + B_{T,2}(k)z_T(\eta) \int_{-\infty}^{\eta} \frac{d\bar{\eta}}{z_T^2(\bar{\eta})}, \quad (71)$$

with

$$B_{T,1}(k) = -\sqrt{8} \frac{k^{3/2}}{\rho_c} \quad \text{and} \quad B_{T,2}(k) = i\sqrt{\frac{1}{8}} \frac{\rho_c}{2k^{3/2}}. \quad (72)$$

Then, the power spectrum for tensor perturbations is

$$\mathcal{P}_T(k) \equiv \frac{k^3}{2\pi^2} \left| \frac{v_{T,k}(\eta)}{z_T(\eta)} \right|^2 = \frac{\rho_c^2}{\rho_{pl}} R_T^2, \quad (73)$$

where $R_T = \int_{-\infty}^{\infty} \frac{d\bar{\eta}}{z_T^2(\bar{\eta})}$. Consequently using (66) and (73), one deduces that the quotient $r = \frac{\mathcal{P}_T(k)}{\mathcal{P}_\zeta(k)}$ is equivalent to the formula (70).

A more involved problem appears when one deals with cosmological perturbations in the framework of holonomy corrected LQC. As we have already pointed out in the introduction, in the super-inflationary phase the speed of sound becomes imaginary implying Jeans instabilities, and then, the use of linear perturbation equations is questionable in this approach. From our viewpoint, this fact, shows that one cannot have too much confidence in the results obtained using the perturbation equations coming from holonomy corrections, which is not the case of the theoretical results obtained from the perturbation equations in $F(T)$ coming from the model given by equation (12).

5.2 Numerical results for the current model

Performing the change of variable $\tilde{t} = \sqrt{\rho_c} t$ in equation (70) we can see that r does not depend on the value of the critical density. Then, to show the viability of the model one has to evaluate numerically the tensor/scalar ratio for all the orbits that satisfy the constrain $\mathcal{P}_\zeta(k) \cong 2 \times 10^{-9}$, and to check if, for that set of orbits, there is a subset which satisfy either r is smaller than 0.11 (the latest *Planck's* data constrain) or $r = 0.20^{+0.07}_{-0.05}$ (last BICEP2 data).

Our numerical study shows that [26]:

1. In holonomy corrected LQC, the minimal value of $\mathcal{P}_\zeta(k)$ is obtained for the orbit that at the bouncing time satisfies $\bar{\varphi} \cong -0.9870$, for that orbit we have obtained $\mathcal{P}_\zeta(k) \cong 23 \times 10^{-3} \frac{\rho_c}{\rho_{pl}}$.
2. In teleparallel LQC the orbit which gives the minimal value of the power spectrum satisfies $\bar{\varphi} \cong -0.9892$ and the value of the power spectrum is approximately the same as in holonomy corrected LQC $\mathcal{P}_\zeta(k) \cong 40 \times 10^{-3} \frac{\rho_c}{\rho_{pl}}$.

For these orbits, in order to match with the current result, in both theories, one has to choose $\rho_c \sim 10^{-7} \rho_{pl}$ which is 2 orders greater than the value needed using the analytical solution.

We have also calculated the ratio of tensor to scalar perturbations using formula (70), and obtaining as a minimal value $r = 0$ for the orbits that, at the bouncing

time, satisfy $\bar{\varphi} \cong -1.205$ and $\bar{\varphi} \cong 1.205$. On the other hand, its maximal value $r \cong 6.7187$ is attained by the solution (59) bouncing at $\bar{\varphi} = 0$. Then, since the value of the tensor/scalar ratio in admissible solutions ranges continuously from the minimal value $r = 0$, to the maximal value $r \cong 6.7187$, one can deduce that there is a set of solutions which matches correctly with BICEP2 data and another one with *Planck's* result.

Then, the confidence interval $r = 0.20_{-0.05}^{+0.07}$ derived from BICEP2 data is realized by solutions bouncing when $\bar{\varphi}$ belongs in $[-1.162, -1.144] \cup [1.144, 1.162]$, and the bound $r \leq 0.11$ provided by *Planck's* experiment is realized by solutions bouncing when $\bar{\varphi}$ is in the interval $[-1.205, -1.17] \cup [1.17, 1.205]$. Moreover, subtracting various dust models the tensor/scalar ratio in BICEP2 experiment could be shifted to $r = 0.16_{-0.05}^{+0.06}$ with $r = 0$ disfavored at 5.9σ . Then, this confidence interval is realized by solutions bouncing when $\bar{\varphi} \in [-1.17, -1.151] \cup [1.151, 1.17]$.

On the other hand, in holonomy corrected LQC, numerical results show that the allowed orbits provide values of r in the interval $[0, 0.1114]$, matching only correctly with *Planck's* constrain $r \leq 0.11$. In figure 4, we have drawn the tensor/scalar ratio for teleparallel and holonomy corrected LQC.

Finally, we have checked numerically that the functions z''/z and z''_T/z_T are increasing in the contracting phase and decreasing in the expanding one, for teleparallel and holonomy corrected LQC. This means that all our formulae are correct, that is, all modes that leave the Hubble radius at early time satisfy the long wavelength relation $|c_s^2|k^2 \ll |z''/z|$ and $|c_s^2|k^2 \ll |z''_T/z_T|$ up to late times, and thus, we can safely disregard the Laplacian term in the M-S equations, giving validity to our approximation.

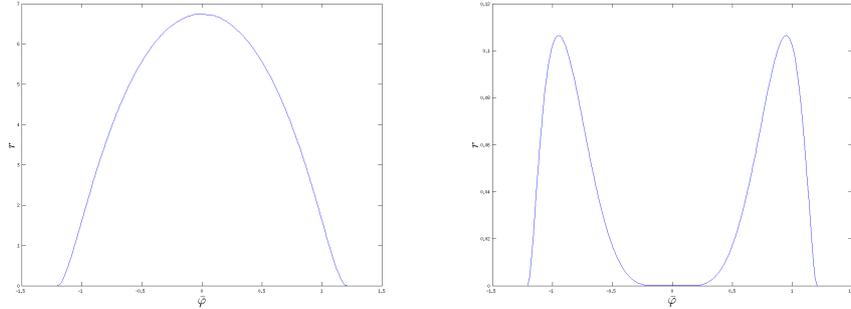


Figure 4: Tensor/scalar ratio for different orbits in function of the bouncing value of $\bar{\varphi}$. In the first picture for teleparallel LQC, and in the second one for holonomy corrected LQC.

