Study of Chaotic Behavior in Automatic Tuning Loops for Continuous–Time Filters

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***Abstract*— Continuous–time filters (CTFs) with automatic tuning loops are nonlinear feedback systems with potential instability. Thus, their appropriate linear dynamic modeling should be obtained to assure stability in case an improved design of the loop controllers is to be carried out. A systematic approach using a small signal model would allow obtaining these controllers. However, bifurcations and nonlinear phenomena may appear which cannot be predicted by this analysis. This leads to potential instability, semiperiodic or chaotic behavior and, thus, circuit malfunction. The aim of this paper is to show by means of simulations and experimental results that nonlinear phenomena, which cannot be predicted by the common small signal analysis, may appear in this kind of circuits when circuit parameters are varied.**

I. INTRODUCTION

CTFs with tuning capabilities are adaptive filtering stages with control loops that incorporate tuning input signals aimed to directly modify the parameters of the original circuit structure. The conventional tuning strategy (Fig. 1) consists of an indirect adjustment based on the so–called master–slave scheme [1], [2]. The auxiliary (master) filter is usually a copy of one of the cells that are part of the main (slave) filter. In general, the tuned parameters are the quality factor (Q) and the central frequency (ω_O) of each one of the cells that compose the main filter. Thus, on the one hand, the main filter performs the filtering process for the incoming signal. On the other, the master filter, which is embedded within the ω_O and Q tuning control loops, receives a reference sine waveform $v_{REF}(t)$, whose frequency (which must be as stable as possible) should ideally be tracked by the filter, hence indirectly setting the central frequency ω_O of the slave filter.

In order to improve the performance of the two control loops (tuning speed, transient response, etc.), and predict the behavior of the tuning system, the first required step consists of modeling the filter, considering the master filter as the system plant. The control loops (consisting of the master filter, the ω_O and Q control blocks) typically exhibit a non–linearity of the bilinear type, which is well–known in several system modeling areas as in average models for switching power converters [3]. This bilinear behavior is caused by terms containing the product of state variables and control inputs in equations that describe their behavior. An additional source of modeling complexity of filters with tuning capability is caused by the sinusoidal nature of the reference input signal to the control loops, which requires the dynamic model to be obtained for low frequency base-band equivalent signals (envelope and phase shift).

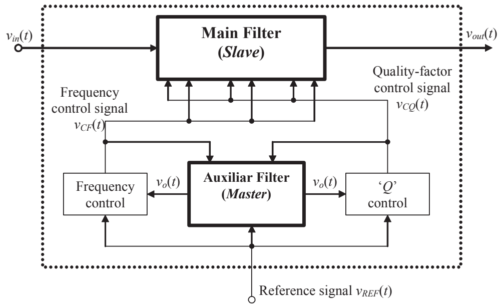

Fig. 1: Block diagram of a tuning system for CTF based on a master-slave strategy.

In particular, starting from the modeling technique proposed in [1] for filters with automatic tuning, the corresponding transfer functions from the quality factor control signal to the amplitude of the output signal, as well as from the central frequency control signal to the phase–shift of the output signal are required. As a result, these transfer functions allow designing both enhanced non–adaptive and adaptive controllers that improve the performance of previous controllers based in dominant pole compensation for tuning both Q and ω_O. Note that the design of loop controllers is, without the knowledge of the involved transfer functions, "*blind*" in relation to the considered system. In addition to this, the aim of the self–tuning subsystem might consist not only in correcting component tolerances and drift DC errors, but also in dynamically varying Q and ω_O parameters of the CTF. In such applications the loop bandwidth is critical.

However, nonlinear phenomena that can not be predicted by a design-oriented small signal modeling approach are observed in this kind of tuning systems. As a matter of fact, this bilinear behavior and time-varying nature of the reference input signal to the control loops make the system prone to exhibit nonlinear phenomena that cannot be predicted by a design-oriented small signal modeling approach. The prediction of such behaviors can only be predicted by a suitable model that retains the nonlinearity and the time-variance of the system. That is, while an appropriate *small signal linear dynamic modeling* of the tunable filter should be obtained for design purpose, its ability to predict the real *large-signal nonlinear dynamic behavior* of the system is very limited. To overcome this problem, a general and systematic procedure has to be used in order to obtain a large signal nonlinear model.

