A Rolling Horizon Optimization Framework for the Simultaneous Energy Supply and Demand Planning in Microgrids

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Abstract

This work focuses on the development of optimization-based scheduling strategies for the coordination of microgrids. The main novelty of this work is the simultaneous management of energy production and energy demand within a reactive scheduling approach to deal with the presence of uncertainty associated to production and consumption. Delays in the nominal energy demands are allowed under associated penalty costs to tackle flexible and fluctuating demand profiles. In this study, the basic microgrid structure consists of renewable energy generators (photovoltaic panels, wind turbines) and energy storage units. Consequently, a Mixed Integer Linear Programming (MILP) formulation is presented and used within a rolling horizon scheme that periodically updates input data information.

Keywords: Energy planning, Rolling horizon, Scheduling, Mathematical programming, Microgrid, MILP.

1. Introduction

The development of sustainable energy supply chains has led to advances in the area of Energy Systems Engineering, which involve all the decision-making procedures from the primary energy source to the final energy delivery to the costumer. The main objectives and challenges of managing energy systems are to reduce costs associated to the exploitation of the energy network, to reduce the environmental impact caused by the production and transmission of energy and to satisfy the energy demand subjected to unexpected internal and external disturbances.

Traditionally, power grids are based on centralized networks where large power plants generate electricity that is used posteriorly at industrial or domestic level (Wang and Singh, 2009). This kind of energy supply chains involves energy losses in power transmission due to the physical distance between the electricity generation and consumption sites. Furthermore, the generation of energy in centralized networks usually exploits non-renewable sources (i.e., fossil fuels), which has a negative environmental impact (i.e., pollution, climate change).

Microgrids are based on the decentralized energy supply chain concept, which can integrate several renewable and non-renewable energy sources that are usually close to energy consumers. A major drawback of renewable energy systems is the apparent mismatch of the volatility of energy production from renewable sources and energy demand. Natural energy resources though, have
the disadvantage of intermittent and unpredictable production, due to their dependence on natural phenomena, the forecast techniques of which are far from reliable. Thus, the simultaneous consideration of energy production and demand is essential to manage appropriately a microgrid, in order to match energy production and demand. Moreover, the use of energy storage systems is a common way to alleviate this mismatch and tackle the uncertainty in energy demand forecasts. Furthermore, energy storage provides the necessary tools to schedule the flexible energy demand according to time-of-use market base pricing, introducing enough operational flexibility to efficiently exploit periods of low prices, avoiding pick prices and reducing energy costs.

As mentioned above, the behavior of natural energy sources involves the consideration of uncertainty in microgrids, in order to ensure the generation of good quality and practical interest management decisions. Particularly, the operations management of microgrids is affected by several types of uncertainty, such as energy demand variations and weather conditions which affect the availability and production capacity of renewable energy generators. The decision making process becomes more complex if the consideration of different sources of uncertainty in the models is essential to ensure solution quality or even its practical feasibility. Different types of uncertainty sources can be found, including:

(i) External sources, including uncertainty in demand, prices and availability of resources.
(ii) Internal sources, like fluctuations in process parameters.
(iii) Other uncertainty sources, such as measurements errors or strikes.

The approaches to address problems under uncertainty can be classified into reactive and preventive. On one hand, reactive approaches are focused on modifying a nominal plan obtained by a deterministic formulation in order to adjust it to different alterations, modifications or updated system data. On the other hand, preventive approaches are based on the consideration of all possible cases, and finding a good solution for all these cases. These approaches have the advantage that a feasible solution is found for all considered scenarios. However, the obtained solutions may be too conservative, since the model must take into account all the possibilities. The most frequently adopted preventive procedure are the stochastic programming (Shapiro et al., 2013) and the robust optimization (Li et al., 2011).

In this work, a new discrete-time Mixed Integer Linear Programming (MILP) mathematical formulation is presented to cope with the underlying uncertainty through a rolling horizon approach with the purpose to optimally manage a microgrid (i.e., schedule the energy production and consumption). This approach will allow to update all input parameters and to react to any variation from the nominal or expected conditions.

The paper is organized as follows. In the next section, the literature review on the energy management models and the use of rolling horizon approach is presented. Then, section 3 describes the problem statement of this study. In section 4, the proposed reactive scheduling approach for the simultaneous energy production and energy demand side management is described in detail, and some remarks on the proposed method are provided. In section 5, a case study is presented. Then, section 6 provides a discussion on the obtained results. Finally, some concluding remarks are drawn and future research directions are revealed in section 7.
2. Literature review

The growing interest in energy microgrids has led to the development of several mathematical models and representation schemes related to their management, as well as to their design, including the energy production management, the energy demand side management and the simultaneous management of energy production and demand.

Regarding the energy production management, different mathematical models were developed in order to minimize the operational cost of a given network. In this field, Zamarrripa et al. (2011) have developed a mathematical model to determine the production and storage levels to satisfy a deterministic energy demand, by minimizing the operational cost. More recently, a MILP model for the energy production planning related to an energy supply chain network based on a residential microgrid, which consists of a number of interconnected combined heat and power systems, under the objective of minimizing the total operational cost, has been presented by Kopanos et al. (2013). These mathematical formulations take into account energy generation constraints, such as production limits, ramping limits and minimum up and down times, which involve the presence of non-linear constraints. The aim of these approaches is to find how a given set of energy generators should satisfy a given demand in order to minimize the total operational costs. This is a very challenging optimization problem in the area of Energy Systems Engineering because of the huge number of possible combinations of the status of the generating units in the network (Bhardwaj et al., 2012). Several works were focused on the management of energy production of fixed and given demand. Carrión and Arroyo (2006) presented a discrete-time MILP formulation which minimizes the energy cost of a given network. More recently, Zondervan et al. (2010) developed a Mixed Integer Non-Linear Programming (MINLP) formulation for process industries, in which the objective was to minimize the operational cost according to the availability and price of energy, by determining the optimal schedule of process tasks.

