

IMPROVED ML CROSS-SPECTRAL ESTIMATION

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ABSTRACT

This work deals with the use of dedicated filters for cross-spectrum estimation.

Basically, the ML cross-spectral estimator can be obtained as the natural extension of the Normalized Maximum Likelihood procedure, reported previously by the authors, to the measurement of cross-power density for two data registers $x(n)$ and $y(n)$.

As an important improvement in present cross-spectrum estimation, the importance of the selection of the cross-correlation matrix estimator used as a starting point is included.

1. INTRODUCTION

The Maximum Likelihood cross-power spectral density estimator is derived from the explanation of the ML procedure in the one channel case as a filter-bank analysis procedure.

From this point of view, an estimation of the power level of a single channel $x(n)$ at frequency ω_0 is obtained from the power of the output of a bandpass filter centered at frequency $\omega_0/1/$.

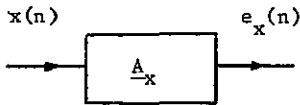


Fig. 1. Diagram of a ML filter in the one channel case.

Fig. 1 depicts the diagram of an ML filter centered at frequency ω_0 with impulsional response represented by the vector \underline{A}_x and whose design can be summarized as follows:

Objective: Minimize $P_e = E |e_x^2(n)| =$
 $= \underline{A}_x^T \underline{R}_{xx} \underline{A}_x$ (1)

Constraint: $\underline{A}_x^T \underline{S} = 1$ (2)

where \underline{S} is the steering vector

$$\underline{S}^T = [1, \exp(-j\omega_0), \dots, \exp(-jQ\omega_0)] \quad (3)$$

$\underline{R}_{xx} = E \{ \underline{X} \underline{X}^T \}$ is the data autocorrelation matrix and $\underline{X}^T = [x(n), x(n-1), \dots, x(n-Q)]$ is the data vector.

By minimizing objective (1) subject to constraint (2) we obtain the following solution for \underline{A}_x :

$$\underline{A}_x = \frac{\underline{R}_{xx}^{-1} \underline{S}}{\underline{S}^T \underline{R}_{xx}^{-1} \underline{S}} \quad (4)$$

The resulting power-level estimate of signal $x(n)$, $\hat{P}_x(\omega)$, is obtained by replacing (4) in the expression (1),

$$\hat{P}_x(\omega) = \frac{1}{\underline{S}^T \underline{R}_{xx}^{-1} \underline{S}} \quad (5)$$

In order to provide a spectral density estimation, we need to compute the effective bandwidth of the filter. As can be seen in /2/ and /3/, assuming that $S_x(\omega)$ is approximately flat over the filter bandwidth, the following expression for the spectral density estimate $\hat{S}_x(\omega)$ is finally obtained:

$$\hat{S}_x(\omega) = \frac{\underline{S}^T \underline{R}_{xx}^{-1} \underline{S}}{\underline{S}^T \underline{R}_{xx}^{-2} \underline{S}} \quad (6)$$

It is important in order to explain the resulting performance that (6) can be viewed as a signal to noise ratio, as can be seen in /2/.

Once we have explained how ML procedure works in the one channel case, in section 2 we

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extend this method to the two channel case.

In section 3 attention is devoted to the estimation of the correlation matrices that appear in the expression of the resulting estimate. Sections 4 and 5 are devoted to conclusions and results which show the good performance of the procedure.

2. CROSS-SPECTRAL ESTIMATION

The new cross-spectrum estimate is obtained, as in the DFT technique, deriving two data dependent F.I.R. filters A_x and A_y and analyzing the cross-correlation between the output residuals $e_x(n)$ and $e_y(n)$.

By following the scheme shown in figure 2, it is clear that the input cross-power density verifies expression (7):

$$P_{e_x e_y} = E |e_x^*(n)e_y(n)| = \frac{1}{2\pi} \int_{-\pi}^{\pi} S_{xy}(w) A_x^*(w) A_y(w) dw \quad (7)$$

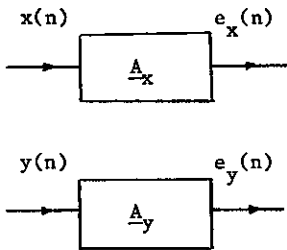


Fig. 2. Basic diagram in the two channel problem.

Where $A_x(w)$ and $A_y(w)$ are the associated transfer responses for both filters with impulsive responses represented by the vectors A_x and A_y .