6 Viable models for the matter bounce scenario

According to the current observational data, in order to obtain a viable model in the matter bounce scenario in LQC, the bouncing model has to satisfy some conditions that we have summarized as follows:

1. The latest Planck data constrain the value of the spectral index for scalar perturbations, namely $n_s \equiv 1 + \frac{d \ln \mathcal{P}_\zeta(k)}{d \ln k}$, to 0.9603 ± 0.0073 [27] (it is scale invariant with a slight red tilt). It is well-known that the ways to obtain a nearly scale invariant power spectrum of perturbations are either a quasi de Sitter phase in the expanding phase or a nearly matter domination phase at early times, in the contracting phase [44]. Then, since for the matter bounce scenario one has $n_s = 1$, if one wants to improve the model to match correctly with that data, one has to consider, at early times in the contracting phase, a Universe with a equation of state $P = \omega\rho$. In that case, the spectral index is given by $n_s = 1 + 12\omega$ [22, 23], and thus one has to choose $\omega = -0.0033 \pm 0.0006$. Then, the model for large values of the field ($\varphi \rightarrow -\infty$ or $\varphi \rightarrow \infty$) must satisfy $V(\varphi) \sim \rho_c e^{-\sqrt{3(1+\omega)}|\varphi|}$, because for this kind of potentials when $|\varphi| \rightarrow \infty$ all the orbits depict a Universe with equation of state $P = \omega\rho$ (this claim could be proved, exactly in the same way, as we have showed, at the beginning of Section 5, the asymptotic behavior of this potential in the case $\omega = 0$).

In this case the formula for the tensor/scalar ratio will become

$$r = \frac{1}{3(1+\omega)} \left(\frac{\int_{-\infty}^{\infty} \frac{1}{az_T^2} dt}{\int_{-\infty}^{\infty} \frac{1}{az^2} dt} \right)^2, \quad (74)$$

however, due to the small value of ω , one can safely choose $\omega = 0$ without changing significantly the results. In fact, numerically we have obtained that in teleparallel LQC, when $\omega = -0.0033$ (which corresponds to $n_s = 0.9603$) the ratio r ranges continuously in the interval $[0, 6.74]$, and thus there is a set of solutions satisfying BICEP2 data, and another one fitting well with *Planck's* results. On the other hand, for holonomy corrected LQC when $\omega = -0.0033$, r ranges continuously in the interval $[0, 0.10]$, meaning that its theoretical results only match with *Planck's* data.

Here an important remark is in order. One could argue that constraining the parameter ω to be $\omega = -0.0033 \pm 0.0006$ to obtain a correct spectral index has to be considered as a fine-tuning. However, in any inflationary scenario that involves a slow rolling scalar field, the same kind of fine-tuning must be done. To show that, we will consider quasi-matter domination, i.e., $\dot{\varphi}^2 \cong 2V \implies \ddot{\varphi} \cong V_\varphi$, at very early times in the contracting phase (see [45] for all the details). In this regime the Friedmann and conservation equations

become

$$H^2 = \frac{2}{3}V, \quad 3H\dot{\varphi} + 2V_\varphi = 0. \quad (75)$$

Introducing, in the same way as in slow roll inflation, a *quasi-matter domination parameter*

$$\bar{\epsilon} = \frac{1}{3} \left(\frac{V_\varphi}{V} \right)^2 - 1, \quad (76)$$

a simple calculation leads to

$$\frac{z''}{z} \cong \frac{1}{\eta^2} \left(\nu^2 - \frac{1}{4} \right), \quad (77)$$

where $\nu \cong \frac{3}{2} - 6\bar{\epsilon}$. And thus, the spectral index for scalar perturbations, namely n_s^{MB} , is given by

$$n_s^{MB} - 1 \equiv 3 - 2\nu = 12\bar{\epsilon}. \quad (78)$$

It is instructive, to compare this result with the one obtained in inflationary cosmology

$$n_s^{SR} - 1 = 2\bar{\eta}_{sr} - 6\bar{\epsilon}_{sr}, \quad (79)$$

where $\bar{\epsilon}_{sr}$ and $\bar{\eta}_{sr}$ are the well known slow-roll parameters, and we have denoted by n_s^{SR} the spectral index in inflationary cosmology. Moreover, in inflationary cosmology, the scalar/tensor ratio, namely r^{SR} , is related with the slow-roll parameter $\bar{\epsilon}_{sr}$, by the relation $r^{SR} = 16\bar{\epsilon}_{sr}$ (Note that in the matter bounce scenario the tensor/scalar ratio (70) does not depend on the quasi-matter domination parameters).

As an example, we will choose the potential $V(\varphi) = V_0 e^{-\sqrt{3(1+\omega)}|\varphi|}$ [46] which, has as Equation of State $P = \omega\rho$, and thus leads to a power law expansion. An easy calculation will provide

$$n_s^{MB} - 1 = 12\omega, n_s^{SR} - 1 = -3(1 + \omega) \text{ and } r^{SR} = 24(1 + \omega). \quad (80)$$

Both theories, matter bounce scenario and inflation, have to match with the current data in order to be viable. In this case, from last *Planck's* data, the spectral index is given by $n_s = 0.9603 \pm 0.0073$, which means that:

- (a) In the matter bounce scenario, for the potential $V(\varphi) = V_0 e^{-\sqrt{3(1+\omega)}|\varphi|}$, to match with experimental data, one will have to choose $\omega = -0.0033 \pm 0.0006$ (nearly matter domination).