The energy demand side management of industrial process represents an emerging challenge. In this area, Della Vedova and Facchinetti (2012) developed a real-time scheduling approach to model and control the electrical availability. The aim was to reduce the presence of peaks of power consumption, which are negative for energy providers, for the grid and for the users, due to the fact that the presence of peaks reduces the grid efficiency and also because energy prices are based in peaks.

Furthermore, the management of both energy production and demand has been studied in a sequential way, by adapting the process schedule to the energy availability from an external power grid. Regarding industrial processes, Nolde and Morari (2010) have developed a continuous-time MILP in order to minimize the total energy cost, by managing the energy consumption. The aim of the proposed formulation was to adapt the schedule of a steel plant, introducing a penalization for any variation from the contracted energy consumption from the plant to the energy supplier. Other works related to the simultaneous management of production and demand were focused on adapting the scheduling of the industrial process to the price of the energy in each period of time. Mitra et al. (2012) developed a discrete-time MILP in order to adjust production planning according to time-dependent electricity pricing schemes for a continuous process. Also, the demand response has been studied by Hadera et al. (2014) for a steel plant, in order to reduce energy costs.
Moreover, Mohsenian-Rad and León-García (2010) developed an offline residential energy consumption scheduling approach based in electricity pricing models.

But, although energy production management has been studied in the last years, the overall scheduling of a microgrid taking into account the simultaneous management of energy production and energy demand (including the possibility to shift energy consumptions) under uncertainty has not been reported and still represents an open challenge to the research community. The development of a new mathematical model to manage a microgrid can be applied to obtain the optimal schedule of energy consumptions, producing several benefits such as the reduction of energy peaks and the reduction of non-renewable energy requirements. Along this line, a discrete-time MILP formulation for the integrated management of energy production and demand was developed by Silvente et al. (2012). More recently, Silvente et al. (2015) presented a comparison between discrete-time and hybrid-time MILP formulations related to a microgrid in order to maximize its profit. Also, a first MILP hybrid approach was presented by Silvente et al. (2013) with the aim of minimizing the operational cost of the overall system, and very recently Marietta et al. (2014) and Silvente et al. (2014) have presented a discrete-time MILP formulation with flexible time windows for the allocation of energy loads into a rolling horizon framework.

The presence of uncertainty is addressed by implementing a rolling horizon approach, which is a reactive scheduling method that solves iteratively the deterministic problem by moving forward the optimization horizon in every iteration; assuming that the status of the system is updated as soon as the different uncertain or not accurate enough parameters became to be known, the optimal schedule for the new resulting scenario (and optimization horizon) may be found. This approach considers: a prediction horizon, in which all the uncertain parameters related to this time horizon are assumed to be known with certainty, due to the fact that the system under study receives feedback related to the unknown parameters, and a control horizon, where the decisions of the optimization for the prediction horizon are applied (Fig. 1). Rolling-horizon has been applied to several scheduling problems under uncertainty, and the interested reader can be referred to Kopanos and Pistikopoulos (2014) for a brief review. As mentioned in the Introduction, the iterative procedure of this formulation allows to update or to modify all input parameters in order to optimize the problem according to the current available information.

Fig. 1. Concepts associated to rolling horizon approach: scheduling, production and control horizon.

3. Problem statement

The system under study consists of a set of interconnected elements (i.e. power generators, energy storages, energy consumptions) as well as a set of decisions (when, where, who, how much) that define a managerial problem (resource allocation and timing) under uncertain conditions.

The proposed formulation takes into account not only the production and storage levels to be managed by the microgrid, but also the possibility to modify the timing of the energy consumption within a certain time window, if this is considered acceptable. The proposed
formulation contemplates energy selling to the main power grid as well as all the involved costs, including generation and storage costs, as well as penalty costs in case of deviations from the energy consumers’ target, in order to maximize the profit. The mathematical model includes both the energy balance constraints required to describe the energy flows (generation, storage and consumption), and the constraints associated to the equipment and technologies involved in the microgrid. Consequently, the problem under study is described in terms of the following items:

(i) A given scheduling horizon $SH$, which is divided into a number of equal-size time intervals $t \in T$. Also, a given Prediction Horizon $PH$ and a Control Horizon $CH$.

(ii) A set of energy generators $i \in I$, characterized by a minimum and maximum energy generation capacity, $P_{i,t}^{\text{min}}$ and $P_{i,t}^{\text{max}}$, and a given operational cost $cpro_{i,t}$.

(iii) A set of energy storage systems $k \in K$, having a minimum and a maximum energy storage capacity, $SE_{k,t}^{\text{min}}$ and $SE_{k,t}^{\text{max}}$, and cost $csto_{k,t}$.

(iv) A given energy demand, given by the amount of energy required by a set of energy consumption tasks $j \in J$, where $f \in F_j$ denotes the number of times that a consumer $j$ can be active. For any energy consumption, its duration $Dur_{0,j,f}$ and a target starting time $T_{s_{j,f}}^{\text{min}}$ are established although, as indicated, consumption tasks can be delayed within certain limits generating a penalty cost $cpen_{j,f}$. The energy demand $Cons_{j,f}$ for consumption tasks is assumed to be constant.

(v) All energy consumption tasks which might be active during each iteration of the rolling horizon approach are included in the dynamic set $F_{j,RH}$.

(vi) A given set of power grids $r \in R$, which can buy the excessive energy production, or sell additional energy to the microgrid $r$ required.

