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ENVELOPE/PHASE REPRESENTATION IN SIGNAL MODELING

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ABSTRACT. In this paper authors show the importance of envelope and instantaneous frequency in analysis and synthesis problems. Signal representation in envelope and instantaneous phase is introduced. Analogy between this representation and modulus/phase of the Fourier Transform is shown in order to prove that well known spectral analysis techniques can be used in envelope parametric representation. Instantaneous phase is sampled not uniformly.

I. INTRODUCTION

Signal characterization in frequency domain have been widely studied by means of Fourier Analysis. Many efforts have been devoted to eliminate redundant information. Considering the Fourier Transform of a real signal as a complex function, two possible representation can be used: modulus/phase or real and imaginary parts. The first mentioned election is the most used as for example in filter design.

In the same way it's possible to characterize a real signal by means its envelope and instantaneous phase. This two signals are obtained from the analytic signal. This representation permits to obtain an analogy between time domain and frequency domain.

In this paper, authors study the properties of this decomposition. Particulary, envelope and instantaneous phase of real and simulated signals are analyced.

Thinking in bandwith reduction, this representation permits to apply parametrics methods of spectral analysis in envelope parametrization of real signals.

This paper is organized as follows: Section II presents basic concepts involved in analytic signal, envelope, and instantaneous phase; Section III reports the relationships between parametric methods of spectral estimation and envelope parametric estimation. Section IV studies instantaneous phase and finally, in Section V, some results are introduced.

II. ANALYTIC SIGNAL

The analytic signal $a_x(t)$ of a real signal $x(t)$ is defined as:

$$a_x(t) = x(t)+j\,h_x(t) \qquad (1)$$

Being $h_x(t)$ the Hilbert Transform of $x(t)$

$$h_x(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(t')}{(t-t')}\,dt' \qquad (2)$$

The Fourier Transform $A_x(w)$ of the analytic signal is:

$$A_x(w) = \begin{cases} 2X(w) & w>0 \\ 0 & w<0 \end{cases} \qquad (3)$$

$A_x(w)$ is a "causal" signal in w and the positive part of the spectrum is proportional to $X(w)$. This property will be very usefull as we'll show forward.

The analytic signal can be expressed in modulus-argument form:

$$a_x(t) = e_x(t)\,\exp(j\phi_x(t)) \qquad (4)$$

Being $e_x(t)$ and $\phi_x(t)$, the envelope and instantaneous phase of the signal $x(t)$.

$$e_x^2(t) = |a_x(t)|^2 = x^2(t)+h_x^2(t) \qquad (5)$$

$$\phi_x(t) = \tan^{-1}(h_x(t)/x(t)) = \qquad (6)$$

$$= \text{phase of } a_x(t)$$

III. PARAMETRIC METHODS IN ENVELOPE REPRESENTATION

The envelope definition as modulus of the analytic signal, permits to establish an analogy between the square of the envelope and the periodogram in time domain. From a known $A_x(w)$ it's possible to obtain the envelope in the same way as periodogram is obtained from a given data signal

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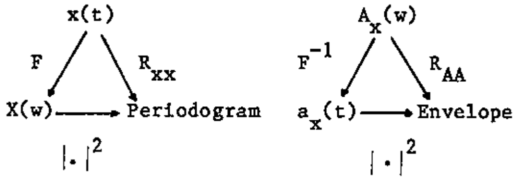

The question is if the well known spectral analysis techniques can be applied over $A_x(w)$ to obtain a parametric representation of a signal envelope.

Suppose a windowed signal $x(n)$ $0\leqslant n\leqslant N-1$ is given, and also is available the sequence $A_x(k)$ $0\leqslant k\leqslant N-1$.

Applying linear prediction over the sequence $A_x(k)$ we design a linear predictor of coefficients $c(q)(q=1...Q)$ minimizing the mean square error E.

The linear predictor is:

$$\tilde{A}_x(k) = -\sum_{q=1}^{Q} c(q)\,A_x(k-q) \qquad (7)$$

Being the quantity to minimize

$$E = \sum_{N_1}^{N_2} (A_x(k)-\tilde{A}_x(k))^2 \qquad (8)$$

The procedure to solve this problem depends on N_1 and N_2 election. In this work correlation method has been chosen. As a consequence of Levinson's Algorithm using, a smoothed envelope version is obtained. Any other methods could had been chosen to estimate $c(q)$.

Once the coefficients $c(q)$ have been obtained, the error will be:

$$\varepsilon(k)=A_x(k)-\tilde{A}_x(k)=\sum_{q=0}^{Q} c(q)\,A_x(k-q);\,c(0)=1 \qquad (9)$$

This equation is equivalent to suppose $\varepsilon(k)$ as the convolution between the sequence $c(q)$ $(0\leqslant q\leqslant Q)$ and the sequence $A_x(k)$ $(0\leqslant k\leqslant N-1)$.

$$\varepsilon(k) = c(k) * A_x(k) \qquad (10)$$

Taking inverse Fourier Transform:

$$e(n) = \beta(n).a_x(n) \qquad (11)$$

where:

$$\beta(n)=N\left[1+\sum_{q=1}^{Q} c(q)\exp(j(2\pi/N)nq)\right] \qquad (12)$$

Supposing $e(n)$ a white noise sequence with average power ko, the following envelope estimator is obtained:

$$e_x^2(n) = |a_x(n)|^2 = k_o/|\beta(n)|^2 \qquad (13)$$

The total error is, applying the Parseval theorem:

$$E = \sum_{k=0}^{N-1} |\varepsilon(k)|^2 = \frac{1}{N}\sum_{m=0}^{N-1} |e(n)|^2 \qquad (14)$$

Substituing (11) and (13) in (14)

$$E = \frac{k_o}{N}\sum_{m=0}^{N-1} \frac{e_x^2(n)}{\hat{e}_x^2(n)} \qquad (15)$$

That means, minimize the error is equivalent to minimize the sum between the quotien of the envelope $e_x(n)$ and its approximation $\hat{e}_x(n)$.

