Design and Implementation of Cycle Slip Detectors for Dual-frequency GNSS Signals

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A mia moglie i a la meva família,
che sono els que resolen
mis verdaderos problemas
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Current satellite navigation systems provide high accuracy positioning by using the most precise ranging information, which is the carrier phase observable. Unfortunately, high dynamics, shadowing and multipath, among others, may cause cycle slips, which represents a jump of the carrier phase observable by an integer number of wavelengths. Any cycle slip, if remained undetected, would deteriorate the high ranging and positioning accuracy. Therefore, before the carrier phase observable can be utilized, the cycle ambiguity must be resolved.

The main objective of this master’s thesis is to design a cycle slip detector for dual-frequency Global Navigation Satellite Systems (GNSS) signals. The results of the research have been implemented in ANSI C within the reference tool of the research group of Astronomy and Geomatics (gAGE), which is called GNSS-Lab (gLAB), developed under the European Space Agency contract No. P1081434. Furthermore, the detector must be able to work in real-time.

An important limitation, regarding data holes, in gLAB’s data structure has been fixed. The single-frequency cycle slip detector has been adapted to the new data structure and improved. The dual-frequency cycle slip detector has been based on both Melbourne-Wübbena and geometry-free combinations, in which several novel features have been included.

The work done in this thesis has been validated by comparing the behaviour of the upgraded gLAB with the original one. Moreover, it has been used real data from geo-referenced stations, which permit calculating the actual positioning error during the entire process. All data analyzed during the validation process cover the year 2014, which is the year of last maximum solar activity. It is worth mentioning that the original gLAB has been used by the group in several publications, university lectures and research projects. Moreover, to ensure the correct behaviour of the upgraded gLAB tool and to exclude any undesired operation due to the new implementation, several tests have been performed.

This master’s thesis has been developed under the European Space Agency contract No. 4000113054/14/NL/HK.
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# Table of Contents

INTRODUCTION ................................................................. 1

CHAPTER 1. STATE OF THE ART .......................................... 3
1.1 Overview ................................................................. 3
1.2 Geometry-free and Melbourne-Wübbena ............................ 4
1.3 Summary of Cycle Slip Detection Techniques .................... 5

CHAPTER 2. BASIC CONCEPTS .............................................. 7
2.1 Overview ................................................................. 7
2.2 GNSS Signals .......................................................... 7
   2.2.1 GPS Signals ..................................................... 7
   2.2.2 GLONASS Signals ............................................. 10
   2.2.3 Galileo Signals ................................................. 12
   2.2.4 BeiDou Signals ................................................. 15
2.3 GNSS Measurements and Data Preprocessing .................... 16
   2.3.1 Combinations of GNSS Measurements ...................... 18
2.4 Carrier Phase Cycle Slip ............................................. 22
   2.4.1 Single-frequency Cycle Slip Detector ....................... 22
   2.4.2 Dual-frequency Cycle Slip Detector ......................... 23
2.5 Carrier Smoothing of Code Pseudoranges ......................... 26
   2.5.1 Code–Carrier Divergence Effect: Single-frequency Smoothing 27
2.6 Solving Navigation Equations ....................................... 29
2.7 Dilution of Precision .................................................. 31

CHAPTER 3. IMPLEMENTATION OF CYCLE SLIP DETECTORS ... 33
3.1 Overview ............................................................... 33
3.2 General Flowchart of the Cycle Slip Detectors Implemented in gLAB 33
3.3 Data Gaps ............................................................. 36
   3.3.1 Original gLAB .................................................. 36
List of Figures

CHAPTER 2. BASIC CONCEPTS

2.1 Spectra of GPS signals before (top) and after modernisation (bottom). Courtesy of Stefan Wallner. .................................................. 10
2.2 Spectra of GLONASS signals. Legacy FDMA signals before and after modernisation (top), and new CDMA signals after modernisation (bottom). Courtesy of Stefan Wallner. .................................................. 11
2.3 Spectra of Galileo signals. Courtesy of Stefan Wallner. .......................................................... 13
2.4 Spectra of BeiDou signals: Phase II (top) and Phase III (bottom). Courtesy of Stefan Wallner. .......................................................... 15
2.5 Determination of the signal travel time. .......................................................... 17
2.6 GPS code and carrier phase measurement features. The geometry-free combinations of code \((R_P^2 - R_P^1)\), in green, and carrier \((\Phi_{L1} - \Phi_{L2})\), in blue, are plotted as function of time for a given satellite. .................................................. 18
2.7 Effect of one-cycle jump in the GPS \(\Phi_1\) carrier phase signal on the ionosphere-free combination. The horizontal axis is seconds of day; the vertical axis is in metres. .......................................................... 24
2.8 Effect of one-cycle jump in the GPS L1 signal in the \(\Phi - R\) (left) and MW (right) combination (raw measurements without smoothing). Vertical axes are in cycles of \(\lambda_1 \approx 19\) cm (left) and \(\lambda_w \approx 86\) cm (right). .......................................................... 26
2.9 Effect of 100 s smoothing during the Halloween storm. The left-hand plot shows the C1–carrier smoothing using equation (2.14), in red (single-frequency smoother). The raw measurements are shown in green. STEC is depicted in the right-side plot. As is shown, the larger temporal ionospheric gradients lead to larger code–carrier divergence-induced error in the single-frequency smoothed solution, which reaches up to about 8 m in this example. .......................................................... 28
2.10 Geometric concept of GNSS positioning: Equations are linearised about the approximate receiver coordinates \((x_0, y_0, z_0)\). The correction \((\Delta x, \Delta y, \Delta z)\) is estimated after solving the navigation equations (2.25). .......................................................... 30
2.11 The DOP effect in positioning: 2D illustration of the variation of the uncertainty region with geometry. .......................................................... 31
2.12 The measurement noise \(\varepsilon\) is translated to the position estimate as an uncertainty region. .......................................................... 31

CHAPTER 3. IMPLEMENTATION OF CYCLE SLIP DETECTORS

3.1 General flowchart of the cycle slip detector implemented in gLAB. .................................................. 34
3.2 Example of measurement tracking lost by a receiver. .......................................................... 35
3.3 Worst case scenario for original gLAB’s data structure. No change in satellites in view after 1 hour of data gap. .......................................................... 36
3.4 Flowchart of the single-frequency cycle slip detector implemented in gLAB. .................................................. 39
3.5 Flowchart of the dual-frequency L1 cycle slip detector implemented in gLAB. .................................................. 41
3.6 Outlier declared (left) and cycle slip detected in the next epoch (right). .................................................. 42
3.7 LI threshold as function of $\Delta t$, with $a_0 = 0.08$ m and $T_0 = 60$ s. 43
3.8 Differences between original and upgraded gLAB's LI detector. 44
3.9 Flowchart of the dual-frequency MW cycle slip detector implemented in gLAB. 45
3.10 Difference between the accumulated mean and its $\bar{m}$ (blue) and the mean$_{300}$ and its $\bar{m}_{300}$ using the 300 s sliding window (black). 46

CHAPTER 4. RESULTS AND VALIDATION

4.1 Effects in data structure of a data gap. 49
4.2 RMS, arithmetic mean and 95th percentile of the 3D positioning error for station CFRM during the year 2014. 50
4.3 NEU positioning errors for station CFRM during the DoY 114 of 2014. 51
4.4 RMS, arithmetic mean and 95th percentile of the 3D positioning error for station IZAN during the year 2014. 52
4.5 NEU positioning errors for station IZAN during the DoY 45 of 2014. 53
4.6 NEU positioning errors for station IZAN during the DoY 110 of 2014. 54
4.7 RMS, arithmetic mean and 95th percentile of the 3D positioning errors for station TRDS during the year 2014. 55
4.8 NEU positioning errors for station TRDS during the DoY 202 of 2014. 56
4.9 NEU positioning errors for station OUS2 during the DoY 1 of 2014. 58
4.10 RMS, arithmetic mean and 95th percentile of the 3D positioning error for station CFRM during the year 2014. 59
4.11 NEU positioning errors for station CFRM during the DoY 74 of 2014. 60
4.12 NEU positioning errors for station CFRM during the DoY 210 of 2014. 61
4.13 NEU positioning errors for station CFRM during the DoY 293 of 2014. 62
4.14 NEU positioning errors for station CFRM during the DoY 346 of 2014. 63
4.15 RMS, arithmetic mean and 95th percentile of the 3D positioning error for station IZAN during the year 2014. 64
4.16 NEU positioning errors for station IZAN during the DoY 157 of 2014. 65
4.17 NEU positioning errors for station IZAN during the DoY 75 of 2014. 66
4.18 NEU positioning errors for station IZAN during the DoY 300 of 2014. 67
4.19 NEU positioning errors for station IZAN during the DoY 353 of 2014. 68
4.20 RMS, arithmetic mean and 95th percentile of the 3D positioning error for station TRDS during the year 2014. 69
4.21 NEU positioning errors for station TRDS during the DoY 247 of 2014. 70
4.22 NEU positioning errors for station TRDS during the DoY 275 of 2014. 71
4.23 NEU positioning errors for station TRDS during the DoY 360 of 2014. 72
4.24 NEU positioning errors for station COCO during the DoY 90 of 2014. 74
# List of Tables

## CHAPTER 1. STATE OF THE ART

1.1 Summary of Cycle Slip Detection Techniques. ........................................... 5

## CHAPTER 2. BASIC CONCEPTS

2.1 Current and new GPS navigation signals. The civil signals are provided free of charge to all users worldwide (Open Services). ........................................... 10
2.2 Legacy GLONASS signal structure. ............................................................... 12
2.3 Galileo navigation signals. The two signals located in the E5a and E5b bands are modulated onto a single E5 carrier frequency of 1191.795 MHz using the Alternate Binary Offset Carrier (AltBOC) technique: AltBOC(15,10). .... 14
2.4 BeiDou Phase II navigation signals. Quadrature Phase-Shifted Keying (QPSK) and BPSK modulation schemes are applied. ............................................. 15
2.5 BeiDou Phase III navigation signals. ............................................................. 16
2.6 Wide and narrow-lane combinations of signals for different frequencies of GPS, GLONASS (only the channel $k = 0$ is given for G1 and G2 signals) and Galileo. The Galileo E5 and E6 signals have not been included to simplify the table. ........ 21
2.7 Computational scheme of differences: a jump in amplitude $\varepsilon$ happens at time $t_4$ and its effect is propagated and amplified by the $n$th-order differences. .... 24

## CHAPTER 4. RESULTS AND VALIDATION

4.1 Station used during the validation of the modifications done in data structure. ... 48
4.2 List of stations used during the validation of the single-frequency detector. .... 48
4.3 Best- and worst-case scenarios for IZAN during year 2014. ......................... 53
4.4 Summary of the Overall Results of the Single-frequency Cycle Slip Detector. .. 57
4.5 Station used during the validation of the single-frequency detector using a file with a data interval of 30 seconds. ...................................................... 58
4.6 Best- and worst-case scenarios for CFRM during year 2014. ....................... 60
4.7 Best- and worst-case scenarios for IZAN during year 2014. ....................... 65
4.8 Best- and worst-case scenarios for TRDS during year 2014. ....................... 70
4.9 Summary of the Overall Results of the Dual-frequency Cycle Slip Detector. .... 73
4.10 Station used during the validation of the dual-frequency detector using a file with a data interval of 30 seconds. ...................................................... 74
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INTRODUCTION

Current satellite navigation systems provide high accuracy positioning by using the most precise ranging information, which is the carrier phase observable. Unfortunately, some phenomena may cause cycle slips, that if remained undetected, would deteriorate the high ranging and positioning accuracy.

This master’s thesis has been developed under the European Space Agency contract No. 4000113054/14/NL/HK. The aim of this work is to improve gLAB [1] in matters of cycle slip detection. gLAB is a GNSS software tool developed under European Space Agency contract No. P1081434 by gAGE research group from the Universitat Politècnica de Catalunya (UPC). It is a multipurpose GNSS Data Processing tool for professional and educational applications, which performs precise modeling of GNSS observables (pseudorange and carrier phase) at the centimetre level, allowing both standalone and precise GPS positioning. Every single error contributor can be assessed independently, which, in turn, provides wide capability for Data Processing and a major educational benefit. gLAB is adapted to a variety of standard formats like RINEX, SP3, ANTEX and SINEX files, among others.

There are three primary aims of this master’s thesis. The first is to fix a current limitation in gLAB, which resides in the data structure in handling data holes. The second objective is to adapt the single-frequency cycle slip detector to the new data structure. The third is to design and implement a new cycle slip detector for dual-frequency GNSS signals. Furthermore, the detectors must be written in ANSI C and must be able to work in real-time.

The first chapter of this master’s thesis introduces the state-of-the-art in the topic of dual-frequency cycle slip detectors.

The second chapter explains the theoretical concepts required to well-understand the cycle slip issue. Furthermore, it presents the current status of GPS, GLONASS, Galileo and BeiDou signals, as well as the possible combinations between their measurements. Finally, the carrier smoothing is also introduced in this chapter with the aim to understand the validation of the single-frequency cycle slip detector.

The third chapter introduces the implementation performed in this master’s thesis. It presents the modifications done in the gLAB’s data structure. Moreover, the entire process of the cycle slip detector is presented through a flow chart diagram, the modifications done in the single-frequency detector are explained in detail as well as the design and implementation of the dual-frequency cycle slip detector.

The validation of the implementation performed in this thesis is presented in the fourth chapter by comparing the results of the upgraded gLAB with the provided by the original gLAB and also with the results provided by an independent software. It has been used real data from several stations during the year 2014, which is the year of last maximum solar activity. Moreover, to ensure the correct behaviour of the upgraded gLAB several tests has been performed.

Finally, the fifth chapter is dedicated to the general conclusions of this works, important aspects of this thesis and possible improvements for the future.
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Chapter 1

STATE OF THE ART

1.1 Overview

The cycle slip detection is being a matter of research in Global Navigation Satellite Systems since the late eighties. The studies can be classified in several categories based on the data used: number of receivers, extra hardware like Inertial Navigation Systems (INS), and number of signals utilized. Additionally, other categories can appear if the detector is not designed to work in real-time.

Cycle slip detectors that use double-differencing (DD) techniques take advantage of the smooth variation of the receiver-to-satellite DD range. However, these methods require two receivers with a short baseline length, what it is not suitable for our purpose of processing single receiver data. Most important studies are [2–5]. Another method is presented in [6] distinguishing from previous ones in the fact that it uses triple-differences with a higher data sampling rates of 10 Hz. This proposed algorithm is even less reliable for our purpose of using only one receiver.

Methods based on the integration of the GNSS signals and INS data take advantage of the improved receiver-to-satellite range in case of a single receiver plus INS, or even an improved true receiver-to-satellite range for more than one receiver, which allows to reach larger baseline lengths. These methods are also not suitable in our case, because our purpose is not to utilize INS. Some relevant studies are [7–11].

More recent methods use triple frequencies to detect cycle slips. In [12,13] the ionospheric residual is ignored, what could suppose an issue in case of rapid variations of the ionosphere. In work [14], the authors proposed to use the geometry-free and the ionosphere-free linear combinations of BeiDou Navigation Satellite System. Some other relevant contributions in triple-frequency are [15,16]. As a matter of fact, the first Global Positioning System (GPS) IIF satellite with a full L5 transmitter was launched on 28 May 2010. In early 2016, 10 GPS satellites are broadcasting L5 but in pre-operational mode. The full GPS constellation broadcasting L5 is expected around 2021 [17]. Consequently, the use of dual-frequency against triple-frequency is still prevailing in the applications.

In brief of all aforementioned, the cycle slip detector implemented in this master’s thesis uses a single-receiver with dual-frequency GNSS signals. In this category, the geometry-free ionospheric residual and the Melbourne-Wübbena combinations are the current state-of-the-art for cycle slip detection [10,18]. Therefore, the methods for this category of cycle slip detection are briefly described in the following subsections.
1.2 Geometry-free and Melbourne-Wübbena

The study done in [20] might be the first effort to detect cycle slips using single receiver data, where it was proposed to use the wide-lane combination and the geometry-free combination simultaneously to detect the ambiguities. The wide-lane combination utilized in [20] is essentially the same as the Melbourne-Wübbena linear combination ([21,22]). This combination uses code and carrier phase measurements, what makes it noisier than the geometry-free combination, but insensitive to ionosphere changes, hence more effective.