The proposed reactive scheduling problem has been introduced into an iterative approach based on a rolling horizon framework (Fig. 2). The rolling horizon algorithm can be applied as shown in Fig. 3:

- Initially, establish the initial conditions of the system, as well as the length of the scheduling, prediction and control horizons.
- Next, establish the first scheduling period and solve the scheduling problem.
- Update the uncertain parameters and solve again the scheduling problem using data from the last optimization, by the use of linking variables (see Section 4.2).
- Then, if this new schedule corresponds to the last period of time, stop. Otherwise, fix the values obtained in the optimization problem for that iteration, re-schedule and update the period of time until the last scheduling period is reached.

The proposed formulation allows to update or modify the different parameters related to the uncertainty, such as the external variability in weather conditions (which affects the energy production) or the variability of the duration of the consumption tasks (which affects the energy demand).

Although the solution obtained in each prediction horizon is optimal for this period of time, the solution of the overall problem could be suboptimal in practice, since future information outside the current prediction horizon is not taken into account. Therefore, the length of the
prediction horizon must be appropriate in order to ensure the quality of the obtained results. The length of the prediction horizon depends on the characteristics of the problem (Kopanos and Pistikopoulos, 2014).

The main decisions to be made in order to maximize the profit of the microgrid, are:

(i) The amount of power $P_t$ to be produced (or purchased) in each time interval $t$.

(ii) The generator to produce energy $X_{i,t}$ in each time interval $t$.

(iii) The energy storage level $SE_{k,t}$ at the end of time interval $t$.

(iv) The specific (nominal) time to execute an energy consumption.

(v) The amount of power $P_{g,t}$ to be sold to the power grid in each time interval $t$.

Fig. 2. Reactive scheduling via a rolling horizon framework (Kopanos and Pistikopoulos, 2014).

Fig. 3. Algorithm for the rolling horizon approach.

The decision variables include the determination of whether or not an energy generator $i$ is switched on/off at a given time interval $t$, according to different energy consumers $j$, as well as decisions associated with the energy demand side management, which determines when to consume.

It is worth noting that the presented formulation is based on a discrete-time representation, in which the scheduling horizon is divided in a finite number of identical time intervals. This formulation forces all tasks to start at the beginning of each time interval (Méndez et al., 2006). The computational time to solve discrete-time models depends on the size of the problem, which is given by the number of time intervals in which the PH is divided, and so in their length. The length of the time interval depends on the characteristics of the problem under consideration, since long time intervals could lead to obtain suboptimal solutions, and short time intervals could imply an unaffordable computational effort in order to obtain the optimal solution (Harjunkoski et al., 2014).

4. Mathematical formulation

4.1. Microgrid schedule

The constraints associated to sequencing, allocating of energy consumptions and generators, and energy distribution of a given microgrid are presented next.

Bounds on energy production for every energy generator (including energy purchases to the external power grid) are specified by eq. 1. For each energy generator $i$ and time interval $t$ included in the prediction horizon the binary variable $X_{i,t}$ indicates if this generator is being used or not. Thus, the total amount of energy produced at each interval $t$ is given by eq. (2):

$$p_{i,t}^{\min} \cdot X_{i,t} \leq P_{i,t} \leq p_{i,t}^{\max} \cdot X_{i,t} \quad \forall i, t \in TRH \quad (1)$$
\[ PT_t = \sum_{t} P_{i,t} \quad \forall t \in TRH \quad (2) \]

The amount of energy in each energy storage \( k \) at each time interval \( t \) is bounded within a minimum and a maximum value, as given by eq. (3). This maximum value corresponds to the full energy storage level. Eq. 4 represents the links among energy flows at a specific storage system and time \( k, t \). This equation considers the storage level \( SE_{k,t} \), the energy requirements covered by the storage \( SP_{k,t} \), the supply flows arriving to the storage \( Ld_{k,t} \) and the input and output efficiency of the energy storage system \( \eta \). The maximum level of load is bounded by eq. (5):

\[
SE_{k,t}^{\text{min}} \leq SE_{k,t} \leq SE_{k,t}^{\text{max}} \quad \forall k, t \in TRH \quad (3)
\]

\[
SE_{k,t} = SE_{k,t-1} + \eta_{in}^{m} \cdot Ld_{k,t} - \frac{SP_{k,t}}{\eta_{out}} \quad \forall k, t \in TRH \quad (4)
\]

\[
\sum_{k} Ld_{k,t} \leq PT_t \cdot DT \quad \forall t \in TRH \quad (5)
\]

The proposed mathematical formulation allows to start the energy consumption tasks within a time window. Thus, the energy demand side management involves the determination of the starting time of each energy consumption task. Hence, some constraints are required regarding the time window within energy consumption task \( jk \) are allowed to consume. According to eq. (6), the starting time is bounded by a minimum (target) and a maximum initial time. Moreover, the finishing time of each energy consumption task \( jf \) is given by the starting time plus its duration, according to eq. (7).

\[
T_{S_{j,f}}^{\text{min}} \leq T_{S_{j,f}} \leq T_{S_{j,f}}^{\text{max}} \quad \forall j, f \in F_{j,RH} \quad (6)
\]

\[
T_{f_{j,f}} = T_{S_{j,f}} + Dur_{j,f} \quad \forall j, f \in F_{j,RH} \quad (7)
\]

