To appear in the *International Journal of Mathematical Education in Science and Technology*, Vol. 00, No. 00, Month 20XX, 1–9

Mathematical Modelling in Engineering: An Alternative Way to Teach Linear Algebra

S. Domínguez-García[a] M.I. García-Planas[b*] and J. Taberna[b]

[a]*Universitat Rovira i Virgili, C. de l'Escorxador, s/n, 43003 Tarragona, Spain*; [b]*Universitat Politècnica de Catalunya, Diagonal 647, 08028 Barcelona, Spain*

(*Received 00 Month 20XX; final version received 00 Month 20XX*)

Technological advances require that basic science courses for engineering, including linear algebra, emphasize the development of mathematical strengths associated with modelling and interpretation of results, which are not limited only to calculus abilities. Based on this consideration, we have proposed a Project-based learning, giving a dynamic classroom approach in which students modeled real-world problems and turn gain a deeper knowledge of the lineal algebra subject. Considering that most students are digital natives, we use the e-portfolio as a tool of communication between students and student-teacher, besides being a good place making the work visible. In this article, we present an overview of the design and implementation of a project-based learning for a linear algebra course taught during the 2014-15 at the "ETSEIB"of Universitat Politècnica de Catalunya (UPC).

Keywords: project-based learning; E-portfolio; linear algebra

1. Introduction

It is well known that Linear Algebra is a basic subject in different areas of sciences. This is due to the multiple problems that can be modeled using linear systems whose solutions can be essentially obtained and discussed through Linear Algebra. Nevertheless, one of the main difficulties that first-year university students who have enrolled in scientific or technical degrees other than Mathematics is that they do not see the importance that mathematics, in general, and Linear Algebra, in particular, may have in their fields of interest. This can seriously affect students' motivation in the course, and prevent their success. This effect appears to be more pronounced in the first year of Linear Algebra due to its abstract factor, whereas Calculus is easier to think that may be more useful.

Since the 90s, there has been a push in order to reform the way in which Linear Algebra is taught. Concretely, in 1993, the Linear Algebra Curriculum Study Group (ICSG) in the USA published a set of recommendations for the Linear Algebra course when they noticed that "the Linear Algebra curriculum at many schools [did] not adequately address the needs of the students" [4]. With these recommendations, there was not only an encouragement of the research teaching Linear Algebra but also a provision of an extensive list of articles on this topic (see [10] and [1], for example). Among these studies, it is worth noting what several researchers say about the use of ICT, and how it affects both students' attitudes and their academic achievements.

Following Piaget's psychological theory of concept development, Harel [8] highlighted the following three "principles" for the teaching of Linear Algebra: the Concreteness, the Necessity and the Generalizability. The Concreteness Principle states that "for students to abstract a

*Corresponding author. Email: maria.isabel.garcia@upc.edu

mathematical structure from a given model of that structure, the elements of that model must be conceptual entities in the student's eyes". Regarding the Necessity Principle, "students must see a[n] [intellectual] need for what they are intended to be taught". Finally, the Generalizability Principle says that "when instruction is concerned with a 'concrete' model, that is to say, a model that satisfies the Concreteness Principle, the instructional activities within this model should allow and encourage the generalizability of concepts".

In the search for alternatives to improve the teaching and learning of Linear Algebra, different experiments have been designed and implemented. It has been concluded that to improve the teaching and learning of Linear Algebra can be useful to include alternatives, such as the use of new technologies, concretely, by means of the creation of e-portfolios and the implementation of projects both individually and in groups. For that reason, the objective of this research is to analyze the benefits of the project-based learning (PBL) in the Linear Algebra curriculum by creating e-portfolios in large groups as a tool for teaching, learning, assessment and self-assessment of student learning. At the end of the paper, we show the results of this study, the satisfaction as well as the usefulness of this tool. The participants used for this project are all students from the first year of Engineering on the subject of Linear Algebra at the ETSEIB of the Universitat Politècnica de Catalunya.