- (b) In power law inflation, for the same potential, to match with the experimental value of the spectral index one has to choose $\omega = -0.9867 \pm 0.0024$ (quasi de Sitter phase). Moreover, for this potential, since the tensor/scalar ratio is given by $24(1 + \omega)$, to fit well with *Planck's* data one has to choose $\omega \leq -0.9954$, which is incompatible with $\omega = -0.9867 \pm 0.0024$, and consequently, *Planck's* data disregard this model. On the other hand, to match the ratio of tensor to scalar perturbations with BICEP2 data, one has to choose $\omega \in [-0.9937, -0.9887]$ that together with the condition $\omega = -0.9867 \pm 0.0024$, restrict the value of the parameter ω to be $\omega = -0.9890^{+0.0001}_{-0.0003}$.

The same will happen for the other models, for instance, in chaotic inflation if one considers the quartic potential $V(\varphi) = \lambda\varphi^4$, one has

$$n_s^{SR} - 1 = -\frac{24}{\varphi_N^2} \quad \text{and} \quad r^{SR} = \frac{128}{\varphi_N^2}, \quad (81)$$

where φ_N is the value of the scalar field N e-folds before the end of the slow-roll phase. Then, in order to match with BICEP2 data one has to choose $\varphi_N = 22^{+0.5973}_{-0.2268}$, i.e., the theoretical value of the spectral index for modes that leave the Hubble radius approximately N e-folds before the end of inflation, matches correctly with the experimental data. This is once again a fine tuning, because for this potential the value of the field N e-folds before the end of the inflationary period must be tuned. In contrast, if one considers, in matter bounce scenario, a potential with the asymptotic form $V(\varphi) = V_0 e^{\sqrt{3}(\varphi + \frac{1}{\varphi})}$ when $\varphi \ll 0$, one also obtains $n_s^{MB} - 1 = -\frac{24}{\varphi_N^2}$, where φ_N is the value of the scalar field, in the contracting phase, N e-folds before the end of the quasi-matter domination period. Then, in order to match with the experimental spectral index, one has to choose $\varphi_N \in [-27.2165, -22.5973]$, that is, the theoretical value of the spectral index for modes that leave the Hubble radius approximately N e-folds before the end of the quasi-matter domination epoch, matches correctly with the experimental data.

To sum up, from our viewpoint, these calculations show that, in order to match the theoretical results with current data, the parameters that appear in both theories (inflation and matter bounce scenario), must be tuned finely.

2. The Universe has to reheat creating light particles that will thermalize matching with a hot Friedmann Universe. The simplest way to do that is with an oscillatory behavior of the field in the expanding phase, because when the field oscillates it decays releasing its energy at the bottom of the potential, where the adiabaticity of the process is strongly violated, producing light particles [49], whose number increases with each oscillation due to this broad parametric resonance regime. To obtain this behavior, we can assume

that in some region the potential has a minimum, i.e., it has a potential well. The simplest potentials with a minimum are the ones used in inflation, for example potentials with the same shape as the power law potentials used in chaotic inflation, i.e., $V(\varphi) = \lambda\varphi^{2n}$.

Another way is the so called *instant preheating*, where no oscillations are required [50]. This mechanism works for potentials with a global minimum, but it is very efficient for potentials which slowly decrease for large values of the scalar field as in the theory of quintessence [51], where, in the expanding phase, an inflationary potential is matched with a quintessence one. Finally, reheating could also be produced due to the gravitational particle creation in an expanding Universe [52]. In this case, an abrupt phase transition (a non adiabatic transition) is needed in order to obtain sufficient particle creation that thermalizes producing a reheating temperature that fits well with current observations. This method was used in the context of inflation in [53, 54], where a sudden phase transition from a quasi de Sitter phase to a radiation domination or a quintessence phase was assumed in the expanding regime. We will show, at the end of this Section, that gravitation particle production could be applied to the matter bounce scenario, assuming a phase transition from the matter domination to an ekpyrotic phase in the contracting regime, and obtaining a reheating temperature compatible with current data.

3. Studies of distant type Ia supernovae [55] (and others) provide strong evidence that our Universe is expanding in an accelerating way. A viable model must take into account this current acceleration, which could be incorporated, in the simplest case, with a cosmological constant, or by quintessence models [56]. Of course, there are other ways to implement the current cosmic acceleration, for example using $f(R)$ or $f(T)$ gravity, but the models that provide this behavior are very complicated, and our main objective in this work is to present the simplest viable models.
4. The numerical results (analytical ones will be impossible to obtain) calculated with the model have to match with experimental data, for instance, the power spectrum of scalar perturbations has to be of order 10^{-9} and the ratio of tensor to scalar perturbations has to be either less than 0.11 or in the range $r = 0.20^{+0.07}_{-0.05}$, depending if one uses *Planck's* or BICEP2 data. The numerical calculations could be performed using formulae (66) and (70), where the quantities appearing in these formulae will be computed after solving numerically the conservation equation (61), where the scale factor, obtained numerically integrating the equation $H = \frac{\dot{a}}{a}$, must satisfy at early times

$$a(t) \cong \left(\frac{3}{4}(1 + \omega)^2 \rho_c t^2 \right)^{\frac{1}{3(1+\omega)}} \cong \left(\frac{3}{4} \rho_c t^2 \right)^{1/3}. \quad (82)$$

Note that, if the potential is proportional to the critical density, performing the change of variable $\tilde{t} = \sqrt{\rho_c}t$, one can show that the tensor/scalar ratio is independent of ρ_c and the power spectrum is proportional to ρ_c , which means that $\mathcal{P}_\zeta(k) = K \frac{\rho_c}{\rho_{pl}}$ where K is a dimensionless quantity independent of ρ_c , and thus, the experimental data $\mathcal{P}_\zeta(k) \sim 10^{-9}$ is easily achieved choosing $\rho_c \sim \frac{10^{-9}}{K} \rho_{pl}$.

5. The model has to be stable, in the sense that, if an orbit depicting the Universe satisfies all the previous requirements, then a small perturbation of this orbit also has to satisfy them. Mathematically speaking, the set of orbits that satisfy all the requirements must have nonzero measure.