The binary variable \( Y_{j,f,t} \) is active when the energy consumption task \( jf \) starts at time interval \( t \). Accordingly, \( Z_{j,f,t} \) is active if the energy consumption \( jf \) finishes its consumption at time interval \( t \). These logical restrictions can be reformulated as a set of big-M constraints, given by eq. (8a)-(9b):

\[
T_{S_{j,f}} \geq T_t - M \cdot (1 - Y_{j,f,t}) \quad \forall j, f \in F_{j,RH}, t \in TRH \quad (8a)
\]

\[
T_{S_{j,f}} \leq T_{t+1} - M \cdot (1 - Y_{j,f,t}) \quad \forall j, f \in F_{j,RH}, t \in TRH \quad (8b)
\]

\[
T_{f_{j,f}} \geq T_t - M \cdot (1 - Z_{j,f,t}) \quad \forall j, f \in F_{j,RH}, t \in TRH \quad (9a)
\]

\[
T_{f_{j,f}} \leq T_{t+1} - M \cdot (1 - Z_{j,f,t}) \quad \forall j, f \in F_{j,RH}, t \in TRH \quad (9b)
\]
Furthermore, energy loads of a given consumption cannot overlap in the same unit time:

\[ T_{f_j,f} \leq T_{s_{j,f}}, \quad \forall j, f \in F_j RH, f < f' \quad (10) \]

Moreover, eq. (11)-(12) ensure a unique starting time for energy consumption \( jf \). The binary variable \( Y_{j,f,t} \) determines if consumption task \( jf \) starts during the current time interval included in the prediction horizon, and \( \hat{Y}_{j,f} \) determines if this consumption starts outside this time interval, forcing that all consumptions must start in the scheduling horizon. In the same way, eq. (13)-(14) determine the finishing time for energy consumption task \( jf \). Also, eq. (15) establishes when each energy consumption task \( jf \) is active at \( t \).

\[ \sum_{t \in T_{s_{j,f}}} Y_{j,f,t} \leq 1 \quad \forall j, f \in F_j RH \quad (11) \]

\[ \hat{Y}_{j,f} + \sum_{t \in TRH} Y_{j,f,t} = 1 \quad \forall j, f \in F_j RH \quad (12) \]

\[ \sum_{t \in T_{s_{j,f}}} Z_{j,f,t} \leq 1 \quad \forall j, f \in F_j RH \quad (13) \]

\[ \hat{Z}_{j,f} + \sum_{t \in TRH} Z_{j,f,t} = 1 \quad \forall j, f \in F_j RH \quad (14) \]

\[ W_{j,f,t} = \hat{Y}_{j,f} + \sum_{t \in TRH} Y_{j,f,t} - \hat{Z}_{j,f} - \sum_{t > t} Z_{j,f,t} \quad \forall j, f \in F_j RH, t \in TRH \quad (15) \]

The total energy demand of the microgrid at time interval \( t \), determined by all the active consumption tasks \( jf \) at this time interval \( t \), is given by eq. (16). The second term of this equation determines the exact energy consumption for those time intervals in which the energy consumption will not take place during the overall time interval.

\[ Dem_t = \sum_{j} \sum_{f} Cons_{j,f} \cdot \left[ W_{j,f,t} \cdot DT - Z_{j,f,t} \cdot (T_{t+1} - T_{f_j,f}) \right] \quad \forall t \in TRH \quad (16) \]

Eq. (17) establishes the overall energy balance of the microgrid, considering the production, the consumption, the charge and discharge of the storage unit (i.e., battery), and the energy sales to the power grids.
\[ \sum_{k=1}^{K} SP_{k,t} + PT_t \cdot DT - Dem_t - \sum_{k=1}^{K} Ld_{k,t} - \sum_{r=1}^{R} Pg_{r,t} \cdot DT = 0 \quad \forall t \in TRH \quad (17) \]

Hence, economic aspects related to the energy management are studied. The production cost is calculated in eq. (18) as the amount of the costs associated to each energy generators, given by the unitary production cost multiplied by the energy units produced. The storage cost for each interval \( t \) is the amount of costs associated to each interval, which are the same as the unitary storage cost multiplied by the energy storage, according to eq. (19). Finally, the penalty cost is determined as a function of the delay in satisfying each energy demand, and given by eq. (20). The total operational cost of the microgrid is given by eq. (21), which considers the previous costs.

\[
\begin{align*}
\text{Cost Pro} &= \sum_{t \in TRH} T \sum_{t=1}^{T} cpro_{t,t} \cdot P_{t,t} \cdot DT \\
\text{Cost Sto} &= \sum_{t \in TRH} T \sum_{k=1}^{K} csto_{k,t} \cdot SE_{k,t} \\
\text{Cost Pen} &= \sum_{j=1}^{J} F_j \sum_{f \in F_{j,t}} \text{cpen}_{j,f} \cdot (T_{s_{j,f}} - T_{i_{j,f}})
\end{align*}
\]

\[
\text{Costs} = \text{Cost Pro} + \text{Cost Sto} + \text{Cost Pen} \quad (21)
\]

Also, energy revenues to the power grid have been taken into account. Thus, incomes are given by eq. (22) considering the energy sales to each power grid.

\[
\text{Incomes} = \sum_{t \in TRH} T \sum_{r=1}^{R} Price_{r,t} \cdot Pg_{r,t} \cdot DT \quad (22)
\]

In this study, the objective function is the profit of the microgrid that includes total incomes and costs, as given by:

\[
\text{Profit} = \text{Incomes} - \text{Costs} \quad (23)
\]

4.2. Rolling horizon approach

In order to use the proposed model in a rolling horizon scheme, the following set of variables and equations are used to link past decisions with the current prediction horizon. Minimum and maximum starting times of energy consumption tasks at each iteration are given by Eq. (24) and (25), considering the minimum/maximum starting time of the previous iteration and...
the duration of the control horizon \((CH)\). The duration of the energy consumption tasks is also
updated at each iteration, as indicated by Eq. (26). And finally, the energy storage level of the
previous control horizon is linked to the initial energy storage level of the current prediction
horizon by Eq. (27). The optimization problem will be iteratively solved according to the rolling
horizon approach. This reactive approach will be used to adjust all input parameters to the current
available information.