Rapid ionospheric variations may cause false cycle slips detections in the geometry-free combination [20]. In order to improve the detection, study [23] proposed a more robust detection based on a polynomial fitting. The philosophy of this method is to smooth the signal and discontinuities (i.e. cycle slips in carrier phase measurements) adjusting a multiple polynomial regression. Despite the effort done, it is worth mentioning that the method is not immune to high ionospheric activities.

A similar method is proposed in [24], where geometry-free and Melbourne-Wübbena combinations are used. Some carrier phase measurements are used to build a low degree polynomial, and a prediction is obtained by extrapolating the polynomial. If the difference is larger than a defined threshold, there is cycle slip in the current epoch. The second combination, which calculates the mean and the standard deviation of the signals combined, is utilized to detect larger (more than one wavelength) cycle slips. This last combination also helps to detect the particular multiple of $\lambda_2 - \lambda_1$ that hide the cycle slip in the geometry-free combination. The authors mentioned that more testing is required in order to further validate the performance of the approach in relation to the levels of ionospheric delay, multipath and receiver noise.

Another algorithm to detect cycle slips using single GNSS data is introduced in [25]. The algorithm uses Total Electron Content Rate (TECR) and Melbourne-Wübbena wide-lane (MWWL) linear combination to detect cycle slips on both L1 and L2 independently. The method essentially detects cycle slips because those will change the MWWL and will amplify the TECR. The TECR is calculated with the phase measurements. Additionally, from the TEC acceleration is calculated the different ionospheric rates between epochs. The calculated TECR is compared with a prediction made with the previous 30 epochs. The authors showed that the algorithm detected and repaired almost all cycle slips except for a few, under very active ionospheric conditions. The limitations of this method come with the number of data required to generate the prediction, with the complexity of the algorithm and with the aforementioned error in the estimation process. Moreover, it is not possible to apply this method with sampling rates greater than 1 second, which is not suitable for our purpose of using also higher data intervals (e.g. 30 seconds).

The research done in [26] proposed an algorithm that made use of high order time-differences to detect and correct large cycle slips. Then, the Lagrange interpolation is used to process these clean observed values to correct small cycle slips, which revises the characteristic that the polynomial method is insensitive to the small cycle slip lying the foundation for the GPS integer ambiguity and high-accuracy measurements. Therefore, this method is designed to work in post-processing, that it is not suitable for our purpose of detecting cycle slips in real-time.
All above research give an idea about the difficulties to design a robust cycle slip detector. Indeed, the geometry-free combination is the least noise combination usable to detect cycle slips using only GNSS data. However, they assume a small ionospheric change (residual) between adjacent epochs, what may cause false detections under ionospheric scintillation. Consequently, some studies propose the polynomial fitting to mitigate moderate to high ionospheric variations. A considerable part of works use Melbourne-Wübbena combination to work simultaneously with the geometry-free combination, because the first combination aids to detect ambiguities that could remain hidden in the second one. All these ingredients will be the starting point of the work presented in this master’s thesis.

1.3 Summary of Cycle Slip Detection Techniques

Table 1.1 presents a summary of the cycle slip detection techniques described in the state-of-the-art.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Year/s</th>
<th>Author/s</th>
<th>Technique</th>
<th>Drawback</th>
</tr>
</thead>
<tbody>
<tr>
<td>[12–16]</td>
<td>2008–2014</td>
<td>Several</td>
<td>Triple-frequency</td>
<td>Triple-frequency</td>
</tr>
<tr>
<td>[23]</td>
<td>2008</td>
<td>Lacy, M.C., Reguzzoni, M.,</td>
<td>Polynomial fitting</td>
<td>Not immune to high ionospheric activities</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sans, F. and Venuti, G.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>and Zhao, Q.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>[26]</td>
<td>2009</td>
<td>Hu, H. and Fang, L.</td>
<td>High order time-differences</td>
<td>Not real-time</td>
</tr>
</tbody>
</table>
Chapter 2

BASIC CONCEPTS

2.1 Overview

With the aim to well-understand the cycle slips phenomena and some important concepts used during the validation process, several theoretical aspects are presented in this chapter.

The chapter, based on [27], is divided as follows: a general overview of the GNSSs signals is presented in section 2.2, some useful combinations between them are shown in section 2.3.1, the cycle slip phenomena is explained in section 2.4, the carrier smoothing of code pseudoranges is presented in section 2.5, the fundamentals of solving navigation equations in section 2.6 and finally the dilution of precision in section 2.7.

2.2 GNSS Signals

GNSS satellites continuously transmit navigation signals at two or more frequencies in L band. These signals contain ranging codes and navigation data to allow users to compute both the travel time from the satellite to the receiver and the satellite coordinates at any epoch. The main signal components are described as follows:

- **Carrier**: Radio frequency sinusoidal signal at a given frequency.
- **Ranging code**: Sequences of zeros and ones which allow the receiver to determine the travel time of the radio signal from the satellite to the receiver. They are called Pseudo-Random Noise (PRN) sequences or PRN codes.
- **Navigation data**: A binary-coded message providing information on the satellite ephemeris (pseudo-Keplerian elements or satellite position and velocity), clock bias parameters, almanac (with a reduced-accuracy ephemeris data set), satellite health status and other complementary information.

2.2.1 GPS Signals

Legacy GPS signals are transmitted on two radio frequencies in the L band, referred to as Link 1 (L1) and Link 2 (L2),\(^1\) or L1 and L2 bands. They are right-hand circularly polarised and their frequencies are derived from a fundamental frequency \(f_0 = 10.23\, \text{MHz}\), generated by onboard atomic clocks.

\[
\begin{align*}
\text{L1} & = 154 \times 10.23\, \text{MHz} = 1575.420\, \text{MHz} \\
\text{L2} & = 120 \times 10.23\, \text{MHz} = 1227.600\, \text{MHz}
\end{align*}
\]

\(^1\)They also transmit two additional signals at frequencies referred to as L3 (associated with the Nuclear Detonations Detection System) and L4 (for other military purposes).
Two services are available in the current GPS system:

**SPS:** The Standard Positioning Service is an open service, free of charge for worldwide users. It is a single-frequency service in the frequency band L1.

**PPS:** The Precise Positioning Service is restricted by cryptographic techniques to military and authorised users. Two navigation signals are provided in two different frequency\(^2\) bands, L1 and L2.

The GPS uses the Code Division Multiple Access (CDMA) technique to send different signals on the same radio frequency, and the modulation method used is Binary Phase Shift Keying (BPSK) (for more details see [28] or [29]). The messages are:

- **Coarse/Acquisition (C/A) code,** also known as civilian code \(C(t)\): This sequence contains 1023 bits and is repeated every millisecond (i.e. a chipping rate of 1.023 Mbps). Then, the duration of each C/A code chip is 1\(\mu\)s, which means a *chip width* or wavelength of 293.1 m. This code is modulated only on L1. The C/A code defines the SPS.

- **Precision code,** \(P(t)\): This is reserved for military use and authorised civilian users. The sequence is repeated every 266 days (38 weeks) and a weekly portion of this code is assigned to every satellite, called the PRN sequence. Its chipping rate is 10 Mbps, which leads to a wavelength of 29.31 m. It is modulated over both carriers L1 and L2. This code defines the PPS.

- **Navigation message,** \(D(t)\): This is modulated over both carriers at 50 bps, reporting on ephemeris and satellite clock drifts, ionospheric model coefficients and constellation status, among other information.

\[
s_{L1}(t) = a_PP_l(t)D_l(t)\sin(\omega_1 t + \phi_{L1}) + acC_l(t)D_l(t)\cos(\omega_1 t + \phi_{L1})
\]
\[
s_{L2}(t) = b_PP_l(t)D_l(t)\sin(\omega_2 t + \phi_{L2})
\]

The index \(i\) stands for the \(i\)-th satellite.

**GPS Signal Modernisation: Introduction of New Signals**

The GPS signal modernisation includes an additional Link 5 (L5) frequency and several new ranging codes on the different carrier frequencies. They are referred to as the civil signals L2C, L5C and L1C and the military M code. All of them are right-hand circularly polarised.

Modernisation of the GPS system began in 2005 with the launch of the first IIR-M satellite. This satellite supported the new military M signal and the second civil signal L2C. This latter signal is specifically designed to meet commercial needs, allowing the development of low-cost, dual-frequency civil GPS receivers.

The L2C code is composed of two ranging codes multiplexed in time: the L2CM code and the L2CL code (for more details see [28]). The L2C code is BPSK modulated onto the

\(^2\)Transmission at two frequencies allows dual-frequency user receivers to cancel out ionospheric refraction, which is one of the main sources of error.
L2 carrier frequency and broadcast at a higher effective power level than the original L1 C/A signal. This, together with its powerful cross-correlation properties, facilitates tracking with large signal-level variations from satellite to satellite, making reception easier under trees and even indoors. This signal will also be interoperable with the Chinese BeiDou system. However, the full GPS constellation broadcasting L5 is expected around 2021 [17]. Therefore, the use of dual-frequency GPS (L1 and L2) receivers and satellites is still prevailing in the applications.

The military M code signals are designed to use the edges of the band with only a minor signal overlap with the pre-existing C/A and P(Y) signals (see Figure 2.1). This military M code is modulated into L1 and L2 carriers using the Binary Offset Carrier (BOC) scheme (for more details see [29]). It has been designed for autonomous acquisition, so that a receiver is able to acquire the M code signal without access to C/A or P(Y) code signals.

The GPS modernisation plan continued with the launch of the Block IIF satellites that include, for the first time, the third civil signal on the L5 band (i.e. within the highly protected Aeronautical Radio Navigation Service (ARNS) band). This new L5C signal has a new modulation type and was designed for users requiring Safety-of-Life (SoL) applications. There are two signal components: the in-phase component (L5-I) with data and ranging code, both modulated via BPSK onto the carrier; and the quadrature component (L5-Q), with no data but also having a ranging code BPSK modulated onto the carrier. This signal has an improved code/cARRIER tracking loop and its high power and signal design provide robustness against interference. Moreover, its higher chipping rate than the C/A code (see Table 2.1) provides superior multipath performance.

The next step involves the Block III satellites, which will provide the fourth civil signal on L1 band (L1C). This signal is designed to enable interoperability between GPS and international satellite navigation systems (such as Galileo). Multiplexed Binary Offset Carrier (MBOC) modulation is used to improve mobile reception in cities and other challenging environments. L1C comprises the L1C-I data channel and L1C-Q pilot channel. The implementation proposed for MBOC is the Time Multiplexed BOC (TMBOC). See [31] and [29] for more details. This signal will be broadcast at the same frequency as the original L1-C/A signal, which will be retained for backward compatibility.

Table 2.1 contains a summary of the current and future GPS signals, frequencies and applied modulations. The ranging code rate and data rate are also given in the table.

Figure 2.1 shows the layout of the different GPS signals and ranging codes for the different modernisation phases.

---

3 C/A code acquisition may be impossible for very weak signals in the presence of a strong C/A signal.
4 The first satellite (PRN25) was launched on 28 May 2010, with full L5 capability; the second (PRN01) on 16 July 2011.
5 Originally, the signal was developed as a common civil signal for GPS and Galileo, but new satellite navigation providers (BeiDou in China, QZSS in Japan) are also adopting L1C as a future standard for international interoperability.
10 Design and Implementation of Cycle Slip Detectors for Dual-frequency GNSS Signals

Table 2.1: Current and new GPS navigation signals. The civil signals are provided free of charge to all users worldwide (Open Services).

<table>
<thead>
<tr>
<th>Link</th>
<th>Carrier freq. (MHz)</th>
<th>Wavelength (cm)</th>
<th>PRN code</th>
<th>Modulation Type</th>
<th>Code rate (Mcps)</th>
<th>Data rate (bps)</th>
<th>Service</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>1 575.420</td>
<td>19.029</td>
<td>C/A</td>
<td>BPSK(1)</td>
<td>1.023</td>
<td>50</td>
<td>Civil</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>P</td>
<td>BPSK(10)</td>
<td>10.23</td>
<td>50</td>
<td>Military</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>M</td>
<td>BOC_{sin}(10,5)</td>
<td>5.115</td>
<td>N/A</td>
<td>Military</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>L1C-I data</td>
<td>MBOC(6,1,1/11)</td>
<td>1.023</td>
<td>50</td>
<td>Civil</td>
</tr>
<tr>
<td>L2</td>
<td>1 227.600</td>
<td>24.421</td>
<td>P</td>
<td>BPSK(10)</td>
<td>10.23</td>
<td>50</td>
<td>Military</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>L2C M L</td>
<td>BPSK(1)</td>
<td>1.023</td>
<td>25</td>
<td>Civil</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>BOC_{sin}(10,5)</td>
<td></td>
<td>N/A</td>
<td>Military</td>
</tr>
<tr>
<td>L5</td>
<td>1 176.450</td>
<td>25.483</td>
<td>L5-I data</td>
<td>BPSK(10)</td>
<td>10.23</td>
<td>50</td>
<td>Civil</td>
</tr>
</tbody>
</table>

2.2.2 GLONASS Signals

Legacy GLONASS signals are right-hand circularly polarised and centred on two radio frequencies in the L band, referred to here as the G1 and G2 bands, \(^6\) see Figure 2.2.

Two services are currently available from GLONASS:

**SPS**: The Standard Positioning Service (or Standard Accuracy Signal Service) is an open service, free of charge to worldwide users. The navigation signal was initially provided only in the frequency band G1, but since 2004 the new GLONASS-M satellites also transmits a second civil signal in G2.

\(^6\)We use G1 and G2 instead of L1 and L2 to better differentiate from GPS. Nevertheless, the ICD uses L1 and L2.
**PPS:** The Precise Positioning Service (or High-Accuracy Signal Service) is restricted to military and authorised users. Two navigation signals are provided in the two frequency bands G1 and G2.

In contrast to GPS satellites that share the same frequencies, each GLONASS satellite broadcasts at a particular frequency within the band. This frequency determines the frequency channel number of the satellite and allows users’ receivers to identify the satellites (with the Frequency Division Multiple Access (FDMA) technique). GLONASS modernisation planning includes the transmission of CDMA signals in the G1, G2 and G3 (L3) bands, and even in the GPS L5 band, in addition to transmitting legacy FDMA signals in the G1 and G2 bands (see Figure 2.2 below).

![Figure 2.2: Spectra of GLONASS signals. Legacy FDMA signals before and after modernisation (top), and new CDMA signals after modernisation (bottom). Courtesy of Stefan Wallner.](image)

The actual frequency of legacy GLONASS signal transmission on G1 and G2 can be derived from the channel number \( k \) by applying the following expressions:

\[
g1(k) = 1602 + k \times 9/16 = (2848 + k) \times 9/16 \text{ MHz}
g2(k) = 1246 + k \times 7/16 = (2848 + k) \times 7/16 \text{ MHz}
\]

Two ranging codes, the coarse acquisition C/A (open civil code) and the precise P (military) code, are modulated onto these frequencies together with a navigation message D, using the BPSK technique. The C/A and P codes have periods of 1 ms and 1 s, and chip widths of 586.7 and 58.67 m, respectively, and are about two times noisier than the GPS ones (see Table 2.2).

As in GPS, the C/A code was initially modulated only on G1, while the military code P is modulated on both carrier frequencies, G1 and G2; however, the new GLONASS-M satellites (from 2004) also transmit the C/A signal in the G2 frequency band. On the other hand, and unlike GPS, in GLONASS the PRN sequences of such codes are common to all satellites, because the receiver identifies the satellite by its frequency.\(^8\)

No Selective Availability (S/A) (i.e. intentional degradation of the standard accuracy signal) is applied in GLONASS, and no P-code encryption has been reported so far.

---

\(^7\)Although code P is not encrypted, its unauthorised use is not recommended by the Russian Ministry of Defence because it may be changed without prior notice.

\(^8\)Note that this applies for legacy signals where the FDMA technique is used. For the new GLONASS signals, the satellites use the same frequency and are identified with different PRN codes using CDMA.
Although the military P code has not been officially published, it has been deciphered by different research groups. Nevertheless, this code may be changed by the Russian Ministry of Defence without prior warning.

**GLONASS Signal Modernisation: Introduction of New Signals and CDMA Usage**

The modernisation of GLONASS added a new third frequency G3 to the ARNS band for the GLONASS-K satellites. This signal will provide a third civil C/A and military P codes, and is especially suitable for SoL applications. The plans for GLONASS signal modernisation are summarised in Figure 2.2 (further details can be found in [32] and [29]).

The addition of CDMA and FDMA signals was initiated first with the GLONASS-K launch in February 2011, providing CDMA signals at a frequency \( f = 1202.025 \) MHz in the G3 band (close to the Galileo E5b carrier).

Table 2.2: Legacy GLONASS signal structure.

