pecially those which involve highly nonstationary environments, the difference in performance exhibited by the LS and SG lattice algorithms may be more pronounced. A comparative study of algorithm performance in these remaining applications should yield interesting and useful results.

ACKNOWLEDGMENT

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Use of the Most Significant Autocorrelation Lags in Iterative ME Spectral Estimation

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Abstract—Many methods of spectral analysis are based either directly or indirectly on a set of autocorrelation values estimated from the available data. A good selection of autocorrelation Lags can improve the quality of the spectral estimate at a given computational cost.

To demonstrate the above possibility, this paper shows how to select Lags corresponding to the most significant values of the autocorrelation. In this way, one obtains better estimates than those found using the standard method, namely, the technique proposed by Lim and Malik [1] for (iterative) ME spectral analysis.

Several examples are considered to illustrate this possibility.

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I. Introduction

In autocorrelation based methods for spectral estimation, we form an estimate $\{\hat{r}_x(0), \cdots, \hat{r}_x(M)\}$ of the autocorrelation function of the signal x from the available data, and then we compute the power spectrum of x by direct or parametric methods using such an estimate.

It seems clear that the quality of the final power spectrum estimate will depend essentially on the amount of signal information which is retained in $\{\hat{r}_x(0), \cdots, \hat{r}_x(M)\}$. This paper attempts to demonstrate how an appropriate selection of lags (different from the standard one, $\{0, \cdots, M\}$), can considerably improve the quality of power spectrum (in a sense which depends on the criterion employed for lag selection).

This possibility is of paramount importance in cases where a high computational effort is necessary; for example, when the criterion is to minimize (or maximize) an objective function under autocorrelation restrictions. (Iterative algorithms or linear programming are then usually required.) Also, the convergence time and properties are better because a good startup of the algorithms is provided.

Although the idea is of general interest, we will focus our attention on ME iterative techniques; first, we show the potential advantage of the proposed procedure in 1-D cases in a clear context; second, we apply this procedure to the more critical 2-D problems, in which memory and computational restrictions force us to use only a very reduced number of autocorrelation values.

We will choose lags which have the largest absolute values of the autocorrelation. This approach shows that, at a fixed computational cost, there are improvements in the resolution and more noise immunity when compared with currently reported works.

II. Lag Selection in Spectral Estimation

The classical selection of the first $M + 1$ autocorrelation lags, $\{0, \cdots, M\}$, is supported by the fact of that this set offers the largest statistical stability among all the possible choices. We claim that using other selections will yield more desirable properties (using entropy as the objective function, the zero lag has to be included to ensure that a maximum exists).

It is clear that for stationary processes and even for nonstationary ones (like sinusoids in noise), the number of samples between highly correlated samples [i.e., number of lags between maxima of $r_x(m)$] provides a good deal of information about periodicities. Then, selecting the M lags (and zero) with largest absolute values will result in an estimate which is better than the classical one in the following sense: the peaks will be reinforced, the noise effects reduced. This approach will be referred to as the modified approach in the rest of this work.

Note that the modified method has a drawback; namely, it reduces the statistical stability since it uses autocorrelation values with lags greater than M; but like in prediction [2] and interpolation problems, the location of the most significant autocorrelation samples can be more important than their concrete values as concerns with some properties, such as resolution in the final spectral estimate, and convergence rate in adaptive or iterative methods for spectral estimation.

Of course, the use of all the lags available (or significant) will result in a better final estimate; but in most 2-D real problems, the use of a large correlation support is not allowed because it will increase the computational load beyond reasonable limits.