2. Objectives

The main objectives of our alternative method of teaching Linear Algebra are the following:

(1) To encourage students to participate and be part of the learning process through interactive teaching and discussions which will lead to students discovering the main concepts themselves.
(2) To encourage cooperative learning. Team projects and reports are excellent vehicles for cooperative learning. Students will work together to solve or resolve problems of importance to them.
(3) To introduce the students to the art of developing and writing correctly mathematical expressions.
(4) To help students to think in an accurate manner and verbalize their thoughts clearly.
(5) To assist in incorporating mathematical software to compute Linear Algebra.
(6) To learn to use the ICT's tools not only as a support when learning linear algebra but also as a way to transmit the knowledge acquired during the course.

3. Methodology

It is important to notice that teaching in large groups does not imply to be only given as a master class. So, under this conviction, we have introduced alternatives to lectures in a core subject in studies of the first year engineering degree; such as including PBL, the e-portfolio and math tools technology. In addition, the e-portfolios are displayed in a common space on the University's virtual campus where students can see their classmates' projects. More concretely, the students must design, plan, and carry out an extended project, whose product must be publicly shown in their e-portfolios that must be visible (at least) to teachers and classmates.

Students are expected to present a model design and the solution of a numerical problem given. To do that, students need to understand what has been taught in class, apply all this knowledge and learn how to perform numeric calculations with a mathematical software tool (Matlab).

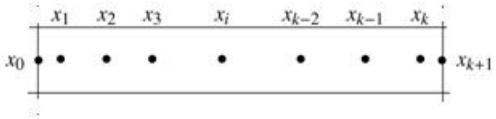

Figure 1. Heat transfer

3.1. *Teaching experiment: Projects*

During the academic year 2014-15, we conducted a pilot test of the project-based collaborative learning by means of e-portfolios in a core mathematical subject. The participants were the undergraduate students of the "Escola Tècnica Superior d'Enginyers Industrials de Barcelona, (ETSEIB)"at the "Universitat Politècnica de Catalunya, (UPC)". The main objective of this project was to figure out if using PBL is much more effective to learn Linear Algebra. Since it uses practical cases, it helps students to create the need for learning this subject that we considered a key point to take into account. Next, we will detail one of the tasks to be performed by students.

3.1.1. Project: Newton's law of heating and the heat equation

This task consists of studying the distribution of heat through a thin bar made of a homogeneous material, discretizing the bar in n equidistant points (see Figure 1). Specifically, it is to analyze the evolution of the temperature in each of the points with the passage of time.

- Step 1: Modeling the evolution as a discrete linear dynamic system.

  Students must come to model the problem of the form

$$T(n+1) = AT(n) + B$$

  more concretely

$$\begin{pmatrix} T_1(n+1) \\ T_2(n+1) \\ \vdots \\ T_{k-1}(n+1) \\ T_k(n+1) \end{pmatrix} = \begin{pmatrix} 1-2\alpha & \alpha & 0 & \dots & 0 & 0 \\ \alpha & 1-2\alpha & \alpha & & & \\ 0 & \alpha & 1-2\alpha & \ddots & & \\ \vdots & \vdots & \ddots & \ddots & & \\ 0 & 0 & 0 & \alpha & 1-2\alpha & \alpha \\ 0 & 0 & 0 & & \alpha & 1-2\alpha \end{pmatrix} \begin{pmatrix} T_1(n) \\ T_2(n) \\ \vdots \\ T_{k-1}(n) \\ T_k(n) \end{pmatrix} + \begin{pmatrix} \alpha b \\ 0 \\ \vdots \\ 0 \\ \alpha c \end{pmatrix}$$

  where $T_i(n)$ is the temperature at the time n at the point x_i, α the Newton's law proportionality constant and b and c the fixed temperatures of the right end and left end points.