\[
T_{sjr}^{\text{min}} = T_{s0,jr}^{\text{min}} - CH \cdot it \quad \forall j, f \in F_j RH \tag{24}
\]

\[
T_{sjr}^{\text{max}} = T_{s0,jr}^{\text{max}} - CH \cdot it \quad \forall j, f \in F_j RH \tag{25}
\]

\[
Dur_{jr} = Dur_{0,jr} - (CH \cdot it - T_{sjr}) \cdot Y_{jr} \quad \forall j, f \in F_j RH \tag{26}
\]

\[
SE_{k,t-1} = SE_{k,t} \quad \forall k, t, t' \in TRH, t' = T_t \cdot DT \tag{27}
\]

This mathematical formulation involves a MILP model. However, this mathematical
formulation can be extended by introducing some features of the typical unit commitment problem
associated to energy problems, such as ramping constraints and minimum up and down time
constraints, as well as non-convex production costs and time-dependent startup costs, which may
introduce non-linearities in the model. Moreover, the discrete-time representation has been chosen
instead of the hybrid or the continuous-time formulation. Although the optimal solution could be
improved by the use of hybrid or continuous-time representations, the resulting models involve
more computational time. Also, the proposed model does not consider the presence of fixed costs
associated to the investment and installation of energy production generators, since the design of
the energy network related to the microgrid has not been taken into account and only short-term
decisions are addressed.

\[
5. \text{ Case study}
\]

The proposed MILP formulation has been applied to a case study. The microgrid of the case
study consists of a photovoltaic panel and a micro-wind turbine. In addition, this microgrid is
connected to the main power grid. The possibility to purchase energy to the power grid ensures the
feasibility of the optimization problem, disregarding weather conditions and anomalous demand
profiles. Also, an energy storage system has been considered.

Moreover, a total number of 30 different energy consumers \(j\) has been taken into account,
with different energy consumption task \(jjf\). The possibility that the same consumer \(j\) will be active
several times is considered. These energy consumptions are modelled allowing a certain delay from
the initial target, depending on the availability and demand of energy. Each energy consumption
task is associated with a penalty cost, in case of deviation from the target.
The underlying scheduling problem includes energy production, storage and consumption tasks to be optimized. Data and decisions related to energy production are given for every 15 min, according to energy demand and the current weather forecast. Also, decisions related to energy demand are considered for every 15 min, as well as production decisions. The total scheduling horizon considered is 24 h, and the duration of each time interval is 15 min (i.e., 96 time intervals in total). Prediction Horizons (PHs) of 5, 10, and 20 time intervals have been considered. It has been considered that input data (e.g., energy demands, weather conditions) are updated at the beginning of each time interval, so the control horizon has been set also equal to 15 min (i.e., one time interval).

Data related to energy production cost for each generator and energy storage cost are presented in Table 1. Also, the same table displays the minimum and maximum values for energy supply and storage. The energy resource availability from different energy generators is displayed in Fig. 4. Data related to maximum power availability can be found in Table A2 in Appendix A. It is important to remark that variable production costs related to the energy production through solar panels and wind turbines are considered to be zero. Also, fixed costs (related to the investment, installation and maintenance of the generators) are not considered since the design of the energy supply chain related to the microgrid has not been taken into account in the operational level. Thus, only short-term decisions are considered. The minimum energy level of the energy storage system is determined by the 80% of the maximum capacity, because forcing lower energy levels shorten the life cycle of the energy storage system.

![Fig. 4. Power availability.](image)

### Table 1. Economic data and capacity constraints.

<table>
<thead>
<tr>
<th>Energy Source</th>
<th>Cost (€/MWh)</th>
<th>Capacity (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar panels</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>Wind turbines</td>
<td>200</td>
<td>15</td>
</tr>
</tbody>
</table>

6. Results and discussion

The resulting MILP model has been implemented in GAMS 24.1 (Rosenthal, 2012) and solved using CPLEX 12, to zero optimality in a Pentium Intel® Core™ i7 CPU 2600 @ 3.40 GHz. The resolution of the model provides the daily optimal schedule for energy generation and consumption in order to maximize the profit of the given microgrid. The rolling horizon approach, used to address the presence of uncertainty, has allowed to update input information related to weather conditions and consumption durations. Fig. 5 displays the daily schedule for energy production and energy consumptions for a prediction horizon of 20 time intervals. Some energy consumption tasks have been delayed (i.e., energy demand has right-shifted). According to this figure, the microgrid produce the maximum energy capacity from energy renewable sources, in order to satisfy the energy demand and to sell this energy to the power grid. The purchases of energy to the power grid are required if the use of both renewable sources and the energy storage system is not enough to guarantee the demand satisfaction.
Fig. 5. Daily schedule for energy production, consumption and storage (PH=20).

The application of the rolling horizon, in which the problem formulation is solved iteratively, can produce constant changes in the expected schedule (Fig. 6). This is produce because the input information is updated and future information is introduced in the system. To highlight these changes, only some consumption tasks have been plotted. At the beginning, some consumption tasks do not appear in the plot because in the current time there is no information related to the mentioned consumptions.

Fig. 6. Evolution of the energy consumption scheduling for PH=20 in iterations 54, 55, 56, 57 and final iteration.

Different lengths of the prediction horizon have been considered, in order to compare, analyse and highlight the characteristics of the proposed rolling horizon and the simultaneous management of energy production and demand. Thus, prediction horizons of 5, 10, 20 and 30 time intervals has been taken into account. Moreover, the perfect information case (PH=96), which corresponds to the situation in which the length of the prediction horizon is equal to the length of the scheduling horizon, has been considered.

