Analysis and dynamical modeling of a piston valve for a wave energy converter

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Abstract

The Ocean Grazer, a novel wave energy converter, has been proposed by the University of Groningen. The system can collect and store multiple forms of ocean energy, with a piston-type hydraulic pump as its core technology.

In this work, the dynamical behavior of a piston valve for use in the piston pump system is studied. In order to gain insight into the dynamical behavior of the piston-type hydraulic pump, a simulation model is developed to describe the movement of the piston valve and the consequent forces acting on the piston. The model accounts for rigid contacts between the valve and the piston, as well as hydrodynamic forces created by the pumped fluid.
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INTRODUCTION

1.1. Renewable energies

1.1.1. Introduction
Considering the human history, the development based on fossil fuels is a recent phenomena, initiated around 200 years ago. At the end of XVIIIth century the first steam machines powered by coal appeared, but it was not until the industrial revolution when the use of fossil fuels became widespread, during the second half of XXth century.

Nowadays, it is usual to talk about renewable and nonrenewable energies as equivalent alternatives. Choosing between them is still an option, and most of the people do not really care about which ones are using. Actually, fuel fossils logistics are easier and much more comfortable; this is because this kind of energies can be accumulated and used whenever they are needed, while renewable energies always depend on the natural cycles and unstable conditions.

![Figure 1.1: Nonrenewable world energy consumption 1751-2011 (GWh)](image)

Figure 1.1: Nonrenewable world energy consumption 1751-2011 [1].
Even though the last generations have been familiarized with the use of fuel fossils, this kind of energy has not always been available. Actually, humanity had been living without nonrenewable energies until the second half of XVIIIth century, as it can be observed in figure 1.1.

The increasing use of coal during the XIXth century, due to the implementation of steam machines, was the first big change into nonrenewable energies. At the beginning of XXth century, the coal standstill opened the way for oil and, far lower, natural gas use. With the economic expansion after the World War II, the use of oil became more common, overtaking the coal use in the first 1960s. The consumption of fuel fossils kept increasing during those decades, reaching its top participation in the worldwide energy mix on 1970s, with an 85,6% contribution. Since then, the consumption of nonrenewable energies lead by oil, coal and natural gas has been increasing only retained by some sporadic political or economical crisis.

In 1973, the first oil crisis gave the base to reconsider the renewable energy as an alternative. Environmentalists promoted its development, and the first electricity generating wind turbines were developed.

1.1.2. Resources and reserves
When talking about energy consumption, it is important to distinguish between resources and reserves. Resources are the amount of fossil organic matter existing in the world, as a result of sedimentation and transformation processes of some animal and vegetal rests. On the other hand, reserves are the fraction of these resources that is possible to extract, taking into account both technical and economical aspects.

Most discoveries of crude oil were between 1925 and 1975, as shown in figure 1.2. Around 1980, the discovery rhythm of new reserves became lower than its consumption. This fact announced the beginning of the decline process, when the reserves could not cover the consumption, yielding to its extinction. The graphic below estimates that the oil zenith was produced around 2010, with 70 Mb/d (million barrels per day).

![Figure 1.2: Annual world production and discovery forecast of crude oil (a) and natural gas (b) [2].](image-url)
Natural gas behavior is similar, with some years lag. Its consumption overtook the discovery rate around 1990, and the zenith forecast is around 2020. Moreover, other estimations with coal and uranium show similar behaviors.

Based on EIA (Energy Information Administration) data, the nonrenewable energy reserves all over the world in 2007 were:

<table>
<thead>
<tr>
<th>Nonrenewable energy reserves in 2007 (GWy)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil</td>
</tr>
<tr>
<td>258.600</td>
</tr>
<tr>
<td>23,0%</td>
</tr>
</tbody>
</table>

Table 1.1: Nonrenewable energy reserves in 2007 [1].

1.1.3. Nonrenewable reserves depletion

Taking into account the reserves values shown above, altogether with 2008 consumption data and a worldwide consumption rise of 1.3% per year, the total depletion of nonrenewable reserves should be around 2060 [1]. Assuming that one kind of energy covers the consumption of another kind when it is finished, the graphic below shows a forecast of the depletion of the different fuel fossils, also including uranium.

![Nonrenewable reserves depletion forecast](image)

Figure 1.3: Progressive deployment of nonrenewable reserves [1].

Actually, from 2009 to 2012 the annual rise of worldwide energy consumption reached a value of 2.3%, one point over the assumption in the graphic. Therefore, if the evolution of the tendency remains similar, the depletion forecast would be even more pessimistic.

1.1.4. Alternative solutions

According to [3] the solution remains on a transition to the renewable energies.

One of the most used arguments to keep producing energy with fossil fuels is that renewable energies are not big enough to produce all the energy needed. In fact, the theoretically
exploitable energy potential combining wind, biomass, geothermal, hydraulic and marine sources is around 140 TW/y (Terawatts per year); while the consumption all over the world in 2012 was around 18 TW/y and the forecast consumption for 2050 is around 28 TW/y. Therefore, renewable resources should be more than enough.

The transition to renewable energies certainly requires a deep technology transformation as well as new infrastructure and consumption habits, but it is completely possible and economical and environmentally advantageous.

1.2. Ocean Grazer

1.2.1. Ocean Grazer

With the aim to extract energy from the sea, some different devices have been patented in recent years, using all kind of design principles, such as tidal current converters or wave energy converters [4]. Regarding this last field, there are some devices that are being commercially used nowadays. However, most of them focus to maximize the system efficiency for very narrow band of wave heights, while ocean waves have a much higher variability and, therefore, are not very efficient [5].

The University of Groningen also started its own project some years ago, conceiving a novel semi-submersible device called Ocean Grazer. The Ocean Grazer project is an innovative long-term project that requires a lot of research and funds for its realization, with academic institutions, research centers and several enterprises working altogether as research partners.

This consortium, established at the end of 2013, is in its very first steps, and most part of the work is being done in the University of Groningen, offering students a possibility for research.

Unlike most investigations in previous wave energy converters (WEC), the Ocean Grazer’s WEC gives a significant role to the adaptability to the extreme variability of ocean waves. Also the energy storage system plays an important role, so this way the energy can be produced when it is needed.

The natural kinetic energy in the sea waves is converted into electrical energy using different power take-off systems. The core technology of the Ocean Grazer is the multi-pump, multi-piston power take-off (MP\textsuperscript{2}PTO), which contributes about 80% of the power generation. Secondary technologies such as oscillating water column, wind turbines and solar energy systems will contribute the rest [5].

A single Ocean Grazer device is projected to produce more than 200 GWh/year and to have a storage capacity of about 800 MWh [5].

As shown in Figure 1.4, the device consists on a large concrete structure with a total height of 220 meters and a diameter of 350 meters, with two reservoirs situated below the water surface (one above the other) connected by a pumping system. This pumping system is actuated by the movement of the buoys floating on the sea surface.
1.2.2. Multi-pump multi-piston power take-off

As shown in figure 1.5, the MP$^2$PTO consists of multiple interconnected string of buoys, termed as floater blanket. Each of these buoys (B$_i$) is connected to a hydraulic multi-piston pump (P$_i$). The first pump unit can potentially extract more energy than the second one, and so on.

The operating principle of the MP$^2$PTO is to create pressure difference, also called hydraulic head (H), in the working fluid circulating between two reservoirs. The working fluid can be stored as lossless potential energy in the upper reservoir, and transformed into electricity via a turbine (T) when the fluid returns to the lower reservoir.

The main achievement of the Ocean Grazer’s WEC is the adaptability to the inherent variability that sea waves have in height and period. Multiple pistons can be activated within each pump in the MP$^2$PTO system to maximize the energy extraction for waves ranging in height from 1 to 12 meters and periods from 4 to 20 seconds [6].

![Figure 1.5: The MP$^2$PTO system (a) and multi-piston pump (b).](image)
As it can be observed in figure 1.5 (b), controlling the coupling between the buoy (B) and the variable-size pistons (P) can optimize the load the buoy has to carry during the upstroke and let uncoupled piston sink due to its own weight.

1.2.3. Current work
As mentioned before, Ocean Grazer project is still on a research phase, and most of these research is being performed at the University of Groningen. At the same time this thesis is being developed, some other projects are in progress, e.g. the design of a check valve for the lower reservoir, the development of the geared transmission system for the piston, the design of the piston-type hydraulic pump, the modeling of the floater blanket or the study of the rod connecting the buoy and the piston.

Also a full-scale prototype of the multi-piston pump system has been built in order to validate the various mathematical models and to measure the performance of the different piston and check valve designs.

1.3. Contributions of the thesis
As commented above, the development of this concept of WEC is in initial stage. The concept of the multi-pump multi-piston power take-off has been described in [7], and a single-piston pump model has been developed in [6]. In order to gain insight on the development of the system, the main goals of this thesis are:

- Analyze the piston valve behavior: Determine and describe the different forces that are involved in the valve movement.
- Reach a dynamical model that describes the movement of the piston valve.
- Determine the reactions acting on the piston during the different configurations of its movement, based on the dynamic response of the valve.

All these contributions will help in order to understand better the dynamical behavior of the piston and its interaction with the working fluid.

1.4. Methodology used

With the aim of achieving the goals described in section 1.3 different tools will be used. Research through literature will be done in order to describe the dynamical system with analytical calculations. If analytical calculations are not found in literature, e.g. in the description of the hydrodynamic force, numerical analysis will be employed using COMSOL software. Finally, simulations will be performed using the software MATLAB SIMULINK, in order to solve the equations of the system and go into the piston valve behavior.
Chapter 2

CONTEXT

2.1. Single piston pump dynamical model

A first dynamical model for the single piston pump was developed in [8] and later improved in [9].

In the system described in [9], the single piston pump is modeled with dynamical elements as follows:

The buoy, with mass $m_1$, is modeled as a rectangular prism which is connected to the piston by a rigid steel rod. The rod is tensioned during the upstroke movement, while the tension decreases during the downstroke. This rod is modeled as a spring with stiffness $K$, combined with a linear damper $C$, that takes into account the friction force at the piston-cylinder interface, the energy losses between the rod and the water, the internal friction of the rod and the frictional contact with secondary interfaces.

The piston is modeled as a mass $m_2$. In order to include the mass of the rod and the mass of the fluid pumped during the upstroke this mass $m_2$ is defined as follows:

$$ m_2 = \begin{cases} m_r + m_p + m_f & \text{in the upstroke}, \\ m_r + m_p & \text{in the downstroke}, \end{cases} \quad (2.1) $$
where \( m_r \) is the mass of the rod, \( m_p \) is the mass of the piston and \( m_f \) is the mass of the pumped fluid.

The dynamical model that describes the movement of the system is described by the following space-state equation:

\[
\dot{q} = Aq + f, \quad q(0) = q_0, \tag{2.2}
\]

where the state vector \( q \) is given by

\[
q = [x_b \quad \dot{x}_b \quad x_p \quad \dot{x}_p \quad p_1 \quad p_4]^T, \tag{2.3}
\]

with \( x_b \) being the position of the buoy’s centre of mass in \( m \), \( \dot{x}_b \) being the velocity of the buoy in \( m/s \), \( x_p \) being the position of the piston’s centre of mass in \( m \), \( \dot{x}_p \) being the velocity of the piston in \( m/s \) and \( p_1 \) and \( p_4 \) being the pressures in \( Pa \) on the upper and lower reservoirs, respectively.