From this point of view, it can be concluded that a narrow band design for A_x and A_y with central frequency w_0 will produce different cross-correlation between both residuals depending on the central frequency.

Thus we can rewrite (7) as

$$P_{e_x e_y} = \frac{1}{2\pi} \int_{-\pi}^{\pi} S_{xy}(w) A_x^*(w_0, w) A_y(w_0, w) dw \quad (8)$$

At this moment, using the narrow band feature of the analysis filters, we may suppose that $P_{e_x e_y}$ is an estimation of the cross-power level of the inputs at frequency w_0 , $\hat{P}_{xy}(w_0)$, and we can also assume that $S_{xy}(w)$ is flat enough in the common band-pass to write,

$$\hat{P}_{xy}(w_0) = \hat{S}_{xy}(w_0) \frac{1}{2\pi} \int_{-\pi}^{\pi} A_x^*(w_0, w) A_y(w_0, w) dw \quad (9)$$

or, using the Parseval relationship

$$\hat{S}_{xy}(w_0) = \hat{P}_{xy}(w_0) / A_x^T A_y \quad (10)$$

Any band-pass design can be used in order to obtain filters A_x and A_y . From these possible choices we select the filters designed as in the ML procedure in the one channel case explained in section 1, this is:

$$A_x = \frac{S_{R_{xx}}^T - 1}{S_{R_{xx}}^T - 1} S \quad (11)$$

$$A_y = \frac{S_{R_{yy}}^T - 1}{S_{R_{yy}}^T - 1} S \quad (12)$$

where R_{xx} and R_{yy} are the autocorrelation matrices for $x(n)$ and $y(n)$.

By using (11) and (12), both the estimators for the cross-power level and the cross-power density are obtained as,

$$\hat{P}_{xy}(w) = \frac{S_{R_{xx}}^T - 1 \quad R_{R_{xy}} \quad R_{R_{yy}} - 1 \quad S}{(S_{R_{xx}}^T - 1) S (S_{R_{yy}}^T - 1) S} \quad (13)$$

$$\hat{S}_{xy}(w) = \frac{S_{R_{xx}}^T - 1 \quad R_{R_{xy}} \quad R_{R_{yy}} - 1 \quad S}{S_{R_{xx}}^T - 1 \quad R_{R_{yy}} - 1 \quad S} \quad (14)$$

where $R_{R_{xy}} = E |X Y^T|$ is the cross-correlation matrix between $x(n)$ and $y(n)$.

It can be shown that, when both inputs $x(n)$ and $y(n)$ are sinusoids of the same frequency in additive gaussian noise, estimator (13) is the maximum likelihood estimator of the cross-power of the two sinusoids.

Also, note that estimator (14) is like a quotient which compares the cross-correlation matrix of the two data records against the case of a common white noise input (i.e. $R_{R_{xy}} = I$ in such a case).

One very important aspect in obtaining a good behaviour for (13) and (14) is the estimator used in the computation of correlation matrices $R_{R_{xx}}$, $R_{R_{yy}}$ and $R_{R_{xy}}$ which appear in both

expressions. This point is dealt with in the next section.

3. NON-PARAMETRIC ESTIMATORS FOR THE CORRELATION MATRICES

In order to evaluate the ML cross-spectral estimators, to compute the $R_{R_{xx}}$, $R_{R_{yy}}$ and $R_{R_{xy}}$

matrices is needed, that is, we have to estimate the $Q+1$ values of the autocorrelation functions of $x(n)$ and $y(n)$, $R_{xx}(q)$ and $R_{yy}(q)$ for $q=0$ to Q , involved in \underline{R}_{xx} and \underline{R}_{yy} and the

$2Q-1$ values of the cross-correlation function, $R_{xy}(q)$ for $q=-(Q-1)$ to $Q-1$, that appear in \underline{R}_{xy} .

There are many well-known non-parametric procedures to estimate the correlation functions, such as: the Blackman-Tukey approach (B-T) /4/, the Weighted, Overlapped, Segment Averaging algorithm (W.O.S.A.) /5/ and the Short-Time Unbiased Spectrum Estimation algorithm (S.T.U.S.E.) /6/, /7/.

The first idea to compute these functions was to use of the Blackman-Tukey algorithm due to its computational simplicity. But, results were not as good as we expected.

Using the S.T.U.S.E. algorithm, estimators of the correlation functions can be obtained, as is shown in /6/, that are unbiased and have less variance than the obtained using B-T-approach.