In all these situations, the control horizon is equal to one time interval. According to the obtained results, longer prediction horizons involve a significant reduction in the use of the power grid to satisfy the demand (Fig. 7), since more future information is received to solve the optimization problem. As a consequence, the profit increases for longer prediction horizons (Fig. 8 and Table 2). In order to compare how the simultaneous energy production and demand management affects the final solution, Table 3 shows the obtained results in the case of managing only the energy production, in which energy consumptions cannot be shifted. As expected, the obtained profit decreases in comparison with that of the simultaneous management.

Fig. 7. Power source in each time interval for different PH lengths.

Fig. 8. Profit for different PHs, perfect information and no demand management case.

Table 2. Comparison of the obtained results for the scenario 1 (demand side management).

Table 3. Comparison of the obtained results for the scenario 2 (demand side management).

Fig. 9 shows the aggregated objective value for the rolling horizon approach under different prediction horizons in comparison with the perfect information case. Not surprisingly, as the length of the prediction horizon increases, the total objective is improved and closes the gap from the perfect information solution. Shorter prediction horizon involve more energy sales to the power
grid (Fig. 10), because there is no information about future energy demand, requiring energy purchases to the power grid and more delays in the energy consumptions. Model statistics for each the proposed model using different prediction horizons can be found in Table 4.

Fig. 9. Aggregated profit for different PHs, perfect information and no demand management case.

Fig. 10. Energy sales for different PHs, perfect information and no demand management case.

Table 4. Comparison of optimal solution and model statistics.

Finally, in order to show how the energy production is managed, a variation from the initial model was solved. This variation consists in minimize the total cost of the microgrid, without taking into account possible energy sales. Thus, the objective function will be to minimize eq. (22) subject to constraints (1)-(21). As example, for a prediction horizon of 20 time intervals, purchases of energy to the power grid are not required to satisfy the demand (Fig. 11), since all the produced energy is to meet the demand as well as to fulfil the energy storage system, because there are no energy sales. For the same reason, the energy produced does not match the maximum production capacity, since only the energy to be consumed or stored is produced, as a difference from the previous solution approach. In this case, 365.8 kWh were produced (30.3 % produced in the photovoltaic panel and 69.7 % in the wind turbine), reducing the energy requirements by 8.0 %.

Fig. 11. Schedule of energy production with no sales to the power grid (PH=20).

Moreover, this reactive approach allows updating input parameters, in order to react to variations from the nominal/initial plan, including alterations in the power availability or in the duration of energy consumptions. As example, Fig. 12 shows an unexpected scenario in which there is a reduction in the power availability as well as an increased duration of some duration consumptions, with the objective of maximizing the profit of the microgrid.

The rolling horizon approach allows to react to any eventual alteration from the initial conditions. Table 5 compares the obtained results for the simultaneous energy supply and demand management, for PH=20 and the perfect information case. According to the results, energy purchases are required in both cases to satisfy the energy demand. However, the use of the rolling horizon approach have allows the benefit to be more than 3% high compared with the perfect information case.

7. Conclusions

In this work, a discrete-time MILP formulation for the simultaneous optimization of energy production and consumption tasks in microgrids has been presented. The objective function considered is the maximization of the total profit. The proposed approach has proven the
advantages of managing the energy demand by optimizing the management of the microgrid, which
allows enhancing its flexibility and autonomy.

A rolling horizon approach has been introduced in the formulation to address the presence of
uncertainty. As expected, longer prediction horizons favor the generation of better solutions, under
the assumption of accurate demand predictions. This approach allows updating input parameters,
in order to react to variations from the nominal schedule, which allows to adapt energy production
and energy demand to update parameters.

The proposed approach has been used to solve a case study of a microgrid and also it could be used
as the basis for solving further problems with higher complexity by combining a reactive and
proactive technique to take into account the presence of uncertainty.

Nomenclature

Indices and sets

- $i \in I$: Energy production generators
- $j \in J$: Energy consumers
- $f \in F_j$: Energy demand included in the overall scheduling horizon
- $f \in F_j^{RH}$: Energy demand included in the current prediction horizon
- $k \in K$: Energy storage systems
- $r \in R$: Power grids
- $t \in T$: Time intervals included in the overall scheduling horizon
- $t \in T^{RH}$: Time intervals included in the current prediction horizon

Parameters

- $CH$: Length of the control horizon [h]
- $C_{Cons_{j,f}}$: Individual energy consumption of each energy consumption task $j$ in [kWh]
- $pen_{j,f,t}$: Penalty cost [m.u./time]
- $pro_{i,t}$: Production energy cost [m.u./kWh]
- $csto_{k,t}$: Storage energy cost [m.u./kWh]
- $DT$: Span of the time interval [h]
- $Dur_{j,f}$: Remaining time consuming of consumption $j$ in the current prediction horizon [h]
- $Dur_{0,j,f}$: Time consuming of energy consumption $j$ in the overall scheduling horizon [h]
- $it$: Iteration
- $p_{i,t}^{\min}$: Minimum power supply of energy generator $i$ at interval $t$ [kW]
- $p_{i,t}^{\max}$: Maximum power supply of energy generator $i$ at interval $t$ [kW]
- $PH$: Length of the prediction horizon [h]
- $Price_{r,t}$: Energy price to be sold to power grid $r$ at interval $t$ [m.u./kWh]
- $SE_{k,t}^{\min}$: Minimum electricity storage of system $k$ at interval $t$ [kWh]
- $SE_{k,t}^{\max}$: Maximum electricity storage of system $k$ at interval $t$ [kWh]
- $SE_{0k,t}$: Initial storage level of system $k$ at interval $t$ [kWh]
- $SH$: Length of the scheduling horizon [h]
- $Ts_{j,f}^{\max}$: Maximum initial time of consumption $j$ in the current prediction horizon [h]
- $Ts_{j,f}^{\min}$: Target initial time of consumption $j$ in the current prediction horizon [h]
$T_{s_{0, jf}}^{\text{max}}$ Maximum initial time of consumption $j f$ in the overall scheduling horizon [h]

