

Figure 3. Performance of Rayleigh Fast-Fading Channel with Synchronization Errors.

CONCLUSION

It is to be expected that the effects of intersymbol interference on fast-fading channels would be more significant than on slowly fading channels. This hypothesis is clearly supported by the data shown in Figures 2 and 3, particularly in the latter case where the intersymbol interference is enhanced by a large synchronization error. When the signal-to-noise ratio is low, the intersymbol interference has little effect on the channel performance, but as the mean signal-to-noise exceeds 10 dB the ratio P_{e_z}/P_e begins to increase rapidly. For mean signal-to-noise ratios in excess of 25 dB, P_{e_z}/P_e approaches unity, i.e., the total bit-error probability is almost entirely due to intersymbol interference. Thus the effect of the intersymbol interference on the fast-fading channel is to introduce an irreducible error probability that limits performance at high signal-to-noise ratios. This irreducible error probability is 7.38×10^{-3} for the Gaussian pulse with no synchronization error. For the Chebyshev pulse it is 3.39×10^{-3} with no synchronization error.

While reliable communication over slowly fading channels can be obtained with large mean signal-to-noise ratios or diversity techniques, the data presented herein clearly indicate that intersymbol interference presents a very serious

problem in Rayleigh fast-fading channels. Application of equalization techniques should also be considered to combat the performance degradation due to intersymbol interference in these cases.

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Comments on and Extensions of Wolf's Signal-to-Channel Noise Formulas for Delta-Modulated Systems

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Abstract—The channel noise effects on linear delta modulation (LDM) systems have not yet been adequately analyzed. This paper presents a new and general formulation of these effects, based on the theoretical work by Wolf [1]. A comparative discussion of our formulas with previous results is also included. Finally, the application of our methods and the validity of our comments are illustrated by some numerical examples.

I. INTRODUCTION

This paper gives a method for calculating the signal-to-channel noise power ratio in a linear delta modulation (LDM) system. Although LDM systems do not offer a valid alternative to PCM for applications requiring a wide dynamic range [2], the technique described can be applied to other robust DM methods, especially digitally syllabic-companded delta modulation (DSCDM) systems.

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In many cases, the channel noise effect is negligible with respect to the quantization noise (granular and excess slope errors), but, in some applications, where the quantization noise is made relatively negligible in an otherwise noisy channel, this effect can be a critical quality parameter. Even in cases where the two noise components are of equal importance, one can calculate the channel noise effects by the techniques described herein.

The quality criterion employed in this paper is the classical mean-squared error measure. Although in most applications of voice or image transmission subjective criteria may be more appropriate, the mean-squared expressions are a first-order indicator. Furthermore, they can be modified easily into a weighted mean-squared error criterion, which is in closer agreement with subjective evaluations.

Finally, it should be pointed out that extensions of the given formulation are easily obtained for those cases in which the independent channel error model cannot be used (e.g., in commercial telephony) in the same way that Wolf indicated [1] for the Gilbert burst-noise model. The only modification necessary would be in the digital autocorrelation function $E(n_i n_{i+m})$ (see Section II).

II. SIGNAL AND CHANNEL NOISE POWERS

Let b_i indicate the i th transmitted symbol, where $b_i = \pm 1$, and n_i the channel noise effect on this symbol, where $n_i = 0$ if there is no error and $n_i = \pm 2$ if there is an error in the decision on b_i .

The receiver detects and regenerates the incoming channel signal; if $p(t)$ is the basic shape of the regenerated pulses, we will have, at the input to the LD demodulator,

$$\text{a signal term: } s(t) = \sum_{i=-\infty}^{\infty} b_i p(t - iT) \quad (1)$$

$$\text{and a noise term: } n(t) = \sum_{i=-\infty}^{\infty} n_i p(t - iT) \quad (2)$$

where T^{-1} is the symbol rate. The LD demodulator can be considered a linear system, having an equivalent impulse response $h(t)$ and a transfer function $H(f)$. Figure 1 shows the general situation just described.

Since the output of the LD demodulator is a continuous process whose power level does not depend on the time origin, we can randomize the reference time for the pulses [3], obtaining the signal and noise terms:

$$s(t) = \sum_{i=-\infty}^{\infty} b_i p(t + \theta - iT) \quad (3)$$

$$n(t) = \sum_{i=-\infty}^{\infty} n_i p(t + \theta - iT) \quad (4)$$

where θ is a random variable, uniformly distributed over $[0, T]$. These signal and noise terms are stationary processes, with respective autocorrelation functions [3]:

$$R_{ss}(t) = \frac{1}{T} \sum_{m=-\infty}^{\infty} E(b_i b_{i+m}) R_{pp}(t + mT) \quad (5)$$

$$R_{nn}(t) = \frac{1}{T} \sum_{m=-\infty}^{\infty} E(n_i n_{i+m}) R_{pp}(t + mT) \quad (6)$$