First, one could deal, for simplicity, with a quadratic potential $V(\varphi) = \frac{1}{2}m^2\varphi^2$, because at early times (in the contracting phase) the Universe is in a matter dominated phase (in fact, it is matter dominated on average over few oscillations, because when $|t| \rightarrow \infty$ one has $H(t) \cong \frac{2}{3t} \left(1 - \frac{\sin(2mt)}{2mt}\right)^{-1}$ [28]). Moreover, at late times in the expanding phase, the field oscillates around the minimum of its potential releasing its energy and creating light particles, that finally thermalize yielding a hot Friedmann Universe that matches with the Standard Model, but does not take into account the current accelerated expansion of the Universe. Note that one cannot add to the model a simple cosmological constant Λ because, in this case, the Universe would start, in the contracting phase, in an anti de Sitter stage which does not lead to a nearly scale invariant spectrum.

A more interesting model that takes into account the current cosmic acceleration is obtained combining $\rho_c e^{-\sqrt{3(1+\omega)}|\varphi|}$ with the quadratic potential and a small cosmological constant, in the following way (subjected to the condition $V(\varphi) < \rho_c$),

$$V(\varphi) = \rho_c \frac{\frac{m^2\varphi^2}{2} + \Lambda}{\frac{m^2\varphi^2}{2} + \rho_0} \frac{1}{\cosh(\sqrt{3(1+\omega)}\varphi)}, \quad (83)$$

where $\omega \approx -0.0033$, Λ is a small cosmological constant and ρ_0 is an energy density parameter. Note that, for large values of $|\varphi|$ one has $V(\varphi) \sim 2\rho_c e^{-\sqrt{3(1+\omega)}|\varphi|}$, meaning that, for large values of $|\varphi|$ the Universe is nearly matter dominated. On the other hand, for small values of $|\varphi|$, one has $V(\varphi) \sim \frac{\rho_c}{\rho_0} \left(\frac{1}{2}m^2\varphi^2 + \Lambda\right)$, then for these values the field will oscillate in the well of the potential releasing its energy and matching with the Λ CDM model, or an instant preheating will occur, before the field starts to oscillate, to match with the Λ CDM model.

From the model we could see that orbits starting at $|\varphi| = \infty$ and ending at $\varphi = 0$, are the ones that, at early times, are in the contracting nearly matter dominated phase and, at late times, match with the Λ CDM model. These orbits are the candidates to describe a viable Universe, the other ones do not accomplish some of the requirements established by observations. Some of these orbits start and end at $|\varphi| = \infty$, giving a matter dominated Universe at early and late times, which

contradicts the current acceleration of the Universe. The other ones start at early times, in the contracting phase, at the bottom of the well, i.e., starting in an anti de Sitter stage (these orbits do not give a nearly invariant spectrum of perturbations). There are two kinds of the latter orbits:

1. The ones that leave the potential well and finish at late times in the expanding phase, in a nearly matter dominated phase.
2. The ones that do not clear the potential well and finish, in the expanding phase, in a de Sitter stage driven by the cosmological constant Λ .

For the orbits that depict a candidate to be a viable Universe one has to compute the ratio of tensor to scalar perturbations, which is given by formula (70), and to check if r is smaller than 0.11 (the last *Planck* data) or it is in the range $r = 0.20^{+0.07}_{-0.05}$ (the last BICEP2 data).

Note finally that, the value of the cosmological constant in Planck units is of the order 10^{-120} (see for instance [57]), which means that, when one makes numerical calculations, its value can be considered zero.

Another way to build models that satisfy the requirements would be to match a potential of the asymptotic form $V(\varphi) \sim \rho_c e^{-\sqrt{3(1+\omega)}|\varphi|}$ which leads to an nearly matter dominated phase at early times with an inflationary potential, for example:

1. Matching with a power law potential

$$V(\varphi) = \kappa \rho_c \begin{cases} \frac{e^{\sqrt{3(1+\omega)}\varphi}}{(1+e^{-\sqrt{3(1+\omega)}\varphi})^2} & \text{for } \varphi < \varphi_0 \\ \lambda(\varphi - \varphi_1)^{2n} + \frac{\Lambda}{\rho_0} & \text{for } \varphi \geq \varphi_0, \end{cases} \quad (84)$$

where $0 < \varphi_0 < \varphi_1$, and the parameters $\lambda > 0$ and $\kappa > 0$ are dimensionless.

One has to impose the continuity of V and its first derivative at $\varphi = \varphi_0$.

In this model the orbits that could be acceptable are the ones that start at $\varphi = -\infty$ and end at the bottom of the potential well ($\varphi = \varphi_1$). For such orbits one has to calculate the corresponding power spectrum and the ratio of tensor to scalar perturbations and choose those whose theoretical values match correctly with observations.

2. Matching with a *plateau* potential

$$V(\varphi) = \kappa \rho_c \begin{cases} e^{\sqrt{3(1+\omega)}\varphi} & \text{for } \varphi < \varphi_0 \\ \lambda \left(1 - \frac{\varphi^2}{\varphi_1^2}\right)^2 + \frac{\Lambda}{\rho_0} & \text{for } \varphi \geq \varphi_0, \end{cases} \quad (85)$$

where $\varphi_0 < 0 < \varphi_1$, with $\lambda > 0$ and $\kappa > 0$ dimensionless parameters.

The last way to build models consists in matching a potential with the asymptotic form $V(\varphi) \sim \rho_c e^{-\sqrt{3(1+\omega)}|\varphi|}$ with a quintessence potential, for example with the potential $V(\phi) = \rho_c \frac{\varphi_0^4}{\varphi^4 + \varphi_0^4}$ introduced by Peebles and Vilenkin in [54].