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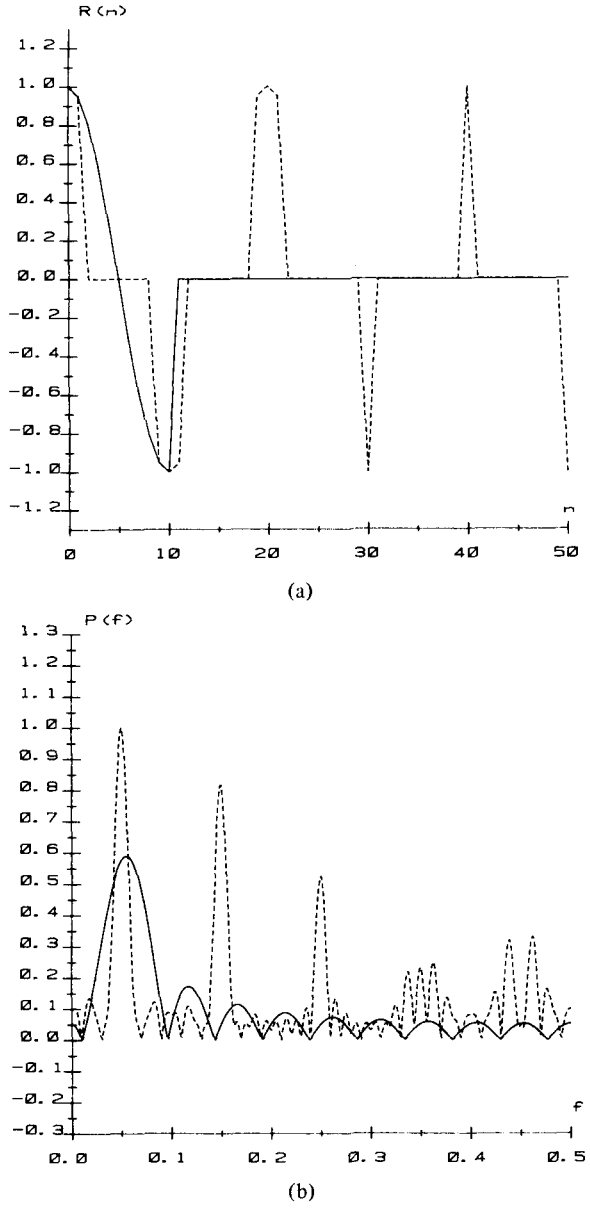

Fig. 1. (a) Classical (solid line) and most significant points selection (broken line) for autocorrelation lags obtained from 128 samples of one sinusoid of normalized frequency 0.05. Ten lags selected in both cases. (b) Corresponding periodograms of 1(a) after padding with zeros both autocorrelation functions.

To show the validity of the previous discussion in a general context, we will consider a nonparametric estimator (the periodogram) with the usual and the modified approaches.

Fig. 1(a) and (b) illustrate the difference between the two choices using a sinusoid with frequency 0.05 as the signal. Fig. 1(a) shows ten selected lags among 50 in both cases; Fig. 1(b) shows the corresponding periodogram.

The examination of this last part of the figure confirms the previous assertion: the "effective window" in the modified approach produces higher resolution than in the classical one.

Of course, the sampling involved in the lag domain will produce extra peaks for very low frequencies when the sampling violates the Nyquist criterion. Nevertheless, it seems that in many cases, it is easier to remove extra lines than to increase resolution from the initial estimate.

III. Results For Iterative ME Algorithms

Recently, Lim and Malik [1] have reported an iterative ME Fourier based method in both 1-D and 2-D problems, that has