The matrix \( A \) corresponding to the upstroke movement is

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
-K/m_1 & -C/m_1 & K/m_1 & C/m_1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
K/m_2 & C/m_2 & -K/m_2 & -C/m_2 & -A_c/m_2 & A_c/m_2 \\
0 & 0 & 0 & \rho g A_c/A_U & 0 & 0 \\
0 & 0 & 0 & -\rho g A_c/A_L & 0 & 0 \\
\end{bmatrix}. \tag{2.4}
\]

And the force vector \( f \) is

\[
f = \begin{bmatrix}
F_b \\
\frac{F_f}{m_1} - g + \frac{K L_r}{m_1} \\
0 \\
-\frac{F_f}{m_2} - g - \frac{K L_r}{m_2} \\
0 \\
0 \\
\end{bmatrix}. \tag{2.5}
\]

During the downstroke movement, the check valves of the cylinder are closed and the flow becomes null. In this case, the pressure from both reservoirs remains constant, \( \dot{p}_1 = \dot{p}_4 = 0 \), and the matrix \( A \) is defined by

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
-K/m_1 & -C/m_1 & K/m_1 & C/m_1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
K/m_2 & C/m_2 & -K/m_2 & -C/m_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}. \tag{2.6}
\]
and the force vector remains the same as in the upstroke movement.

This model is also implemented in [6], where the description of the pumping force acting on the piston is made as follows:

As represented on the free body diagrams of the piston (figure 2.2), the pumping force is acting during the upstroke movement, while is equal to zero during the downstroke.

![Free body diagrams of the piston in the upstroke (a) and downstroke (b).](image)

When the device is pumping water to the upper reservoir, the pumping force expression is described by

\[
F_p = -A_c p_A + \rho (L_c + L_u) A_c (\dot{Z}_p + g) + \rho A_c Z_p^2. \tag{2.7}
\]

This pumping force is plotted in figure 2.3 (c), where higher-frequency oscillations can be observed in the acceleration of the piston, due to the effect of the fluid column mass that goes from zero to several thousand at the instances of switching.

![Piston displacement and velocity (a), acceleration (b) and pumping force (c) [6.](image)
Introducing into the model a flow-dependent function for $F_p$ that takes into account the dynamic response of the piston valve could remove these discontinuities at the top- and bottom-dead-centers of the piston oscillation.

2.2. Piston design

Some different piston designs have been developed based in the first schematics included in [7].

The first design, which can be observed in figure 2.4 (a), is based in a cylindrical structure, that allows the working fluid circulation through its interior section. The rod is connected to the piston by a triangular structure screwed in the cylindrical surface. The flaps are attached in a central beam, and are supported by the cylindrical surface when the piston is pumping the fluid upwards.

The second design, figure 2.4 (b), removes the entire circumference to minimize the contact area between the piston and the cylinder, aiming to reduce the friction on this interface. This design has a central beam connected to the rod, from which other beams protrude to support the flaps during the upstroke movement.

Both designs have been proved to work in the experimental set up. However, in the present day some research is being done in order to improve the design, studying the most efficient geometries and materials to assure a proper movement of the piston, as well as minimizing the leakage and improving the cylinder-piston interface lubrication.

Although the piston structure changes in each design, the piston valve remains faithful to the same concept.
2.3. Piston valve

The main goal of the piston is to hoist the working fluid from the lower to the upper reservoir. This liquid is moved upwards by the piston valve, also referred as flaps. When the piston valve is closed, the working fluid above the flaps is pushed to the upper reservoir, while additional liquid is sucked out of the lower reservoir.

During the downstroke of the piston, the fluid volumes in both reservoirs are constant so the working fluid column remains motionless. The piston valve opens to allow the piston sink through the column, until it reaches its bottom dead center.

2.3.1. Dual Disc Check Valves

The concept of the piston valve used in the Ocean Grazer is similar to the implemented by the Double-Disc Swing Check Valve [10,11]. This type of valves are commonly used in pumping installations, where the check valves are located inside a pipe to prevent backflow.

The dual disc check valves employ two spring-loaded plates hinged on a central hinge pin. For a certain flow, the plates remain open due to the fluid force. When the flow decreases, the torsion spring action makes the plates close without requiring reverse flow. The following figure shows different configurations of these type of valves:

![Figure 2.5: Dual Disc Check Valve configurations [12].](image)

Although the mechanical concept is really similar, there are some big differences with the Ocean Grazer piston valve.

On one hand, the Dual Disc Check Valve is used to prevent backflow. This means that the valve is located in a fixed position in the pipe, and takes a passive role in the fluid movement. Moreover, the fluid is moving as a consequence of an external actuator, e.g. a pumping engine. In this case, the valve is open for a certain flow, while the plates are closed when the fluid is motionless. The interaction between the moving fluid and the open check valve introduces a pressure loss in the pumping system, described by

13
\[ \Delta p = \frac{1}{K_v^2} \Delta p_{kv} \frac{\rho}{\rho_w} Q^2. \]  

(2.8)

Where \( K_v \) is the valve rating which depends on the shape and the size of the valve, \( \Delta p_{kv} \) is the static pressure loss, \( \rho \) is the density of the fluid, \( \rho_w \) is the water density and \( Q \) is the flow rate through the valve [13].

On the other hand, the piston valve of the Ocean Grazer is located on the piston, assuming an active role in the fluid movement, as it is used to pump the working fluid. In this case, the flaps are closed when the system is pumping fluid, and they open when the fluid becomes motionless and the piston sinks.

Therefore, the pressure loss that appears in the first case does not influence in the second one, but is reconsidered as a sinking resistance. Nevertheless, the relative velocity between the valve and the fluid is the same in both cases.

### 2.3.2. Flap characteristics

As commented above, the piston valve is composed by two semicircular flaps attached to a central beam of the piston. Since some research is still being done in order to improve the piston geometry, the connection between the flaps and the piston is not clearly defined yet. In this thesis, this connection is assumed to be a central hinge pin. The radius of the flaps corresponds to the radius of the piston \( R_p \), which is given by the dimension of the cylinder \( R_{cyl} \), and the separation between the cylinder and the piston \( s \):

\[ R_{flap} = R_{cyl} - s. \]  

(2.9)

The thickness of the flap, named as \( t_{flap} \), will depend on the material as well as on the piston design. Due to its semicircular geometry, the volume of one flap is described as

\[ V_{flap} = \frac{\pi R_{flap}^2}{2} t_{flap}. \]  

(2.10)

The piston valve operating in the Ocean Grazer is assumed to be made of aluminum, with a density around 2700 kg/m\(^3\). Therefore, the mass of one flap is defined by

\[ m_{flap} = V_{flap} \rho_{aluminum}. \]  

(2.11)

Although spring-loaded plates are commonly used in Dual Disc Check Valves, no springs are considered in the Ocean Grazer design, in order to prevent eventual failure or maintenance costs. Further research could be done in this field to determine if the use of springs in the piston valve design would introduce any efficiency improvement on the pumping system, and if this improvement would be more significant than the subsequent costs.
2.4. First movement estimations

A first study has been done in [14] in order to determine the angle of the valve for a given flow speed. It studies the smallest piston configuration to be used in the full scale Ocean Grazer, with a cylinder radius of 20 cm. The model is based on a stationary laminar formulation, considering different fixed positions for the flap and different flow speeds. The cylinder is modeled as a cylindrical stationary wall with velocity equal to zero.

The results of the model show the vertical hydrodynamic force acting on the valve. This force, shown in figure 2.6, increases significantly for higher flow speeds and nearly closed valve positions.

![Total Vertical Force Acting on Valve](image)

**Figure 2.6: Vertical force acting on the piston valve [14].**

Comparing this hydrodynamic force with the vertical force given by the weight of the valve, a stable angle can be determined depending on the flow speed.

![Valve Angle for Different Flow Velocities](image)

**Figure 2.7: Valve angle Vs. Flow Velocity [14].**
Chapter 3

FLAP MODELING

3.1. Introduction

The main goal of this thesis is to study and describe the movement of the piston valve. In order to achieve this goal, a dynamical model will be created, including all the forces acting on the flaps. Since the piston valve, as well as all the piston geometry, presents symmetric properties, this model will focus on the movement of one of the flaps.

This flap consists of a semicircular plate, connected to the piston by a hinge. Since the hinge is considered to be fixed in the piston geometry, there is only one degree of freedom between the piston and the flap, named as $\theta$. This way, the flap is allowed to rotate around the hinge, but it cannot move in other directions.

In order to simplify the future calculations, a reference point (O) is located in the symmetrical plane of the flap, at the center of the hinge. This reference point will be considered as the origin of coordinates. Moreover, all the forces acting on the flap will be applied on the center of mass (COM) of the object. Considering the half-disc geometry of the flap and the axes described in figure 3.1, this point is located at
\[
\begin{bmatrix}
X_{COM} \\
Y_{COM} \\
Z_{COM}
\end{bmatrix}
= \begin{bmatrix}
\frac{4R_{flap}}{3\pi} \cos \theta \\
\frac{4R_{flap}}{3\pi} \sin \theta \\
0
\end{bmatrix},
\] (3.1)

or, in polar coordinates

\[
\tau_{COM} = \frac{4R_{flap}}{3\pi}.
\] (3.2)

Two different connections can be established between the piston geometry and the flap. The first one, commented above, is a central hinge that allows the rotational movement. The second one, refers to the piston surfaces which stop the flap in its maximum, \(\theta_{\text{max}}\), or minimum, \(\theta_{\text{min}}\), position.

Therefore, the movement of the flap and the forces acting on it can be represented on a two-dimensional free body diagram, differentiating three configurations.

In figure 3.2 (a) and (c), a solid contact does not allow the flap to exceed its maximum or minimum angle, respectively. On the other hand, the moving configuration is represented in figure 3.2 (b), where the hinge is the only connection between the piston and the flap.

\section{3.2. Solving method}

In order to solve the dynamical behavior of the piston valve, both COMSOL and MATLAB software will be used. The SIMULINK model that is used to simulate the movement of the flap
is described in appendix B, along with detailed explanations in order to allow for potential further works.

The aim of this model is to determine the reactions on the piston, as well as the flap position, during all the piston-pump system movement. As the interaction between the buoy and the piston is not included in this thesis, the movement of the piston will be used as input. In this case, it is supposed that the piston follows the same movement of an incident wave with sinusoidal shape. Its displacement about a zero mean, velocity and acceleration defined as

\[ Z_p = \frac{H_w}{2} \sin \left( \frac{2\pi}{T_w} t + \frac{\pi}{2} \right), \]  

\[ \dot{Z}_p = \frac{\pi H_w}{T_w} \cos \left( \frac{2\pi}{T_w} t + \frac{\pi}{2} \right), \]  

\[ \ddot{Z}_p = -\frac{2\pi^2 H_w}{T_w^2} \sin \left( \frac{2\pi}{T_w} t + \frac{\pi}{2} \right), \]  

where \( H_w \) is the wave height and \( T_w \) is the wave period.

This way, all the movement of the piston through the cylinder will be modeled, and the results will be obtained for both upstroke and downstroke configurations.

A closed position of the flap is considered as initial condition, coinciding with the top dead-center of the piston, and the beginning of the downstroke movement.

The hydrodynamic force will be calculated, depending on the piston velocity and the flap position. Then, this force will be combined with all the other forces acting on the flap, obtaining its acceleration, \( \ddot{\theta} \), and the piston reactions. This acceleration will be used to determine the new position of the flap, \( \theta \), and, hence, a new distribution of the forces. This process will be repeated several times, with a time-step small enough to describe the dynamical behavior of the flap properly.