<table>
<thead>
<tr>
<th>Atomic clock frequency</th>
<th>( f_0 = 0.511 ) MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequencies G1</td>
<td>( f = \frac{9}{16}(2848 + k) = 1602.000 + 0.5625k ) MHz</td>
</tr>
<tr>
<td>Wavelength G1</td>
<td>18.7 cm (( k = 0 ))</td>
</tr>
<tr>
<td>Frequencies G2</td>
<td>( f = \frac{7}{16}(2848 + k) = 1246.000 + 0.4374k ) MHz</td>
</tr>
<tr>
<td>Wavelength G2</td>
<td>24.1 cm (( k = 0 ))</td>
</tr>
<tr>
<td>P code frequency (chipping rate)</td>
<td>10( f_0 = 5.11 ) Mcps</td>
</tr>
<tr>
<td>P code wavelength</td>
<td>58.67 m</td>
</tr>
<tr>
<td>P code period</td>
<td>1 s</td>
</tr>
<tr>
<td>C/A code frequency (chipping rate)</td>
<td>( f_0 = 0.511 ) Mcps</td>
</tr>
<tr>
<td>C/A code wavelength</td>
<td>586.7 m</td>
</tr>
<tr>
<td>C/A code period</td>
<td>1 ms</td>
</tr>
<tr>
<td>Navigation message frequency</td>
<td>50 bps</td>
</tr>
<tr>
<td>Frame length</td>
<td>30 s (on CA), 10 s (on P)</td>
</tr>
<tr>
<td>Total message length</td>
<td>2.5 min (on CA), 12 min (on P)</td>
</tr>
</tbody>
</table>

### 2.2.3 Galileo Signals

In Full Operational Capability (FOC) phase, each Galileo satellite will transmit 10 navigation signals in the frequency bands E1, E6, E5a and E5b, each right-hand circularly polarised. These signals are designed to support the different services that will be offered by EGNOS,\(^9\) based on various user needs as follows:

**OS:** The Open Service (OS) is free of charge to users worldwide. Up to three separate signal frequencies are offered within it. Single-frequency receivers will provide performances similar to GPS C/A. In general, OS applications will use a

---

\(^9\)The European Geostationary Navigation Overlay Service (EGNOS) is a Satellite-Based Augmentation System (SBAS) that enhances the US GPS satellite navigation system to make it suitable for safety-critical applications such as flying aircraft or navigating ships through narrow channels. More details can be found in [33] and at [http://www.esa.int/esaNA/egnos.html](http://www.esa.int/esaNA/egnos.html).
combination of Galileo and GPS signals, which will improve performance in severe environments such as urban areas.

**PRS:** The Public Regulated Service (PRS) is intended for the security authorities (police, military, etc.) who require a high continuity of service with controlled access. It is under governmental control. Enhanced signal modulation/encryption is introduced to provide robustness against jamming and spoofing. Two PRS navigation signals with encrypted ranging codes and data will be available.

**CS:** The Commercial Service (CS) provides access to two additional signals protected by commercial encryption (ranging data and messages). Higher data rates (up to 500 bps) for broadcasting data messages are introduced.

**SAR:** This service contributes to the international Cospas–Sarsat system for Search and Rescue Service (SAR). A distress signal will be relayed to the Rescue Coordination Centre and Galileo will inform users that their situation has been detected.

**SoL:** The SoL Service is already available for aviation to International Civil Aviation Organization (ICAO) standards thanks to EGNOS; Galileo will further improve the service performance.

As in GPS, all satellites share the same frequencies, and the signals are differentiated by the CDMA technique [34]. As mentioned earlier, these signals can contain data and pilot channels. Both channels provide ranging codes, but the data channels also include navigation data. Pilot channels (or pilot tones) are data-less signals, so no bit transition occurs, thus helping the tracking of weak signals. The spectra of Galileo signals are given in Figure 2.3, where the data and pilot channels are plotted in orthogonal planes.

![Figure 2.3: Spectra of Galileo signals. Courtesy of Stefan Wallner.](image)

A brief description of each signal follows:

**E1** supports the OS, CS, SoL and PRS services. It contains three navigation signal components in the L1 band. The first one, E1-A, is encrypted and only accessible to authorised PRS users; it contains PRS data. The other two components, E1-B and E1-C, are open access signals with unencrypted ranging codes accessible to all users. E1-B is a data channel and E1-C a pilot (or data-less) channel. The E1-B

---

10 That is, where the spread spectrum codes enable the satellite to transmit at the same frequencies simultaneously.

11 Mainly from the Galileo ICD [34].
data stream, at 125 bps, also contains unencrypted integrity messages and encrypted commercial data. The MBOC modulation is used for the E1-B and E1-C signals, which is implemented by the Composite Binary Offset Carrier (CBOC), see Table 2.3 and Figure 2.3. More details can be found in [31] and [35]. (Note that the E1 band is shared with GPS L1 and BeiDou B1.)

**E6** is a dedicated signal for supporting the CS and PRS services. It provides three navigation signal components transmitted in the E6 band. As with E1, the first one, E6-A, is encrypted and only accessible to authorised PRS users, carrying PRS data. The other two, E6-B and E6-C, are commercial access signals and include a data channel E6-B and a pilot (or data-less) channel E6-C. The E6 ranging codes and data are encrypted. A data rate of 500 bps allows the transmission of added-value commercial data. (Note that the E6 band is shared with BeiDou B3.)

**E5a** supports OS. It is an open access signal transmitted in the E5a band and includes two signal components, a data channel, E5a-I, and a pilot (or data-less) channel, E5a-Q. The E5a signal has unencrypted ranging codes and navigation data, which are accessible to all users. It transmits the basic data to support navigation and timing functions, using a relatively low 25 bps data rate that enables more robust data demodulation. (Note that the E5a band is shared with GPS L5, BeiDou B2a and future GLONASS L5 signals.)

**E5b** supports the OS, CS and SoL services. It is an open access signal transmitted in the E5b band and includes two other signal components: the data channel E5b-I and the pilot (or data-less) channel E5b-Q. It has unencrypted ranging codes and navigation data accessible to all users. The E5b data stream also contains unencrypted integrity messages and encrypted commercial data. The data rate is 125 bps. (Note that the E5b band is shared with BeiDou B2b and GLONASS G3 (slightly shifted).)

A summary of Galileo signals, frequencies and applied modulations is presented in Table 2.3. The ranging code rate and data rate are also given in the table.

**Table 2.3:** Galileo navigation signals. The two signals located in the E5a and E5b bands are modulated onto a single E5 carrier frequency of 1191.795 MHz using the Alternate Binary Offset Carrier (AltBOC) technique: AltBOC(15,10).

<table>
<thead>
<tr>
<th>Band</th>
<th>Carrier freq. (MHz)</th>
<th>Wavelength (cm)</th>
<th>Channel or sig. comp.</th>
<th>Modulation type</th>
<th>Code rate (Mcps)</th>
<th>Data rate (bps)</th>
<th>Services</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>1575.420</td>
<td>19.029</td>
<td>E1-A data</td>
<td>BOCcos(15,2.5)</td>
<td>2.5575</td>
<td>N/A</td>
<td>PRS</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>E1-B data</td>
<td>MBOC(6,1,1/11)</td>
<td>1.023</td>
<td>125</td>
<td>OS, CS, SoL</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>E1-C pilot</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E6</td>
<td>1278.750</td>
<td>23.444</td>
<td>E6-A data</td>
<td>BOCcos(10,5)</td>
<td>5.115</td>
<td>N/A</td>
<td>PRS</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>E6-B data</td>
<td>BPSK(10)</td>
<td>10.23</td>
<td>500</td>
<td>CS</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>E6-C pilot</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E5a</td>
<td>1176.450</td>
<td>25.483</td>
<td>E5a-I data</td>
<td>BPSK(10)</td>
<td>10.23</td>
<td>25</td>
<td>OS</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>E5a-Q pilot</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E5b</td>
<td>1207.140</td>
<td>24.835</td>
<td>E5b-I data</td>
<td>BPSK(10)</td>
<td>10.23</td>
<td>125</td>
<td>OS, CS, SoL</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>E5b-Q pilot</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2.2.4 BeiDou Signals

BeiDou Phase II/III satellites will transmit right-hand circularly polarised signals centred on three radio frequencies in the L band, referred to here as the B1, B2 and B3 bands, see Figure 2.4.

Two services are foreseen for the BeiDou system (in Phase II as a regional service and Phase III as a global service):

**Open Service:** The SPS (or Standard Accuracy Signal Service) is an open service, free of charge to all users.

**Authorised Service:** This service will ensure very reliable use, providing safer positioning, velocity and timing services, as well as system information, for authorised users [36].

Like GPS, Galileo or the new GLONASS signals, BeiDou ranging signals are based on the CDMA technique. The different navigation signals, structure and supported services, according to the current signal plan for Phase II and Phase III, are summarised in Tables 2.4 and 2.5 and illustrated in Figure 2.4.

![Figure 2.4: Spectra of BeiDou signals: Phase II (top) and Phase III (bottom). Courtesy of Stefan Wallner.](image)

Table 2.4: BeiDou Phase II navigation signals. Quadrature Phase-Shifted Keying (QPSK) and BPSK modulation schemes are applied.

<table>
<thead>
<tr>
<th>Band</th>
<th>Carrier freq. (MHz)</th>
<th>PRN code</th>
<th>Modulation type</th>
<th>Code rate (Mcps)</th>
<th>Service</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>1561.098</td>
<td>B1-I</td>
<td>QPSK(2)</td>
<td>2.046</td>
<td>Open</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B1-Q</td>
<td>BPSK(2)</td>
<td></td>
<td>Authorised</td>
</tr>
<tr>
<td>B2</td>
<td>1207.14</td>
<td>B2-I</td>
<td>BPSK(2)</td>
<td>2.046</td>
<td>Open</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B2-Q</td>
<td>BPSK(10)</td>
<td>10.23</td>
<td>Authorised</td>
</tr>
<tr>
<td>B3</td>
<td>1268.52</td>
<td>B3</td>
<td>QPSK(10)</td>
<td>10.23</td>
<td>Authorised</td>
</tr>
</tbody>
</table>

In late December 2011, an English version of the ICD for BeiDou [37] was published. This is an 11-page document that covers the open B1 civil signal centred at 165.098 MHz (see
The official ICD [38] was published one year after, and the second version of the ICD [39] in December 2013. This 82-page document provides details of the navigation message, including parameters of the satellite almanacs and ephemerides that were in the "test version" of the ICD.

<table>
<thead>
<tr>
<th>Band</th>
<th>Carrier freq. (MHz)</th>
<th>PRN code</th>
<th>Modulation type</th>
<th>Code rate (Mcps)</th>
<th>Data rate (bps)</th>
<th>Service</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>1575.42</td>
<td>B1-C_D</td>
<td>MBOC(6,1,1/11)</td>
<td>1.023</td>
<td>50</td>
<td>Open</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B1-C_P</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>B1</td>
<td>BOC(14,2)</td>
<td>2.046</td>
<td>50</td>
<td>Authorised</td>
</tr>
<tr>
<td>B2</td>
<td>1191.795</td>
<td>B2-a_D</td>
<td>AltBOC(15,10)</td>
<td>10.23</td>
<td>25</td>
<td>Open</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B2-a_P</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>B2-b_D</td>
<td></td>
<td></td>
<td>50</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>B2-b_P</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B3</td>
<td>1268.52</td>
<td>B3</td>
<td>QPSK(10)</td>
<td>10.23</td>
<td>500</td>
<td>Authorised</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B3-A_D</td>
<td>BOC(15,2,5)</td>
<td>2.5575</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>B3-A_P</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### 2.3 GNSS Measurements and Data Preprocessing

The basic GNSS observable is the travel time $\Delta T$ of the signal to propagate from the phase centre of the satellite antenna (at the emission time) to the phase centre of the receiver antenna (at the reception time). This value multiplied by the speed of light gives the apparent\textsuperscript{12} range $R = c \Delta T$ between them.

As mentioned in section 2.2, the GNSS signals contain ranging codes to allow users to compute the travel time $\Delta T$. Indeed, the receiver determines $\Delta T$ by correlating the received code ($P$) from the satellite with a replica of this code generated in the receiver, so this replica moves in time ($\Delta T$) until the maximum correlation is obtained (see Figure 2.5).

The measurement $R = c \Delta T$ is what is known as the pseudorange. It is called pseudorange, because it is an ‘apparent range’ between the satellite and the receiver which does not match its geometric distance because of, among other factors, synchronisation errors between receiver and satellite clocks. Taking explicitly into account possible synchronisation errors between such clocks, the travel time between transmission and reception is obtained as the difference in time measured on two different clocks or time scales: the satellite ($t_{sat}$) and the receiver ($t_{rcv}$). Thus, considering a reference time scale $T$ (i.e. GNSS time), the measured pseudorange (using the code $P$ for the frequency signal $f$) for the satellite and receiver may be expressed as

$$ R_P = c \left[ t_{rcv}(T_2) - t_{sat}(T_1) \right] $$  \hspace{1cm} (2.1)

where: $c$ is the speed of light in a vacuum; $t_{rcv}(T_2)$ is the time of signal reception, measured on the time scale given by the receiver clock; and $t_{sat}(T_1)$ is the time of signal transmission, measured on the time scale given by the satellite clock.

\textsuperscript{12}It is called apparent, or pseudorange, to distinguish it from the true range, since different effects cause them to differ.
The pseudorange $R_P$ measurement obtained by the receiver using this procedure includes, besides the geometric range $\rho$ between the receiver and the satellite and clock synchronisation errors, other terms due to signal propagation through the atmosphere (ionosphere and troposphere), relativistic effects, instrumental delays (of satellite and receiver), multipath and receiver noise. Taking explicitly into account all these terms, the previous equation can be rewritten as follows, where $R_{P_f}$ represents any GNSS code measurement at frequency $f$ (from GPS, GLONASS, Galileo or BeiDou):

$$R_{P_f} = \rho + c(dt_{rcv} - dt_{sat}) + Tr + \alpha_f STEC + K_{P_f, rcv} - K_{P_f, sat}^s + M_{P_f} + \varepsilon_{P_f} \tag{2.2}$$

Here:

- $\rho$ is the geometric range between the satellite and receiver Antenna Phase Centres (APCs) at emission and reception time, respectively. Note: The APC is frequency dependent, but it is neglected this effect here for simplicity.
- $dt_{rcv}$ and $dt_{sat}$ are the receiver and satellite clock offsets from the GNSS time scale, including the relativistic satellite clock correction.
- $Tr$ is the tropospheric delay, which is non-dispersive.
- $\alpha_f STEC$ is a frequency-dependent ionospheric delay term, where $\alpha_f$ is the conversion factor between the integrated electron density along the ray path (STEC), and the signal delay at frequency $f$. That is, $\alpha_f = \frac{40.3}{f^2} 10^{16}$ m(signal delay at frequency$f$)/TECU, where the frequency $f$ is in Hz and 1 TECU$=10^{16}$ e$^{-}$/m$^2$.
- $K_{P_f, rcv}$ and $K_{P_f, sat}$ are the receiver and satellite instrumental delays, which are dependent on the code and frequency (section 2.3.1).
- $M_{P_f}$ represents the effect of multipath, also depending on the code type and frequency, and $\varepsilon_{P_f}$ is the receiver noise.

Besides the code, the carrier phase itself is also used to obtain a measure of the apparent distance between satellite and receiver. These carrier phase measurements are much more precise than the code measurements (typically two orders of magnitude more...
precise), but they are ambiguous by an unknown integer number of wavelengths ($\lambda N$). Indeed, this ambiguity changes arbitrarily every time the receiver loses the lock on the signal, producing jumps or range discontinuities (i.e. cycle slips).

The carrier phase measurements ($\Phi_{L_f} = \lambda_{L_f} \phi_{L_f}$) can be modelled as

$$\Phi_{L_f} = \rho + c(dt_{rcv} - dt_{sat}) + Tr - \alpha_f STEC + k_{L_f,rcv} - k_{L_f,sat} + \lambda_{L_f} N_{L_f}$$

$$+ \lambda_{L_f} w + m_{L_f} + \varepsilon_{L_f}$$

(2.3)

where this equation, besides the terms in equation (2.2), includes the wind-up ($\lambda_{L_f} w$) due to the circular polarisation of the electromagnetic signal$^{13}$ and the integer ambiguity $N_{L_f}$ (see Figure 2.6 below). The terms $k_{L_f,rcv}$ and $k_{L_f,sat}$ are frequency dependent and correspond to carrier phase instrumental delays associated with the receiver and satellite, respectively. The $m_{L_f}$ and $\varepsilon_{L_f}$ terms are the carrier phase multipath and noise, respectively.