The purpose of this work is to highlight that, when control parameters are varied, the system could present different kinds of dynamical nonlinear phenomena such as bifurcations and chaotic behavior, which cannot be predicted by the small signal design-oriented model. In addition, these nonlinear phenomena are also shown in high efficiency filters with tuning capabilities. Thus, the study of this behavior could be used to improve tuning control loops in electronically tunable switch-mode high-efficiency adaptive band-pass filters intended for energy harvesting applications [4].

The rest of the paper is organized as follows. Section II presents the system description and the mathematical dynamic model of CTFs with tuning capability. Section III shows the description of and application example, in which a discrete implementation of CTF with tuning capability is considered. Finally, in Section IV, non-chaotic and chaotic behaviors are observed. Different tools are combined to identify the dynamical behavior of the system. Finally, some concluding remarks are drawn in the last section.

II. Generic Dynamic Modeling of Tunable Filters

The design and implementation of improved controllers for Q and ω_O loops (Fig. 2.*a*) requires first the dynamic modeling of the *master* filter considering control signals $v_{CF}(t)$ and $v_{CQ}(t)$ as inputs, and the phase–shift of the filter output signal for the ω_O–control loop, and the amplitude of the filter output signal for the Q–control loop as outputs. Note that both control loops are nonlinear systems with non–DC steady–state.

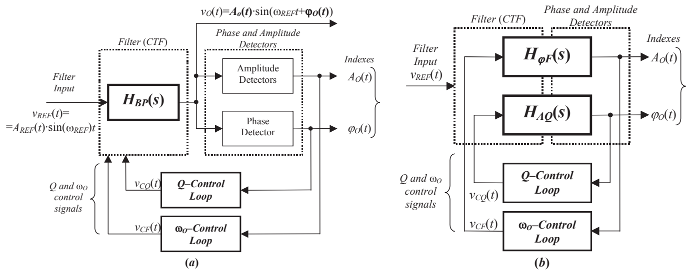

Fig. 2: (*a*) Basic block diagram of an on–chip tuning system for a CTF. (*b*) CTF model with amplitude and phase detectors and control loops.

In order to analytically study the local stability of the tuning system, the first step consists of modeling the filter, considered as the control system plant, yielding the transfer functions of the equivalent small signal circuit. Generally [4], [5], most analog filtering structures, including automatic tuning, consider second order filters with three input signals as a master cell. Namely: The original input signal, $v_{REF}(t)$ in Fig. 2.*a* and Fig. 2.*b*, and two control inputs, represented as $v_{CF}(t)$ and $v_{CQ}(t)$, that tune, respectively, central frequency (ω_O) and quality factor (Q) of the filter. In addition, this second order *master* filter has two state variables $v_1(t)$, $v_O(t)$, one of which is usually the output signal of the circuit. Therefore, the system can be expressed in a space–state representation form as [1]:

$$\mathbf{x}(t)=\begin{bmatrix} v_O(t) \\ v_1(t) \end{bmatrix} \qquad \mathbf{u}(t)=\begin{bmatrix} v_{REF}(t) \\ v_{CF}(t) \\ v_{CQ}(t) \end{bmatrix} \tag{1}$$

$$\dot{\mathbf{x}}(t)=\mathbf{A}\left(v_{CF}(t),v_{CQ}(t)\right)\cdot\mathbf{x}(t)+\mathbf{B}\left(v_{CF}(t),v_{CQ}(t)\right)\cdot v_{REF}(t)$$

It can be argued that phase and amplitude detectors (Fig. 2.*a*) provide a circuit approximation to the *baseband demodulation* operation, with the objective of extracting from the state phasor $v_O(t)$ (*modulated carrier* signal), the phase and amplitude slow information that is required to properly tune both frequency and quality factor parameters, respectively (Fig. 2.*b*).