  At this moment the students must wonder about the equation's solution and to do so, they should find inductively the following equation.

$$T(n) = A^n T(0) + \sum_{i=0}^{n-1} A^i B$$

  And now, students need to learn to handle matrix operations and especially need to learn how to calculate the powers of the matrix. When students get there, on the following classes, the professor explains the matrices theory, introduces the vector spaces and the linear maps. What is more, the professor asks students to start using MATLAB.
- Step 2: Explain in detail the linear algebra information required to solve the equation, such as (see [7] for more details):

a) Analyze the type of matrix describing the system.
Students observe that the matrix is tridiagonal.

b) How to calculate powers of a matrix.
Students observe that if the matrix is diagonal is very easy to obtain the powers of the matrix, and professors expect that students relate what has been explained in class, in this case, the similarity of matrices ($A = SDS^{-1}$), with their tridiagonal matrix problem.
From that moment on, the teacher explains diagonalization of endomorphisms and the reduced form of Jordan.

c) Obtaining of eigenvalues and analysis of stability study.
Then, students compute the eigenvalues:

$$\lambda_i = (1-2\alpha) + 2\alpha \cos \frac{i\pi}{k+1} \; i = 1, \ldots, k$$

- Step 3: Analyze the evolution of solutions.
After collecting all the information obtained so far, students are supposed to observe that

$$\lim_n T(n) = \lim_n A^n T(0) + \lim_n \sum_{i=0}^{n-1} A^i B = \lim_n SD^n S^{-1} T(0) + \lim_n \sum_{i=0}^{n-1} SD^i S^{-1} B$$

and that if $|\lambda_i| < 1$, then

$$\lim_n A^n = 0,$$

and also notice that the matrix $I - A$ is invertible and $\lim_n \sum_{i=0}^{n-1} A^i B = (I-A)^{-1} B$.
So,

$$\lim_n T(n) = (I-A)^{-1} B.$$

- Step 4: Solve a practical case applying all theoretical results. It is necessary to indicate at all times the theoretical side that has been applied and explain why.
At this time, students should use MATLAB that was taught how to use it during the theoretical course.

It is important to put all references used (books, websites, and comments with other groups...) in all the steps of the project

The final delivery took place during the last week of the course. In this final phase, students also deliver the e-portfolio which must contain their résumé, their own overview of the subject and all the references.

Obviously, every single piece of work that has been updated in the e-portfolio has been reviewed and given feedback by the teacher and even by the owner's classmates.

Until date, we have proposed and carried out four different projects which are:

- Leontief economic open model.
- Application of Markov chains to the study of the optimum number of beds in ICU, intermediate care in a hospital room for decision making in hospital management.
- Population models age-structured.

They have been organized in a similar way in order to learn the same topics.

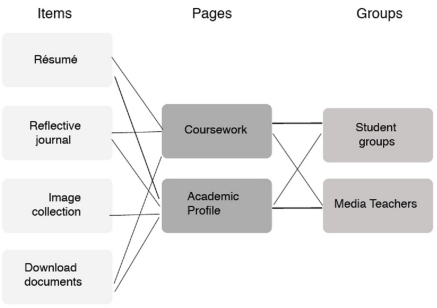

Figure 2. Organizing a portfolio: Schema

3.2. *E-portfolio*

The E-portfolio is a valuable tool for teaching, learning and assessment. Remember that, an e-portfolio is a digitized collection of evidence joined and managed by a user, usually on the Web. Such electronic evidence may include entering text, electronic files, images, multimedia, blog entries, and hyperlinks, among others. E-portfolios are both a process of showing of the user's abilities and platforms for self-expression, and, if they are online, they can be maintained dynamically over time. In order to clarify the structure of an e-portfolio, we show a schematic overview of the artifacts constituting the portfolio in Figure 2, as well as the pages that integrate it and the groups that compose it, [6].

In our particular setup, the e-portfolio has the sense of "assessment folder"or a broader form of "learning portfolio". More specifically in the context of this work, the e-portfolio is a method of teaching, learning and assessment of different types of productions by students through which they can evaluate their abilities in the context of a discipline or field of study. Moreover, staff reviews the student productions with evaluation criteria previously established, allowing the student (and others) to see their efforts and achievements into the learning objectives.