![Figure 3.3: Solving method diagram.](image-url)
3.3. Dynamical model description

As commented above, in the following system the movement of one flap will be studied in order to determine the reactions on the piston. All the forces involved in the flap’s dynamics are transmitted to the piston through the piston-flap connections. This connections are considered to be a hinge in the central point, as well as two surfaces modeled in the piston geometry which stop the flap in its maximum, \( \theta_{\text{max}} \), and minimum, \( \theta_{\text{min}} \), position.

Both vertical and horizontal reactions are described for the hinge connection. On one hand, the reaction on the \( x \) direction can be described as

\[
F_x = F_{\text{hydro}} \sin \theta .
\]  

(3.6)

For the full geometry model, the forces on the \( x \) direction acting on both flaps have to be considered. Taking into account the symmetrical properties of the full geometry model, this reaction becomes zero:

\[
F_{x_{\text{total}}} = F_{x_1} + F_{x_2} = 0 .
\]  

(3.7)

On the other hand, the reaction on the \( y \) direction is described as follows

\[
F_y = W - F_{\text{buoyancy}} - F_{\text{hydro}} \cos \theta .
\]  

(3.8)

Considering the forces acting on the two flaps of the full geometry model, the total force acting on the piston is

\[
F_{y_{\text{total}}} = F_{y_1} + F_{y_2} = 2F_y .
\]  

(3.9)

In future work, this vertical force acting on the piston could be implemented as the pumping force \( F_p \) in the single-piston pump model described in [6]. This way, the force acting on the flaps would be considered not only during the upstroke, but also when the piston is sinking.

For the connections that fix the position of the flap in its maximum and minimum configurations, the reaction is described as a torque located in the center of the hinge, yielding to

\[
T_{\text{piston}} = \begin{cases} 
(W - F_{\text{buoyancy}}) \cos \theta - F_{\text{hydro}} r_{\text{COM}} - T_{\text{hinge}} & \text{when } \theta = \theta_{\text{max}} \text{ or } \theta = 0 \\
0 & \text{otherwise.}
\end{cases}
\]  

(3.10)

As happens with the horizontal force \( F_{x_{\text{total}}} \), this torque becomes zero due to the symmetrical properties in the full geometry model.

\[
T_{\text{piston total}} = T_{\text{piston 1}} + T_{\text{piston 2}} = 0 .
\]  

(3.11)
Even though $F_{\text{total}}$ and $T_{\text{piston, total}}$ become zero when considering the action of both flaps in the full geometry model, this forces will play an important role in the piston-flap connection design.

In order to determine all these piston reactions, the dynamical model of one flap is described as follows. The central point of the hinge is taken as the reference point for the conservation of momentum, yielding to

$$I_x \ddot{\theta} = \left( (-W + F_{\text{buoyancy}}) \cos \theta + F_{\text{hydro}} \right) r_{\text{COM}} + T_{\text{hinge}} + T_{\text{piston}}. \quad (3.12)$$

$\ddot{\theta}$ is the angular acceleration of the flap, and $I_x$ is the inertia tensor, described as

$$I_x = \frac{1}{4} (m_{\text{flap}} + m_a) R_{\text{flap}}^2, \quad (3.13)$$

and $m_a$ is the added mass. This concept takes into account the inertia of the fluid surrounding the volume of the flap. This fluid will be accelerated with the movement of the flap since the object and the fluid cannot occupy the same physical space simultaneously. The added mass term can be described with an added mass coefficient $C_a$, as follows:

$$m_a = C_a \rho_{\text{water}} V_{\text{flap}}. \quad (3.14)$$

The velocity of the flap $\dot{\theta}$ is defined as the time integral of the acceleration $\ddot{\theta}$, and becomes zero when the flap reaches its maximum and minimum positions.

$$\dot{\theta} = \begin{cases} \int \ddot{\theta} \, dt, \\ 0 \text{ when } \dot{\theta} = 0 \text{ or } \theta = \theta_{\text{max}}. \end{cases} \quad (3.15)$$

This velocity $\dot{\theta}$ is also integrated in order to obtain the displacement of the flap.

$$\theta = \int \dot{\theta} \, dt. \quad (3.16)$$

$W$ is the weight of the flap, defined as

$$W = m_{\text{flap}} g. \quad (3.17)$$

$F_{\text{buoyancy}}$ is the buoyancy force. According to Archimedes' principle any immersed object experience a net force that tends to accelerate the object upwards. This force is equal to the magnitude of the weight of the fluid displaced by the body. In this case, as the flap will always be submerged in the working fluid, the volume of the fluid displaced is the entire volume of the flap. Then,

$$F_{\text{buoyancy}} = \rho_{\text{water}} g V_{\text{flap}}. \quad (3.18)$$
Assuming that the flap is connected to the piston by the central hinge, there is also a friction force, named as $F_{hinge}$, that should be taken into account. Moreover, if this connection is a point contact, the friction force can be described as a normal force times the friction coefficient:

$$F_{hinge} = N \mu_{\text{steel-aluminium}}, \quad (3.19)$$

where $N$ is the module of the reactions on the piston

$$N = \sqrt{F_x^2 + F_y^2}. \quad (3.20)$$

This force can also be described as a torque in the reference point,

$$T_{hinge} = \frac{t_{flap}}{2} \sqrt{F_x^2 + F_y^2} \mu_{\text{steel-aluminium}}, \quad (3.21)$$

where $t_{flap}$ is the thickness of the flap, as commented in section 2.3.2.

The hydrodynamic force $F_{\text{hydro}}$, however, is more difficult to describe. Even though there are some analytical calculations to describe the force that a fluid does on some basic geometries, they cannot be applied in this case. Therefore, the interaction between the fluid and the flap requires a deeper study, which will be made in chapter 4.
Chapter 4

FLUID-FLAP INTERACTION

4.1. Introduction

The movement of the piston valve is mainly produced by the interaction between the working fluid and the flaps. When the piston is sinking through the fluid column, the flaps open as a consequence of the hydrodynamic forces acting on the piston. This hydrodynamic forces are also opposing the relative movement of the piston, due to the resistance of the fluid to shear. The weight and inertia of the fluid also plays an important role during the upstroke, keeping the flaps closed and preventing fluid leakage through the piston.

As commented in section 2.1, the pumping force acting on the piston during the upstroke movement is defined in [6] as

\[ F_p = \rho A_c \bar{z}_p^2 - A_c p_4 + \rho (L_c + L_U) A_c (\bar{z}_p + g) \]. \hspace{1cm} (4.1)

Dividing the equation in two main terms, it could be said that the first term relates to the imparting of momentum to the stationary fluid surrounding the flap, and the second term takes into account the inertia and weight of the fluid column.

In order to study the hydrodynamic force acting on the piston valve during all the configurations of the piston movement, a similar concept is applied: The hydrodynamic force is described as a combination of pressure difference acting on the flaps, and fluid inertial terms, so that

\[ F_{\text{hydro}} = F_{\text{pressure}} + F_{pf} \cos \theta \]. \hspace{1cm} (4.2)

Since analytical calculations and experimental data are only available for simplified bodies, the pressure gradient acting on the flaps is studied using numerical analysis.

Both components of the hydrodynamic force, \( F_{\text{pressure}} \) and \( F_{pf} \), are described in sections 4.7 and 4.8, respectively.
4.2. Flow properties. Reynolds number

One of the first steps when modeling an hydrodynamic system is the characterization of the flow. It is possible to classify flows, but there is no general agreement on how to do it. In [15], a general classification is presented, saying that a given flow can be either

- Gas or liquid
- Inviscid or viscous
- Incompressible or compressible
- Steady or unsteady

In this case, the object of study is the working fluid contained in the cylinder, which interacts with the piston valve. This fluid is assumed to be liquid water at 20ºC [6]. Even though the effects of temperature are considered, the variation of temperature within the flow is assumed to be relative low. Therefore, incompressible flow conditions are applied. The fluid is also considered viscous, in order to include its interaction with the surfaces.

The Reynolds number is commonly used to determine if a flow is either laminar or turbulent. It can be calculated as

\[ Re = \frac{\rho UL}{\mu}, \]  

(4.3)

where \( \rho \) is the density of the fluid in \( \text{kg/m}^3 \), \( U \) refers to the velocity in \( \text{m/s} \), \( L \) is a characteristic linear dimension in \( \text{m} \) and \( \mu \) is the dynamic viscosity of the fluid in \( \text{Pa·s} \). Although critical Reynolds numbers are defined in literature for standard scenarios such as flow in ducts, these representative ranges vary somewhat with flow geometry, surface roughness and other parameters [15]. For uncommon geometries, such as the one studied in this thesis, the flow patterns are unknown, and can only be determined experimentally.

4.3. Computational requirements

The computational requirements are usually a difficult obstacle to overcome when solving numerical analysis models. More complicated models offer more accuracy in their results, but need higher computational resources. The main consideration in the COMSOL models used in this thesis is the availability of memory, which should be enough to store all the matrices created by the solver.

Actually, the computational resources available in this thesis can be considered as limited for complex calculations, such as fluid dynamics. The more complicated models require either longer time to solve or larger memory to converge.
The first attempts were to create a time dependent model which would take into account both fluid and structure interaction. However, due to the complexity of the models and the limited memory, the models did not converge in good results. So a simplified model was created in order to study the hydrodynamic forces.

4.4. Laminar flow interface

The laminar flow interface [16] is used to determine the velocity $u$ and pressure $p$ fields for the flow comprised in the model domain, assuming a single-phase fluid in a laminar flow regime. The laminar flow interface uses vectors for the velocity and pressure fields, altogether with the volume force vector $f$, to solve the Navier-Stokes equations for conservation of momentum,

$$\rho \left( \frac{du}{dt} + u + \nabla u \right) = -\nabla p + \mu \nabla^2 u + f, \quad (4.4)$$

and the continuity equation for conservation of mass

$$\frac{dp}{dt} + \nabla \cdot (pu) = 0. \quad (4.5)$$

Due to the working conditions of the Ocean Grazer, the variation of temperature within the working fluid is relatively low, so the flow is modeled as incompressible. The fluid for the entire domain is assumed to be water, with constant density $\rho$ and viscosity $\mu$. Taking into account the assumptions commented above, the continuity equation is defined as

$$\nabla \cdot (pu) = 0, \quad (4.6)$$

and the conservation of momentum

$$\rho (u \cdot \nabla) u = \nabla \cdot [-pI + \mu \left( \nabla u + (\nabla u)^T \right)] + F, \quad (4.7)$$

where $I$ is the identity matrix and $F$ is the volume force vector.

The laminar flow interface is also capable of solving turbulent flows, but the numerical solutions are usually unstable, so turbulent flow interfaces are recommend in those conditions. However, this turbulent interfaces, which are focused on solving fully developed turbulent flows, cannot visualize turbulence in its transient form. Therefore, the laminar flow interface will be used in the following models, and the cell Reynolds number will be taken as a reference to assure the stability of the solutions.
4.5. Model definition

4.5.1. Geometry and mesh
To determine the hydrodynamic forces acting on the piston valve, a simplified geometry of the piston is considered. It consists of a central squared-section beam that holds the two semicircular flaps of the valve in a certain position in the cylinder. Symmetrical properties can be observed in the entire geometry, so the full three-dimensional model can be reduced using appropriate boundary conditions. In order to reduce the amount of elements and degrees of freedom to solve in the model, only a quarter of the geometry is considered in the model.