Here an important remark is in order: In the expanding phase, reheating occurs when holonomy effects are negligible. Then, the M-S variable z will be given by

$$z(t) = \frac{a(t)\sqrt{-2\dot{H}(t)}}{H(t)} = \frac{a(t)\sqrt{\dot{\varphi}^2 + \rho_\chi + P_\chi}}{H(t)}, \quad (86)$$

where $\rho_\chi(t)$ and $P_\chi(t)$ are the energy density and pressure of the produced light χ -particles when adiabaticity is strongly violated. If reheating occurs via broad parametric resonance (the potential has a minimum), between 15 and 25 oscillations are needed to complete the reheating [28], and during these oscillations it is nearly impossible to describe analytically this process, i.e., it is nearly impossible to have a formula for the evolution of ρ_χ and P_χ , and consequently, since is impossible to know the value of $z(t)$ during the reheating one cannot calculate the ratio of tensor to scalar perturbations. For this reason, we will assume that reheating could only be done instantaneously [50, 51] in potentials with a minimum. To be more precise, in the case of potentials with a global minimum, assuming instant reheating, we will use the formula $z(t) = \frac{a(t)\dot{\varphi}(t)}{H(t)}$ up to when the field arrives at the minimum of the potential where adiabaticity is violated and particles are created giving, nearly instantaneously, a radiation dominated Universe. Then, after reaching the minimum, since the Universe is radiation dominated, we will use the formula $z(t) = \frac{a(t)\sqrt{-2\dot{H}(t)}}{H(t)} = 2a(t)$. In contrast, for potentials without a minimum (for example, $V \sim e^{-\sqrt{3}|\varphi|}$ matched with a quintessence potential), gravitational particle creation explains the reheating of the Universe. In the case of inflationary cosmology, gravitational reheating is produced via an abrupt transition, in the expanding phase, from a quasi de Sitter regime to a quintessence one [54], but as we will see, in a bouncing scenario gravitational reheating could also be produced before the bounce, i.e., in the contracting phase.

6.1 Numerical results

In figure 5 we have depicted the potential (84) with $n = 1$, where the matching has been done imposing the continuity of the first derivative.

For this quadratic potential we have obtained the following results:

1. In teleparallel LQC, the confidence interval $r = 0.20_{-0.05}^{+0.07}$ derived from BICEP2 data is realized by solutions bouncing when $\bar{\varphi}$ belongs in the interval $[-1.17, -1.145] \cup [1.107, 1.14]$, and the bound $r \leq 0.11$ provided by *Planck's* experiment is realized by solutions bouncing when $\bar{\varphi}$ is in the interval $[-1.2039, -1.1685] \cup [1.1432, 1.2081]$.

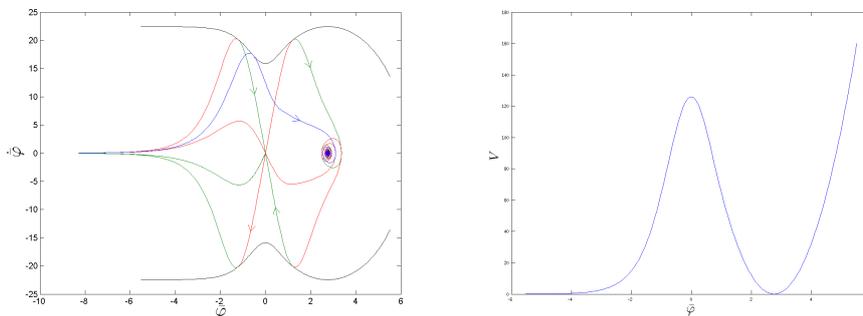


Figure 5: In the first picture we have the phase portrait for the potential (84) with $n = 1$: black curves defined by $\rho = \rho_c$ depict the points where the Universe bounces. The point $(0, 0)$ is a saddle point, red (resp. green) curves are the invariant curves in the contracting (resp. expanding) phase. The blue curve corresponds to an orbit of the system. Note that the allowed orbits are those that touch the black curve in the region delimited by an unstable red curve and a stable green curve. Those orbits start in the contracting phase depicting a matter dominated Universe and, when they touch the black curve, the Universe enters in the expanding phase, oscillating around the minimum of the potential. In the second picture we have drawn the shape of the matched quadratic ($n = 1$) potential (84).

2. In holonomy corrected LQC, the confidence interval $r = 0.20^{+0.07}_{-0.05}$ is realized by solutions bouncing when $\bar{\varphi}$ belongs in $[0.125, 0.42] \cup [0.895, 1.04]$, and the bound $r \leq 0.11$ is realized by solutions whose value at bouncing time is in the interval $[-1.2039, 0.1] \cup [1.05, 1.2081]$. Note that, contrary to the simplest case, i.e., to the current model provided by the potential (60), there are orbits whose theoretical results agree with BICEP2 data.

The graphics of the tensor/scalar ratio, for teleparallel and holonomy corrected LQC, are depicted in figure 6.

We have also studied numerically the potential (84) for $n = 2$ whose phase portrait and shape are given in figure 7. For this model, we have calculated numerically the corresponding ratio of tensor to scalar perturbations for different orbits, and the results are depicted in figure 8. From this last figure, we will see that in teleparallel LQC there are orbits whose theoretical results match correctly with BICEP2 data and others that fit well with last *Planck's* data. In contrast, in holonomy corrected LQC all the orbits provide a tensor/scalar ratio smaller than 0.11 what means that, for this model, holonomy corrected LQC only matches correctly with *Planck's* data.

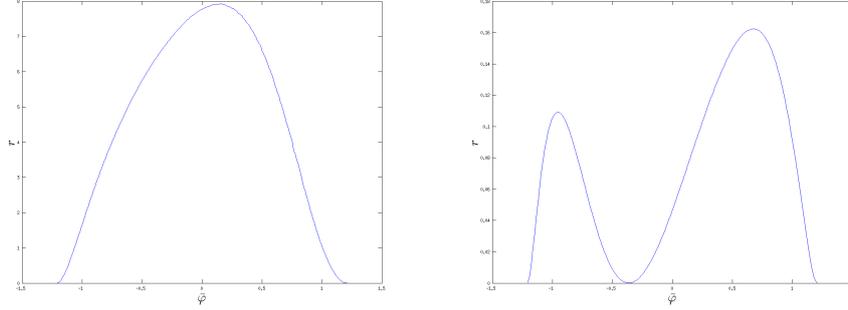


Figure 6: Tensor/scalar ratio for different orbits in function of the bouncing value of $\bar{\varphi}$ for the potential (84) with $n = 1$. In the first picture for teleparallel LQC, and in the second one for holonomy corrected LQC.