Results using the S.T.U.S.E. algorithm in the computation of the cross-correlation function utilized as a starting point in the ML cross-spectrum estimator are shown in the next section and it can be seen that its behaviour is better than the obtained using the B-T approach.

4. SIMULATION EXAMPLES

The performance of the reported procedure was checked out by means a simple simulation example.

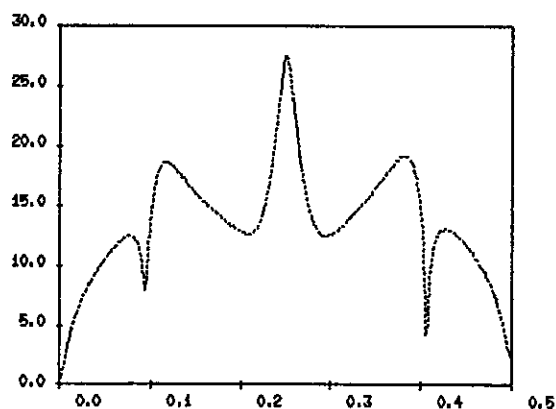


Fig. 3. ML cross-power density estimator using B-T in the computation of correlation matrices.

Two data records of 32 samples each were under consideration. Record $x(n)$ consists in two

real sinusoids with normalized frequencies of 0.1 and 0.25 buried in white gaussian noise with a signal to noise ratio of 20 dB. each. Record $y(n)$ was essentially of the same structure but with normalized frequencies of 0.25 and 0.4.

Fig. 3 shows the ML cross-power density estimator using 10 order filters and matrices \underline{R}_{xx} , \underline{R}_{yy} and \underline{R}_{xy} calculated using the

Blackman-Tukey algorithm.

Fig. 4 shows the same estimator using in the estimation of the cross-correlation matrix \underline{R}_{xy} the S.T.U.S.E. algorithm.

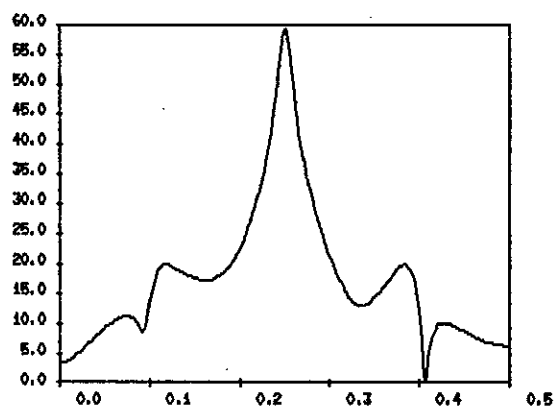


Fig. 4. ML cross-spectral estimator using S.T.U.S.E. to calculate \underline{R}_{xy} matrix.

Finally, Fig. 5 shows the cross-power density estimator obtained using directly the S.T.U.S.E. approach.

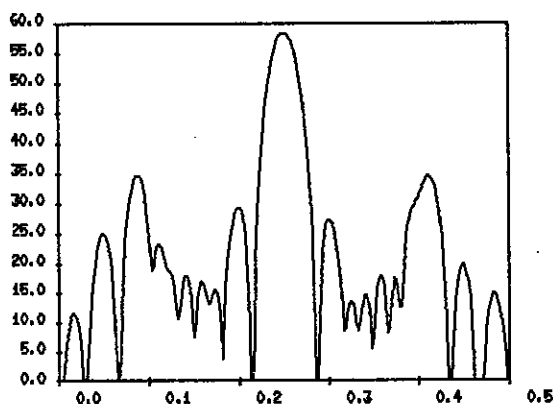


Fig. 5. S.T.U.S.E. cross-power density.

We can see the good performance as to resolution and side lobe level of the ML cross-spectral estimate using in the computation of matrix \underline{R}_{xy} the S.T.U.S.E. approach.

5. CONCLUSIONS

It has been reported in this paper how ML filters can be used in cross-spectrum estimation problems to obtain both cross-power levels and cross-spectral density, with the same degree of quality we get in auto-spectrum analysis.

The potential of ML filters in this kind of problems allows the design of spectrum analyzers, to obtain auto-spectral and cross-spectral density, auto and cross-power levels, time delay estimation, coherence, signal to noise ratio and all the functions of interest in multichannel time series analysis.

The importance of the selection of the correlation functions used as a starting point in the ML procedure has been shown and the good results obtained using the S.T.U.S.E. algorithm has been presented.

6. REFERENCES

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