![Figure 2.6: GPS code and carrier phase measurement features. The geometry-free combinations of code ($R_{P_2} - R_{P_1}$), in green, and carrier ($\Phi_{L_1} - \Phi_{L_2}$), in blue, are plotted as function of time for a given satellite.](image)

Note that the ionospheric term has opposite signs for code and phase. This means that the ionosphere produces an advance in the carrier phase measurement equal to the delay on the code measurement.

### 2.3.1 Combinations of GNSS Measurements

Starting from the basic observables as described previously, the following combinations can be defined (where $R_i$ and $\Phi_i$, $i = 1, 2$, indicate measurements in the frequencies $f_1$ and $f_2$ and $P$ and $L$ are omitted for simplicity):

- **Ionosphere-free combination:** This removes the first-order (up to 99.9%) ionospheric effect, which depends on the inverse square of the frequency ($\alpha_i \propto 1/f_i^2$)

$$\Phi_c = \frac{f_1^2 \Phi_1 - f_2^2 \Phi_2}{f_1^2 - f_2^2}, \quad R_c = \frac{f_1^2 R_1 - f_2^2 R_2}{f_1^2 - f_2^2}$$

(2.4)

$^{13}$A rotation of $360^\circ$ of the receiver antenna, keeping its position fixed, would mean a variation of one wavelength in the phase-obtained measurement of the apparent distance between receiver and satellite.
Satellite clocks are defined relative to the $R_c$ combination.

- **Geometry-free (or ionospheric) combination:** This cancels the geometric part of the measurement, leaving all the frequency-dependent effects (i.e. ionospheric refraction, instrumental delays, wind-up) as it is shown later in section 2.4. It can be used to estimate the ionospheric electron content or to detect cycle slips in the carrier phase, as well. Note the change in the order of terms in $\Phi_i$ and $R_i$:

$$\Phi_i = \Phi_1 - \Phi_2, \quad R_i = R_2 - R_1 \quad (2.5)$$

- **Wide-laning combinations:** These combinations are used to create a measurement with a significantly wide wavelength. This longer wavelength is useful for carrier phase cycle slip detection and fixing ambiguities:

$$\Phi_w = \frac{f_1 \Phi_1 - f_2 \Phi_2}{f_1 - f_2}, \quad R_w = \frac{f_1 R_1 - f_2 R_2}{f_1 - f_2} \quad (2.6)$$

- **Narrow-laning combinations:** These combinations create measurements with a narrow wavelength. The measurement in this combination has a lower noise than each separate component:

$$\Phi_n = \frac{f_1 \Phi_1 + f_2 \Phi_2}{f_1 + f_2}, \quad R_n = \frac{f_1 R_1 + f_2 R_2}{f_1 + f_2} \quad (2.7)$$

$\Phi_w$ and $R_w$ have the same ionospheric dependence, which is exploited by the MW combination (see equations in subsection 2.3.1.1) to remove the ionospheric refraction.

### 2.3.1.1 Combinations of Measurements Written in Closed Form

By replacing the expressions for $R_i$ and $\Phi_i, i = 1, 2$, in definitions (2.4) to (2.7), the following expressions can be found. Remark: the APC effect is neglected here for simplicity.

**Input measurements: $R_i$ and $\Phi_i (i = 1, 2)$:**

$$R_i = \rho + c(\delta t_{rcv} - \delta t_{sat}) + Tr + \bar{\alpha}_i(I + K_{21}) + M_i + \varepsilon_i$$

$$\Phi_i = \rho + c(\delta t_{rcv} - \delta t_{sat}) + Tr - \bar{\alpha}_i(I + K_{21}) + B_i + \lambda_i w + m_i + \varepsilon_i$$

where the ambiguity $B_i$ is given by

$$B_i = b_i + \lambda_i N_i, \quad \bar{\alpha}_1 = 1/(\gamma_1 - 1), \quad \bar{\alpha}_2 = \gamma_2 \bar{\alpha}_1 = 1 + \bar{\alpha}_1,$$

$$\gamma_1 = (f_1/f_2)^2$$

with the bias $b_i$ a real number and $N_i$ an integer ambiguity.

Note that $K_{21} = K_{21,rcv} - K_{21}^{sat}$, $K_{21,rcv} = K_{2,rcv} - K_{1,rcv}$, $K_{21}^{sat} = K_{2}^{sat} - K_{1}^{sat}$ and $b_i = b_{i,rcv} - b_{i}^{sat}$.

**Ionosphere-free combination:**

$$R_c = \rho + c(\delta t_{rcv} - \delta t_{sat}) + Tr + M_C + \varepsilon_C$$

$$\Phi_c = \rho + c(\delta t_{rcv} - \delta t_{sat}) + Tr + B_C + \lambda_N w + m_C + \varepsilon_C$$

where the bias $B_C$ is given by

$$B_C = b_C + \lambda_N (N_1 + (\lambda_W / \lambda_2) N_W)$$
Geometry-free combination:
\[ \Phi_f = I + K_{21} + M_I + \varepsilon_I \]
\[ \Phi_w = I + K_{21} + B_I + (\lambda_1 - \lambda_2)w + m_I + \varepsilon_I \]

where the bias \( B_I \) is given by
\[ B_I = b_I + \lambda_1 N_1 - \lambda_2 N_2 \]

Wide-lane (phase) and narrow-lane (code) combinations:
\[ R_N = \rho + c(\delta t_{\text{rcv}} - \delta t_{\text{sat}}) + T r + \tilde{\alpha}_W (I + K_{21}) + M_N + \varepsilon_N \]
\[ \Phi_w = \rho + c(\delta t_{\text{rcv}} - \delta t_{\text{sat}}) + T r + \tilde{\alpha}_W (I + K_{21}) + B_W + m_W + \varepsilon_W \]

where the bias \( B_W \) is given by
\[ B_W = b_w + \lambda_W N_W \]

Other combinations involving code and phase measurements:

*The Melbourne–Wübben combination*
\[ \Phi_w - R_N = b_w + \lambda_W N_W + M_{MW} + \varepsilon_{MW} \]

*The Group and Phase Ionospheric Calibration (GRAPHIC) combination*
\[ \frac{1}{2} (R_N + \Phi_I) = \rho + c(\delta t_{\text{rcv}} - \delta t_{\text{sat}}) + T r + \frac{1}{2} B_I + \lambda_W \lambda - \lambda W + M_G + \varepsilon_G \]

Definitions and relationships (where \((\cdot)_x \equiv (\cdot)_{x_{12}}\):
\[ N_W \equiv N_1 - N_2 \]
\[ \lambda_W \equiv c/(f_1 - f_2), \quad \lambda_N \equiv c/(f_1 + f_2), \]
\[ \bar{\alpha}_W \equiv \sqrt{\alpha_1 \alpha_2} = f_1 f_2/(f_1^2 - f_2^2) = \sqrt{\gamma_{12}^2/(\gamma_{12}^2 - 1)}, \quad \gamma_{12} = (f_1/f_2)^2 \]
\[ b_w \equiv (f_1 b_1 - f_2 b_2)/(f_1 - f_2), \quad B_C \equiv (f_1^2 b_1 - f_2^2 b_2)/(f_1^2 - f_2^2), \]
\[ b_I \equiv b_1 - b_2, \quad B_W - B_C = \bar{\alpha}_W b_I, \]

the same expressions for \( B_X \) as \( b_X \). (2.8)

The effect of a jump in the integer ambiguities in terms of \( \Delta N_1, \Delta N_2 \) and \( N_W \) is given next:

<table>
<thead>
<tr>
<th>Variations</th>
<th>( \Delta \Phi_w )</th>
<th>( \Delta \Phi_f )</th>
<th>( \Delta \Phi_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \Phi_w = \lambda_W \Delta N_W = \lambda_W (\Delta N_1 - \Delta N_2) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta \Phi_f = \lambda_1 \Delta N_1 - \lambda_2 \Delta N_2 = (\lambda_2 - \lambda_1) \Delta N_1 + \lambda_2 \Delta N_W (N = \text{integer ambig.}) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Delta \Phi_c = \lambda_N \left( \frac{\lambda_W}{\lambda_1} \Delta N_1 - \frac{\lambda_W}{\lambda_2} \Delta N_2 \right) = \lambda_N \left( \Delta N_1 + \frac{\lambda_W}{\lambda_2} \Delta N_W \right) )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The different wavelengths for the wide and narrow-lane combinations of frequencies in GPS, GLONASS and Galileo, as well as the values for the associated parameters, are given in Table 2.6.
Table 2.6: Wide and narrow-lane combinations of signals for different frequencies of GPS, GLONASS (only the channel \( k = 0 \) is given for G1 and G2 signals) and Galileo. The Galileo E5 and E6 signals have not been included to simplify the table.

<table>
<thead>
<tr>
<th>System</th>
<th>Signal</th>
<th>Frequency (MHz)</th>
<th>Wavelength ( \lambda_i ) (m)</th>
<th>Signals combined ( \lambda_{W} ) (m)</th>
<th>Narrow lane ( \lambda_{N} ) (m)</th>
<th>Cycle slip hidden ( \gamma_{it} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS</td>
<td>L1</td>
<td>1575.420</td>
<td>( \lambda_{L1} = 0.190 )</td>
<td>L1,L2</td>
<td>0.862</td>
<td>0.107</td>
</tr>
<tr>
<td></td>
<td>L2</td>
<td>1227.600</td>
<td>( \lambda_{L2} = 0.244 )</td>
<td>L1,L5</td>
<td>0.751</td>
<td>0.109</td>
</tr>
<tr>
<td></td>
<td>L5</td>
<td>1176.450</td>
<td>( \lambda_{L5} = 0.255 )</td>
<td>L2,L5</td>
<td>5.861</td>
<td>0.125</td>
</tr>
<tr>
<td>GLONASS</td>
<td>G1</td>
<td>1602.000</td>
<td>( \lambda_{G1} = 0.187 )</td>
<td>G1,G2</td>
<td>0.842</td>
<td>0.105</td>
</tr>
<tr>
<td></td>
<td>G2</td>
<td>1246.000</td>
<td>( \lambda_{G2} = 0.241 )</td>
<td>G1,G3</td>
<td>0.750</td>
<td>0.107</td>
</tr>
<tr>
<td></td>
<td>G3</td>
<td>1202.025</td>
<td>( \lambda_{G3} = 0.249 )</td>
<td>G2,G3</td>
<td>6.817</td>
<td>0.122</td>
</tr>
<tr>
<td>Galileo</td>
<td>E1</td>
<td>1575.420</td>
<td>( \lambda_{E1} = 0.190 )</td>
<td>E1,E5b</td>
<td>0.814</td>
<td>0.108</td>
</tr>
<tr>
<td></td>
<td>E5b</td>
<td>1207.140</td>
<td>( \lambda_{E5b} = 0.248 )</td>
<td>E1,E5a</td>
<td>0.751</td>
<td>0.109</td>
</tr>
<tr>
<td></td>
<td>E5a</td>
<td>1176.450</td>
<td>( \lambda_{E5a} = 0.255 )</td>
<td>E5b,E5a</td>
<td>9.768</td>
<td>0.126</td>
</tr>
</tbody>
</table>

Remarks on the previous equations:

- Although, for simplicity, the equations (in previous subsection 2.3.1.1) are written in terms of two signals at frequencies \( f_1 \) and \( f_2 \), they are valid for any pair of frequencies \( f_k \) and \( f_m \) (see Table 2.6).

- The code DCBs have been included in the equations of carrier phase measurements, joining the ionospheric term, to provide a closed expression. Nevertheless, they could be included in the unknown bias \( B(\cdot) \). That is, \( \tilde{\alpha}(\cdot)(I + K_{21}) + B(\cdot) \equiv \alpha(\cdot)I + B(\cdot) \).

- Note that the wind-up term does not appear in the wide-lane carrier phase combination. This is because the signals are subtracted in cycles, see equation (2.3). That is, \( \Phi_{W} = (f_{1}\Phi_{1} - f_{2}\Phi_{2})/(f_{1} - f_{2}) = c(\phi_{1} - \phi_{2})/(f_{1} - f_{2}) \), and both signals are affected by the same fraction of cycle by the wind-up.

- The GRAPHIC combination [40] provides an ionosphere-free single-frequency measurement with reduced noise (half the code noise), but contains the unknown ambiguity of the carrier phase. This combination is used, for example, for GPS single-frequency orbit determination for Low Earth Orbiting (LEO) satellites; see, for instance, [41].

- **Final remark**: the equations in subsection 2.3.1.1 are based on the redefinition of the clock to cancel out the instrumental code delays in the ionosphere-free combination of codes \( R_{C12} \); see [27] for further details.
2.4 Carrier Phase Cycle Slip

As already mentioned, receiver losses of lock cause discontinuities in the phase measurements (cycle slips) that are seen as jumps of integer numbers of wavelengths $\lambda$ (i.e. the integer ambiguity $N$ changes by an arbitrary integer value).

There are three main causes of cycle slips [44]:

1. Obstruction of the satellite signal. When the satellite signal are obstructed and a receiver (temporarily) loses lock, all integer ambiguities are reset causing a cycle slip on all frequencies. If the occurrence of this event is registered by the receiver the loss of lock indicator is set and, when the interruption is longer then the measurement interval, is accompanied by missing data.

2. Failure of the receiver tracking loop. When a receiver fails to track a carrier wave correctly this can lead to a cycle slip. A receiver tracks each carrier wave on a separate channel. Therefore, the occurrence of cycle slips on different frequencies can be considered as independent events. Given the relatively small probability of a cycle slip occurring at a certain epoch under normal conditions, the probability of multiple cycle slip occurring simultaneously is quite small.

3. Low carrier-to-noise density ratio. When a receiver is tracking a satellite with a carrier-to-noise density ratio e.g. a satellite with low elevation, the receiver may not be able to track the carrier waves correctly, which can lead to a lot of cycle slips. The probability of cycle slip occurring on multiple frequencies simultaneously also increases.

Different heuristic methods are used for cycle slip detection, operating over undifferenced, single-differenced or double-differenced measurements between pairs of satellites, or pairs of satellites and receivers.

The methods presented in this section are oriented towards single-receiver positioning, and thus do not require any differencing of data between receivers, being suitable for implementation in real time. Moreover, they are based on using only combinations of measurements at different frequencies, or just one frequency measurement. That is, they do not need any geometric delay modelling.

2.4.1 Single-frequency Cycle Slip Detector

The single-frequency detector considered in this master’s thesis is based only on data measurements of a single receiver and do not use any geometric delay model. In fact, it is a simple algorithm, suitable for operating in real time, but with a worse performance than the two-frequency detector.

The non-dispersive delays (geometry, clocks, troposphere, etc.) are cancelled when forming the code pseudorange and carrier phase combination for a given satellite and receiver measurement, that is $\Phi - R = \lambda N - 2I + K + \epsilon$, where the ionospheric refraction $I$ is affected by a factor of 2. The terms $N$, $K$ and $\epsilon$ indicate the ambiguity, instrumental delays and measurement noise, respectively. Likewise, the ionospheric term $I$ varies...
slowly with time, with small changes between consecutive epochs (typically less than $1\text{--}2$ cm in $30$ s).

The detection is based on computing the mean and sigma values of the code pseudorange and carrier phase ($\Phi - R$) differences over a sliding window of $N$ sample (e.g. $N = 100$ with $1$ Hz data). A cycle slip is declared when a measurement differs from the mean bias value over a predefined threshold.

### 2.4.2 Dual-frequency Cycle Slip Detector

With two-frequency signals (or multifrequency signals in general) it is possible to build combinations of measurements to enhance the reliability of cycle slip detection. The target is to remove the geometry, which is the largest varying effect, the clocks and the other non-dispersive delays, as well as ionospheric delays.

Two types of detectors are presented in this section: detectors based on carrier phase measurements only; and detectors based on code and carrier phase data. In the first type, carrier phase measurements of signals at two different frequencies are subtracted in order to remove the geometry and all non-dispersive effects. This provides a very precise test signal (multipath and noise less than $1$ cm), although it is affected by the ionospheric refraction. However, this effect varies as a smooth function and can be modelled by a low-degree polynomial fit. Nevertheless, high ionospheric activity conditions can degrade the performance of this detector, mainly with low sampling rate data (e.g. $30$ s).