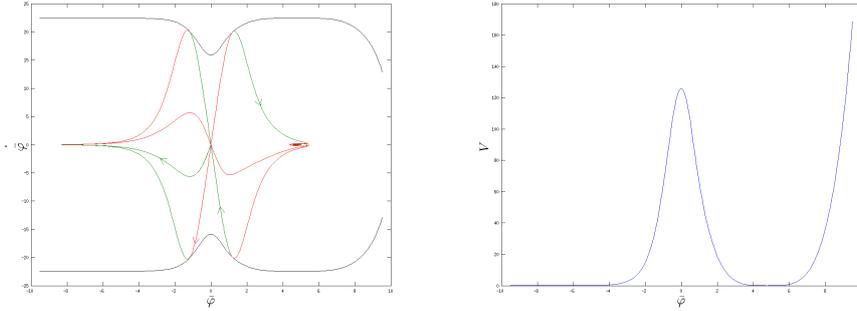


Figure 7: Shape and phase space portrait of a potential that has matter domination at early times in the contracting phase and is matched with a quartic potential.

Finally, we have studied the model of quintessence matching, with continuous derivative, a potential with the asymptotic form $V(\varphi) \sim \rho_c e^{-\sqrt{3(1+\omega)}|\varphi|}$ with a quintessence potential for example, with the potential $V(\varphi) = \rho_c \frac{\varphi_0^4}{\varphi^4 + \varphi_0^4}$. In figure 9, we show the shape of this potential and its phase portrait.

For this potential we have obtained the following results:

1. In teleparallel LQC, the confidence interval $r = 0.20^{+0.07}_{-0.05}$ derived from the BICEP2 data is realized by solutions bouncing when at bouncing time $\bar{\varphi}$ belongs in the interval $[-1.162, -1.145] \cup [1.135, 1.155]$, and the bound $r \leq 0.11$ provided by *Planck's* experiment is realized by solutions whose value at bouncing time is in the interval $[-1.208, -1.17] \cup [1.164, 1.204]$. Moreover, if one considers BICEP2 subtracting various dust models the tensor/scalar ratio, as we have already explained, is shifted to $r = 0.16^{+0.06}_{-0.05}$. Then, theoretical results fit well when at bouncing time the value of the field

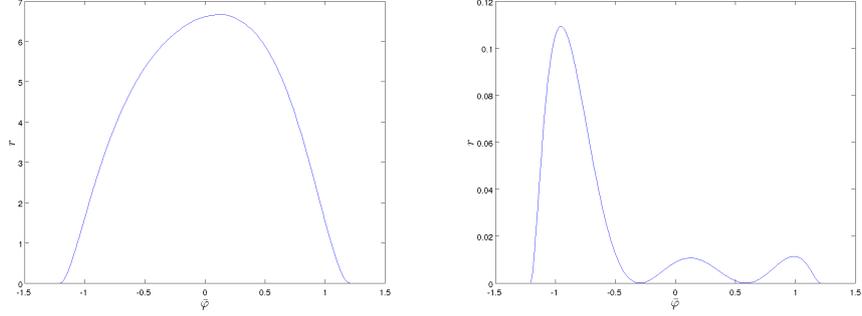


Figure 8: Tensor/scalar ratio for different orbits in function of the bouncing value of $\bar{\varphi}$ for the potential (84) with $n = 2$. In the first picture for teleparallel LQC, and in the second one for holonomy corrected LQC.

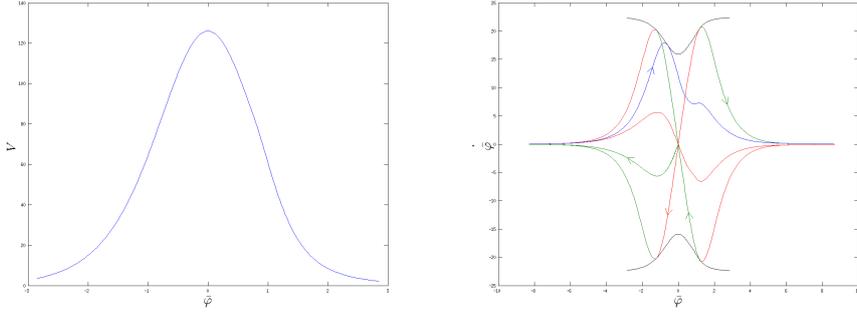


Figure 9: Shape and phase space portrait of a potential that has matter domination at early times in the contracting phase and quintessence at late times in the expanding phase.

belongs in $[-1.17, -1.152] \cup [1.143, 1.164]$.

2. In holonomy corrected LQC, the confidence interval $r = 0.20_{-0.05}^{+0.07}$ is never realized because the maximum value of r is 0.115, and the bound $r \leq 0.11$ is realized by solutions whose value at bouncing time belongs in the interval $[-1.208, -0.995] \cup [-0.865, 1.204]$.

The graphics of the tensor/scalar ratio are depicted in figure 10

As we will see from these numerical results, the shape of the tensor/scalar ratio in teleparallel LQC is very robust, in the sense that, it is potential independent (is nearly the same for all the viable potentials we have studied). However, dealing with holonomy corrected LQC, one can see that the shape of the ratio of tensor to scalar perturbations and the theoretical results obtained from it, change completely depending on the potential chosen.

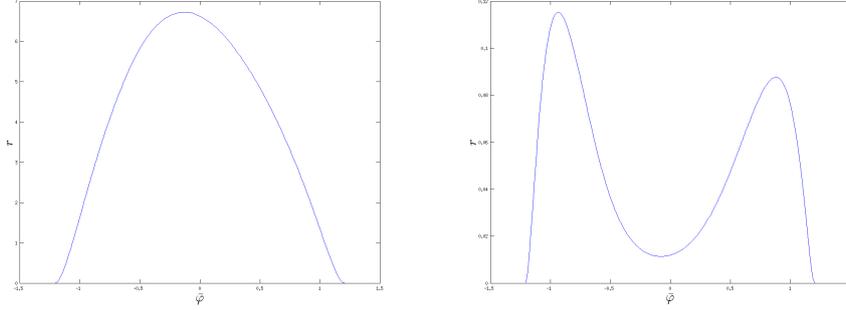


Figure 10: Tensor/scalar ratio for different orbits in function of the bouncing value of $\bar{\varphi}$ for the quintessence potential. In the first picture for teleparallel LQC, and in the second one for holonomy corrected LQC.