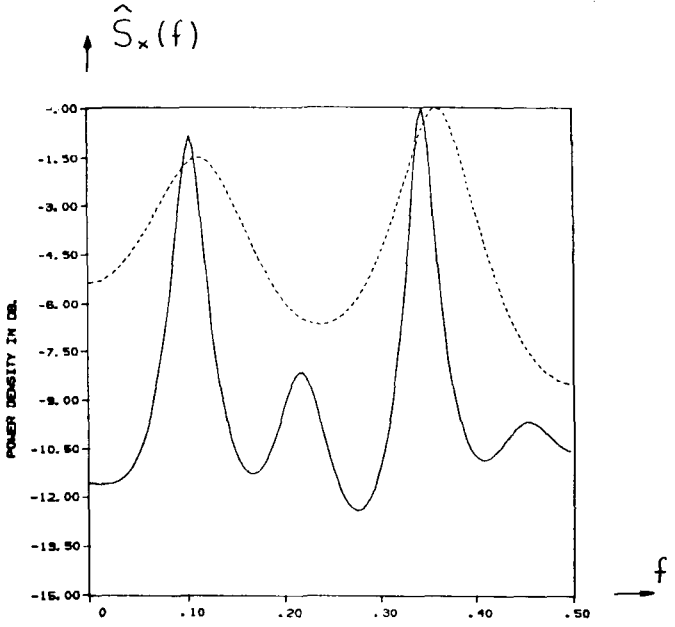

Fig. 2. Iterative ME algorithm after four iterations. Broken line: classical approach (equally spaced lags). Solid line: Most significant lags approach (lags corresponding to maximum absolute value of the autocorrelation function). Nine lags selected (positive and negative) in the two approaches.

the extra advantage of including (without relevant modifications) cases in which some points of the autocorrelation are not available [3]. It is also possible to introduce without modifications the proposed approach in this algorithm.

We will now show results from simulated and real experiments using Lim and Malik's algorithm with the classical and the modified approaches.

A. 1-D Case

Using

$$r_x(n) = \sigma^2\delta(n) + \cos(2\pi n\ 0.1) + \cos(2\pi n\ 0.3456) \qquad (1)$$

with $\sigma^2 = 1$ and $r_x(n)$ known for $-9 < n < 9$, Fig. 2 displays the results after four iterations of Lim and Malik's algorithm for the two approaches.

It can be seen that, using the 0 and four additional more significant lags (largest absolute values), the resolution is better than that exhibited by the classical procedure (lags 0–4). Some additional comments and conclusions can be obtained form this example; they can be useful not only in 2-D cases (in which we have not the easy option of using all the lags as in this 1-D problem, since a typical 64×32 data set cannot be completely used, because less than 100 constraints have to be used to keep computing time and memory under adequate limits in each iteration), but also in other nonlinear procedures of spectral estimation.

We can check that the improvement in quality is accompanied by better extrapolated (interpolated) values when the modified approach is used. This is very reasonable, since we are working with the most representative values of the autocorrelation. Table I shows the actual values of the autocorrelation (second column) and the induced ones in both approaches (third and fourth column) after the fourth iteration. Each approach approximates better the values corresponding to the selected lags; but the global result is better for the modified procedure. This last approach increases also the dynamic range of the estimated spectrum.

B. 2-D Case

In the first example, we have selected

TABLE I
LAGS SELECTED IN CLASSICAL AND MODIFIED PROCEDURES FROM a −9 TO 9 INITIAL AUTOCORRELATION FUNCTION. ACTUAL, CONSTRAINED (MARKED WITH ARROWS), INTERPOLATED AND EXTRAPOLATED VALUES OF THE AUTOCORRELATION SEQUENCE FOR BOTH PROCEDURES.

Lag	Actual Correlation Values	ME extrapolation Classical	Modified
0	3.00	→ 3.16	→ 3.47
1	0.24	→ 0.13	0.21
2	−0.04	→ −0.07	0.09
3	0.66	→ 0.53	0.46
4	−1.55	→ −1.14	→ −1.38
5	−1.12	−0.15	→ −1.01
6	0.07	0.14	0.08
7	−1.19	−0.30	→ −1.01
8	0.42	0.27	0.24
9	1.56	0.11	→ 1.37

Selected lags:
classical approach: −4, −3, −2, −1, 0, 1, 2, 3, 4
modified approach: −9, −7, −5, −4, 0, 4, 5, 7, 9.