Since any deformation of the flaps is not to be expected due to isothermal conditions and relatively low pressure differences, the central beam and the flaps are only modeled as a restriction to the fluid flow. The angle of the flap is varied between 90° to 5°, that corresponds to fully open and almost closed configurations, respectively.

As shown in figure 4.2, free tetrahedral mesh is used for the entire fluid domain, while a boundary layer mesh with coarser elements is chosen for the walls of the central beam.
The resultant mesh consists of 58291 domain elements. A more complex mesh with finer elements would lead to more accurate results, at the cost of introducing more elements and, therefore, an increased demand on computational resources. Since the model should not only be accurate, but also practical, this mesh is considered enough.

4.5.2. Boundary conditions

In order to explain the boundary conditions applied in the different surfaces, the model domain with labeled boundaries is displayed in figure 4.3.

Figure 4.2: Free tetrahedral meshed domain.

Figure 4.3: Model domain with labeled surfaces.
Boundaries 5, 6, 7, 8, 9 and 10 correspond to the flap and central beam surfaces. As stationary solid walls, the fluid is not moving on these surfaces, so the condition prescribes zero velocity:

\[
\mathbf{u} = 0. \quad (4.8)
\]

Boundary 1 corresponds to the cylinder wall. Since the model considers that the central beam and the flap are in a fixed position, this surface has a relative velocity. Therefore, the adjacent fluid must move at the same velocity of the wall, yielding to

\[
\mathbf{u} = \mathbf{u}_w. \quad (4.9)
\]

In reality, the cylinder wall is motionless, while the piston is moving up and downwards within the fluid column. During the upstroke, the fluid reaches the same velocity than the piston, as it is pumped to the upper reservoir. During the downstroke, the fluid remains motionless and the piston sinks to the bottom. In the model, the movement of the piston is translated to the inlet velocity of the fluid. Hence, the relative velocity between the cylinder, the fluid and the piston is the same as in reality, even though the piston has a fixed position. Therefore, the velocity of the wall is also the velocity of the inlet flow \(\phi\),

\[
\mathbf{u}_w = \phi. \quad (4.10)
\]

Boundaries 2, 11 and 12 correspond to the symmetry planes, used to reduce the full three-dimensional geometry. The boundary condition for symmetry boundaries prescribes no penetration and vanishing shear stresses. It uses a Neumann boundary condition for the velocity

\[
\mathbf{u} \cdot \mathbf{n} = 0, \quad (4.11)
\]

where \(\mathbf{n}\) is the unit vector normal to the surface where the boundary condition is applied.

It also uses a Dirichlet condition for the solution of the Navier-Stokes equation, simplified as

\[
\mathbf{K} - (\mathbf{K} \cdot \mathbf{n}) \mathbf{n} = 0, \quad (4.12)
\]

where

\[
\mathbf{K} = \mu (\nabla \mathbf{u} + (\nabla \mathbf{u})^T) \mathbf{n}. \quad (4.13)
\]

Two different models have been created in order to consider the upstroke and downstroke configurations. In the downstroke model, boundary 3 corresponds to the inlet, which is prescribed by a given flow velocity \(\phi\) as

\[
\mathbf{u} = -\phi \mathbf{n}. \quad (4.14)
\]
The flow speed $\phi$ is varied between 0.1 m/s and 1.2 m/s for every flap position, obtaining a velocity dependent flow distribution for every piston valve configuration.

In the same model, boundary 4 corresponds to the outlet, prescribed by a zero pressure condition with normal flow and suppressed backflow,

$$[-p] + \mu(\nabla u + (\nabla u)^T) = 0.$$ \hspace{1cm} (4.15)

In the upstroke model, boundary 3 corresponds to the outlet, while boundary 4 corresponds to the inlet. The mathematical conditions are the same explained above.

4.6. Model results

4.6.1. Stability

A solution is numerically stable if errors are insignificant, relative to a threshold value. The cell Reynolds number, defined below, is one of the indicators of the stability of a solution.

$$\text{Re}^C = \frac{\rho|u|h}{2\mu}.$$ \hspace{1cm} (4.16)

In the cell Reynolds number, the element size $h$ becomes the characteristic length, and the local velocity vector $u$ is used. As it is dependent on the geometry and mesh of the model, it cannot be compared to other models, so there is no generic value to decide whether the solution is stable or not [17].

In figure 4.4, it can be observed that the maximum value of the cell Reynolds number increases with the inlet velocity, as well as for lower positions of the flap: The cell Reynolds number is significantly bigger in configuration (d) than in (a). In each configuration, this maximum values appear near the flap, where the fluid velocity is bigger. Although the initial condition for the inlet flow speed is the same in all the domain, the fluid velocity becomes bigger in the zones where the cross sectional area available for the fluid is lower, such as the zones near the flap. This fact is related to the conservation of mass principle, commented on section 4.4.

Since $\text{Re}^C$ is calculated for each element of the mesh, the different values observed in each configuration could be explained not as unstable solutions, but as a difference in the fluid velocity distribution. Moreover, the areas where the fluid velocity is similar for different flap positions, e.g. the top of the cylinder, have also similar cell Reynolds numbers.

As commented in section 4.4, the use of a turbulent solver is not expected to introduce any improvements on the stability field, since the low velocities of the fluid are not enough to achieve a fully developed turbulent flow.
In the downstroke model, figure 4.5, the cell Reynolds number does not show significant variations.

Figure 4.4: Upstroke cell Reynolds number. $\theta = 70^\circ, \varphi = 0.1 \text{ m/s} \ (a); \ \theta = 55^\circ, \varphi = 0.4 \text{ m/s} \ (b); \ \theta = 30^\circ, \varphi = 0.7 \text{ m/s} \ (c); \ \theta = 5^\circ, \varphi = 1.1 \text{ m/s} \ (d)$.

Figure 4.5: Downstroke cell Reynolds number. $\theta = 5^\circ, \varphi = 0.1 \text{ m/s} \ (a); \ \theta = 30^\circ, \varphi = 0.3 \text{ m/s} \ (b); \ \theta = 55^\circ, \varphi = 0.4 \text{ m/s} \ (c); \ \theta = 70^\circ, \varphi = 1.1 \text{ m/s} \ (d)$. 

4.6.2. Pressure and velocity distribution

The velocity field for the downstroke model can be observed in figure 4.6 (a). In the first steps of the downstroke movement, the velocity of the fluid is relatively small in the entire fluid column. As the flap is almost closed, the fluid within the piston and cylinder separation reaches higher velocities, due to conservation laws. When the piston is sinking and the flaps are fully opened, the flap leaves a large wake behind it, with lower velocities.

On the other hand, the pressure field is shown in figure 4.6 (b). Between closed or open configurations, not significant differences can be observed in the values of the pressure field, due to the increase of velocity during the opening steps. However, there is a difference regarding the distribution of the pressure. When the flaps are closed, the fluid is divided into two different areas, with higher pressures in the bottom. When the flaps are open there are not such different areas, but gradual changes of pressure.

Figure 4.7 (a) shows the velocity distribution of the upstroke model. When the flaps are open, the fluid is relatively accelerated after the piston valve. When the flaps are closed, the fluid is stopped by the valve, and pumped upwards to the top reservoir. The fluid behind the flaps is also pumped, so only the leakage within the piston and cylinder separation increases its velocity to large values. The model considers that all the flow rate has to go through this little separation, reaching the outlet in the bottom of the cylinder. However, in the reality the leakage is expected to be relatively small, while almost all the fluid is being pumped upwards. Since the velocity values in the leakage zone consider all the flow rate, this values should not be considered as real when the flap is almost closed, but as effect of continuity laws in the model.
Figure 4.7 (b) shows the pressure field. As in the downstroke model, two different areas can be observed when the flaps are closed, while gradual distribution appears when the flaps are open. However, there is a big difference in the values of the pressure field. When the valve is open and the flow velocity is small, the pressure differences are practically negligible in the entire fluid column. When the valve is closed and the water is being pumped big pressure differences can be observed between the two areas separated by the flaps.

### 4.7. Pressure force

The pressure force is evaluated by taking the surface integral of the pressures acting on the boundaries of the flap as follows:

\[
F_{\text{pressure}}(i,j) = \int p_{ij} \, dS.
\]  

(4.17)

Since the resultant of each surface integration can be described as a vector normal to the surface, the pressure acting on the thickness of the flap is considered to be negligible on the movement of the valve. The top and bottom boundaries where the hydrodynamic force is evaluated are highlighted on figure 4.8.

Figure 4.7: Upstroke velocity (a) and pressure distribution (b).
As described in the model geometry section, the model only consists on a quarter of the piston, so the surface integral only considers a half of the flap. Therefore, a corrector factor is introduced in order to obtain the pressure force acting on one flap, so equation 4.17 becomes

$$F_{pressure}(i,j) = 2 \int p_{ij} \, dS.$$  \hfill (4.18)

A value of the force is obtained for every flap position \((i)\) and inlet velocity configuration \((j)\), yielding to a matrix of data points for each model. Considering this data points as a sample of the different forces acting on the flap for every configuration, nonlinear regression can be used to find an equation that describes this force. Multiple candidate models can fit to the data set, so goodness of fit have to be evaluated to determine the best model. It can be done using a combination of descriptive statistics, visual inspection and validation [18].

Therefore, the coefficient of determination \(R^2\) is taken into account. It is defined as

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}},$$  \hfill (4.19)

where \(SS_{res}\) is the sum of squares of residuals, which evaluates the difference between the data set value \(y_i\) and the predicted one \(f_i\). It is described by

$$SS_{res} = \sum_i (y_i - f_i)^2.$$  \hfill (4.20)

\(SS_{tot}\) is the total sum of squares, which evaluates the difference of the dependent variable and its mean:

$$SS_{tot} = \sum_i (y_i - \bar{y})^2.$$  \hfill (4.21)

An \(R^2\) of 1 indicates that the regression perfectly fits the data. However, it is monotone increasing with the number of variables included in the model, so a model with more variables would have a better \(R^2\) even though those added variables were unnecessary. Hence, and adjusted \(R^2\), written as \(\bar{R}^2\), is also used to determine the goodness of fit. Unlike \(R^2\), the \(\bar{R}^2\)
increases when a new term is included only if the new term improves the $R^2$ more than would be expected by chance. It is described by

$$ \tilde{R}^2 = R^2 - (1 - R^2) \frac{p}{n - p - 1}, $$

where $p$ is the total number of regressors in the model and $n$ is the sample size.

The level at which $\tilde{R}^2$ reaches a maximum, and decreases afterwards, indicates the regression that has the best fit without unnecessary terms.

Taking into account the descriptive statistics commented above, the downstroke fit yields to:

![Figure 4.9: Pressure force regression for downstroke model.](image)

While upstroke regression is represented in figure 4.10.

![Figure 4.10: Pressure force regression for upstroke model.](image)
Both forces for the upstroke and downstroke movement can be described by a polynomial regression as

\[ F_{\text{pressure}} = p_{00} + p_{10}x + p_{01}y + p_{20}x^2 + p_{11}xy + p_{02}y^2 + p_{30}x^3 + p_{21}x^2y + p_{12}xy^2 + p_{40}x^4 + p_{31}x^3y + p_{22}x^2y^2 + p_{50}x^5 + p_{41}x^4y + p_{32}x^3y^2, \]

(4.23)

where \( x \) represents the angle \( \theta \), and \( y \) refers to the inlet velocity \( \dot{z}_p \).