From the baseband–equivalent linear equations, the transfer function that relates the amplitude of the state variables to the control voltage that tunes the quality factor $\tilde{v}_{CQ}(t)$, as well as the transfer function that relates the phase–shift of these state variables to the control voltage $\tilde{v}_{CF}(t)$ that tunes the central frequency ω_O of the master (and thus slave) filter can be obtained [1]. These transfer functions are required for studying loop (local) stability and, in turn, design enhanced compensators.

III. Application Example: Discrete of Continuous-Time Filter with Tuning Capability

As an example of a CTF with tuning capability that shows dynamical nonlinear phenomena that cannot be predicted by the small signal design-oriented model, a particular 2nd order continuous-time filter with tuning capability is considered in this work (Fig. 3). This circuit structure consists in a 2nd order state–variable filter, in particular, the so–called TQE (transimpedance Q–enhancement) filter [6], [7]. In order to perform the automatic tuning of the CTF, resistors must be implemented by means of electronically tunable circuits. In an on chip implementation, there are several available structures in order to perform the tuning capability such as the he cell known as MOS Resistive Circuit (MRC) [8]. However, as a matter of example, and in order to reveal these dynamical nonlinear phenomena (such as the aforementioned bifurcations and chaotic behavior) in automatic tuning of CTFs, analog multipliers are used as electronically tunable cells in order to implement a discrete implementation of the CTF (Fig. 4). Notice that, in this case, control voltages $v_{CF}(t)$ and $v_{CQ}(t)$ tune, respectively, the central frequency (ω_O) and the quality factor (Q) of the TQE structure. On the one hand, equivalent resistances $R'_1=R'_3=R'_{in}$ (all equal) that set the central frequency ω_O of the structure are given by:

$$R'_1=R'_3=R'_{IN}=\frac{R_1}{K_M V_{CF}}\ , \tag{2}$$

where constant K_M is the same for all analog multipliers (1/10 for commercial model AD633). Control voltage $v_{CF}(t)$ that tunes the central frequency (ω_O) is applied to analog multipliers M_1, M_3 y M_{in}. Therefore, the central frequency ω_O of the 2nd order topology is given by:

$$\omega_O=\frac{1}{R'_1C}=\frac{K_M}{R_1C}V_{CF} \tag{3}$$

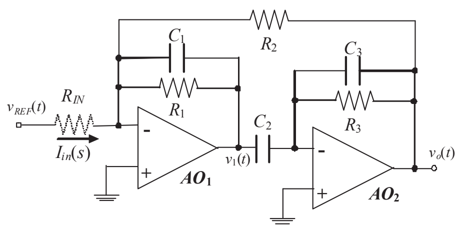

Fig. 3: 2nd order band-pass filter with TQE (transimpedance Q–enhancement) topology.

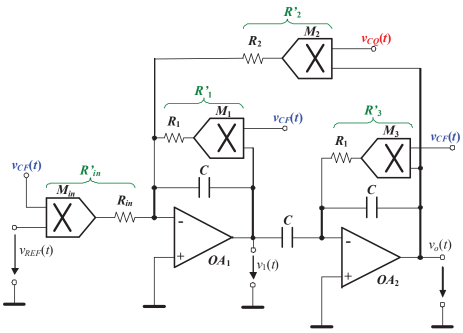

Fig. 4: 2nd order analog multiplier-based band-pass filter with TQE topology.

On the other, the equivalent resistance R'_2 that fixes the quality factor Q of the structure is:

$$R'_2 = \frac{R_2}{K_M V_{CQ}} \tag{4}$$

Considering that Q is:

$$Q = \frac{1}{\left(2 - \frac{R'_1}{R'_2}\right)} \Rightarrow Q = \frac{1}{2 - \frac{R'_1}{R'_2}} = \frac{1}{2 - \frac{R_1 V_{CQ}}{R_2 V_{CF}}} \tag{5}$$

IV. Simulation and Experimental Results

In order to corroborate and validate the chaotic behavior, simulations and experimental trials have been carried out with CTF shown in Fig. 4, operating with its two control loops.