As the cycle slips can occur in each of the signals independently, two independent combinations must be used to ensure that all possible jumps are taken into account. In this way, the simultaneous use of two independent detectors protects against those situations where the combination of $\Delta N_1$ and $\Delta N_2$ cycle slips would produce inappreciable jumps in the geometry-free combination.

The second type of detector is based on the Melbourne-Wübbena (MW) combination of code and carrier phase measurements [42]. This combination cancels not only the non-dispersive effects, but also the ionospheric refraction. Nevertheless, the resulting test signal (i.e. MW combination) is affected by the code multipath, which can reach up to several metres. The impact of this noise is partially reduced by the increased ambiguity spacing of the wide-lane combination of carrier phases, and the noise reduction due to the narrow-lane combination of code measurements, on the other hand (both of which are involved in the MW combination). Nevertheless, and in spite of these benefits, the performance is worse than in the carrier-phase-based detector due to the code noise and it is used as a secondary test.

#### 2.4.2.1 Detector Based on Carrier Phase Data: The Geometry-Free Combination

With two-frequency signals it is possible to obtain the carrier phase geometry-free combination, in order to remove the geometry, including clocks, and all non-dispersive effects.

---

14 The range $\rho$ varies up to hundreds of metres in $1$ s.

15 For instance, with GPS signals, $\Delta N_1 / \Delta N_2 = 9/7$ or $18/14$ or $68/53$... produces jumps of few millimetres in the geometry-free combination. In particular, no jump happens when $\Delta N_1 = 77$ and $\Delta N_2 = 60$, but this event produces a jump of $17\lambda W \simeq 15$ m in the wide-lane combination.
effects in the signal. As commented previously, in non disturbed conditions, this very precise (i.e. with very low-noise) test signal performs as a smooth function, driven by the ionospheric refraction, with very few changes between close epochs. Indeed, although, for instance, the jump produced by a simultaneous one cycle slip in both signal components is smaller in this combination than in the original signals, it can provide reliable detection, even for small jumps.

![Cycle-slip detection with the Geometry-free combination, PRN18](image)

Figure 2.7: Effect of one-cycle jump in the GPS $\Phi_1$ carrier phase signal on the ionosphere-free combination. The horizontal axis is seconds of day; the vertical axis is in metres.

The easiest way to build a cycle slip detector is to consider the differences in time of consecutive sample (see Figure 2.7). A refinement of this procedure is the use of $n$th-order differences to take advantage of the jump amplitude enlargement produced by the differencing process (see Table 2.7).

<table>
<thead>
<tr>
<th>$t_i$</th>
<th>$y(t_i)$</th>
<th>$\Delta y$</th>
<th>$\Delta^2 y$</th>
<th>$\Delta^3 y$</th>
<th>$\Delta^4 y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1$</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_2$</td>
<td>0</td>
<td>0</td>
<td>$\varepsilon$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_3$</td>
<td>0</td>
<td>$\varepsilon$</td>
<td>$-2\varepsilon$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_4$</td>
<td>$\varepsilon$</td>
<td>$-\varepsilon$</td>
<td>$3\varepsilon$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_5$</td>
<td>$\varepsilon$</td>
<td>0</td>
<td>$\varepsilon$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$t_6$</td>
<td>$\varepsilon$</td>
<td>0</td>
<td>0</td>
<td>$-\varepsilon$</td>
<td></td>
</tr>
<tr>
<td>$t_7$</td>
<td>$\varepsilon$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This approach allows us to make a reasonable enough detector for many applications. Nevertheless, it must be taken into account that, as the jumps are enlarged, also the

---

16For GPS signals, this jump is $\lambda_2 - \lambda_1 = 5.4$ cm, which is about 3–4 times shorter than $\lambda_1 = 19.0$ cm or $\lambda_2 = 24.4$ cm (see Table 2.6).
signal noise (i.e. signal instabilities) is amplified, which can lead to false detections in some scenarios (for instance, with low signal-to-noise ratios, large ionospheric gradients, etc.).

One way to mitigate the impact of these effects is to use a low-order polynomial fit, reducing the test signal noise. This concept is the basis of the detector implemented in this master’s thesis.

### 2.4.2.2 Detector Based on Code and Carrier Phase Data: The MW Combination

The MW combination provides a noisy estimate of the wide-lane ambiguity $B_W$, according to the equation

$$B_W = \Phi_W - R_N = \lambda_W N_W + b_W + \varepsilon_{MW}$$

(2.10)

where $N_W = N_1 - N_2$ is the integer wide-lane ambiguity, $b_W$ accounts for the satellite and receiver instrumental delays and $\varepsilon$ is the measurement noise, including carrier phase and code multipath (see equations in subsection 2.3.1.1).

This combination has a double benefit; the wide-lane combination has a larger wavelength $\lambda_W = c/(f_1 - f_2)$ than each signal individually (see Table 2.6), which leads to an enlargement of the ambiguity spacing. On the other hand, the measurement noise is reduced by the narrow-lane combination of code measurements, reducing the dispersion of values around the true bias.

The effect of the ambiguity space widening produced by the MW combination is shown in Figure 2.8 and compared with the single-frequency phase minus code combination (see section 2.4.1) as a reference. As in Figure 2.7, a jump of one cycle is introduced in the $\Phi - R$ carrier phase measurement at time 5000 s. This jump cannot be identified from the $\Phi - R$ combination shown in the left plot of Figure 2.8, due to the receiver code noise and multipath, and to the ionospheric drift. On the contrary, it is clearly seen in the MW combination plot shown on the right of the figure, which has a lower noise and is not affected by the ionospheric refraction.

Figure 2.8 (right) shows a clear example of cycle slip detection with the MW combination, where the jump is well defined. Unfortunately, this is not the case on many occasions, because the detection threshold is ‘fussier’ due to the code receiver noise and multipath. This noise can be smoothed by filter averaging (i.e. by computing the mean bias $B_W$), but small jumps can still escape from the detector in the first epochs following a filter reset.

---

17The noisy measurements are concentrated around discrete levels of separated multiples of $\lambda_W$ units (see Figure 2.8, right). That is, the jumps are integer numbers of $\lambda_W$.

18Namely, $\sigma_W^2 = (f_1^2 \sigma_1^2 + f_2^2 \sigma_2^2)/(f_1 + f_2)^2 \approx 1/2\sigma_1^2$.

19This combination ($\Phi - R$) cancels all non-dispersive effects (geometry, clocks, etc.) so only the ionospheric refraction remains (among the instrumental delays), producing the drift seen in the figure.

20The measurements shown in this figure are unsmoothed. They were collected under $A/S$ off conditions (IGS station CASA, California, USA, 18 October 1995).
2.5 Carrier Smoothing of Code Pseudoranges

The noisy (but unambiguous) code pseudorange measurements can be smoothed with the precise (but ambiguous) carrier phase measurements. A simple algorithm (Hatch filter) is given as follows.

Let $R(s;n)$ and $Φ(s;n)$ be the code and carrier measurements of a given satellite $s$ at time $n$. Then, the smoothed code $^\hat{R}(s;n)$ can be computed as

$$^\hat{R}(s;k) = \frac{1}{n}R(s;k) + \frac{n-1}{n}\left[^\hat{R}(s;k-1) + (Φ(s;k) - Φ(s;k-1))\right]$$ (2.11)

The algorithm is initialised with $^\hat{R}(s;1) = R(s;1)$, where $n = k$ when $k < N$ and $n = N$ when $k \geq N$.

This algorithm must be initialised every time that a carrier phase cycle slip occurs.

The algorithm can be interpreted as a real-time alignment of the carrier phase with the code measurement. That is,

$$^\hat{R}(k) = \frac{1}{n}R(k) + \frac{n-1}{n}\left[^\hat{R}(k-1) + (Φ(k) - Φ(k-1))\right]$$

$$= Φ(k) + \frac{n-1}{n}\left[^\hat{R}(k-1) + Φ(k-1)\right] + \frac{1}{n}(R(k) - Φ(k))$$

$$= Φ(k) + \langle R - Φ\rangle_{(k-1)} + \frac{1}{n}(R(k) - Φ(k))$$ (2.12)

where the mean bias$^{21}$ $\langle R - Φ\rangle$ between the code and carrier phase is estimated in real time and used to align the carrier phase with the code.

$^{21}$The mean value of a set of measurements $\{x_1, \ldots , x_n\}$ can be computed recursively as $\langle x\rangle_k = (1/k)x_k + [(k-1)/k]\langle x\rangle_{k-1}$. Equation (2.12) is a variant of the previous expression and provides an estimate of the moving average over a window of $N$ sample. Note that, when $k \geq N$, the weighting factors $1/N$ and $(N-1)/N$ are used instead of $1/k$ and $(k-1)/k$. 

Figure 2.8: Effect of one-cycle jump in the GPS L1 signal in the $Φ–R$ (left) and MW (right) combination (raw measurements without smoothing). Vertical axes are in cycles of $λ_1 \simeq 19$ cm (left) and $λ_w \simeq 86$ cm (right).
2.5.1 Code–Carrier Divergence Effect: Single-frequency Smoothing

The time-varying ionosphere induces a bias in the single-frequency smoothed code when it is averaged in the smoothing filter. This effect is analysed as follows.

The single-frequency code \( R_1 \) and carrier \( \Phi_1 \) measurements given by the first two equations of subsection 2.3.1.1 can be written in a simplified form as

\[
R_1 = r + I_1 + \varepsilon_1 \\
\Phi_1 = r - I_1 + B_1 + \varepsilon_1
\]  

where \( r \) includes all non-dispersive terms such as geometric range, satellite and receiver clock offset and tropospheric delay. \( I_1 \) represents the frequency-dependent terms as the ionospheric and instrumental delays. \( B_1 \) is the carrier phase ambiguity term, which is constant along continuous carrier phase arcs. \( \varepsilon_1 \) and \( \varepsilon_1 \) account for the code and carrier thermal noise and multipath. Since the ionospheric term has opposite sign in code and carrier measurements, it does not cancel in the \( R–\Phi \) combination, but, on the contrary, its effect is twofold. That is,

\[
R_1 - \Phi_1 = 2I_1 - B_1 + \varepsilon_1
\]  

The term \( 2I_1 \) is often called code–carrier divergence, because it results from the fact that the ionosphere affects code and carrier in different ways, that is, the ionosphere delays the code and advances the carrier by the same amount.

Substituting equation (2.14) into (2.12) results in

\[
\hat{R}_1(k) = \Phi_1(k) + \langle R_1 - \Phi_1 \rangle(k) = r(k) - I_1(k) + B_1 + 2\langle I_1 - B_1 \rangle(k)
\]  

(2.15)

Since the carrier ambiguity term \( B_1 \) has a constant bias and the average \( \langle \cdot \rangle \) is a linear operator, \( B_1 \) cancels in the previous equation (2.15), which can be rewritten as

\[
\hat{R}_1(k) = \Phi_1(k) + \langle R_1 - \Phi_1 \rangle(k) = r(k) + I_1(k) + 2\left( \langle I_1 \rangle(k) - I_1(k) \right)
\]  

(2.16)

(where the ionosphere is a time-varying term). If the ionosphere were constant, the averaged value \( \langle I_1 \rangle(k) \) would coincide with the instantaneous value \( I_1(k) \) and, hence, the bias \( \text{bias}_I \) would cancel. However, the time-varying ionosphere will result in a bias that depends on the magnitude of the temporal gradient.

That is, the time-varying ionosphere produces a bias in the single-frequency carrier-smoothed code due to the code–carrier divergence effect, in such a way that the first equation of (2.13) becomes, for the smoothed code \( \hat{R}_1 \),

\[
\hat{R}_1 = r + I_1 + \text{bias}_I + \nu_1
\]  

(2.17)

where \( \nu_1 \) is the noise term after the filter smoothing. The magnitude of this bias is a function of the smoothing time window \( N \).

In order to assess such an effect, let us assume a simple model where STEC varies linearly over time:

\[
I_1(t) = I_{01} + \dot{I}_1 t
\]  

(2.18)

\(^{22}\)Where the carrier term \( \varepsilon_1 \) is negligible compared to the code noise and multipath \( \varepsilon_1 \).
Assuming equation (2.18), the bias in the smoothed code, in the steady state, is given by

\[
\text{bias}_I = 2 \left( \langle I_1 \rangle(k) - I_1(k) \right) = -2\tau \dot{I}_1
\] (2.19)

where \(\tau\) is the filter smoothing time constant (i.e. \(\tau \equiv N\) in equation (2.11)).

Figure 2.9 shows an example of the error induced by the ionosphere in the single-frequency smoothed code. Figure 2.9 corresponds to the Halloween storm (on 30 October 2003) with high ionospheric temporal gradients.

Figure 2.9: Effect of 100 s smoothing during the Halloween storm. The left-hand plot shows the C1–carrier smoothing using equation (2.14), in red (single-frequency smoother). The raw measurements are shown in green. STEC is depicted in the right-side plot. As is shown, the larger temporal ionospheric gradients lead to larger code–carrier divergence-induced error in the single-frequency smoothed solution, which reaches up to about 8 m in this example.
2.6 Solving Navigation Equations

The aim is to determine the receiver coordinates \( \mathbf{r} = (x, y, z) \) and clock offset \( \delta t \) from pseudorange measurements \( R \) of at least four satellites in view. The positioning principle is based on solving a geometric problem from the measured ranges to the satellites, with known coordinates. The satellite coordinates can be computed from the broadcast message, which also provides all the necessary information for modelling the measurements for the Standard Positioning Service (i.e. the SPP).

From the code pseudorange measurements \( R^j \) for \( n \geq 4 \) satellites,

\[
R^j = p^j + c(\delta t - \delta t^j) + T^j + \alpha_1I^j + TGD^j + M^j + \varepsilon^j, \quad j = 1, \ldots, n
\]  

(2.20)

where \( \text{TGD}^j = \alpha_1 \cdot K_{21} \) (see equations in subsection 2.3.1.1), the following measurement equation system can be written,\(^{23}\) neglecting the multipath and receiver noise terms:

\[
R^j - D^j \approx \sqrt{(x^j - x)^2 + (y^j - y)^2 + (z^j - z)^2} + c \delta t, \quad j = 1, \ldots, n
\]  

(2.21)

where the left-hand side contains the measurements \( R^j \) and all modelled terms \( D^j = -c \delta t^j + T^j + \alpha_1I^j + \text{TGD}^j \). The right-hand side contains the four unknown parameters: the receiver coordinates \((x, y, z)\) and the receiver clock offset \( \delta t \).

Equations (2.21) define a nonlinear system, whose usual resolution technique consists of linearising the geometric range \( p \) in the neighbourhood of a point \((x_0, y_0, z_0)\) corresponding to the approximate position of a receiver (see Figure 2.10).

Then, linearising the satellite–receiver geometric range

\[
p^j(x, y, z) = \sqrt{(x^j - x)^2 + (y^j - y)^2 + (z^j - z)^2}
\]  

(2.22)

gives, for the approximate solution \( r_0 = (x_0, y_0, z_0) \),

\[
p^j = p_0^j + \frac{x_0 - x^j}{p_0^j} \Delta x + \frac{y_0 - y^j}{p_0^j} \Delta y + \frac{z_0 - z^j}{p_0^j} \Delta z
\]  

(2.23)

with \( \Delta x = x - x_0, \quad \Delta y = y - y_0, \quad \Delta z = z - z_0 \)

Substituting (2.23) in (2.21), we can rewrite the measurement equations as a linear system (where \( R^j \) can be either smoothed or unsmoothed code)

\[
R^j - p_0^j - D^j = \frac{x_0 - x^j}{p_0^j} \Delta x + \frac{y_0 - y^j}{p_0^j} \Delta y + \frac{z_0 - z^j}{p_0^j} \Delta z + c \delta t, \quad j = 1, \ldots, n
\]  

(2.24)

The previous system for the navigation equations is written in matrix notation as

\[
\begin{bmatrix}
R^1 - p_0^1 - D^1 \\
\vdots \\
R^n - p_0^n - D^n
\end{bmatrix} = 
\begin{bmatrix}
\frac{x_0 - x^1}{p_0^1} & \frac{y_0 - y^1}{p_0^1} & \frac{z_0 - z^1}{p_0^1} & 1 \\
\vdots & \vdots & \vdots & \vdots \\
\frac{x_0 - x^n}{p_0^n} & \frac{y_0 - y^n}{p_0^n} & \frac{z_0 - z^n}{p_0^n} & 1
\end{bmatrix}
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z \\
c \delta t
\end{bmatrix}
\]  

(2.25)

In general, an over-determined system is obtained (for \( n > 4 \)), which can be solved using the least squares adjustment.