6.2 Reheating via gravitational particle production

Gravitational particle production in the matter bounce scenario has been recently introduced in [58]. The idea is the same as in inflationary models with potentials without a minimum (the so-called non oscillatory models): to have an efficient reheating one needs a non-adiabatic transition, in the expanding regime, between two different phases in order to have enough gravitational particle creation. In inflation, there is an abrupt transition from a quasi de Sitter regime to a radiation-dominated one, during this transition light particles are created and its density evolves like $\rho_r \sim a^{-4}$. On the other hand, after the end of the quasi de Sitter phase, the inflaton field, namely ϕ , enters a kinetic-dominated period where the energy density of the inflaton field evolves like $\rho_\phi \sim a^{-6}$ [53, 54, 59], which means that the inflation energy density decreases faster than that of radiation, and thus, the Universe becomes radiation dominated and matches with the hot Friedmann universe. Only at very late times, if the inflation potential is matched with a quintessence one, the Universe enters in an accelerated regime from which it never recovers.

In the matter bounce scenario the non-adiabatic transition could be produced in the contracting phase. In fact, a transition from matter-domination to an ekpyrotic phase with equation of state $P = \omega\rho$ where $\omega > 1$ could be assumed in the contracting regime. The obtained model is called matter-ekpyrotic bounce scenario [60], and since in the ekpyrotic phase the energy density of the field evolves like $\rho_\varphi \sim a^{-3(1+\omega)}$, which in the contracting phase increases faster than a^{-6} , anisotropies become negligible (note that the energy density in the anisotropies grows in the contracting phase as a^{-6} which is faster than the energy density of radiation, and thus, without an ekpyrotic transition the isotropy of the bounce is destroyed; this is the so-called Belinsky-Khalatnikov-Lifshitz instability [61]). Moreover, the energy density of the field also increases faster than that of radiation, this means that the field dominates the evolution of the Universe in the contract-

ing phase, but when the Universe bounces its energy density eventually dominates, because in the expanding phase $a^{-3(1+\omega)}$ decreases faster than a^{-4} . Then, the Universe will become radiation-dominated, and only at late times, if the potential is matched with an quintessence one, the Universe will accelerate forever.

To be more specific, we will study reheating via massless χ -particles nearly conformally coupled with gravity, using the method developed in [62]. It is well known that the number density of created particles and their energy density is related via the β -Bogoliubov coefficient as follows [53]

$$n_\chi = \frac{1}{2\pi^2 a^3} \int_0^\infty |\beta_k|^2 k^2 dk, \quad \rho_\chi = \frac{1}{2\pi^2 a^4} \int_0^\infty |\beta_k|^2 k^3 dk, \quad (87)$$

where

$$\beta_k = \frac{i(1-6\xi)}{2k} \int_{-\infty}^\infty e^{-2ik\eta} \frac{a''(\eta)}{a(\eta)} d\eta, \quad (88)$$

being $\xi \cong \frac{1}{6}$ the coupling constant.

In this approach, the number density of produced particles could be easily calculated using the properties of the Fourier transform which lead to the following simple formula

$$n_\chi = \frac{(1-6\xi)^2}{2\pi^2 a^3} \int_{-\infty}^\infty \left(\frac{a''(\eta)}{a(\eta)} \right)^2 d\eta. \quad (89)$$

To perform the calculation we consider the simplest model of an abrupt transition from matter to ekpyrotic phase [60]

$$a(t) = \begin{cases} a_E \left(\frac{t-t_0}{t_E-t_0} \right)^{2/3} & \text{for } t \leq t_E \\ \left(\frac{3}{4} \rho_c (1+\omega)^2 t^2 + 1 \right)^{\frac{1}{3(1+\omega)}} & \text{for } t \geq t_E, \end{cases} \quad (90)$$

where $\omega \gg 1$, $t_0 = t_E - \frac{2}{3H_E}$, t_E is the time at which the transition occurs and $H_E = H(t_E)$. Note that t_0 has been chosen in order that $a(t)$ has continuous first derivative at the transition time t_E .

For this scale factor, formula (89) leads to following density of created particles

$$n_\chi \cong \frac{3\sqrt{3}}{\pi^2} (1-6\xi)^2 \omega \left(\frac{a_E}{a} \right)^3 \rho_c^{3/2} \int_{-\infty}^\infty \frac{(1-x^2)^2}{(1+x^2)^4} dx, \quad (91)$$

and performing the integral one finally obtains

$$n_\chi \cong \frac{3\sqrt{3}}{4\pi} (1-6\xi)^2 \omega \left(\frac{a_E}{a} \right)^3 \rho_c^{3/2}, \quad (92)$$

The calculation of the energy density of the produced particles is more involved. First of all, to remove ultra-violet divergences one has to assume that the

second derivatives of the scale factor are continuous all the time. In this case, after integration by parts the β -Bogoliubov coefficient becomes

$$\beta_k = -\frac{(1-6\xi)}{4k^2} \int_{-\infty}^{\infty} e^{-2ik\eta} \left(\frac{a''(\eta)}{a(\eta)} \right)' d\eta. \quad (93)$$

Now, for the sake of simplicity, we will assume as in [53] that the third derivative of the scale factor is discontinuous at the transition time η_E . Then, one has

$$\beta_k \cong \frac{(1-6\xi)}{8ik^3} e^{-2ik\eta_E} \frac{a'''(\eta_E^+)}{a(\eta_E)} \cong \frac{9(1-6\xi)}{16ik^3} e^{-2ik\eta_E} \omega^2 H_E^3 a_E^3, \quad (94)$$

where $a'''(\eta_E^+)$ is the value of the third derivative of the scale factor at the beginning of the ekpyrotic phase.