A Matlab-Simulink® system model has been obtained in order to study the performance of the two control loops [7], and, in addition, the possible observation of the aforementioned dynamical nonlinear phenomena such as bifurcations and chaotic behavior when circuit parameters are varied. On the one hand, Fig. 5 and 6 show the steady-state waveforms of the system when the 2nd order analog multiplier-based band-pass filter is tuned with a reference frequency equal to 2.5 *kHz* and reference Q of 4. In particular, in Fig. 5 both control signals ($v_{CF}(t)$ and $v_{CQ}(t)$) can be observed, whereas Fig. 6 shows output voltage $v_o(t)$. The phase-space diagram of the tuned CTF that shows the two output signals (V_1, V_o) is shown in Fig. 7. Clearly, it can be seen that the system is stable and there is not chaotic behavior.

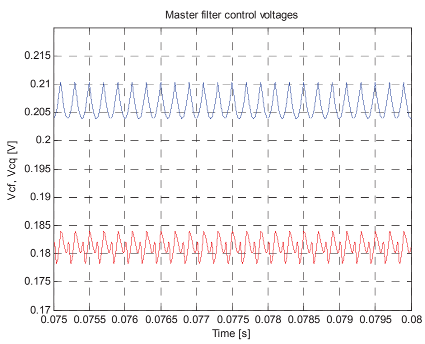

Fig. 5: Example of steady-state control voltages waveforms $v_{CF}(t)$ (upper trace) and $v_{CQ}(t)$ of the control loops when there is not caothic behavior.

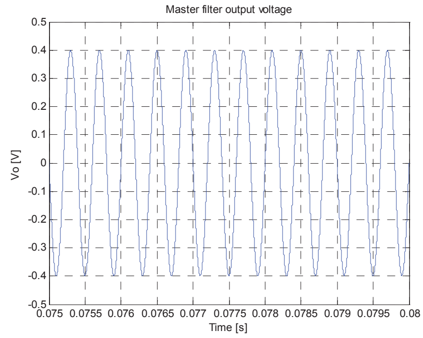

Fig. 6: Example of steady-state output voltage waveform at the output terminal $v_o(t)$ of the master filter when there is not caothic behavior.

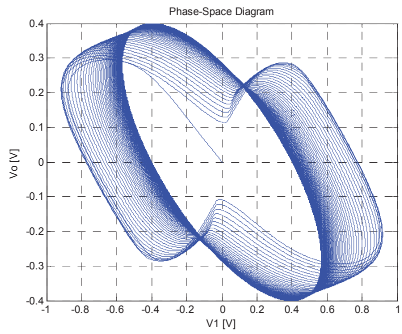

Fig. 7: Phase-space diagram of the tuned CTF that shows the two output signals (plot(V_1, V_o)) without chaotic behavior.

However, on the other hand, if the gain in the frequency control loop increases, control signals ($v_{CF}(t)$ and $v_{CQ}(t)$) and filter output voltage $v_o(t)$ can be appreciated in Fig. 8 and 9, respectively. In this case, the phase-space diagram of the tuned CTF that shows the two output signals (V_1, V_o) is shown in Fig. 10. Notice that a chaotic orbit takes place.

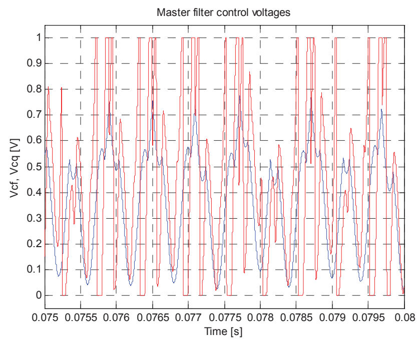

Fig. 8: Example of control voltages waveforms $v_{CF}(t)$ and $v_{CQ}(t)$ of the control loops when caotic behavior is present.

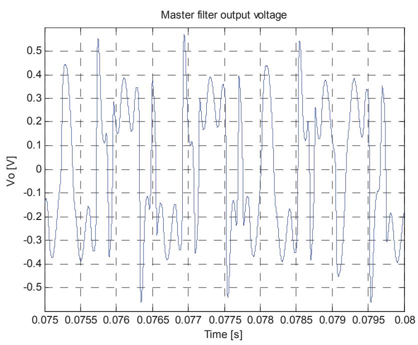

Fig. 9: Example of output voltage waveform at the output terminal $v_o(t)$ of the master filter when caotic behavior is present.