\(^{23}\)The unknown receiver DCB \( K_{21,\text{rcv}} \) is included in the receiver clock term \( \delta t \).
After solving the equation system (2.25), the estimate of the receiver coordinates is

\[(x, y, z) = (x_0, y_0, z_0) + (\Delta x, \Delta y, \Delta z)\] (2.26)

Equations (2.21) can be linearised again about these new estimates (2.26) of the receiver’s position, and the solution can be iterated until the change between two consecutive iterations is below a given threshold. Typically, the iterations converge quickly, in a few iterations, even if starting with \((x_0, y_0, z_0) = (0, 0, 0)\), that is Earth’s centre.

Equations (2.25) are called the navigation equations system and can be written in compact form as

\[y = Gx\] (2.27)

where the vectors and matrix involved can be defined as follows:

**Prefit residuals:** \(y\) is an \((n \times 1)\) vector containing the residuals between the measured and predicted pseudoranges, ‘before fitting’ the linear model.

**Geometry matrix:** \(G\) is an \((n \times 4)\) matrix containing the receiver–satellite geometry.\(^{24}\) Notice that the first three elements of each row \((j = 1, \ldots, n)\) are the components of an unitary vector in the line-of-sight direction (\(\hat{\rho}\)) to the \(j\)-th satellite.

\[\hat{\rho}_0^j = -(x_0 - x^j, y_0 - y^j, z_0 - z^j)/\| (x^j - x_0, y^j - y_0, z^j - z_0)\|\]

**Unknown parameters:** \(x\) is a \((4 \times 1)\) vector containing the deviation \((\Delta x, \Delta y, \Delta z)\) between the true and approximate coordinates, and the receiver clock offset \(\delta t\).

---

\(^{24}\)The matrix \(G\) can be computed in East North Up (ENU) coordinates instead of XYZ as in equation (2.25). In this local system the rows are \([-\cos e^i \sin a^i, -\cos e^i \cos a^i, -\sin e^i, 1]\), where \(e^i\) and \(a^i\) are the elevation and azimuth angles of satellite \(i\) observed from the receiver’s position.
2.7 Dilution of Precision

The geometry of the satellites (i.e. how the user sees them) affects the positioning error. This is illustrated in Figure 2.11, where the size and shape of the region change depending on their relative positions. This effect is called Dilution of Precision (DOP).

In order to clarify the explanation, consider the situation of determining a ship position by means of two lighthouse clocks, assumed fully synchronised. If the range measurements were perfect, a ship could determine its position as the intersection point of the two circles centred on lighthouses F1 and F2. However, the measurements are not exact, and have some measurement error \( \varepsilon \). Figures 2.11 and 2.12 illustrate how this measurement error is translated to the coordinate estimate as an uncertainty region, which depends on the geometry defined by the relative positions of the ship and lighthouses.

As the matrix \( G \), from equation (2.27), does not depend on the measurements, but only on the geometry, it can be computed from the almanac (because accurate satellite positions are not needed); that is, it does not require receiver measurements.

On the basis of this simple approach, the following DOP parameters are defined:
\[ Q \equiv (G^T G)^{-1} = \begin{bmatrix}
  q_{xx} & q_{xy} & q_{xz} & q_{xt} \\
  q_{xy} & q_{yy} & q_{yz} & q_{yt} \\
  q_{xz} & q_{yz} & q_{zz} & q_{zt} \\
  q_{xt} & q_{yt} & q_{zt} & q_{tt}
\end{bmatrix} \] (2.28)

- **Geometric Dilution Of Precision:**

\[ \text{GDOP} = \sqrt{q_{xx} + q_{yy} + q_{zz} + q_{tt}} \] (2.29)

- **Position Dilution Of Precision:**

\[ \text{PDOP} = \sqrt{q_{xx} + q_{yy} + q_{zz}} \] (2.30)

- **Time Dilution Of Precision:**

\[ \text{TDOP} = \sqrt{q_{tt}} \] (2.31)

As in the previous case, using the proper translation and rotations, the submatrix \( Q_{xyz} \) of \( Q \) can be transformed to ENU coordinates as \( Q_{enu} = R^T Q_{xyz} R \), in order to define the following:

- **Horizontal Dilution Of Precision:**

\[ \text{HDOP} = \sqrt{q_{ee} + q_{nn}} \] (2.32)

- **Vertical Dilution Of Precision:**

\[ \text{VDOP} = \sqrt{q_{uu}} \] (2.33)

Hence, estimations of the expected accuracy are given by:

- GDOP \( \sigma \) geometric precision in position and time
- PDOP \( \sigma \) precision in position
- TDOP \( \sigma \) precision in time
- HDOP \( \sigma \) precision in horizontal positioning
- VDOP \( \sigma \) precision in vertical positioning

where, basically, DOP represents an approximate ratio factor between the precision in the measurements (\( \sigma \)) and that in positioning. This ratio is computed only from the satellite–receiver geometry.
Chapter 3

IMPLEMENTATION OF CYCLE SLIP DETECTORS

3.1 Overview

The work carried out in this master’s thesis is presented in this chapter. The chapter is divided as follows: a general flowchart of the cycle slip detector is presented in section 3.2, and an important limitation in the original gLAB’s data structure has been fixed and is presented in section 3.3. The modifications done over the single-frequency cycle slip detector are explained in section 3.4 and, finally, the design and implementation of the dual-frequency cycle slip detector is presented in section 3.5.

3.2 General Flowchart of the Cycle Slip Detectors Implemented in gLAB

Figure 3.1 shows a general flowchart of the cycle slip detectors implemented in this master’s thesis. The decision blocks are explained in the following paragraphs.

N-consecutive Epochs:

*N-consecutive Epochs* is the first decision block presented in Figure 3.1 and is a new feature included in gLAB. It has been observed that measurements after a single data hole or a short data gap are of low quality. Therefore, the users can define the period of time\(^1\) until where the observables will be excluded. Thus, they will not enter in the detector, or in the smoother or in the navigation solution.

Measurements Pre-Check:

Frequently, GNSS receivers can lose the tracking of some measurements (see Figure 3.2),\(^2\) this fact is more usual at high data sampling rates (1 Hz or higher). Original gLAB was designed to declare cycle slip after losing any measurement, which does not allow data gap detector to work properly. Therefore, and independently of the sampling rate, the upgraded gLAB in case of losing any measurements, will ignore that epoch and the next two seconds.

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\(^1\)Set as default to 2 s.

\(^2\)In case of single-frequency cycle slip detector either C1 or L1, in case of dual-frequency cycle slip detector either C1, L1, P2 or L2.
Figure 3.1: General flowchart of the cycle slip detector implemented in gLAB.
Implementation of Cycle Slip Detectors

Figure 3.2: Example of measurement tracking lost by a receiver.

Loss of Lock Indicator (LLI):

The LLI parameter (see section 2.4) is included in the Receiver Independent Exchange Format (RINEX) file and when it is odd; i.e. 1, 3, 5 or 7, indicates that the measurement may have a cycle slip. Therefore, during this third block decision and if the option has been enabled, gLAB will look for LLI flags; when detected, a cycle slip will be declared, skipping the other detectors and resetting all the cycle slip variables such as the arc length, a flag to mark that has been cycle slip, flag to mark that has been outlier, the number of arc, the mean, the sigma, the accumulated wind up, the smoothed measurement, the pre-alignment and the previous time.

The rest of decision blocks, which are the main cycle slip detectors, require a more detailed description due to their complexity, thus they are presented in the following sections.
3.3 Data Gaps

The original gLAB was designed to look for differences in observables with respect to the previous epoch, independently of the reception time, what implies that if one satellite was not in view for only one epoch (e.g. 1 second), it would be declared as a new satellite, resetting all the cycle slip variables. This approach is very conservative and can be relaxed.

With aim to fix this limitation, the original gLAB's data structure has been redesigned. The structure of both original and upgraded gLAB is presented in the following subsections.

3.3.1 Original gLAB

In the original gLAB, there was a couple of functions, which were executed before and after reading one epoch of observables, with the aim to evaluate changes: either new or left satellites with respect to the previous epoch and its indices. The assigned index to all satellites corresponded to the chronological order when they appeared in that particular epoch. This approach required a large set of auxiliary variables and a complex algorithm to update the associated parameters when they had a different order with respect to the previous epoch.

Notice that no matter the longitude of the gap was, if the satellites were exactly the same before and after the data gap (Figure 3.3), original gLAB had treated them as they were adjacent. Furthermore, if during a short data gap one epoch contained just one of the previous epoch satellites, all the accumulated information from the others satellites would be lost.

![Figure 3.3: Worst case scenario for original gLAB's data structure. No change in satellites in view after 1 hour of data gap.](image)
3.3.2 Upgraded gLAB

The data gap limitation has been fixed in the upgraded gLAB by generating a satellites’ dictionary where indices are given according to their chronological and historical apparition in the observables. Therefore, these indices are always maintained, and all their cycle slip variables stored in its corresponding index.

These modifications, brief to describe, have implied changes in almost all the modules of gLAB, due to many functionalities were based on that structure. The affected modules are: data handling, pre-processing, input, filter and model (all except the output module). Indeed, the modified data structure supposes a common improvement in both single- and dual-frequency cycle slip detectors.

Note that if any epoch is ignored due to N-consecutive or Consistency Check (see subsection 3.4.1), that particular epoch will not declare data gap cycle slip. In other words, the data gap detector takes into account times between usable measurements. The maximum period of time allowed without declaring cycle slip is a user configurable parameter, set as default to 40 s.

It is worth recalling that when a data gap cycle slip is declared, all its associated variables will be reseted.

Important Considerations about LLI and Data Gaps

Although both LLI and Data Gaps are considered two cycle slip detectors in independent blocks, they are complementary to the main detectors and thus, they can work together with the single-frequency cycle slip detector in case of using the Standard Point Positioning (SPP) approach or with the dual-frequency cycle slip detector in case of the Precise Point Positioning (PPP) approach. They are put within independent blocks because despite being trivial, can declare cycle slip by themselves.

3.4 Single-frequency Cycle Slip Detector

This section is devoted to explain the single-frequency cycle slip detector implemented, whose approach is based on the concepts presented in subsection 2.4.1.

3.4.1 Consistency Check

Before the measurement can be examined by the detector, a previous check is performed. It takes into account the consistency between time differences of both C1 and L1 signals by using the following relationship

\[ |(C_{1i} - C_{1i-1}) - (L_{1i} - L_{1i-1})| > 20.0 \, m \] (3.1)

where \( i \) represents the current epoch.

Equation 3.1 targets large jumps by means of time differences between code and phase. In case of a jump larger than 20 m, the epoch is ignored. Notice that if there was a large
cycle slip in the previous epoch (with a carrier jump) this check can detect an inconsistency between code and phase and, thence, set the current epoch measurement to be ignored. But this is not critical, as it only affects to a single epoch (and already mentioned, the measurements after a cycle slip are of low quality).

### 3.4.2 Detector Algorithm Description

The detection is based on computing the mean and sigma values of the code pseudorange and carrier phase ($\Phi - R$) differences over a sliding window of $N$-sample. A cycle slip is declared when a measurement differs from the mean bias value over a threshold, which is computed with the sigma.

#### Remarks

The code of the single-frequency cycle slip detector has been adapted to the new data structure. Moreover, the new feature implemented in the upgraded gLAB is to apply the Hatch filter over the sigma, its effects can be seen in Figure 3.10.

This detector is affected by the pseudorange noise and multipath, as well as the divergence of the ionosphere. Thus, higher sampling rates can improve detection performance, but the shortest jumps can still escape from this detector. On the other hand, a minimum number of samples ($\text{min}_{\text{Als f}}$) are needed to ensure a reliable value of $S_d$ for the detection threshold.

A Hatch filter is used instead of the previously mentioned sliding window to compute the mean and quadratic mean values $m_d$, $m_{d^2}$ in order to simplify the code:

$$m_d(s; n) = \frac{a - 1}{a} m_d(s; n - 1) + \frac{1}{a} d(s; n)$$
$$m_{d^2}(s; n) = \frac{a - 1}{a} m_{d^2}(s; n - 1) + \frac{1}{a} d^2(s; n)$$

(3.2)

where $d = \Phi - R$, $a = n$ when $n < N$ and $a = N$ when $n \geq N$. These equations allow computation of a sequential estimate of the mean and sigma values, but this filter has infinite memory, propagating forward the divergence of the ionospheric refraction (see subsection 2.5.1). Nevertheless, such an accumulated effect, despite biasing the ambiguity estimate, should not affect the cycle slip detection, because it varies smoothly and the detector looks for large jumps.

To avoid unrealistic estimates of sigma during the first iterations of the filter, the following weighted average with an initial value of $S_0^2$ is used:

$$\tilde{S}_d^2(s; n) = \frac{n - 1}{n} S_d^2(s; n) + \frac{1}{n} S^2_0$$

(3.3)

where $S_0$ is an user configurable value, set as default to 1 m. Furthermore, an upper bound for the sigma threshold is included to avoid unreliable values. That is, to take $th = n_T \times S_d$, with $th \leq th_{\text{max}}$. Furthermore, when using higher data sampling rates, the factor $n_T^5$ can be relaxed.

---

3. $N$-sample is an user configurable parameter, set as default to 300 s.
4. $\text{min}_{\text{Als f}}$ is an user configurable parameter, set as default to 3 samples.
5. $n_T$ is an user configurable parameter, set as default to 5.
Figure 3.4 shows the flowchart of the single-frequency cycle slip detector.

Figure 3.4: Flowchart of the single-frequency cycle slip detector implemented in gLAB.
3.5 Design and Implementation of the Dual-frequency Cycle Slip Detector

This section is devoted to explain the dual-frequency cycle slip detector implemented, whose approach is based on the concepts presented in subsection 2.4.2.

As it can be seen in section 3.2, the dual-frequency detector is divided in two blocks: Geometry-free and Melbourne-Wübbena. The reason is to give flexibility to the user to activate or deactivate any detector, although the best results are achieved when both detectors are enabled.

The geometry-free combination is only based on the carrier phase measurements, what provides a precise behaviour. The combination removes the geometry but not the ionosphere. On the other hand, while in the geometry-free combination, cycle slips under some particular \((\lambda_2 - \lambda_1)\) signal combinations will remain undetected, in the Melbourne-Wübbena detector those particular combinations are easily detectable (produce big jumps). Therefore, this last detector complements the geometry-free (see the concepts in section 2.4.2).

The Melbourne-Wübbena detector uses both code measurements and carrier phase, what provides a noisy behaviour in front of the second combination that only uses carrier phase. The key point is that Melbourne-Wübbena removes both geometry and ionosphere and it only multipath remains in its combination.

The research done in this master’s thesis has been focussed on how to make the detector more accurate than the classical method, with the same ingredients.

3.5.1 Geometry-free Algorithm Description

The detection is based on fitting a second-degree polynomial over a sliding window of \(N\)-sample. The predicted value from this polynomial is compared with the observed value to detect the cycle slip. As the geometry-free combination is affected by the ionospheric refraction, a sampling-rate-dependent threshold is considered.

Remarks

The geometry-free detector in the original version of gLAB was using 3-point Lagrange interpolation to predict the next epoch. The formula of 3-point Lagrange interpolation is:

\[
P(x) = \frac{(x - x_2)(x - x_3)}{(x_1 - x_2)(x_1 - x_3)} y_1 + \frac{(x - x_1)(x - x_3)}{(x_2 - x_1)(x_2 - x_3)} y_2 + \frac{(x - x_2)(x - x_1)}{(x_3 - x_2)(x_3 - x_1)} y_3
\]

(3.4)

where \(y_i\) are observed points, \(x_i\) are epochs and \(P(x)\) is the predicted point at epoch \(x\).

The most distinguishing feature in upgraded geometry-free detector is that the prediction is done by fitting a second-degree polynomial using Least Square (LS) over a sliding window of \(\text{min}_{ALDf}\)-sample, which is an user configurable parameter, set as default to 7 samples.

\[6\] The use of code in the combination implies two orders of magnitude of larger noise than using only carrier phase.
Figure 3.5 shows the flowchart of the LI detector.

Figure 3.5: Flowchart of the dual-frequency LI cycle slip detector implemented in gLAB.

Legend
LI = geometry-free combination
$\Delta t$ = min. detectable jump
$\Delta t$ = sampling rate dependent term
$T_0$ = iono time correlation
$\min_{ALdf}$ = min. arc length dual-fq
$Liest$ = LI prediction
$th$ = threshold
$S_{BW}$ = sigma of Bw
$nL$ = factor
$\text{res}$ = RMS residual
The novel feature implemented in this detector is to ignore outliers. When the geometry-free detector finds a cycle slip for the first time, this is marked as outlier and will not enter in the second-degree polynomial for the fitting of the next epoch.\(^7\) Then, during the next epoch, if the measured LI is still over the threshold, a cycle slip is declared (Figure 3.6). Conversely, if the measured LI is under the threshold, no cycle slip is declared.