The values of these coefficients can be found in appendix A.

4.8. Pumping force


During the upstroke movement, the pumped volume of water increases gradually while the flaps are closing, reaching a maximum on \( \theta = 0 \). After that, when the flaps are completely closed, the volume of pumped water above the flaps starts to decrease again as a consequence of the position of the piston, which is closer to the top of the cylinder.

Taking into account that the pumped water must have the same velocity of the piston, i.e. relative velocity equal to zero in the model shown below (figure 4.11), it can be observed that the volume of pumped water will be major for lower angles.

![Figure 4.11: Pumped volume for different flap configurations.](image)
In order to determine an equation for this changing volume as a function of the flap’s angle, the areas shadowed in figure 4.11 has been revolved about the z-axis, obtaining a solid of revolution for every situation. To find the volume of those solids of revolution, the cylinder method [19] has been applied. This method is used when the drawn slice is parallel to the axis of revolution. In this case, the volumes of the solid formed by rotating the areas shadowed above are given by

$$V = 2\pi \int_{a}^{b} x |f(x)| \, dx,$$  \hspace{1cm} (4.23)

where $f(x)$ is the equation that describes the shadowed area in each case.

Obtaining the following distribution, depending on the position of the flaps

![Figure 4.12: Pumped volume Vs. Flaps' angle.](image)

Theoretically, the volume of the pumped fluid located above the flaps, when the piston valve is closed can be calculated as

$$V = L_{21} \pi R_{flap}^2 z.$$ \hspace{1cm} (4.24)

So considering a cylinder radius of 10 cm, $R_{cyl} = 10 \text{ cm}$, and the piston located in its central point, $L_{21} = 80 \text{ m}$, the value obtained equals to $2.5 \text{ m}^3$. However, the volume obtained with the model simulation for an almost closed configuration, shown in figure 4.12, is around $0.015 \text{ m}^3$.

This can be explained because the model does not consider the entire fluid column, due to the huge amount of computational resources it would require. It only considers the proximity area of the piston valve and, therefore, the volume outside the model domain is not considered, even though it is the most influent one.
In order to consider the entire fluid column, the total volume above the flaps is described as an addition of three different volumes, shadowed in figure 4.13.

![Figure 4.13: Pumped volumes for different flap configurations.](image)

In this case, an increasing cylindrical volume is described, dependent on the position of the flap as follows

\[
V_1 = L_{21} \pi \left( R_{\text{flap}} (1 - \cos c \theta) \right)^2,
\]

(4.25)

where \( L_{21} \) is the distance between the top surface of the piston and the upper reservoir, shown in figure 2.1, and \( C \) is an exponential parameter to smooth the increase of the cosine definition. In future work, an experimental fluid study or a transient numerical analysis model, could lead to a better definition of the pumped fluid during the closure time of the piston valve.

This cylindrical volume can be observed in the top left, while the other two volumes take into account the zero velocity areas located around this first one.
The total volume obtained, considering both flaps and the entire fluid column, is shown in figure 4.14.

![Diagram of volume above flaps](image)

Figure 4.14: Total pumped volume Vs. Flaps’ angle.

Hence, the volume located above one flap can be described by the following rational equation:

\[
V_{af} = \frac{12.4}{\theta^3 - 1.66\theta^2 + 4.21\theta + 10.06},
\]

(4.25)

with an adjusted R-square value of \( \tilde{R}^2 = 0.9992 \).

### 4.8.2. Pumping force description

The pumping force takes into account all the inertial forces produced by the movement of the pumped fluid during the upstroke movement.

On one hand, there is the force produced by the fluid contained above the flaps, taking into account both weight and inertial components. It depends on the position of the flaps as discussed on the last point. This component can be described as follows,

\[
F_{af} = V_{af} \rho_{water} (g + \dot{Z}_p).
\]

(4.26)

On the other hand, there is the inertia of the fluid situated below the flaps, which is moving upwards as a consequence of the piston movement. It is considered that the fluid below the flaps starts to move only when the flaps are fully closed, otherwise it remains motionless.

\[
F_{bf} = A_{cyl} \rho_{water} L_{43} \dot{Z}_p,
\]

(4.27)

where \( A_{cyl} \) is described as the cross sectional area of the cylinder that corresponds to one flap.
In this case, the weight of the fluid is not taken into account as it is not supported on the flaps but on the lower reservoir structure.

Combining both formulae, the pumping force can be described as follows,

\[
F_{pf} = \begin{cases} 
V_a f \rho_{water} (g + \ddot{Z}_p) & \text{when } z \neq 0, \\
A_{cyl} \rho_{water} (L_c \ddot{Z}_p + L_{z1} g) & \text{when } z = 0.
\end{cases}
\]
Chapter 5

RESULTS VALIDATION & MODEL IMPROVEMENTS

5.1. Results validation

The parameters and nomenclature that are used in the simulation of the model explained in chapter 3 are summarized in the following table.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
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</thead>
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<td>m</td>
</tr>
<tr>
<td>Wave period</td>
<td>$T_w$</td>
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<td>s</td>
</tr>
<tr>
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<td>m/s²</td>
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<td>kg/m³</td>
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<tr>
<td>Aluminium density</td>
<td>$\rho_{Al}$</td>
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<td>kg/m³</td>
</tr>
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<td>Friction coefficient</td>
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<td>-</td>
</tr>
<tr>
<td>Added mass coefficient</td>
<td>$C_a$</td>
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<td>-</td>
</tr>
<tr>
<td>Volume above flaps</td>
<td>$C$</td>
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<td>-</td>
</tr>
<tr>
<td>Radius of the flap</td>
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<td>m</td>
</tr>
<tr>
<td>Thickness of the flap</td>
<td>$t_{flap}$</td>
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<td>m</td>
</tr>
<tr>
<td>Piston cylinder separation</td>
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<td>m</td>
</tr>
<tr>
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<td>°</td>
</tr>
<tr>
<td>Minimum angle of the flap</td>
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<tr>
<td>Piston central position</td>
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<td>m</td>
</tr>
</tbody>
</table>

Table 5.1: Simulation parameters.

In order to validate the first model explained in chapters 3 and 4, a first comparison between the results and the expected values is made.

In the single-piston pump model defined in [6], and commented in chapter 2, the pumping force $F_p$ was described as the vertical force acting on the piston during the upstroke. This force appeared as a consequence of the fluid being pumped, including its weight, inertia and pressure difference on top and bottom of the piston.
As shown in figure 5.1, this model predicted a convergence value for $F_p$ around 22440 N. Taking into account that this was the force acting on the entire piston, it is assumable that the force acting on just one flap should be half of it, around 11220 N.

The results of the vertical reaction on the piston ($F_y$) for the first model can be observed below.

It is clear that the value is much bigger than expected, reaching a maximum of almost 20,000 N for just one flap, during the upstroke movement. It is important to mention that $F_y$ is not the equivalent of the pumping force $F_p$, but a combination of the hydrodynamic force and other vertical forces acting on the flap, such as the weight of the flap $W$ and the buoyancy force $F_{buoyancy}$, which were not included in [6].

When the flaps are closed and the piston is pumping water upwards, the hydrodynamic force becomes large due to the quantity of fluid that is being moved, increasing also the reaction on the piston. On the other hand, the weight and the buoyancy force remain constant, since their definition only depends on the geometry of the flap. During this configuration, the results shown in figure 5.2 reach a maximum that significantly exceeds the expected value. Due to the important role played by the hydrodynamic force during this configuration, it is thought that $F_{hydro}$ might be wrongly defined.

Therefore, some hypothesis will be made in order to discern the shortcomings of this model and introduce a better definition for the hydrodynamic force.
5.2. 1st improvement hypothesis: $F_{\text{pressure}}$ overlaps with $F_{pf}$

The first hypothesis that could justify the high results presented in section 5.1, could be an overlap between $F_{\text{pressure}}$ and $F_{pf}$. In the COMSOL model explained in chapter 4, only a part of the cylinder was included: the nearest part to the piston valve. On the other hand, $F_{pf}$ included the entire fluid column, moving during the upstroke. It can be thought that the volume calculated in the COMSOL model to extract the pressure force, was taken into account two times, as it was also included in the fluid column defined in $F_{\text{pressure}}$. Therefore, a new definition for the hydrodynamic force will be done.

In order to simulate the pressure acting on the flaps as real as possible, the weight of the fluid column above the piston is included in the numerical analysis. Hence, some boundary conditions described in section 4.5.2 have to be changed. A new fixed pressure is described in boundary 4 as follows

$$p_c = L_{21\text{top}} \rho g.$$  \hspace{1cm} (5.1)

This way, the static pressure produced by the fluid located above the piston valve is included in the COMSOL model. On the other hand, the flow velocity is described in boundary 3. The flow speed $\phi$ is varied between 0.1 m/s and 1.2 m/s for every flap position, obtaining a velocity dependent flow distribution for every piston valve configuration.

$F_{\text{pressure}}$ is also defined as the difference between the force acting on the lower surface of the flap and the force acting on the upper surface:

$$F_{\text{pressure}}(i,j) = 2 \left( \iint p_{ij\text{low}} dS_{\text{low}} - \iint p_{ij\text{up}} dS_{\text{up}} \right).$$  \hspace{1cm} (5.2)

This simulation yields to the following results for the downstroke movement:

Figure 5.3: Pressure force regression for downstroke model.
And for the upstroke configuration:

![Graph showing pressure force regression for upstroke model.]

Conceptually, the force was expected to be positive during the downstroke and negative during the upstroke. It should increase with the velocity of the fluid, and the closure of the flap. The maximum value should appear with the highest flow speed and the lower angle. However, in figures 5.3 and 5.4, this behavior can only be observed in low angles, while the results are quite similar in high angles for both configurations. Large variations on the values of the pressure force can be observed in both regressions, when the flaps are completely open, with angles between 60 and 90 degrees. On the other hand, the velocity of the fluid does not influence the result, only when the angle of the flap is lower than 20 degrees.

The discontinuities on the pressure force values can be explained due to the changing areas of the upper and lower surfaces. In the COMSOL model, a central beam is considered to hold the flaps in a fixed position and to prevent the fluid simulation near the area of the flap-piston connection (See section 4.5.1). Therefore, for large angles, the geometry of the central beam overlaps with the geometry of the flap, changing the integrated area for each surface. This fact introduces wrong values in the pressure force results, since the evaluated areas are not the same for the upper and lower surfaces.

Since the integration method is not good enough, only the average pressure is calculated in the COMSOL model, obtaining the pressure force acting on the flaps as follows

\[
F_{\text{pressure}}(i,j) = A_{\text{flap}} \left( \bar{p}_{\text{low},ij} - \bar{p}_{\text{top},ij} \right)
\]  

(5.3)

Introducing these new values, the regression of all the data points yields to the following results for the downstroke model:
While for the upstroke model the regression is plotted in figure 5.6.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure55.png}
\caption{Redefined pressure force regression for downstroke model.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure56.png}
\caption{Redefined pressure force regression for upstroke model.}
\end{figure}

Since \( F_{\text{pressure}} \) includes the weight of the fluid column, the second component of \( F_{\text{hydro}} \) only takes into account the inertia of the fluid:

\[
F_{\text{inertia}} = \begin{cases} 
V_{mf} \rho \bar{Z}_p^\theta & \text{ during } US, \\
0 & \text{ during } DS, 
\end{cases} \tag{5.4}
\]

where \( V_{mf} \) is the volume of moving fluid which changes gradually with its movement, as follows:

\[
V_{mf} = \begin{cases} 
V_{af} & \text{ when } \theta \neq 0, \\
V_{af} + A_{cil}(L_c - L_{21}) & \text{ when } \theta = 0.
\end{cases} \tag{5.5}
\]

\( V_{af} \) is the volume located above the flaps, defined in section 4.8.1.
Introducing these changes in the dynamical model, the following results for the vertical reaction on the piston are obtained.