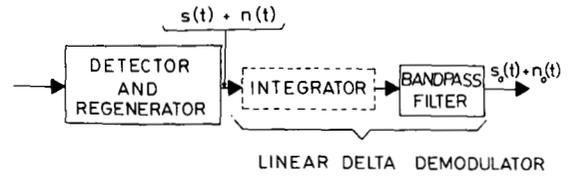


Fig. 1: Linear delta-modulation receiver.

and the power spectral densities:

$$G_{ss}(f) = \frac{1}{T} \sum_{m=-\infty}^{\infty} E(b_i b_{i+m}) S_{pp}(f) \exp(j2\pi mTf) \quad (7)$$

$$G_{nn}(f) = \frac{1}{T} \sum_{m=-\infty}^{\infty} E(n_i n_{i+m}) S_{pp}(f) \exp(j2\pi mTf). \quad (8)$$

Here $R_{pp}(t)$ and $S_{pp}(f)$ are the autocorrelation function and the energy spectral density of $p(t)$, respectively. Then, the signal and noise powers at the output can be calculated with the help of the following formulas:

$$\begin{aligned} P(s_0) &= \frac{1}{T} \sum_{m=-\infty}^{\infty} E(b_i b_{i+m}) [R_{pp}(t) * R_{hh}(t)]_{t=mT} \\ &= \frac{1}{T} \sum_{m=-\infty}^{\infty} E(b_i b_{i+m}) \int_{-\infty}^{\infty} S_{pp}(f) |H(f)|^2 \\ &\quad \cdot \exp(j2\pi mTf) df; \end{aligned} \quad (9)$$

$$\begin{aligned} P(n_0) &= \frac{1}{T} \sum_{m=-\infty}^{\infty} E(n_i n_{i+m}) [R_{pp}(t) * R_{hh}(t)]_{t=mT} \\ &= \frac{1}{T} \sum_{m=-\infty}^{\infty} E(n_i n_{i+m}) \int_{-\infty}^{\infty} S_{pp}(f) |H(f)|^2 \\ &\quad \cdot \exp(j2\pi mTf) df; \end{aligned} \quad (10)$$

where * indicates convolution, and $R_{hh}(t)$ is the autocorrelation function of the impulse response of the LD demodulator.

We have thus found the signal-to-channel noise formula:

$$\frac{P(s_0)}{P(n_0)} = \frac{\sum_{m=-\infty}^{\infty} E(b_i b_{i+m}) I(mT)}{\sum_{m=-\infty}^{\infty} E(n_i n_{i+m}) I(mT)} \quad (11)$$

where

$$I(mT) \triangleq \int_{-\infty}^{\infty} S_{pp}(f) |H(f)|^2 \exp(j2\pi mTf) df. \quad (12)$$

This signal-to-channel noise formula is absolutely general for LDM systems. It can also be extended to adaptive (variable step) DM systems, using an appropriate redefinition of the signal and noise terms, although the digital source model will have to be changed. It is also possible to consider line coding effects, but we will restrict our discussion to direct transmission.

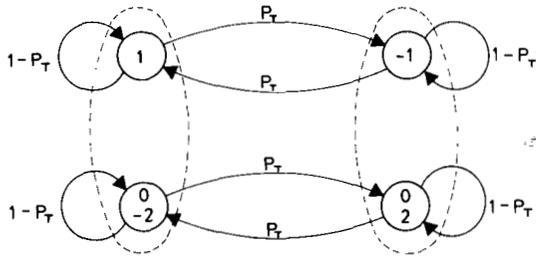


Fig. 2: Markov system model.

III. WOLF'S MODEL FOR INDEPENDENT CHANNEL ERRORS

The only problem remaining to be solved is the evaluation of $E(b_i b_{i+m})$ and $E(n_i n_{i+m})$. Wolf's model [1] is appropriate for evaluating these mathematical expectations assuming independent channel errors. This model consists of two synchronized Markov chains for the digitized source and the channel errors; Figure 2 shows the model. The transition probabilities P_T are equal, since we assume symmetry in this regard. Only P_T is required for the evaluation of the digital autocorrelation functions, and we can obtain it from more structured (and realistic) Markov models for the quantized signal.

We derive the signal-to-channel noise formula in Appendix I. The obtained result is:

$$\frac{P(s_0)}{P(n_0)} = \frac{1}{4P_e} \frac{\int_{-\infty}^{\infty} \frac{P_T (1 - P_T) S_{pp}(f) |H(f)|^2}{P_T^2 + (1 - 2P_T) \sin^2 \pi T f} df}{\int_{-\infty}^{\infty} \left[1 + P_e \frac{P_T(1 - 2P_T) - (1 - 2P_T) \sin^2 \pi T f}{P_T^2 + (1 - 2P_T) \sin^2 \pi T f} \right] S_{pp}(f) |H(f)|^2 df} \quad (13)$$