In the context of teaching and learning of the subject of Linear Algebra, the e-portfolio can be used as a repository of learning experiences as through the teachers and students work the teaching and learning activities from the Internet. More specifically:

a) The students use the e-portfolio for:
   i) Collecting systematically achievements in learning,
   ii) Self-evaluating how to acquire and develop the skills required in the subjects they are studying,
   iii) Self-evaluating their learning results.
b) The teacher uses the e-portfolio for:
   i) Collecting and placing the work done by students in their courses
   ii) Evaluating the acquisition and development of skills of students through the implementation of activities.
   iii) Assessing the outcome of learning.

Thus, a clear interaction between teacher and student is obtained.

During the first week of the course, the professor explained comprehensively what is an e-portfolio and the project that students had to perform both individually and collectively.

3.2.1. Software for creating e-portfolio

There are many online platforms that allow creating a personal e-portfolio in general and educational e-portfolio in particular. Several authors (see [2] for example), analyze some of these educational platforms to facilitate the decision about which platform will be choosen.

The platform models chosen for our experience were Mahara, Exabis, WordPress and Google Sites. We assigned each student a different platform. We wanted also to analyze which platform was more understandable and easy to work with for the students.

3.3. *Evaluation of projects*

One way to tell if students have acquired the required training skills to ensure that they have achieved the learning objectives is by means of an evaluation. E-portfolio evaluation criteria should take into account operational, appearance, the evidence and the reflection Indicators. Considering all these aspects and in order to respond to such needs, we have chosen to create a rubric to assess the different e-portfolios. As it is well known, rubrics are descriptive scoring schemes that are developed by teachers or other evaluators in order to guide the analysis of the results or the efforts in the processes of students, (see [3] and [9] for more details about rubrics). For the Linear Algebra course, we proposed two rubrics, the first one for the student self-assessment as well as for peer assessment, and the second one, for the teacher assessment of the student's work .

We decided to evaluate different aspects of the final task introduced at the e-portfolio, the most important one corresponds to the problem-solving process of the project, the second one is related to the formal appearance of the site, and the last part is reserved for the self-assessment and peer assessment of the student's work. The formal aspect was evaluated depending on the assigned platform (Mahara, Exabis, Google Sites or WordPress), because we knew the different limitations that each platform presented.

There is always a need to evaluate students' skills in order to know whether they have achieved the learning objectives or not.

For that reason, the E-Portfolio Evaluation Criteria takes into account operational, appearance, evidence and reflection Indicators.

Considering all these aspects and to respond to such needs, the creation of a rubric was decided to be the best tool to assess the student's work. Remember that scoring rubrics are descriptive scoring schemes that are developed by teachers or other evaluators to guide the analysis of the products or processes of students' efforts, (see [3], [5] and [9] for more details).

The rubrics used are for the student self-assessment, for peer assessment, and for the teachers to assess the student's work. Moreover, it is important to emphasize that the students know the rubrics from the first day of course. However, there was not existing rubrics that provide the information required for Algebra Linear course. For that reason we decided to perform new rubrics in such a way that they were perfectly adapted to the course that we wanted to evaluate.

1- Student's rubric
   1. the work has coherence between the template and the result proposed.
   2. the e-portfolio has been written in, at least, two languages.
   3. bibliographical resources have been enough and adequate.
   4. the elements have been selected adequately.
   5. the site is readable
   6. the site has a coherent graphic design.
   7. the e-portfolio's language is competent and proficient.
   8. the tasks have worked on data analyse
   9. the tasks have worked on unknowns
   10. the tasks have worked on modelling
   11. the problem has been solved
   12. the solution has been verified
   13. a reflection and critique of the work has been made
   14. previous knowledge about the topic of the task
   15. something new has been learnt by doing these tasks