As mentioned previously, the ionospheric refraction between two consecutive epochs depends on the elapsed time interval between them, due to the variation in ionospheric refraction. Thus, a \(\Delta t\)-dependent threshold is considered to account for the measurement sampling rate. Furthermore, \(T_0\) is the ionosphere time decorrelation, set as 60 s. The equation of the threshold (\(th\)) implemented is:

\[
th = \frac{a_0}{1 + \exp\left(-\frac{\Delta t}{T_0}\right)}
\]  

(3.5)

Note that \(a_0\) is the maximum threshold, which is an user configurable parameter, set as default to 0.08 m. Considering the default value of \(a_0\), Figure 3.7 shows the possible thresholds as function of \(\Delta t\). Then, the minimum threshold can be calculated by doing \(\lim_{\Delta t \to 0} \exp\left(-\frac{\Delta t}{T_0}\right) = 1\), which corresponds to \(a_0/2\). The \(\Delta t\) takes the same value as the maximum data gap permitted, seen in section 3.3.2 (user configurable parameter, set as default to 40 s). In this way, the minimum detectable jump between two contiguous measurements is 5.3 cm.\(^8\)

\(^7\)Notice that the outlier will not enter in the smoothing neither in the navigation solution, because it would be declared as cycle slip, like the classical geometry-free detector would do.

\(^8\)Note that, from equation (2.9), \(\lambda_2 - \lambda_1\) is the jump produced on the geometry-free combination \(\Phi_I\) when a jump of one cycle occurs simultaneously in both carriers. This jump, for instance for the GPS L1 and L2 signals, is \(\lambda_2 - \lambda_1 = 5.4\) cm (see Table 2.6).
The geometry-free detector declares cycle slip when any of the following conditions is fulfilled:

1. A new feature implemented in the geometry-free detector is to take into account the size of the $LI$ jump, remember that $LI = L_1 - L_2$, between consecutive epochs, if it is greater than 1 m, a cycle slip will be declared. The condition is

$$|LI_t - LI_{t-1}| > 1$$

2. This other statement consists of two conditions and both must be fulfilled to declare a cycle slip:

   - The first one is when the difference between $LI$ and $LI_{est}$ (prediction of $LI$) is greater than the aforementioned threshold. The condition is

     $$|LI - LI_{est}| > th$$

   - The second condition refers to the accuracy of the prediction. In other words, the same difference as before $(LI - LI_{est})$ must be greater than the factor $(n_L)_9$ times the RMS residual $(res)$. This residual is the error of the LS adjustment of the second-degree polynomial. The condition is

     $$|LI - LI_{est}| > 2.0 \times res$$

Figure 3.8 shows the differences between original and upgraded versions, under high ionospheric activity. Original gLAB (black) detected two false positive cycle slips and did not detect the real one, whilst upgraded gLAB (blue) detected only the real cycle slip (outlier + confirmation).

---

9The $n_L$ parameter has been hardcoded to 2 (double the residual of the RMS).
3.5.2 Melbourne-Wübbena Algorithm Description

The detection is based on the real-time computation of mean and sigma values of the measurement test data $B_W$ (see equation in subsection 2.4.2.2). A cycle slip is declared when a measurement differs from the mean bias value over a predefined number of standard deviations ($S_{B_W}$), that is the threshold.

Remarks

The original gLAB was only contemplating the condition ($|d| > th$) to declare cycle slip, where $d = B_w - m_{B_w}$ and $th = K_{factor} \cdot \sqrt{S_{B_w}^2}$. However, the upgraded gLAB requires a triple condition, and all of whom must be fulfilled at the same epoch to declare cycle slip:

- $[|d| > th]$ : This condition takes into account the accumulated sigma from the last cycle slip. The threshold ($th$) contains the $K_{factor}$ that is an user configurable parameter, set as default to 5.
- $[\sqrt{S_{B_w}^2} \leq \lambda_W]$ : This condition forces the standard deviation $S_{B_w}$ to be lower or equal to the wavelength of the wide lane combination in order to declare cycle slip.\(^{10}\)
- $[|d_{300}| > \lambda_W]$ : This condition takes into account the last 300 s sliding window ($d_{300}$) accumulated sigma, which is related to the most recent part of the code measurement (see Figure 3.10).

The mean bias estimate $m_{B_w}$ can be greatly affected by strong code multipath at the beginning of the data arc (due to low-elevation rising satellites), but, as the number of

\(^{10}\)Currently, due to L5 is is expected around 2021 [17] and gLAB is fully processing only GPS constellation, the $\lambda_W$ is set to 0.8 m, regarding L1 and L2. Check Table 2.6 to see all possible wavelengths.
averaged sample increases, this estimate becomes more stable and robust. This is the reason why at least \( \min_{ALd_f} \text{-sample} \) are required before to use the detector.\(^{11}\)

Figure 3.9 shows the flowchart of the MW detector.

---

\(^{11}\) \( \min_{ALd_f} \) is an user configurable parameter, set as default to 5.
Figure 3.10 shows the difference between the accumulated mean and its $th$ from the last cycle slip (blue) and the accumulated $mean_{300}$ and its $th_{300}$ using the 300 s sliding window (black). Indeed, as the number of samples increases, the mean and its $th$ are frozen, becoming more insensitive to the measurement noise variations than the $mean_{300}$ and its $th_{300}$.

![Figure 3.10: Difference between the accumulated mean and its $th$ (blue) and the $mean_{300}$ and its $th_{300}$ using the 300 s sliding window (black).](image)
Chapter 4

RESULTS AND VALIDATION

4.1 Overview

This chapter is devoted to show the results and to explain the methodology carried out to validate the new data structure and the detectors implementation.

Due to the lack of an actual reference of cycle slips, the philosophy of the validation resides in comparing the behaviour of the original gLAB against the upgraded one. Additionally, during the validation of the single-frequency cycle slip detector an independent software developed by Professor Jaume Sanz has been used as an additional comparison. Real data from geo-referenced stations have been used to calculate the actual error during the entire process.

The chapter is divided as follows: the results about the modifications done in the data structure are presented in section 4.2, the results and validation of the single-frequency cycle slip detector are shown in section 4.3, whereas the results and validation of the dual-frequency cycle slip detector are displayed in section 4.4.

With the aim to validate the updates done in both single- and dual-frequency cycle slip detectors, several shell scripts (bash) have been written, which automatically download all the required input files, run both the original and upgraded gLAB and generate the results by means of figures.

The input files aforementioned depend on the approach utilized and are listed as follows:

**SPP input files:**

- **RINEX observation:** file that includes three fundamental measurements: time, phase and pseudorange (code) collected by the antenna of one station. Examples of RINEX observation files at a data interval of 30 seconds and 1 second, respectively; can be found in the following links:
  - ftp://cddis.gsfc.nasa.gov/pub/gps/data/daily
  - ftp://cddis.gsfc.nasa.gov/pub/gps/data/hourly

- **RINEX navigation:** file that contains the navigation message broadcasted by the satellites. Examples of RINEX navigation files can be found in the following link:
  - ftp://cddis.gsfc.nasa.gov/pub/gps/data/daily

- **SINEX:** file with precise geo-referenced antenna coordinates for each station. Examples of SINEX files can be found in the following link:
  - ftp://cddis.gsfc.nasa.gov/pub/gps/products

**PPP input files:**

- **RINEX observation:** file that includes three fundamental measurements: time, phase and range collected by the antenna of one station. Examples of RINEX files
at a data interval of 30 seconds can be found in the following link:
ftp://cddis.gsfc.nasa.gov/pub/gps/data/daily

- **ANTEX**: standard to exchange Phase Center Offsets (PCOs) and Phase Center Variations (PCVs) of geodetic GNSS antennae. Examples of ANTEX files can be found in the following link:
ftp://igscb.jpl.nasa.gov/igscb/station/general/pcv_archive

- **SP3**: file to exchange precise satellite orbits and clocks information. Examples of SP3 files can be found in the following link:
ftp://cddis.gsfc.nasa.gov/pub/gps/products

- **CLK**: file to exchange precise satellite clocks information with a higher frequency rate. Examples of CLK files can be found in the following link:
ftp://cddis.gsfc.nasa.gov/pub/gps/products

- **SINEX**: file with precise geo-referenced antenna coordinates for each station. Examples of SINEX files can be found in the following link:
ftp://cddis.gsfc.nasa.gov/pub/gps/products

It is worth mentioning that during the validation process it is used real data from geo-referenced stations, which permit calculating the actual error during the entire process.

The coordinates of the station used during the validation about the modifications done in data structure are shown in Table 4.1.

Table 4.1: Station used during the validation of the modifications done in data structure.

<table>
<thead>
<tr>
<th>Station name</th>
<th>Latitude (degrees)</th>
<th>Longitude (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TERU</td>
<td>40.2</td>
<td>-1.1</td>
</tr>
</tbody>
</table>

The stations and its latitude and longitude coordinates used during the validation process of both single- and dual-frequency cycle slip detectors are presented in Table 4.2. The station selection has been done with the aim to cover high, mid and low latitudes.

Table 4.2: List of stations used during the validation of the single-frequency detector.

<table>
<thead>
<tr>
<th>Station name</th>
<th>Latitude (degrees)</th>
<th>Longitude (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFRM</td>
<td>49.5</td>
<td>18.4</td>
</tr>
<tr>
<td>IZAN</td>
<td>28.1</td>
<td>-16.5</td>
</tr>
<tr>
<td>TRDS</td>
<td>63.2</td>
<td>10.3</td>
</tr>
</tbody>
</table>

All available data during the year 2014 have been assessed. This particular year has been chosen because it is the year of the last maximum solar activity, which makes the validation more challenging. The data sampling rate used during the validation is 1 Hz.

Finally, during the validation process for the SPP approach, gLABs have been run using smoothing with 100-sample window (see section 2.5), which holds false positive (non actual cycle slip detected) and false negative (actual cycle slip not detected) detections, making them more visible.
4.2 Results about the Modifications done in the Data Structure of gLAB using the SPP Approach

The aim of this section is to present the effects of allowing or not data gaps when using the SPP approach. Considering data gaps is not critical in SPP, due to the accuracy of the approach, however, it takes a relevant role with the PPP approach. The effect of the modifications done in the data structure is frequently demonstrated during the validation of the dual-frequency cycle slip detector and, thus, omitted in this section.

Figure 4.1 shows the North East Up (NEU) errors of TERU station during the entire day 213 of 2014. Both original and upgraded gLABs have applied smoothing. Furthermore, an intentional data gap of 10 seconds has been “manually” introduced within the observables.

Figure 4.1: Effects in data structure of a data gap.

Results show that the upgraded gLAB holds the smoothing, whilst the original one presents a bias between the last epoch before the data gap and the first epoch after the gap.

Notice that zero implies no error (precise station coordinates). Therefore, the effect of resetting the smoother in original gLAB, in this particular case, has a positive effect over the Up error because it is closer to zero than the upgraded. Unlike the effect over the North error, which is negative.

The same resetting effect experienced by the smoothing, occurs in all the satellite’s associated parameters: arithmetic mean, arithmetic quadratic mean, standard deviation, arc length, accumulated wind up, pre-alignment, etc. This strongly affects the behaviour of the cycle slip detector, specially during the first epochs after the reset.
4.3 Results and Validation of the Single-frequency Cycle Slip Detector

The results and validation are presented in the following subsections through two columns, where the left one contains the RMS, the arithmetic mean and the 95th percentile of the 3D positioning error. Due to the results of the left column are rather similar between original gLAB (red) and upgraded gLAB (blue), differences between them are shown in the right column (blue), as well as the average (red) of those differences along the year.

4.3.1 CFRM Station

Figure 4.2: RMS, arithmetic mean and 95th percentile of the 3D positioning error for station CFRM during the year 2014.
With regard to the RMS results, the best improvement corresponds to the day of year (DoY) 114 of 2014, where the upgraded gLAB has reduced 66.57 cm of RMS 3D error compared to the original. This day is analyzed in detail in the next subsection.

### 4.3.1.1 CFRM Best-case Scenario: DoY 114

Figure 4.3 presents two columns, where the left one shows the DoY 114 of 2014 for the original gLAB (top), the upgraded gLAB (middle) and an independent software (bottom), whilst the right column shows a zoom of the period of time when discrepancies occurred.

![Figure 4.3: NEU positioning errors for station CFRM during the DoY 114 of 2014.](image)

Figure 4.3 shows that the navigation solution of both upgraded gLAB and independent software have better results than the original gLAB thanks to ignore the two following epochs after a data gap between 1 and 30 seconds. The observed data gap is comprised...
between 33982 and 33999 seconds of the day (17 seconds). The affected PRN numbers are 8, 16, 3, 27. The observed error in original gLAB is not only due to use 4 satellites but also because the measurements are not robust.

Furthermore, the results demonstrate that gLABs are more accurate than the independent software with regard to the Up error, which is always plotted in red colour (Figure 4.3).

### 4.3.2 IZAN Station

![IZAN RMS of the 3D error (Year 2014)](image1)

![IZAN Mean of the 3D error (Year 2014)](image2)

![IZAN 95 percentile of the 3D error (Year 2014)](image3)

![IZAN RMS difference (Original minus upgraded gLAB-Year 2014)](image4)

![IZAN Mean difference (Original minus upgraded gLAB-Year 2014)](image5)

![IZAN 95 percentile difference (Original minus upgraded gLAB-Year 2014)](image6)

Figure 4.4: RMS, arithmetic mean and 95th percentile of the 3D positioning error for station IZAN during the year 2014.

With regard to the RMS results, Table 4.3 summarizes the best- and worst-case scenarios for station IZAN during the year 2014, which are analyzed in detail.
Table 4.3: Best- and worst-case scenarios for IZAN during year 2014.

<table>
<thead>
<tr>
<th>DoY</th>
<th>RMS: Original gLAB (m)</th>
<th>RMS: Upgraded gLAB (m)</th>
<th>Original – Upgraded (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>45</td>
<td>7.5541</td>
<td>7.5910</td>
<td>-3.69</td>
</tr>
<tr>
<td>110</td>
<td>6.1771</td>
<td>6.0548</td>
<td>12.23</td>
</tr>
</tbody>
</table>

4.3.2.1 IZAN Worst-case Scenario: DoY 45

Figure 4.5 presents two columns, where the left one shows the DoY 45 of 2014 for the original gLAB, the upgraded gLAB and an independent software, whilst the right column shows a zoom of the period of time when the discrepancies occurred.

Figure 4.5: NEU positioning errors for station IZAN during the DoY 45 of 2014.
Results show that the original gLAB is continuously resetting the smoother, due to its limitation on data structure (see the effects in section 4.2). This phenomenon is easy to see in the *Up error* during the first three jumps, where the original gLAB error is close to -10 metres, whilst the upgraded gLAB and the independent software are close to -13 metres. Referring to the fourth jump, original gLAB is under -15 metres, while both upgraded gLAB and independent software are over -15 metres.

### 4.3.2.2 IZAN Best-case Scenario: DoY 110

Figure 4.6 presents two columns, where the left one shows the DoY 110 of 2014 for the original gLAB, the upgraded gLAB and the independent software, whilst the right column shows a zoom of the period of time when the discrepancies occurred.

![Graphs showing NEU positioning errors for station IZAN during the DoY 110 of 2014.](image-url)

*Figure 4.6: NEU positioning errors for station IZAN during the DoY 110 of 2014.*
Results show the effect, in the original gLAB, of non declaring cycle slip after a data gap, from 81899 to 82800 seconds of the day (901 seconds), which is strongly necessary. The original gLAB did not detect any difference with the previous epoch, likewise the detector is only able to declare cycle slip in PRN numbers 11, 27, 16, 3, 22 and 19 over a total amount of 11 satellites. However, all satellites should be reset.

4.3.3 TRDS Station

Figure 4.7: RMS, arithmetic mean and 95th percentile of the 3D positioning errors for station TRDS during the year 2014.

With regard to the RMS results, the best improvement corresponds to the DoY 202 of 2014, where the upgraded gLAB has reduced 45.08 cm of RMS 3D error compared to the original. This day is analyzed in detail in the next subsection.
4.3.3.1 **TRDS Best-case Scenario: DoY 202**

Figure 4.8 presents two columns, where the left one shows the DoY 202 of 2014 for the original gLAB, the upgraded gLAB and an independent software, whilst the right column shows a zoom of the period of time when the discrepancies occurred.