$T_{s_{0, jf}}^{\text{min}}$ Target initial time of consumption $j f$ in the overall scheduling horizon [h]

$r_{k}^{n}$ Charging efficiency of energy storage system $k$

$r_{k}^{u t}$ Discharging efficiency of energy storage system $k$

**Continuous variables**

- **Benefit** Microgrid benefit [m.u.]
- **Cost Pen** Total penalty cost [m.u.]
- **Cost Pro** Total energy production cost [m.u.]
- **Cost Sto** Total storage cost [m.u.]
- **Cost s** Total operating cost of the microgrid [m.u.]
- **Dem}_{t}$ Total energy consumption at interval $t$ [kWh]
- **Incomes** Microgrid incomes [m.u.]
- **Ld}_{k,t}$ Energy supplied to load system $k$ during interval $t$ [kWh]
- **P}_{i,t}$ Power supply of energy generator $i$ at interval $t$ [kW]
- **P}_{g,r,t}$ Power supplied to power grid $r$ at interval $t$ [kW]
- **Profit** Total profit along the time horizon (objective function) [m.u.]
- **P}_{t}$ Total power supply at interval $t$ [kW]
- **SE}_{k,t}$ Electricity storage level of system $k$ at the end of the interval $t$ [kWh]
- **SE}_{k,t}$ Linking variable determining the storage level of energy storage system $k$ at the end of interval $t$ in the current prediction horizon [kWh]
- **SP}_{k,t}$ Energy supplied by storage system $k$ during interval $t$ [kWh]
- **Tf}_{j,f}$ Final time of each consumption $j f$ [h]
- **Ts}_{j,f}$ Initial time of each consumption $j f$ [h]
- **T}_{t}$ Time corresponding to time interval $t$ [h]

**Binary variables**

- $X_{i,t}$ = 1, if energy generator $i$ is used at interval $t$
- $Y_{j,f,t}$ = 1, if consumption $j f$ starts at interval $t$ during the current prediction horizon
- $Y_{j,f,t}$ = 1, if consumption $j f$ starts outside the current prediction horizon
- $Z_{j,f,t}$ = 1, if consumption $j f$ finishes at interval $t$ during the current prediction horizon
- $Z_{j,f,t}$ = 1, if consumption $j f$ finishes outside the current prediction horizon
- $W_{j,f,t}$ = 1, if consumption $j f$ is active at interval $t$ during the current prediction horizon

434

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436
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References


Appendix A

Table A1. Input parameters.

Table A2. Maximum availability of power for each energy generator $i$ at interval $t$. 
Fig. 1. Concepts associated to rolling horizon approach: scheduling, production and control horizon.
Fig. 2. Reactive scheduling via a rolling horizon framework (Kopanos and Pistikopoulos, 2014).
Initialization step
Define the initial state of the system, the total scheduling horizon length (SH), the control horizon (CH). For the given prediction horizon (PH), calculate the total number of iterations (tot).

Update step
Update the uncertain parameters (e.g. energy demands), and the current state of the system.

Calculate solution step
Calculate the variable values by solving the deterministic problem for the PH.

Save solution step
Save the values of the variables for the predefined CH.

iter ≤ tot

① iter = iter + 1

no

END

Fig. 3. Algorithm for the rolling horizon approach.
Fig. 4. Power availability.
Fig. 5. Daily schedule for energy production, consumption and storage ($PH=20$).
Fig. 6. Evolution of the energy consumption scheduling for PH=20 in iterations 54, 55, 56, 57 and final iteration.
Fig. 7. Power source in each time interval for different $PH$ lengths.
Fig. 8. Profit for different $PH$s, perfect information and no demand management case.
Fig. 9. Aggregated profit for different PHs, perfect information and no demand management case.
Fig. 10. Energy sales for different PHs, perfect information and no demand management case.
Fig. 11. Schedule of energy production with no sales to the power grid ($PH=20$).
a) Nominal conditions

![Nominal conditions graph]

b) Unexpected scenario

![Unexpected scenario graph]

Fig. 12. Power availability and target schedule for the nominal conditions scenario and the unexpected scenario.
<table>
<thead>
<tr>
<th>Unit</th>
<th>Description</th>
<th>Cost (m.u./ kWh)</th>
<th>$p_i^{\min}$ (kW)</th>
<th>$SE_k^{\min}$ (kWh)</th>
<th>$SE_k^{\max}$ (kWh)</th>
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<tbody>
<tr>
<td>i1</td>
<td>Photovoltaic panel</td>
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<td>-</td>
</tr>
<tr>
<td>i2</td>
<td>Wind turbine</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>i3</td>
<td>Power grid</td>
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<td>0</td>
<td>-</td>
<td>-</td>
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<td>k1</td>
<td>Energy storage system</td>
<td>$1 \cdot 10^{-6}$</td>
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<td>13.44</td>
<td>16.80</td>
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Table 2. Comparison of the obtained results for the scenario 1 (demand side management).