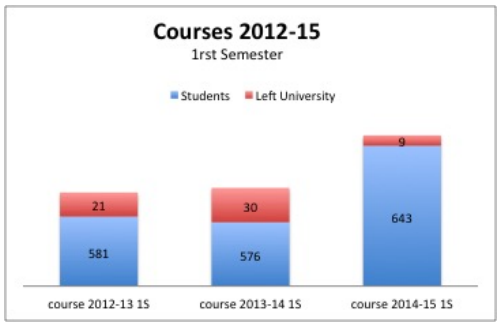

Figure 3. Number of students that left the University

Scale: Inadequate, limited, adequate, proficient, excellent

2- Teacher's rubric
   1. Selection of Items 5%
   2. Graphic Design, Multimedia, Navigation, Readability 5%
   3. Citation, bibliography 5%
   4. Quality of Writing and Proofreading 5%
   5. Exercise comprehension, data analysis, unknowns, modelling, resolution, verification of solution, Reflection/Critique 70%
   6. Coherence between the template and the result proposed 10%

   Scale: Inadequate, limited, adequate, proficient, excellent

4. Results

We have compared the different qualifications of the students during three periods which correspond to the courses 2012-13, 2013-14 and 2014-15, this last one is the period where we have introduced the e-portfolio as part of the curriculum and there has been a change in the evaluation methodology using the rubrics for the e-portfolio. We show the results of the first semester because that is the moment when we have more new students; in the second semester, the most important amount of students are the ones that have failed the subject in the first semester, so they already knew the material. As we can see in Figure 3, if we compare the number of students that left Linear Algebra during the first semester of the university over the past three years, we can see that despite the number of students have increased considerably this last year, just nine students have left the course. This last course 2014-15 just a 1,40% of the total amount of students of Linear Algebra left the subject, the years before this average was 5,21% in course 2013-14 and 3,61% in course 2012-13.

We have also analyzed the qualifications between this last academic year 2014-15 and the academic year 2013-14 (see Figures 4,5,6,7). Thanks to that, we have observed that fewer students failed this last course (33,43%) in comparison to the first semester from the course 2013-14 (52,38%). This is a reduction of the 18,95%. What is more, although the second semester was not taken into account for this project, it is noteworthy that there was also a decrease of failures of the 39,18%. The good marks have also increased a 3% this last year.

5. Conclusions

The use of the project-based learning and the e-portfolio let us see the improvement of the autonomy of the students. They have been working individually and in groups, so they have also been able to cooperate and to work together to solve the different tasks related to the project.