Figure 4.8: NEU positioning errors for station TRDS during the DoY 202 of 2014.

Results show the negative effect of taking into account the following two epochs after a single data gap lower than the maximum defined in measurements.
4.3.4 Summary of the Overall Results of the Single-frequency Cycle Slip Detector

Table 4.4 shows a summary of the overall results regarding the averages (arithmetic means) along the year of the RMS, the arithmetic mean and the 95th percentile of the 3D positioning error.

Table 4.4: Summary of the Overall Results of the Single-frequency Cycle Slip Detector.

<table>
<thead>
<tr>
<th>Station name</th>
<th>RMS average (cm)</th>
<th>Mean average (cm)</th>
<th>95th %ile average (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFRM</td>
<td>0.26</td>
<td>0.09</td>
<td>0.02</td>
</tr>
<tr>
<td>IZAN</td>
<td>0.32</td>
<td>0.05</td>
<td>0.21</td>
</tr>
<tr>
<td>TRDS</td>
<td>0.65</td>
<td>0.11</td>
<td>0.38</td>
</tr>
</tbody>
</table>
4.3.5 Results Using Measurements with an Interval of 30 Seconds

The aim of the present subsection is to demonstrate that the updates applied in the single-frequency cycle slip detector do not negatively affect the performance of the detector when it uses a file with a data interval of 30 seconds.

The coordinates of the station used in these results are presented in Table 4.5.

Table 4.5: Station used during the validation of the single-frequency detector using a file with a data interval of 30 seconds.

<table>
<thead>
<tr>
<th>Station name</th>
<th>Latitude (degrees)</th>
<th>Longitude (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OUS2</td>
<td>-45.7</td>
<td>170.5</td>
</tr>
</tbody>
</table>

Figure 4.9 shows that there are no visible differences between the original and upgraded gLAB. Indeed, this is not a surprise because using code measurements, which are noisy, and an interval between samples of 30 seconds, produce the sigmas of the detector become large and, thus, the threshold as well.

Figure 4.9: NEU positioning errors for station OUS2 during the DoY 1 of 2014.
4.4 Results and Validation of the Dual-frequency Cycle Slip Detector

The results and validation are presented in the following subsections through two columns, where the left one contains the RMS, the arithmetic mean and the 95th percentile of the 3D positioning error. Due to the results of the left column are rather similar between original gLAB (red) and upgraded gLAB (blue), differences between them are shown in the right column (blue), as well as the average (red) of those differences along the year.

4.4.1 CFRM Station

![Graphs showing RMS, arithmetic mean and 95th percentile of the 3D positioning error for CFRM station during year 2014.]

Figure 4.10: RMS, arithmetic mean and 95th percentile of the 3D positioning error for station CFRM during the year 2014.
With regard to the RMS results, Table 4.6 summarizes the best- and worst-case scenarios for station CFRM during the year 2014. These particular days are analyzed in detail through the following subsections.

Table 4.6: Best- and worst-case scenarios for CFRM during year 2014.

<table>
<thead>
<tr>
<th>DoY</th>
<th>RMS: Original gLAB (cm)</th>
<th>RMS: Upgraded gLAB (cm)</th>
<th>Original – Upgraded (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>74</td>
<td>54.04</td>
<td>73.16</td>
<td>-19.12</td>
</tr>
<tr>
<td>210</td>
<td>38.34</td>
<td>21.05</td>
<td>17.29</td>
</tr>
<tr>
<td>293</td>
<td>34.64</td>
<td>23.65</td>
<td>10.99</td>
</tr>
<tr>
<td>346</td>
<td>21.52</td>
<td>54.10</td>
<td>-32.58</td>
</tr>
</tbody>
</table>

4.4.1.1 CFRM Worst-case Scenario: DoY 74

Figure 4.11 presents two columns, where the left one shows the results of the original gLAB and the right column the results of the upgraded gLAB.

Figure 4.11: NEU positioning errors for station CFRM during the DoY 74 of 2014.

The bottom results in Figure 4.11 have been generated omitting the calculated positions when GDOP (see the theory in section 2.7) was greater than 30 from both original and upgraded gLABs. Therefore, those differences between both gLABs were only due to have had 4 satellites with bad geometries.
4.4.1.2 CFRM Best-case Scenario: DoY 210

Figure 4.12 presents two columns, where the left one shows the results of the original gLAB and two zooms of the periods of time when discrepancies occurred, whilst the right column shows the corresponding results of the upgraded gLAB. The results correspond to the DoY 210 of 2014.

Results show in the first zoom that original gLAB did not tolerate the short data gap (lower to 40 s, in epoch 46000 s) and it was not able to detect the 4 cycle slips, positively detected by the upgraded gLAB. Moreover, original gLAB detected 7 cycle slips during the second 27870, whereas upgraded gLAB proved that only 4 were real cycle slips. This happened again in the second zoom, where upgraded gLAB was able to detect 4 cycle slips after the large data gap and coped the short data gap.
4.4.1.3 CFRM Best-case Scenario: DoY 293

Figure 4.13 presents two columns, where the left one shows the results of the original gLAB and the right column the results of the upgraded gLAB.

Figure 4.13: NEU positioning errors for station CFRM during the DoY 293 of 2014.

Results show the correct effect of tolerating short data gaps and justifies the necessity of modifying the data structure of the original gLAB. Notice that during the second short data gap 1 cycle slip was detected by the upgraded gLAB.
4.4.1.4 CFRM Worst-case Scenario: DoY 346

Figure 4.14 presents two columns, where the left one shows the results of the original gLAB and the right column the results of the upgraded gLAB.

The bottom results of Figure 4.14 show the same behaviour experienced in DoY 74. Upgraded gLAB detected 7 cycle slips after the data gap and only 4 satellites were available to compute the positioning. Therefore, the worse performance of the upgraded gLAB was only due to have had a higher DOP than the original gLAB (not caused by a malfunction of the cycle slip detector).
4.4.2 IZAN Station

Figure 4.15: RMS, arithmetic mean and 95th percentile of the 3D positioning error for station IZAN during the year 2014.

With regard to the RMS results, Table 4.7 summarizes the the best- and worst-case scenarios for station IZAN during the year 2014. These particular days are analyzed in detail through the following subsections.
Table 4.7: Best- and worst-case scenarios for IZAN during year 2014.

<table>
<thead>
<tr>
<th>DoY</th>
<th>RMS: Original gLAB (cm)</th>
<th>RMS: Upgraded gLAB (cm)</th>
<th>Original – Upgraded (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>75</td>
<td>29.97</td>
<td>31.34</td>
<td>-1.37</td>
</tr>
<tr>
<td>157</td>
<td>76.36</td>
<td>46.17</td>
<td>30.19</td>
</tr>
<tr>
<td>300</td>
<td>59.87</td>
<td>34.26</td>
<td>25.61</td>
</tr>
<tr>
<td>353</td>
<td>58.94</td>
<td>80.46</td>
<td>-21.52</td>
</tr>
</tbody>
</table>

4.4.2.1 IZAN Best-case Scenario: DoY 157

Figure 4.16 presents two columns, where the left one shows the results of the original gLAB and the right column the results of the upgraded gLAB.

Figure 4.16: NEU positioning errors for station IZAN during the DoY 157 of 2014.

Results show how upgraded gLAB was able to detect at least 5 cycle slips that original gLAB could not detect.
4.4.2.2 IZAN Worst-case Scenario: DoY 75

Figure 4.17 presents two columns, where the left one shows the results of the original gLAB and the right column the results of the upgraded gLAB.

![Figure 4.17: NEU positioning errors for station IZAN during the DoY 75 of 2014.](image)

The top results in Figure 4.17 show a rather similar behaviour between gLABs. On the other hand, middle results show that since second 76600 until second 76790, upgraded gLAB detected more cycle slips than the original one, leaving only 4 satellites with bad geometries to compute the positioning. Finally, bottom results show the same as in the middle ones but omitting positions when the DOP was larger than 30.
4.4.2.3  **IZAN Best-case Scenario: DoY 300**

Figure 4.18 presents two columns, where the left one shows the results of the original gLAB and the right column the results of the upgraded gLAB.

![Figure 4.18: NEU positioning errors for station IZAN during the DoY 300 of 2014.](image-url)

Results show how at the beginning of the DoY 300, the upgraded gLAB is able to detect all the cycle slips and maintain the convergence, whereas the original gLAB did not cope the short data gaps and was not able to detect the cycle slips.
4.4.2.4 IZAN Worst-case Scenario: DoY 353

Figure 4.19 presents two columns, where the left one shows the results of the original gLAB and the right column the results of the upgraded gLAB.

Figure 4.19: NEU positioning errors for station IZAN during the DoY 353 of 2014.

The bottom results in Figure 4.19 have been generated omitting the positions calculated when GDOP was larger than 30 from both original and upgraded gLABs. Upgraded gLAB detected 5 cycle slips, so its position was calculated with only 4 satellites with bad geometries. Notice that after removing the positions with large DOPs, results from both gLABs are almost identical.
4.4.3 TRDS Station

Figure 4.20: RMS, arithmetic mean and 95th percentile of the 3D positioning error for station TRDS during the year 2014.

With regard to the RMS results, Table 4.8 summarizes the the best- and worst-case scenarios for station TRDS during the year 2014. These particular days are analyzed in detail through the following subsections.
### Table 4.8: Best- and worst-case scenarios for TRDS during year 2014.

<table>
<thead>
<tr>
<th>DoY</th>
<th>RMS: Original gLAB (cm)</th>
<th>RMS: Upgraded gLAB (cm)</th>
<th>Original – Upgraded (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>247</td>
<td>27.98</td>
<td>32.91</td>
<td>-4.93</td>
</tr>
<tr>
<td>275</td>
<td>172.69</td>
<td>47.56</td>
<td>125.13</td>
</tr>
<tr>
<td>360</td>
<td>27.60</td>
<td>8.79</td>
<td>18.81</td>
</tr>
</tbody>
</table>

#### 4.4.3.1 TRDS Worst-case Scenario: DoY 247

Figure 4.21 presents two columns, where the left one shows the results of the original gLAB and the right column the results of the upgraded gLAB.

![Figure 4.21: NEU positioning errors for station TRDS during the DoY 247 of 2014.](image)

The bottom results in Figure 4.21 show in detail how, after a large data gap in epoch 26594, the upgraded gLAB was not affected by cycle slips. In second 27500 there was a short data gap of 32 s, where upgraded gLAB was able to maintain the variables, whereas the original gLAB did not, causing, in this particular case, larger errors appear in upgraded gLAB.
4.4.3.2 TRDS Best-case Scenario: DoY 275

Figure 4.22 presents two columns, where the left one shows the results of the original gLAB and the right column the results of the upgraded gLAB.

Results show that part of the better performance in upgraded gLAB was not only attributable to the DOP effects over the 4 satellites used to compute the positioning, but also thanks to have detected 3 cycle slips.

Figure 4.22: NEU positioning errors for station TRDS during the DoY 275 of 2014.
4.4.3.3 TRDS Best-case Scenario: DoY 360

Figure 4.23 presents two columns, where the left one shows the results of the original gLAB and the right column the results of the upgraded gLAB.

Figure 4.23: NEU positioning errors for station TRDS during the DoY 360 of 2014.

Results show that upgraded gLAB correctly detected 4 cycle slips at second 78410, allowing to maintain the estimation of the filter ambiguities and a better performance than the original gLAB.
4.4.4 Summary of the Overall Results of the Dual-frequency Cycle Slip Detector

Table 4.9 shows a summary of the overall results regarding the averages (arithmetic means) along the year of the RMS, the arithmetic mean and the 95th percentile of the 3D positioning error.

Table 4.9: Summary of the Overall Results of the Dual-frequency Cycle Slip Detector.

<table>
<thead>
<tr>
<th>Station name</th>
<th>RMS average (cm)</th>
<th>Mean average (cm)</th>
<th>95th %ile average (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFRM</td>
<td>1.10</td>
<td>4.89</td>
<td>3.82</td>
</tr>
<tr>
<td>IZAN</td>
<td>2.03</td>
<td>5.44</td>
<td>6.17</td>
</tr>
<tr>
<td>TRDS</td>
<td>2.43</td>
<td>9.70</td>
<td>6.63</td>
</tr>
</tbody>
</table>
4.4.5 Results Using Measurements with an Interval of 30 Seconds

The aim of the present subsection is to demonstrate that the updates applied in the dual-frequency cycle slip detector do not negatively affect the performance of the detector when it uses a file with a data interval of 30 seconds.

The coordinates of the station used in these results are presented in Table 4.10.

Table 4.10: Station used during the validation of the dual-frequency detector using a file with a data interval of 30 seconds.

<table>
<thead>
<tr>
<th>Station name</th>
<th>Latitude (degrees)</th>
<th>Longitude (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>COCO</td>
<td>-12.2</td>
<td>96.8</td>
</tr>
</tbody>
</table>

Figure 4.24 shows that both results are quite similar. However, upgraded gLAB presents a more accurate positioning from the second 50200 until the end of the day. Although the improvements are more visible in the Up error, North error and East error also depict better accuracies after the second 70000 than the original gLAB.

Figure 4.24: NEU positioning errors for station COCO during the DoY 90 of 2014.
CONCLUSIONS

This chapter is devoted to present the principal achievements of this master's thesis and some of the steps that can be performed as future work.

4.5 Present work

It is worth mentioning that a big effort has been done to understand the behaviour of the cycle slip phenomenon, as well as to find and know how the modulus of gLAB in charge of them were handling the data flux.

The original gLAB was extremely conservative designed and did not allow data gaps in the input observables. This limitation, consequence of its data structure, has been fixed. Nevertheless, modifying the data structure has implied a great programming effort because almost the entire gLAB was based on it.

Several new features have been implemented in gLAB, such as to handle data gaps, to manage the LLI flag provided in the RINEX files and to check the consistency of measurements by comparing the sizes of the jumps between contiguous measurements of both carrier and code.

The single-frequency cycle slip detector, based on the L1-C1 combination, has been adapted to the new data structure. Furthermore, applying the Hatch filter over the sigma of the aforementioned combination has been the new feature implemented, the result of which is a weighted and stable sigma that better characterizes the detection threshold.

The dual-frequency cycle slip detector has been designed and implemented through two detectors: Geometry-free and Melbourne-Wübben, which are the current state-of-the-art in cycle slip detection. The most important feature implemented in the last detector has been a complex logic that considers not only the entire mean of the Bw combination, but also the mean of a sliding windows of the 300 last seconds. The novel feature implemented in the geometry-free detector has been ignoring outliers and taking into consideration the goodness of a polynomial fit to the carrier samples.

The validation has been performed by comparing the original gLAB with the upgraded one and also considering the real coordinates of the stations’ antennae as an absolute reference. It has revealed that the single-frequency cycle slip detector, despite having a rather similar behaviour, has achieved a slight improvement. On the other hand, the validation of the dual-frequency cycle slip detector has shown a considerable enhancement.

Finally, a set of tests to check all the capabilities of gLAB have been performed to ensure the correct behaviour of the program, in despite of having upgraded its data structure.
4.6 Future work

Although gLAB is planned to be a GNSS multi-constellation tool in a near future, currently it is only able to fully process GPS data. Therefore, the cycle slip detectors implemented and presented in this master’s thesis are configured to work properly with the American constellation. Nevertheless, the detectors are based on analyzing any combinations of measurements, thus the detectors can be replicated in any constellation.

In conclusion, the detectors implemented in this master’s thesis are able to work with any constellation by adjusting only a few parameters: thresholds and wavelengths, which depend on the frequency of the signals used in their combination. Therefore, the future work here proposed is to automatically (no user interaction) adjust these parameters to all possible combinations from whatever constellation.
<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>AltBOC</td>
<td>Alternate Binary Offset Carrier</td>
</tr>
<tr>
<td>ANTEX</td>
<td>ANTenna EXchange format</td>
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<tr>
<td>APC</td>
<td>Antenna Phase Centre</td>
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<td>ARNS</td>
<td>Aeronautical Radio Navigation Service</td>
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<td>Anti-Spoofing</td>
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<td>Code Division Multiple Access</td>
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<td>CS</td>
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<td>DOP</td>
<td>Dilution of Precision</td>
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<td>DoY</td>
<td>Day of Year</td>
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<tr>
<td>FOC</td>
<td>Full Operational Capability</td>
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<td>gLAB</td>
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<td>GLONASS</td>
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<tr>
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