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<th>PH=20</th>
<th>PH=30</th>
<th>PH=96</th>
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<td>359.0</td>
<td>359.0</td>
<td>359.0</td>
<td>359.0</td>
</tr>
<tr>
<td>Total delays (h)</td>
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<td>12.75</td>
<td>12.50</td>
<td>11.75</td>
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<td>397.8</td>
<td>394.5</td>
<td>391.9</td>
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<td>112.7</td>
<td>112.7</td>
<td>112.7</td>
<td>112.7</td>
</tr>
<tr>
<td>Energy from wind turbines (kWh)</td>
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<td>279.2</td>
<td>279.2</td>
<td>279.2</td>
<td>279.2</td>
</tr>
<tr>
<td>Energy from power grid (kWh)</td>
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<td>6.1</td>
<td>5.9</td>
<td>2.6</td>
<td>0.0</td>
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<td>Energy from energy storage systems (kWh)</td>
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<td>16.9</td>
<td>18.0</td>
<td>19.3</td>
</tr>
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<td>Energy sold to the power grid (kWh)</td>
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<td>35.3</td>
<td>35.3</td>
<td>31.7</td>
<td>28.5</td>
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<td>Energy to load the storage system (kWh)</td>
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<td>21.6</td>
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<td>0.3</td>
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Table 3. Comparison of the obtained results for the scenario 2 (non-demand side management).

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<td>359.0</td>
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<tr>
<td>Total delays (h)</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<tr>
<td>Energy produced or purchased (kWh)</td>
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<td>419.0</td>
<td>419.0</td>
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</tr>
<tr>
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<td>112.7</td>
<td>112.7</td>
<td>112.7</td>
<td>112.7</td>
</tr>
<tr>
<td>Energy from wind turbines (kWh)</td>
<td>279.2</td>
<td>279.2</td>
<td>279.2</td>
<td>279.2</td>
<td>279.2</td>
</tr>
<tr>
<td>Energy from power grid (kWh)</td>
<td>28.4</td>
<td>27.0</td>
<td>27.0</td>
<td>27.0</td>
<td>27.0</td>
</tr>
<tr>
<td>Energy from energy storage systems (kWh)</td>
<td>18.2</td>
<td>19.5</td>
<td>19.5</td>
<td>19.5</td>
<td>19.5</td>
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<td>57.2</td>
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<td>55.5</td>
<td>55.5</td>
<td>55.5</td>
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<td>24.1</td>
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<td>24.1</td>
<td>24.1</td>
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<td>Energy loses (kWh)</td>
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<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
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Table 4. Comparison of optimal solution and model statistics.

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<th>PH=20</th>
<th>PH=30</th>
<th>PH=96</th>
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<tr>
<td>Number of iterations</td>
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<td>87</td>
<td>77</td>
<td>67</td>
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<td>Equations per iteration</td>
<td>481</td>
<td>1,216</td>
<td>3,726</td>
<td>8,016</td>
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<tr>
<td>Continuous variables per iteration</td>
<td>360</td>
<td>855</td>
<td>2,424</td>
<td>5,004</td>
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<td>Discrete variables per iteration</td>
<td>80</td>
<td>260</td>
<td>1,001</td>
<td>2,411</td>
<td>33,468</td>
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<tr>
<td>Computational time per iteration (CPU, s)</td>
<td>0.1</td>
<td>0.3</td>
<td>0.3</td>
<td>0.4</td>
<td>0.8</td>
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<tr>
<td>Relative gap (%)</td>
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<td>0.0</td>
<td>0.0</td>
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Table 5. Comparison of the obtained results for the demand side management for the unexpected scenario.

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<th>Unit</th>
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<tr>
<td>Profit (m.u.)</td>
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<td>Consumed energy (kWh)</td>
<td>368.6</td>
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<tr>
<td>Total delays (h)</td>
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<td>Energy produced or purchased (kWh)</td>
<td>405.0</td>
<td>399.4</td>
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<td>Energy from photovoltaic panels (kWh)</td>
<td>112.7</td>
<td>112.7</td>
</tr>
<tr>
<td>Energy from wind turbines (kWh)</td>
<td>269.2</td>
<td>269.2</td>
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<tr>
<td>Energy from power grid (kWh)</td>
<td>23.0</td>
<td>17.5</td>
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<td>Energy from energy storage systems (kWh)</td>
<td>16.9</td>
<td>19.3</td>
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<tr>
<td>Energy sold to the power grid (kWh)</td>
<td>34.2</td>
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<td>Energy to load the storage system and energy loses (kWh)</td>
<td>19.0</td>
<td>23.1</td>
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</table>
## Appendix A

Table A1. Input parameters.

<table>
<thead>
<tr>
<th>Energy consumer $j$</th>
<th>Energy demand $f$</th>
<th>Power (kW)</th>
<th>$T_{s_{j,f}}^{\min}$ (h)</th>
<th>$Dur_{j,f}$ (h)</th>
<th>$T_{s_{j,f}}^{\max}$ (h)</th>
<th>Penalty cost (m.u./h)</th>
</tr>
</thead>
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<tr>
<td>j1</td>
<td>f1</td>
<td>2.557</td>
<td>0.00</td>
<td>0.250</td>
<td>0.00</td>
<td>40</td>
</tr>
<tr>
<td>j1</td>
<td>f2</td>
<td>1.012</td>
<td>0.25</td>
<td>0.250</td>
<td>0.25</td>
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<tr>
<td>j1</td>
<td>f3</td>
<td>0.957</td>
<td>0.50</td>
<td>0.250</td>
<td>0.50</td>
<td>40</td>
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<tr>
<td>j1</td>
<td>f4</td>
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<td>0.75</td>
<td>0.250</td>
<td>0.75</td>
<td>40</td>
</tr>
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<td>j1</td>
<td>f5</td>
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<td>1.00</td>
<td>0.250</td>
<td>1.00</td>
<td>40</td>
</tr>
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<td>j1</td>
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<td>j1</td>
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<td>j1</td>
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<td>0.250</td>
<td>4.50</td>
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<td>4.75</td>
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<td></td>
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<td>5.00</td>
<td>0.250</td>
<td>5.00</td>
<td>40</td>
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Table A2. Maximum availability of power for each energy generator $i$ at interval $t$.

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