As a consequence can be observed:

a) Global adaptation of the envelope: As the contributions to the total error are determined by a quotien, the adaptation is uniform over the all segment, its so good in the hight energy zones as in the low energy ones.

b) Local adaptation: From (15), the minimum error is k_o and:

$$\frac{1}{N}\sum_{n=0}^{N-1} \frac{e_x^2(n)}{\hat{e}_x^2(n)} = 1 \qquad (16)$$

(16) indicates that the arithmetic mean of $e_x^2(n)/\hat{e}_x^2(n)$ is equal to 1, and as a consequence, will be values of this quotien bigger or lower than 1 and in average equal to one.

That means that $\hat{e}_x(n)$ will be bigger than $e_x(n)$ in some parts and lower in others to (16) follows. The contribution to the total error is more significant when $e_x^2(n)$ is bigger than $\hat{e}_x^2(n)$ than when is lower and as a consequence, we hope after error minimization, the parametric envelope be more similar to the original when $e_x(n)$ is bigger than $\hat{e}_x(n)$ (in average).

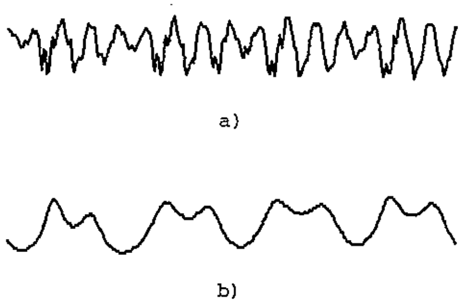

Fig.1. a) Voiced speech signal, b) parametric envelope.

The estimator obtained in (14) have been used in speech signals with very promissing results Fig. 1a) shows a 32 msec voiced segment, Fig 1b) shows the parametric envelope with 12 coefficients.

IV. INSTANTANEOUS PHASE

In the above section a parametric version of a signal envelope have been obtained. The correlation method chosen to solve the problem assumes an instantaneous phase asociated to the envelope that is the minimum phase. With the parameters founded it's possible to recover a signal whose envelope coincides with the signal envelope but with a minimum phase associated.

In order to recover the original signal, it's necessary to send not only the parametric envelope but also its instantaneous phase.

Instantaneous phase can be separated in three terms:

$$\phi_x(t)=\theta_{min}(t)+w_o t+\theta(t) \qquad (17)$$

where $\theta_{min}(t)$ is the minimum phase associated to the signal envelope, $w_o t$ is a linear phase term and $\theta(t)$ is a phase term associated to an analytic signal with unity modulus.

Paley Wiener theorem states that the minimum phase associated to an envelope $e_x(t)$ is given by the equation:

$$\theta_{min}(t)= \frac{2t}{\pi}\int_0^{\infty} \frac{\ln e_x(\tau)}{\tau^2-t^2}\, d\tau \qquad (18)$$

Some easy manipulatins give:

$$\theta_{min}(t)= \frac{1}{T}\int_{-\infty}^{\infty} \frac{d \ln e_x(\tau)}{dy} \ln \left|\frac{e^y+1}{e^y-1}\right| dy \quad (19)$$

Where $y=\ln(\tau/t)$.

Minimum phase is a pondered version of the derivative of $\ln e_x(\tau)$. Supossing that the ponderation coefficient tends to infinite in the interest instant and decay exponentially to zero for $y \to \infty$, it's possible to suppose

$$\theta_{min}(t) \cong \frac{1}{e_x(t)} \frac{d\, e_x(t)}{dt} .t \qquad (20)$$

This approximation minimum phase have been used to sample no uniformly the unwrapped instantaneous phase.

This method permits to reduce the phase sampling rate under 1/8 of the original with a mean square error normalized lower than 10%.

In order to reduce the total error between the original signal and the recovered one, an analogous procedure could be applyed over the error signal. In this case few parameters are needed to represent the envelope of the error.

Fig 2) shows the synthesized signal obtained whith this method. The original signal is shown in Fig 1a) and the envelope used in the synthesis is in Fig 1b).

Fig.2. Synthesized signal.

V. CONCLUSIONS

Main contribution of this work is to show the importance of associated functions as envelope and instantaneous phase of a given signal.

Most of the works related with this functions are dedicated to modulated signals. Authors believe that this functions should be studied under this point of view because of the intrinsic information that envelope and instantaneous phase have.

In this paper is reported how envelope is suitable for parametric methods which are familiar in spectral estimation problems.

Instantaneous phase is also studied and adequately sampled in order to recover properly the synthesized signal.

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