$F_y$ is plotted in figure 5.7, where the blue line represents the result of the first model and the red one represents the actual one. It is clear that the changes applied in the model have not contribute with the expected results. The pressure force has been incremented when fixing the pressure above the piston, but it has not converged to the expected value. Far from that, it has reached a maximum around $50.000 \, N$. Therefore, this first hypothesis can be rejected.

Despite this assumption is not useful to describe the dynamical behavior of the flaps, it shows an interesting fact: Observing the piston movement, it can be noticed that the maximum value of $F_{pressure}$ coincides with the maximum velocity of the piston.
5.3. 2nd Improvement hypothesis: Wrong flow speed definition

In chapter 4, the flow speed $\phi$ was described as equal to the velocity of the piston $Z_p$, since the piston valve was modeled as motionless. Therefore, the pressure term on the hydrodynamic force was dependent on the piston velocity. However, $\dot{Z}_p$ is not the real velocity between the piston and the fluid. When the flaps are closed, the fluid should reach the same velocity as the piston, so the relative velocity between them should become zero after a short period of time. When the piston is sinking, the fluid column is motionless, so the relative velocity should converge to the value of the piston velocity.

In order to take into account this concept, the flow speed $\phi$ is redefined as the relative velocity between the piston and the fluid:

$$\phi = \dot{Z}_p - \dot{Z}_{fluid}.$$  \hfill (5.6)

This fact will only introduce a change in the input of calculating $F_{\text{pressure}}$, so there is no necessity of solving the numerical analysis another time. However, some equations have to be changed. On first $F_{pf}$ definition, it was assumed that acceleration of the pumped fluid was the same as the piston. Since the piston and the fluid do not have the same velocity anymore, their accelerations neither. Therefore, equation (4.29) becomes:

$$F_{pf} = \begin{cases} V_{af} \rho_{\text{water}} (g + \dot{Z}_{fluid}) & \text{when } \theta \neq 0, \\ A_{cil} \rho_{\text{water}} (L_c \dot{Z}_{fluid} + L_{21} g) & \text{when } \theta = 0. \end{cases}$$  \hfill (5.7)

To define $\dot{Z}_{fluid}$, the upstroke and downstroke movements will be studied separately. During the downstroke of the piston, the fluid backflow is prevented with the actuation of the check valves situated in the lower and upper reservoirs. Hence, the fluid column is motionless, the fluid velocity is described as

$$\dot{Z}_{fluid, \text{downstroke}} = 0.$$  \hfill (5.8)

In the upstroke movement, the check valves open, and the fluid is pumped upwards. Therefore, the description of the fluid velocity is less trivial.

In [20] a solution for the viscous, low-Reynolds-number flow generated by a moving piston is presented. The authors represent a two-dimensional analog of flow in a syringe for the case that the tube length is several times the radius. In that case, a fully developed Poiseuille flow [21] evolves, as shown in figure 5.9.
In this case, the piston moves at a constant velocity, and the fluid is pushed through the pipe, reaching a parabolic velocity distribution due to shear forces in the walls. This Poiseuille flow distribution only appears when the tube is long enough, at some distance from the piston. The fluid near the piston does not reach this distribution, and its velocity can be described as the same as the piston, even for the particles close to the walls.

This concept shows that the fluid near the piston should reach the same velocity as the piston as soon as the flaps are closed. Therefore, the relative velocity between the fluid and the piston would be zero. However, in the Ocean Grazer system, the piston velocity is not constant, and the flaps are open at the beginning of the upstroke movement.

Since there are no analytical calculations to determine the relative velocity between the flaps and the fluid, and a time-dependent model cannot be solved using numerical analysis due to computational requirements, the movement of the fluid column is observed in the experimental setup [22] in order to understand its behavior. It is observed that, while the flaps are open, the fluid moves upwards with lower velocity than the piston. The difference between both velocities decreases with the angle of the flaps. Finally, the fluid velocity reaches the piston velocity when the flaps are closed.

In order to describe the fluid velocity during the closure of the flap, some assumptions and simplifications will be made.

The conservation of mass principle [23] of incompressible fluid flow, for a fixed control volume without any fluid mass accumulation with time or internal fluid sources, is defined as

$$\sum Q_{in} = \sum Q_{out}. \quad (5.9)$$

Therefore, the flow rate inside the cylinder of the single piston pump system can be described as

$$Q_{fluid} = Q_{af} + Q_{leak}.\quad (5.10)$$
In section 4.8.1, $V_{af}$ was described as the volume above the flap that has the same velocity as the piston. This moving volume generates a flow rate, named $Q_{af}$. As this flow rate is referring to the portion of fluid that is being pumped upwards, the remaining part of the fluid can be described as a leakage flow rate, $Q_{leak}$.

The rate of change of the flow-rates described above should also be conserved, so that

$$\dot{Q}_{fluid} = \dot{Q}_{af} + \dot{Q}_{leak}. \quad (5.11)$$

Since the exact velocity distribution may not be known, it is more convenient to use an average velocity [24], given by the volume flow rate definition

$$Q = AV. \quad (5.12)$$

Then, equation (5.11) can be written as follows

$$A_{cyl} \dot{Z}_{fluid} = A_{af} \dot{Z}_{af} + A_{leak} \dot{Z}_{leak}. \quad (5.13)$$

Considering that backflow is prevented by the check valves located in the lower reservoir even during the upstroke movement, this leakage volume cannot move downwards the cylinder and, therefore, its velocity is assumed to be zero:

$$\dot{Z}_{leak} = 0. \quad (5.14)$$
while, as commented above, the velocity of the pumped fluid is

$$\dot{z}_{af} = \dot{z}_{piston}. \quad (5.15)$$

Combining the above formulae into the expression for the fluid velocity yields

$$\dot{z}_{fluid_{upstroke}} = \frac{\dot{A}_f}{\dot{A}_{cyl}} \dot{z}_p, \quad (5.16)$$

where $\dot{A}_f = \dot{V}_{af}/L_{21}$ is the average cross sectional area of the pumped volume.

Summarizing both upstroke and downstroke configurations, the fluid velocity is

$$\dot{z}_{fluid} = \begin{cases} \dot{z}_{fluid} = \frac{\dot{A}_f}{\dot{A}_{cyl}} \dot{z}_p & \text{if } \dot{z}_p > 0, \\ 0 & \text{if } \dot{z}_p \leq 0. \end{cases} \quad (5.17)$$

The fluid velocity can be observed in the following figure, where $\dot{z}_{fluid}$ is plotted in red and $\dot{z}_p$ is plotted in blue.

![Figure 5.11: Piston and fluid velocities.](image)

As commented before, the velocity of the fluid is lower than the velocity of the piston while the flap is open, during the first steps of the upstroke movement. It increases rapidly while the flap is closing, until it reaches the same velocity as the piston when the flap is closed. During the downstroke, the fluid velocity becomes zero due to the action of the check valves.

This new flow speed definition yields to the following results for $R_f$: 

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The plot shows negative values during the downstroke, when the piston is sinking and the working fluid offers a drag force. During the upstroke movement, its value becomes significantly bigger, due to the inertial components of the hydrodynamic force. In this case, the velocity of the fluid depends on the flap position, and so does the acceleration. Since $V_{af}$ takes big values during the last steps of the closure, the acceleration of the flap increases significantly in this period. This yields to a fast increase of $V_{af}$ until it reaches its maximum value. Since the fluid velocity, $\dot{\gamma}_{fluid}$, directly depends on $V_{af}$ (see equation 5.17), it also increases rapidly during the last times of the closure, yielding to big accelerations of the fluid. Therefore, some spikes appear on the reactions on the piston, as shown in figure 5.12.

Comparing these results with the results of the first model the following plot is obtained:

![Figure 5.12: Piston vertical reaction.](image)

![Figure 5.13: 2nd improvement hypothesis validation for $F_y$.](image)
In this case, the results for $F_y$ match with the expected values. At the beginning of the upstroke, $t = 5$ s, the reaction on the piston increases with the closure of the flap. When the flap is totally closed the value decreases, due to the higher position of the piston and the deceleration of the fluid.

### 5.4. Results discussion

Figure 5.14 is a plot of the vertical movement of the piston for the harmonic wave input of equations (3.3), (3.4) and (3.5) over two and a half wave periods. It can be observed that the simulation starts when the piston is situated on its top-dead center position. Then it moves downwards, simulating the downstroke configuration, until it reaches its minimum position in $t = 5$ s. After that, the velocity becomes positive, and the upstroke movement is simulated.

This piston movement is translated to the hydrodynamic force by the working fluid. The combination of the pressure force and the pumping force acting on the flap yields to the results for $F_{\text{hydro}}$ shown in figure 5.15.

At the beginning of the downstroke, the force increases due to the positive acceleration of the piston. While the flap opens the force decreases, and becomes steady when the flap reaches its maximum position. In this part of the simulation, the hydrodynamic force is mostly caused by the shear between the fluid and the flap surface, since the working fluid column is motionless, and its inertial component does not play a significant role.
On the other hand, the absolute values become significantly bigger during the upstroke. The force increases gradually while the flap closes. When the piston is pumping water to the upper reservoir, the weight of the water column above the piston is supported by the flap. As commented before, $F_{pf}$ increases while the flap is closing, reaching its maximum value when the piston valve is totally closed. As the height of the fluid column above the flap, $L_{21}$, is considerably large, this weight is the principal cause of the hydrodynamic force acting on the piston. After the closure, when $\theta = 0$, its absolute value decreases, due to the deceleration of the fluid.

When the piston reaches the top-dead-center, $t = 10\, s$, the velocity of the fluid becomes almost zero, and the check valves of the upper and lower reservoir start to close. The location of this valves is shown in figure 5.16.

When these check valves are totally closed, the fluid within the cylinder cannot move anymore. Therefore, the weight of the entire fluid column is no longer supported by the flaps but by the check valves. In the current model, it is assumed that both check valves close instantaneously at the beginning of the downstroke movement. This yields to a significant discontinuity when switching from upstroke to downstroke movement, which can be observed in the hydrodynamic force model results, figure 5.15.
Introducing the movement of these check valves, which have been partly studied in [25], and their interaction with the different piston valves acting on the cylinders, would describe a gradual change in the weight supported by the flaps during the very first steps of the downstroke.

The hydrodynamic force, altogether with the other forces acting on the flaps and explained in chapter 3, cause the movement of the flap, plotted below.

Figure 5.17: Flap displacement (a), velocity (b) and acceleration (c).
During the first steps of the simulation, the working fluid is motionless and the piston is sinking due to its own weight. Therefore, the flap opens until it reaches its maximum position. When the flap contacts with the piston surface, the acceleration and the velocity become zero. The piston continues its downstroke movement with the flap fixed in $\theta = \theta_{\text{max}}$. After reaching the dead-bottom center, the upstroke configuration starts. The flap close with relative slow velocity at the beginning, due to the slow velocity of the piston. However, after a short period of time, its velocity increases (in absolute terms) and the movement is faster. This is caused by the acceleration peak that can be observed in figure 5.17 (c), which can be explained by the increasing volume situated above the flaps that yields to big hydrodynamic force values. Again, when the flap contacts with the piston surface, the position of the flap is fixed in $\theta = \theta_{\text{min}}$, and the velocity and acceleration become zero.