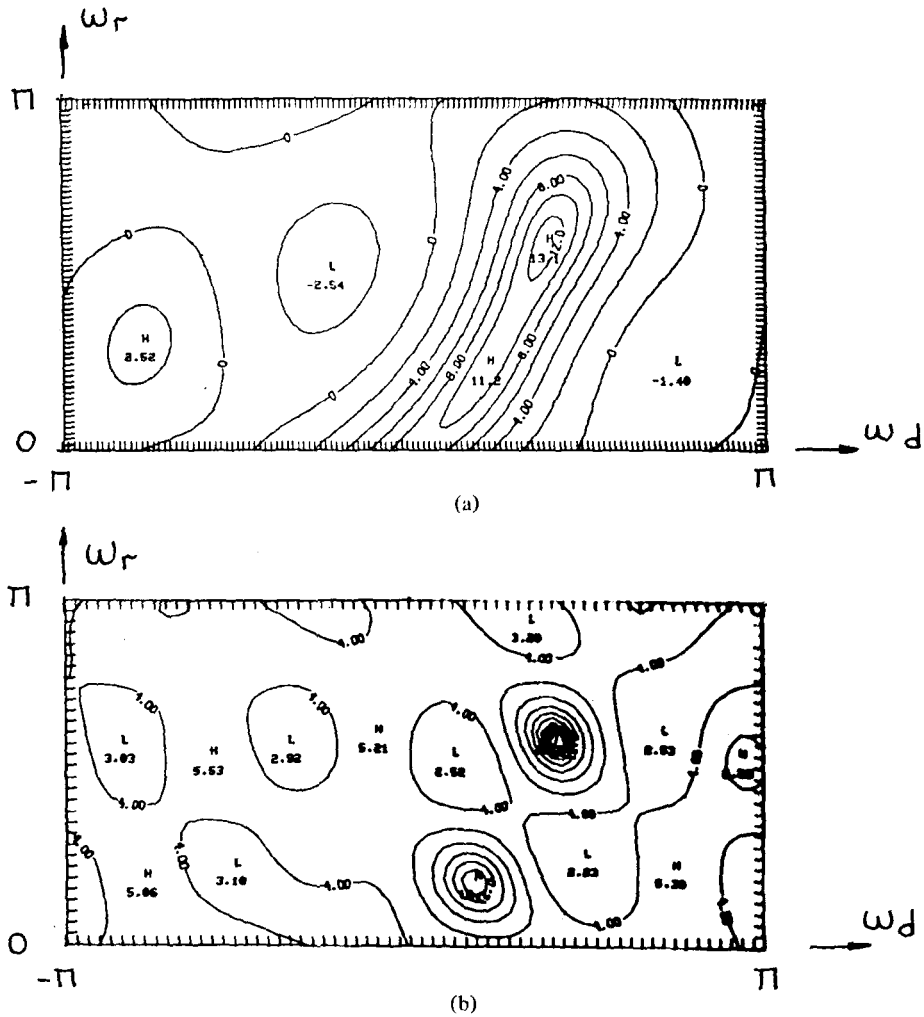

Fig. 3. Iterative ME algorithm. (a) Classical approach after 35 iterations. (b) Most significant lags approach after 10 iterations.

$$r_x(n,m) = 1 + \cos\{2\pi(0.1n + 0.1m)\} + \cos\{2\pi(0.2n + 0.3215m)\} \quad (2)$$

and the use of 25 (total) lags from a basic 10 × 10 mask. The results of the classical and the modified approaches appear in Fig. 3(a) (35 iterations) and Fig. 3(b) (10 iterations), respectively. The reduction of computational burden is clear (the 70 percent obtained is a typical value; similar percentages have been obtained in other tested cases). The classical procedure does not allow the detection of the peaks; the modified version shows a well defined low-frequency peak, regardless of the number of iterations (this allows the considerable reduction of

computing time). Additionally, the dynamic range increases about 10 dB.

Another intersting experimental result, obtained in these and other cases, is that the convergence rate is slow and nearly constant under the classical approach; under the modified one, the error (measured only over the constrained lags) decreases very quickly in the first steps, and, beyond a certain point, no significant improvement appears. This seems to be the cause of the simultaneous reduction of computing time and improvement of resolution usually observed.