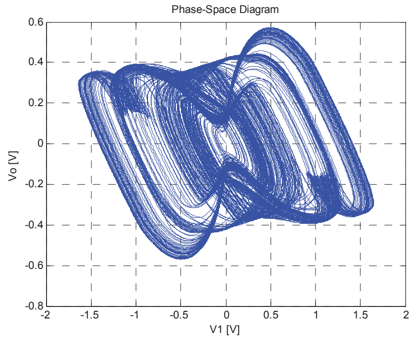

Fig. 10: Phase-space diagram of the tuned CTF that shows the two output signals (plot(V_1, V_o)) with chaotic behavior.

In addition, Fig. 11 and 12 show some experimental results. In Fig. 11 (up) circuit signals can be observed when the system is not tuned. In this case, with an input signal (CH1) with amplitude of 3 *V*, and frequency of 7 *kHz*, and a *Q* set point of 1, the output signal (CH2) is not in phase with the reference input. However, if the filter is tuned (bottom), input and output signal signals of the filter are *in phase*, and the output amplitude of the output signal is a value that agrees with the desired *Q* factor (*Q*=1 in this case). Finally, if the input reference frequency of the master filter decreases (Fig. 12), the tuned filter leaves its stable behavior and shows chaotic behavior.

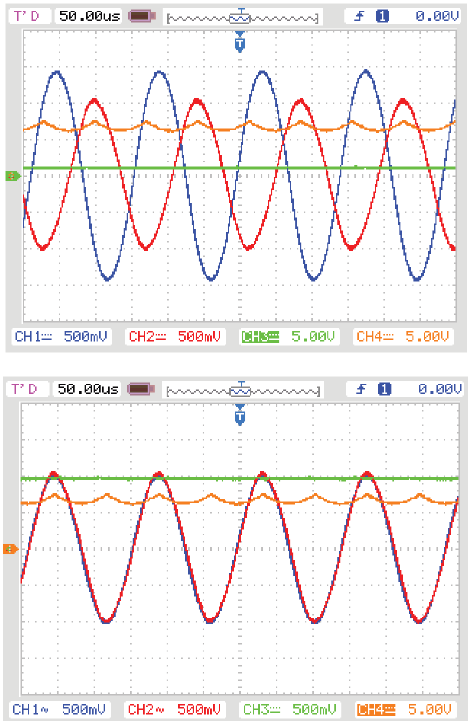

Fig. 11: Experimental waveforms when the filter is not tuned (up) and tuned (bottom). Channel I: Input signal $v_{REF}(t)$, channel 2: Output signal $v_o(t)$; channel 3: Q-control voltage $v_{CQ}(t)$; and channel 4: ω_O-control voltage $v_{CF}(t)$.

V. Conclusions

In this paper, we have presented a continuous-time filter with two automatic tuning loops and its dynamic behavior that may present nonlinear phenomena. Results are presented by means of Matlab-Simulink® simulations and also by means of experimental results of a discrete implementation.

The traditional small signal models used to design these controllers fail to predict the real behavior of the system. This paper shows that nonlinear phenomena as chaos may appear in this kind of tuned systems. To conclude, results obtained with this work may provide some help in order to advance in the study of the aforementioned phenomena and avoid unstable behavior in automatic tuned filters.

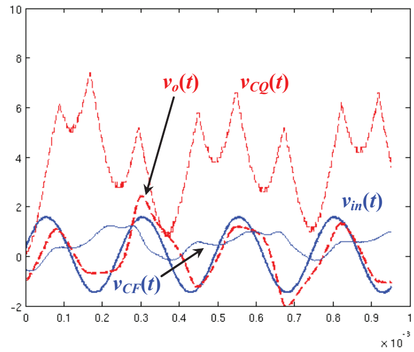

Fig. 12: Experimental waveforms when the filter shows chaotic behavior.

Acknowledgments

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