Finally, the reactions on the piston can be observed in the following figure:

![Graphs showing reactions on the piston](image)

**Figure 5.18:** Horizontal (a), torque (b) and vertical (c) piston reactions.

In figure 5.18 (a), the horizontal reaction produced by one flap can be observed. It reaches its maximum value during the upstroke movement, when the flap is almost closed. Looking at its definition, equation (3.6), it would be normal to expect its higher value when the flap is open, so the sinus component would be bigger. However, the net force acting on the flap is much bigger during the upstroke, when the fluid weight and inertia is acting on the flap, and so the horizontal force.

The torque produced by the contact between the piston and the flap, equation (3.10), is shown in figure 5.18 (b). Again, it is bigger during the upstroke, when the flap is closed.

The vertical reaction, figure 5.18 (c), has been already commented in figure 5.12.
As commented on section 3.1, only the dynamical behavior of one flap has been modeled. Considering the full geometry of the piston valve, the horizontal component of the piston reaction and the piston torque become zero, equations (3.7) and (3.11), while the total vertical force, equation (3.9) is:

\[ \text{Total Fy} \]

![Figure 5.19: Total vertical reaction.](image)

In future work, this vertical reaction could be introduced in the single-piston pump model described in section 2.1. in order to remove the discontinuities at the bottom-dead-center of the piston oscillation, and provide a more nuanced description for the pumping force.
Chapter 6

CONCLUSIONS & FUTURE WORK

During this thesis a mathematical model to describe the dynamical behavior of the piston valve has been reached. In the development of the model, the forces that are involved in the movement of the valve have been determined and described using different tools. This model has been used to determine the forces acting on the pumping piston, due to the interaction between the valve and the working fluid. Therefore, a gradual description of this forces during the movement of the piston valve has been introduced.

Based on the limitations of the current research, future work is necessary to validate the assumptions used in the present model:

- Experimental investigation of fluid behavior during the movement of the piston valve.
- Extension of COMSOL models to include transient effects.
- Validation of the developed model to determine the accuracy of the results.
- Combination of the piston valve model with other models accounting for the behavior of the check valves located in the lower and upper reservoirs.
- Inclusion of the current model in the SPP model developed in [6] and [9].

The present model can be used in the optimization process of the piston geometry, in order to describe the connection between the piston valve and the piston itself. Moreover, the simulation of the movement of the flap can be used to determine the optimal value for $\theta_{\text{max}}$, regarding the pumping and sinking efficiency of the piston.
APPENDIX A

This appendix includes all the values that have been used in the description of the pressure force for each model. It also refers to the descriptive statistics used to choose the fitting regression. All the regressions explained in this appendix has been obtained using the Curve Fitting application of MATLAB software.

As it was explained in section 4.7, the results obtained in COMSOL are processed using non-linear regression, in order to find an equation to describe $F_{pressure}$. The regression that yields to more accurate results consists on a polynomial with 15 coefficients.

Two main configurations, upstroke and downstroke movement, have to be studied separately for each model. Since the resultant forces increase significantly for angles below 50° in both movements, two different regressions are made for each configuration.

A.1. Basic model

The coefficients obtained for the downstroke movement are shown below.

<table>
<thead>
<tr>
<th>$\theta &gt; 50$</th>
<th>$p_{00}$</th>
<th>$p_{10}$</th>
<th>$p_{01}$</th>
<th>$p_{20}$</th>
<th>$p_{11}$</th>
<th>$p_{02}$</th>
<th>$p_{30}$</th>
<th>$p_{21}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-789.8</td>
<td>62.01</td>
<td>167.6</td>
<td>-1.911</td>
<td>-12.01</td>
<td>574.8</td>
<td>2.89e-2</td>
<td>0.314</td>
<td></td>
</tr>
<tr>
<td>-22.28</td>
<td>-2.15e-4</td>
<td>-3.52e-3</td>
<td>0.288</td>
<td>6.28e-7</td>
<td>1.43e-5</td>
<td>-1.24e-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta \leq 50$</td>
<td>$p_{00}$</td>
<td>$p_{10}$</td>
<td>$p_{01}$</td>
<td>$p_{20}$</td>
<td>$p_{11}$</td>
<td>$p_{02}$</td>
<td>$p_{30}$</td>
<td>$p_{21}$</td>
</tr>
<tr>
<td>69.33</td>
<td>-34.01</td>
<td>-1301</td>
<td>3.119</td>
<td>188</td>
<td>2943</td>
<td>-0.102</td>
<td>-7.863</td>
<td></td>
</tr>
<tr>
<td>-171.1</td>
<td>1.36e-3</td>
<td>0.1227</td>
<td>3.09</td>
<td>-6.31e-6</td>
<td>-6.35e-4</td>
<td>-1.75e-2</td>
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</table>

Table A.1: Downstroke coefficients.

The coefficients obtained for the upstroke configuration are included in the following table:

<table>
<thead>
<tr>
<th>$\theta &gt; 50$</th>
<th>$p_{00}$</th>
<th>$p_{10}$</th>
<th>$p_{01}$</th>
<th>$p_{20}$</th>
<th>$p_{11}$</th>
<th>$p_{02}$</th>
<th>$p_{30}$</th>
<th>$p_{21}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-90.58</td>
<td>9.621</td>
<td>-606.2</td>
<td>-0.3663</td>
<td>35.27</td>
<td>-391.9</td>
<td>6.511e-3</td>
<td>-0.762</td>
<td></td>
</tr>
<tr>
<td>15.05</td>
<td>-5.51e-5</td>
<td>7.24e-3</td>
<td>-0.2005</td>
<td>1.79e-7</td>
<td>-2.55e-5</td>
<td>9.07e-4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta \leq 50$</td>
<td>$p_{00}$</td>
<td>$p_{10}$</td>
<td>$p_{01}$</td>
<td>$p_{20}$</td>
<td>$p_{11}$</td>
<td>$p_{02}$</td>
<td>$p_{30}$</td>
<td>$p_{21}$</td>
</tr>
<tr>
<td>-109.6</td>
<td>29.39</td>
<td>-602.3</td>
<td>-2.624</td>
<td>138.1</td>
<td>-3587</td>
<td>0.0891</td>
<td>-6.913</td>
<td></td>
</tr>
<tr>
<td>217</td>
<td>-1.23e-3</td>
<td>0.1182</td>
<td>-4.02</td>
<td>5.89e-6</td>
<td>-6.47e-4</td>
<td>0.0231</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A.2: Upstroke coefficients.
The descriptive statistics of the regressions explained above are shown in table A.3.

<table>
<thead>
<tr>
<th>Fit type</th>
<th>( \theta &gt; 50 )</th>
<th>( \theta \leq 50 )</th>
<th>( SS_{res} )</th>
<th>( R^2 )</th>
<th>( \bar{R}^2 )</th>
<th>( s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downstroke</td>
<td>Poly52</td>
<td>Poly52</td>
<td>22.6125</td>
<td>0.9984</td>
<td>0.9981</td>
<td>0.4641</td>
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<tr>
<td>Upstroke</td>
<td>Poly52</td>
<td>Poly52</td>
<td>14.7771</td>
<td>0.9990</td>
<td>0.9989</td>
<td>0.3751</td>
</tr>
</tbody>
</table>

Table A.3: Descriptive statistics.

Since the definition of the pressure force, \( F_{pressure} \), in the 2\textsuperscript{nd} improvement hypothesis model is the same as in the basic model, the values of the coefficients commented above do not change, but are also used in that improvement model.

### A.2. 1\textsuperscript{st} improvement hypothesis model

In this case, the changes applied in the COMSOL model, and explained in section 5.2, yield to a new definition of \( F_{pressure} \) and, therefore, new coefficients have to be used in the improvement model.

Table A.4 refers to the coefficients used in the downstroke configuration.

<table>
<thead>
<tr>
<th>( \theta &gt; 50 )</th>
<th>( p_{00} )</th>
<th>( p_{10} )</th>
<th>( p_{01} )</th>
<th>( p_{20} )</th>
<th>( p_{11} )</th>
<th>( p_{02} )</th>
<th>( p_{30} )</th>
<th>( p_{21} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_{12} )</td>
<td>-707.9</td>
<td>52.7</td>
<td>-922.3</td>
<td>-1.53</td>
<td>58.73</td>
<td>815.8</td>
<td>0.0217</td>
<td>-1.369</td>
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<tr>
<td>( p_{40} )</td>
<td>-27.84</td>
<td>-1.51e-4</td>
<td>0.0139</td>
<td>0.3225</td>
<td>4.10e-7</td>
<td>5.16e-5</td>
<td>1.23e-3</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \theta \leq 50 )</th>
<th>( p_{00} )</th>
<th>( p_{10} )</th>
<th>( p_{01} )</th>
<th>( p_{20} )</th>
<th>( p_{11} )</th>
<th>( p_{02} )</th>
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<th>( p_{21} )</th>
</tr>
</thead>
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<tr>
<td>( p_{12} )</td>
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<td>614.8</td>
<td>7.982</td>
<td>171.6</td>
<td>7541</td>
<td>-0.2521</td>
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<td>( p_{40} )</td>
<td>-484.9</td>
<td>3.39e-3</td>
<td>0.2465</td>
<td>9.329</td>
<td>-1.60e-5</td>
<td>-1.44e-3</td>
<td>-0.0549</td>
<td></td>
</tr>
</tbody>
</table>

Table A.4: Downstroke coefficients.

The coefficients obtained for the upstroke movement are shown below.

<table>
<thead>
<tr>
<th>( \theta &gt; 50 )</th>
<th>( p_{00} )</th>
<th>( p_{10} )</th>
<th>( p_{01} )</th>
<th>( p_{20} )</th>
<th>( p_{11} )</th>
<th>( p_{02} )</th>
<th>( p_{30} )</th>
<th>( p_{21} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_{12} )</td>
<td>-5973</td>
<td>438.7</td>
<td>6480</td>
<td>-12.56</td>
<td>-423</td>
<td>-4081</td>
<td>0.1749</td>
<td>10.27</td>
</tr>
<tr>
<td>( p_{40} )</td>
<td>129.5</td>
<td>-1.18e-3</td>
<td>-0.1088</td>
<td>-1.459</td>
<td>3.09e-6</td>
<td>4.24e-4</td>
<td>5.81e-3</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \theta \leq 50 )</th>
<th>( p_{00} )</th>
<th>( p_{10} )</th>
<th>( p_{01} )</th>
<th>( p_{20} )</th>
<th>( p_{11} )</th>
<th>( p_{02} )</th>
<th>( p_{30} )</th>
<th>( p_{21} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_{12} )</td>
<td>-600.4</td>
<td>264.6</td>
<td>-8385</td>
<td>-24.38</td>
<td>1372</td>
<td>-2.55e4</td>
<td>0.8014</td>
<td>-59.56</td>
</tr>
<tr>
<td>( p_{40} )</td>
<td>1442</td>
<td>-0.0107</td>
<td>0.9428</td>
<td>-25.71</td>
<td>4.97e-5</td>
<td>-4.91e-3</td>
<td>0.1444</td>
<td></td>
</tr>
</tbody>
</table>

Table A.5: Upstroke coefficients.
Finally, the descriptive statistics of the regressions used in this model are shown in table A.6.