IV. Conclusions

The idea of selecting with a certain criterion lags to be used in autocorrelation based spectral analysis has been shown to be useful in some general examples. This possibility can be very interesting in parametric methods that have to be applied under iterative algorithms, such as many objective function minimization (maximization) procedures and 2-D situations. The advantage seems to be mainly in computational cost and resolution.

The usefulness of a particular application of the proposed idea, choosing the largest absolute values of the autocorrelation in a 2-D iterative ME method, has been tested. More work is needed to obtain a general perspective of the possible advantages in other cases.

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On the Computation of Autocorrelation Using Polynomial Transforms

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Abstract—**Polynomial transforms (PT's) are known to be efficient for computing convolutions. In this correspondence, the use of polynomial transforms for computing autocorrelation has been considered. A method has been described for using polynomial transforms instead of the FFT in the Rader's algorithm for autocorrelation, and the number of arithmetic operations compared; it is found that the use of PT's does not result in the expected computational advantage over the FFT method.**

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I. Introduction

The autocorrelation function, very frequently encountered in many spectral analysis methods, can be computed by means of an FFT algorithm as first suggested by Stockham [1]. In many instances, like the estimation of power spectra, the data for which the autocorrelation is desired form a very long (per haps indefinitely long) sequence; also, the autocorrelation is desired for only a number of lag values which is a small fraction of the length of data sequence. The improved algorithm of Radar [2] is advantageous under such circumstances.

Reddy and Reddy [3] have adapted this method to rectangular transforms (RT) and shown that the use of RT's is advantageous as compared to FFT's.

Nussbaumer [4] has shown that polynomial transforms (PT's) offer about 50 percent savings in computation if used for the computation of radix-2 type convolution. Hence, it is interesting to examine whether PT's prove efficient when used for computing autocorrelation functions also. First let us briefly review the Rader's method of computing autocorrelation. The autocorrelation $R_x(m)$ of a real data sequence $x(n)$, $n = 0, 1, \cdots, N' - 1$, is defined by ($x(n)$ is zero for $n \geqslant N'$)

$$R_x(m) = \frac{1}{N'} \sum_{n=0}^{N'-1} x(n)\, x(n+m). \tag{1}$$

Define a series of subsequences x_i and y_i of length M' as

$$x_i(n) = \begin{cases} x\left(n + \dfrac{iM'}{2}\right) & 0 \leqslant n < \dfrac{M'}{2} \\ 0 & \dfrac{M'}{2} \leqslant n < M', \quad i = 0, 1, 2, \cdots \end{cases} \tag{2}$$

$$y_i(n) = x\left(n + \frac{iM'}{2}\right) \qquad 0 \leqslant n < M' \quad i = 0, 1, 2, \cdots. \tag{3}$$

Now consider the cyclic correlation $Z_i(m)$ of x_i and y_i:

$$Z_i(m) = \sum_{n=0}^{M'-1} x_i(n)\, y_i[(n+m) \bmod M'], \qquad m = 0, 1, \cdots, M' - 1. \tag{4}$$

From (2) and (3)

$$Z_i(m) = \sum_{n=0}^{M'/2-1} x\left(n + \frac{iM'}{2}\right) x\left(n + \frac{iM'}{2} + m\right), \qquad m = 0, 1, \cdots, \frac{M'}{2}. \tag{5}$$

The relation in (5) is true for $m = 0, 1, \cdots, (M'/2)$ only because of the nature of cyclic correlation.

Now consider

$$\begin{aligned} \omega_\lambda(m) &= \sum_{i=0}^{\lambda} Z_i(m) \\ &= \sum_{i=0}^{\lambda} \sum_{n=0}^{M'/2-1} x\left(m + \frac{iM'}{2}\right) x\left(n + \frac{iM'}{2} + m\right) \\ &= \sum_{n=0}^{\lfloor(\lambda+1)M'/2\rfloor - 1} x(n)\, x(n+m), \qquad m = 0, 1, \cdots, \frac{M'}{2} \end{aligned} \tag{6}$$