As a consequence, for modes well outside of the Hubble radius at the transition time, i.e., for modes satisfying $a_E H_E < k < \infty$, using (94) one obtains the following radiated energy

$$\rho_\chi \cong \frac{9}{16} (1-6\xi)^2 \omega^3 \frac{\rho_E^2}{\rho_{pl}} \left(\frac{a_E}{a} \right)^4, \quad (95)$$

where $\rho_E = 3H_E^2$ is the energy density of the background at the transition time.

On the other hand, for modes well inside of the Hubble radius, i.e., satisfying $0 < k < a_E H_E$, one can approximate the β -Bogoliubov coefficient by

$$\beta_k = \frac{i(1-6\xi)}{2k} \int_{-\infty}^{\infty} \frac{a''(\eta)}{a(\eta)} d\eta \cong -\frac{9i}{4} (1-6\xi) \frac{a_E H_E}{k}, \quad (96)$$

which means, because we have assumed $\omega \gg 1$, that the energy density of those modes is smaller than that of the ones that are well outside of the Hubble radius, and consequently, the total energy density of the produced particles is

$$\rho_\chi \cong \frac{9}{16} (1-6\xi)^2 \omega^3 \frac{\rho_E^2}{\rho_{pl}} \left(\frac{a_E}{a} \right)^4. \quad (97)$$

To end the Section, we will calculate the reheating temperature T_R . To calculate that quantity for our model, first of all one has to define the reheating time t_R as the time when the radiated energy density equals the background energy density. Since the background energy in the ekpyrotic phase is given by

$$\rho(t) = \frac{\rho_c}{\frac{3}{4}\rho_c(1+\omega)^2 t^2 + 1}, \quad (98)$$

for large values of t one has $\rho(t) = \frac{4}{3\omega^2 t^2}$. And thus, the reheating time is of the order

$$t_R \sim \sqrt{\frac{\rho_{pl}}{\rho_E}} \frac{1}{\omega^{5/2} |1-6\xi| H_E}. \quad (99)$$

Consequently, the reheating temperature is of the order

$$T_R \sim \rho_r^{1/4}(t_R) \sim \sqrt{|1 - 6\xi|} \omega^{3/4} \frac{\rho_E^{1/2}}{\rho_{pl}^{1/4}} \frac{a_E}{a(t_R)}. \quad (100)$$

Finally, writing ρ_E in terms of Planck's density as follows $\rho_E \equiv \lambda^2 \rho_{pl}$, and approximating $\frac{a_E}{a(t_R)}$ by 1 because we are considering the case $\omega \gg 1$, one has

$$T_R \sim \rho_\chi^{1/4}(t_R) \sim \sqrt{|1 - 6\xi|} \omega^{3/4} \lambda M_{pl}, \quad (101)$$

where we have introduced the Planck mass $M_{pl} \equiv \rho_{pl}^{1/4}$. This theoretical value will coincide with current observations provided that one chooses $\sqrt{|1 - 6\xi|} \omega^{3/4} \lambda \sim 10^{-7}$ [58].

Remark 6.1. *Reheating via an interaction between the field φ and light particles, of the form $\epsilon^2 \varphi^2 \chi^2$, has been recently studied in the context of bouncing scenarios in [58]. The calculation of the energy density is very complicated, and only an upper bound has been obtained. Moreover, this energy density has been compared with the energy density of produced light particles minimally coupled with gravity, obtaining that the energy density of gravitational particle production dominates over the energy density from the interaction for small values of ϵ .*

7 Conclusions

In this work we have shown that the matter bounce scenario in holonomy corrected LQC and in its teleparallel version (the $F(T)$ model whose modified Friedmann equation coincides with the holonomy corrected one of LQC, when one considers the flat FLRW geometry) given by the simplest potential (the potential given in equation (60)) must be improved in order to reproduce our Universe. Although, when one considers our teleparallel version of LQC the theoretical results predicted by some solutions of the potential (60) match correctly either with BICEP2 or *Planck's* current data, this potential neither provides a reheating mechanism in order to match with a hot Friedman Universe nor takes into account the current acceleration of the Universe. For these reasons we introduce more phenomenological potentials that take into account the reheating process and the current cosmic acceleration, and we show that, in the teleparallel version of LQC, they have sets of solutions whose theoretical results fit well with current observational data, concluding that the matter bounce scenario in teleparallel LQC is a viable alternative to the inflationary paradigm. On the other hand, holonomy corrected LQC provides theoretical results that always match with *Planck's* data, but in general, its theoretical results do not fit well with BICEP2 data, only for some models with a potential well (for example, for a quadratic potential there are solutions whose theoretical results match correctly with BICEP2 data, but for a quartic one theoretical results only fit well with *Planck's* data). In fact, our numerical results show

that the teleparallel version of LQC is not potential dependent in our models, that is, we have shown that the shape of the tensor/scalar ratio is practically the same for the models that we have studied. This does not happen in holonomy corrected LQC where we have shown that it changes completely depending of the potential matched with (60). From our viewpoint this is a great advantage of our $F(T)$ model with respect holonomy corrected LQC, but what really disfavours holonomy corrected LQC with respect our teleparallel model, is that in this theory, the speed of sound becomes imaginary in the super-inflationary phase, which implies Jeans instabilities, and these may invalidate the use of the linear perturbation equations in this regime. This problem does not arise in our $F(T)$ version of LQC, where the velocity of sound is always positive.

We have also studied in detail the reheating process via gravitational particle production: we have considered the gravitational reheating in the matter-ekpyrotic matter bounce scenario, where a phase transition from the matter domination to an ekpyrotic epoch occurs in the contracting phase of the Universe. We have assumed that the reheating is due to the creation of massless nearly conformally coupled particles, and we have applied the method introduced by [62], for the first time, to our matter-ekpyrotic model, obtaining a very simple expression for the reheating temperature. Concluding that, when one considers potentials without a minimum, the production of gravitational particles via a phase transition in the contracting regime leads to an efficient, i.e. compatible with the latest experimental observations, reheating. On the other hand, when one considers potentials with a minimum as in inflationary cosmology, the instant reheating introduced in [50] gives rise to a reheating temperature compatible with current results.

Finally, we expect that more precise unified PLANCK-BICEP2 data (the B2P collaboration), which are going to be issued in the future, may be helpful to improve or rule out some of the bouncing models under consideration.

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