<table>
<thead>
<tr>
<th>Fit type</th>
<th>$SS_{res}$</th>
<th>$R^2$</th>
<th>$\bar{R}^2$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Downstroke $\theta &gt; 50$</td>
<td>Poly52</td>
<td>27.5616</td>
<td>0.9997</td>
<td>0.9996</td>
</tr>
<tr>
<td>Downstroke $\theta \leq 50$</td>
<td>Poly52</td>
<td>9.64e6</td>
<td>0.94</td>
<td>0.9358</td>
</tr>
<tr>
<td>Upstroke $\theta &gt; 50$</td>
<td>Poly52</td>
<td>1.003e4</td>
<td>0.9973</td>
<td>0.9970</td>
</tr>
<tr>
<td>Upstroke $\theta \leq 50$</td>
<td>Poly52</td>
<td>3.2602e7</td>
<td>0.9915</td>
<td>0.9909</td>
</tr>
</tbody>
</table>

Table A.6: Descriptive statistics.
APPENDIX B

This appendix includes all the figures related with the code and the SIMULINK MATLAB models that have been used in the development of this thesis.

B.1. Basic model

As it was mentioned during the description of the dynamical model, the following SIMULINK models have been created in order to be easily understandable by someone else in future work. Therefore, the three models used in the development of this thesis share the same structure.

The comprehensive SIMULINK model is shown in figure B.1. The position and velocity of the piston are used as input, obtaining the reactions on the piston and the flap position. As it can be observed, the acceleration of the flap is integrated firstly to get the velocity and lately to get its displacement. In the first integration block, an external reset is used to assure that the velocity becomes zero when the flap reaches its maximum or minimum positions. Without this reset condition, the acceleration of the flap would become zero when it is stopped by the piston geometry, but the velocity would remain constant and, therefore, the flap displacement would keep increasing. This fact is mathematically described in equation (3.15).

Two main blocks can be observed. On one hand, the 'Fluid interaction' block contains all the equations that are need to obtain the hydrodynamic force. Since the description of this force is different in every model, this block also changes in each model, and will be explained below.
On the other hand, the equations of motion for the flap are the same in each model. These equations have been implemented using a MATLAB Function block, named 'Flap Movement', with the following code:

```matlab
function [AcFlap,Fx,Fy,Tpiston]=FlapMovement(Fhydro,Zeta)

% PARAMETERS
g = 9.81; % Gravity constant (m/s^2)
rho = 1000; % Water density (kg/m^3)
rhoAl = 2700; % Aluminium density (kg/m^3)
muc = 0.16; % Friction coefficient (steel-aluminium lubricated)
Ca = 0.1; % Added mass coefficient

% Flap
Rf = 0.1; % Flap radius (m)
Rcom = 4*Rf/(3*pi); % COM radius (m)
dflap = 0.003; % Flap thickness (m)
vflap = pi*Rf^2*dflap/2; % Flap volume (m^3)
ma = Ca*rho*vflap; % Added mass (kg)
Iz = 1/4*(mflap+ma)*Rf^2*1000; % Inertia tensor (kg*m^2)
zetamax = 60*pi/180; % Zeta max value (rad)
Zeta = Zeta*pi/180; % Change degrees to radians

% FORCES
W = mflap*g; % Flap weight
Fb = vflap*rho*g; % Flap buoyancy
Fx = Fhydro*sin(Zeta); % Piston reaction x
Fy = -Fhydro*cos(Zeta)+W-Fb; % Piston reaction y
Thinge = sqrt(Fx^2+Fy^2)*muc*dflap/2; % Hinge friction torque

% DYNAMICAL MODEL
if Zeta <= 0
    if Fhydro <= W-Fb+Thinge/Rcom
        Tpiston = (-Fhydro+W-Fb)*Rcom+Thinge;
        AcFlap = 0;
    else
        AcFlap = (((-W+Fb)*cos(Zeta)+Fhydro)*Rcom+Thinge)/Iz;
        Tpiston = 0;
    end
elseif Zeta >= zetamax
    if Fhydro <= (W-Fb)*cos(zetamax)+Thinge/Rcom
        AcFlap = (((-W+Fb)*cos(zetamax)+Fhydro)*Rcom+Thinge)/Iz;
        Tpiston = 0;
    else
        Tpiston = ((W-Fb)*cos(zetamax)-Fhydro)*Rcom-Thinge;
        AcFlap = 0;
    end
else
    AcFlap = (((-W+Fb)*cos(Zeta)+Fhydro)*Rcom+Thinge)/Iz;
    Tpiston = 0;
end
```
Inside the 'Fluid interaction' block of the basic model, the following subsystem have been implemented:

A MATLAB Function block has been created in order to include the pressure force obtained with the COMSOL simulations. It contains the following code:

```matlab
function Fpressure=PressureForce(VelPiston,Zeta)
    x = Zeta;
    y = VelPiston;
    if y > 0
        if Zeta>50
            p00 = -90.58;
            p10 = 9.621;
            p01 = -606.2;
            p20 = -0.3663;
            p11 = 35.27;
            p02 = -391.9;
            p30 = 0.006511;
            p21 = -0.7618;
            p12 = 15.05;
            p40 = -5.514e-05;
            p31 = 0.1182;
            p22 = -4.02;
        else
            p00 = -109.6;
            p10 = 29.39;
            p01 = -602.3;
            p20 = -2.624;
            p11 = 138.1;
            p02 = -3587;
            p30 = 0.08906;
            p21 = -6.913;
            p12 = 217;
            p40 = -0.001232;
            p31 = 0.1182;
            p22 = -4.02;
        end
    end
end
```
p30 = 5.886e-06;
p41 = -0.0006464;
p32 = 0.0231;
end

else
    if Zeta > 50
        p00 = -789.8;
p10 = 62.01;
p01 = 167.6;
p20 = -1.911;
p11 = -12.01;
p02 = -1.911;
p30 = 0.02891;
p21 = 0.3135;
p12 = -22.28;
p40 = -0.0002149;
p31 = -0.003516;
p22 = 0.2878;
p50 = 6.282e-07;
p41 = 1.429e-05;
p32 = -0.001237;
    else
        p00 = 69.33;
p10 = -34.01;
p01 = -1301;
p20 = 3.119;
p11 = 188;
p02 = 2943;
p30 = -0.1021;
p21 = -7.863;
p12 = -171.1;
p40 = 0.1227;
p22 = 3.09;
p50 = -6.311e-06;
p41 = -0.0006353;
p32 = -0.01747;
    end
end

Fpressure = (p00 + p10*x + p01*y + p20*x^2 + p11*x*y + p02*y^2 + p30*x^3 + ... + p21*x^2*y + p12*x*y^2 + p40*x^4 + p31*x^3*y + p22*x^2*y^2 + ... + p50*x^5 + p41*x^4*y + p32*x^3*y^2);
else
end
if Zeta<=0
    Fpf = Acyl*rho*(LC*AcPiston + L21*g);
else
    Fpf = Vaf*rho*(g + AcPiston);
end
if VelPiston>0
else
    Fpf = 0;
end

% HYDRODYNAMIC FORCE
Fhydro = Fpressure - Fpf*cos(Zeta);
end

B.2. 1st improvement hypothesis model

Although the structure of this model is the same as the shown in figure B.1, and the 'Fluid interaction' block is also implemented as in figure B.2, some differences appear in the 'Pressure Force' and 'Hydrodynamic Force' MATLAB Function blocks, regarding the basic model.

In the first one, only the values of the coefficients \( p_{ij} \) change, so there is no need to show the code again. These values are presented in appendix A, figures A.4 and A.5.

The code of the 'Hydrodynamic Force' MATLAB Function block of this model is:

```matlab
function Fhydro=HydrodynamicForce(Zpiston,VelPiston,AcPiston,Fpressure,Zeta)

% PARAMETERS
rho = 1000;  % Water density (Kg/m^3)

% CYLINDER
Rcyl = 0.1;  % Cylinder radius (m)
Acyl = pi*Rcyl^2/2;  % HALF Cylinder area (m^2)
L21=80-Zpiston;  % Pumped water above piston
LC=100;  % Pumped water column
Zeta = Zeta*pi/180;  % Change degrees to radians

% MOVING VOLUME
Vaf = 0.01187/( Zeta^4-0.5844*Zeta^3+ 0.1672 *Zeta^2+ -0.007068*Zeta+ 0.009709);
if Vaf>1.25
    Vaf=1.25;
elseif Vaf<0
    Vaf=0;
else
end
if Zeta<=0
    Vmf = Vaf+Acyl*(LC-L21);
else
    Vmf = Vaf;
end

% FLUID INERTIA
if VelPiston>0
    Finertia = -Vmf*rho*AcPiston;
else
    Finertia = 0;
end
```
B.3. 2nd improvement hypothesis model

Again, the comprehensive SIMULINK model follows the same structure shown in figure B.1. In this case, the 'Fluid interaction' block is changed in order to introduce the new definition of the flow speed. This subsystem is implemented as follows:

As commented in section 5.3, the pressure force is redefined to be the same as in the basic model, so the 'Pressure Force' MATLAB Function block contains the same code shown in the explanation of the basic model. Only the input is changed, replacing the piston velocity with the flow speed.

In figure B.3, it can be observed that a new MATLAB Function block, named 'Fluid Velocity', is introduced in the subsystem. This block aims to define the fluid velocity described in the 2nd improvement hypothesis model description, using the following code:

```matlab
function VelFluid = FluidVelocity(Zpiston,VelPiston,Zeta)
Rflap = 0.1; % Flap radius (m)
L21 = 80-Zpiston; % Water column above piston (m)
Acyl = pi*Rflap^2/2; % Cylinder area above one flap (m^2)
Zeta = Zeta*pi/180; % Change degrees to radians
Vaf = 0.01187/(( Zeta^4-0.5844*Zeta^3+ 0.1672 *Zeta^2+ -0.007068*Zeta+ 0.009709);
if Vaf>1.25
    Vaf=1.25;
end
```
elseif Vaf<0
    Vaf=0;
else
end
if VelPiston>0
    VelFluid=VelPiston*Vaf/(L21*Acyl);
else
    VelFluid=0;
end

The content of the MATLAB Function used for the calculation of the hydrodynamic force is:

```matlab
function Fhydro=HydrodynamicForce(Zpiston,VelPiston,AcFluid,Fpressure,Zeta)
% PARAMETERS
g = 9.81;
rho = 1000; % Water density (Kg/m^3)
% Cylinder
Rcyl = 0.1; % Cylinder radius (m)
Acyl = pi*Rcyl^2/2; % HALF Cylinder area (m^2)
L21=80-Zpiston; % Water column above piston (m)
LC=100; % Cylinder length (m)
Zeta = Zeta*pi/180; % Change degrees to radians
% PUMPED FLUID
Vaf = 0.01187/( Zeta^4-0.5844*Zeta^3+0.1672*Zeta^2-0.007068*Zeta+0.009709);
if Vaf>1.25
    Vaf=1.25;
elseif Vaf<0
    Vaf=0;
else
end
if Zeta<=0
    Fpf = Acyl*rho*(LC*AcFluid + L21*g);
else
    Fpf = Vaf*rho*(g + AcFluid);
end
if VelPiston>0
else
    Fpf = 0;
end
% HYDRODYNAMIC FORCE
Fhydro = Fpressure - Fpf*cos(Zeta);
end
```
BIBLIOGRAPHY