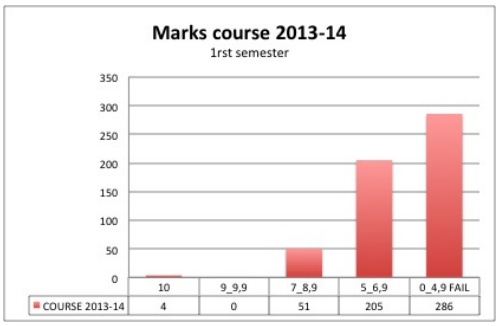

Figure 4. Qualifications course 2013-14 first semester

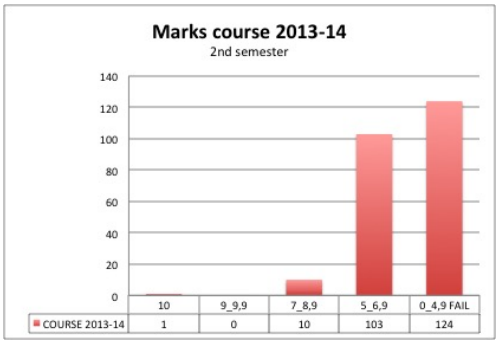

Figure 5. Qualifications course 2013-14 second semester

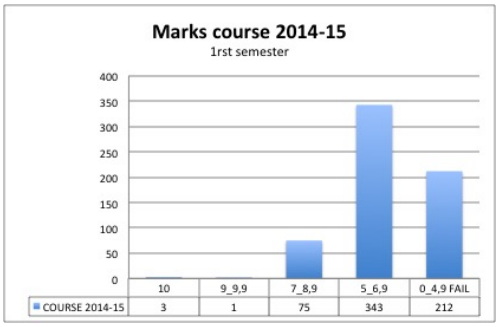

Figure 6. Qualifications course 2014-15 first semester

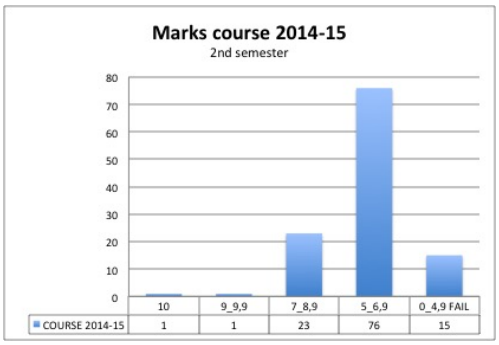

Figure 7. Qualifications course 2014-15 second semester

For students to create a need to study Linear Algebra, we decided to do real-life projects. This fact has motivated the students to ask questions and understand that Linear Algebra, which they thought was useless at the beginning, has a practical application.

Working with the four different platforms made us realize that WordPress is the best tool to use to create e-portfolios for students and also for teachers.

With this new experience, the qualifications of the students have been improved and also, they have achieved specific and general skills of Linear Algebra. The students had used the e-portfolio as a tool for improving the comprehension of the subject through solving a project, and for learning to structure, organize, communicate and show the work they have done properly.

Acknowlegments

We are indebted to Maria Isabel Domínguez García for her help in the structure of the paper.

References

[1] Aydin, A. The factors effecting teaching linear algebra. World Conference on Educational Sciences. Procedia Social and Behavioral Sciences, 1, 2009. p. 1549–1553.

[2] Bri, D., García, M., Coll, H. LLoret, J. A Study of Virtual Learning Environments. Wseas Transactions on Advances in Engineering Education, 6(1), 2009. p. 33–43.

[3] Brookhart, S.M. The Art and Science of Classroom Assessment: The Missing Part of Pedagogy. ASHE-ERIC Higher Education Report Washington, DC: The George Washington University, Graduate School of Education and Human Development, 27(1). 1999.

[4] Carlson, D. C.R. Johnson, D.C. Lay, A.D. Porter. The linear algebra curriculum Study group recommendations for the first course in linear algebra. The College Mathematics Journal, 24(1), 1993. p. 41–46.

[5] Domínguez-García, S. Creació d'una rúbrica per avaluar la competència docent en l'ús de la Pissarra Digital Interactiva, TFG. Universitat Rovira i Virgili. 2014.

[6] Domínguez-García, S., García-Planas, M.I., Palau, R., Taberna, J. Using the e-portfolio for large groups of students Proceedings of INTED2015 Conference 2015, p. 1352–1360.

[7] García-Planas, M.I., Domínguez, J.L. Introducción a la teoría de matrices positivas. Aplicaciones. Ed. Iniciativa Digital Politècnica, Barcelona, 2013.

[8] Harel, G. Principles of Learning and Teaching Mathematics, With Particular Reference to the Learning and Teaching of Linear Algebra: Old and New Observations. In J-L. Dorier Ed. On the Teaching of Linear Algebra, Kluwer Academic Publishers, Dordrecht. 2000. p. 177–189,

[9] Koh, J.H.L. A rubric for assessing teachers' lesson activities with respect to TPACK for meaningful learning with ICT. Australasian Journal of Educational Technology, 29(6), 2013. p. 887–900.

[10] Yildiz, A.U. Teaching the diagonalization concept in linear algebra with technology: A case study at Galatasaray University. The Turkish Online Journal of Educational Technology, 12(1), 2013. p. 119–130,