In practical systems, the final step in the LD demodulator is a bandpass filter, with lower and upper cutoff frequencies f_{c1} , f_{c2} , respectively, and $1/T \gg f_{c2}$. A first approximation would be to assume $\sin^2 \pi T f \approx 0$ in the effective band of integration. Then:

$$\sum_{m=-\infty}^{\infty} E(b_i b_{i+m}) \exp(j2\pi m T f) \approx (1 - P_T)/P_T \quad (14)$$

$$\sum_{m=-\infty}^{\infty} E(n_i n_{i+m}) \exp(j2\pi m T f) \approx 4P_e [1 - 2P_e(1 + P_T)/P_T] \quad (15)$$

and

$$\frac{P(s_0)}{P(n_0)} \approx \frac{1}{4P_e} \frac{1 - P_T}{P_T + P_e(1 - 2P_T)} \quad (16)$$

This is Wolf's formula. Note that Wolf did not consider the final bandpass filter. We note that the approximation is acceptable when $1/T$ is large with respect to f_{c2} . If we accept the approximation, the resulting formula will apply independent of the shape of the regenerated pulses. Wolf assumed a perfect integrator, but his result is also useful for leaky and double integration systems and for delta-sigma systems. In the latter case, P_T would correspond to a model of the digitized integrated message signal. The same model previously indicated would apply to the process resulting from the integration of

the analog message; we will denote the corresponding transition probability by P_T' .

Wolf's formula is exact when $P_T = 0.5$, i.e., when $P(s_0)/P(n_0) = 1/4P_e$, but it will be more and more inaccurate as P_T approaches zero or unity. This appears to be the main reason for the progressive separation of Wolf's theoretical curve from Braun's experimental values [1]: the fitting for low P_e implied deviation for high P_e values.

If P_T approaches unity, $P(s_0)$ will increase appreciably in practice with respect to the value obtained assuming $\sin^2 \pi T f = 0$, due to the term $(1 - 2P_T) \sin^2 \pi T f$ in the denominator of (I.3), considering that, in practice, the effective bandwidth corresponding to $H(f)$ is less than $1/T$. $P(n_0)$ has a small variation because of the presence of the factor P_e in the second term of (I.4). Consequently, $P(s_0)/P(n_0)$ exceeds Wolf's value. Just the contrary occurs when P_T approaches zero. The variations are larger when P_e is small, i.e., in "good" channel cases.

It is easy to see that our formulation gives $P(s_0)/P(n_0) = 0$ when $P_T = 0$ or $P_T = 1$ (signal absent), but it also implies that $P(s_0)/P(n_0) \neq 0$ when $P_e = 1/2$. This incorrect result is due to the definition selected for signal-to-channel noise ratio, as Wolf indicated. Alternative definitions chosen in order to solve this problem present other difficulties [1]. Nevertheless, the cases in which $P_e \approx 1/2$ correspond to transmissions over unusable channels (having a capacity near zero), and the previous $P(s_0)/P(n_0)$ formulas are decreasing functions of P_e which indicate the system performance in all practical situations.

IV. EXAMPLES

When the information consists of a voice signal and the system is approximately optimized with respect to granular and excess slope noises, Kikkert's simulation results [4] allow us to conclude that P_T will not be very far from 0.5. In this case, the application of Wolf's formula will be acceptable. This remains true as long as $1/T$ is large enough to maintain $\sin^2 \pi T f \approx 0$ in the integration band. Nevertheless, we will present more general calculations to illustrate the departure from Wolf's results.

Let us consider a final ideal bandpass filter having cutoff frequencies $f_{c1} = 300$ Hz, $f_{c2} = 3400$ Hz, let us also assume $T = 1/5600$ symbols/s, and:

$$p(t) = V \Pi(t/T) \quad (17)$$

where:

$$\Pi(x) = \begin{cases} 1, & \text{if } |x| < 1/2 \\ 0, & \text{if } |x| > 1/2. \end{cases} \quad (18)$$

From the above expression of $p(t)$:

$$S_{pp}(f) = V^2 T^2 \text{sinc}^2 T f \quad (19)$$

where:

$$\text{sinc } x \triangleq \sin \pi x / \pi x. \quad (20)$$

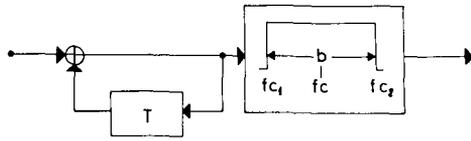


Fig. 3: Perfect integration linear delta demodulator (T : delay; the final block is an ideal bandpass filter).

TABLE I
SIGNAL-TO-CHANNEL NOISE POWER RATIOS (IN dB), PERFECT INTEGRATION (AND WOLF'S FORMULA VALUES)

P_e/P_T	10^{-1}	10^{-3}	10^{-5}	10^{-7}
0.05	11.61 (12.30)	34.86 (36.69)	54.91 (56.77)	74.91 (76.77)
0.20	8.73 (8.86)	29.81 (29.99)	49.82 (50.00)	69.82 (70.00)
0.35	6.28 (6.31)	26.63 (26.66)	46.63 (46.67)	66.63 (66.67)
0.50	3.98 (3.98)	23.98 (23.98)	43.98 (43.98)	63.98 (63.98)
0.65	1.51 (1.50)	21.30 (21.29)	41.30 (41.29)	61.30 (61.29)
0.80	-1.69 (1.70)	17.98 (17.96)	37.97 (37.96)	57.97 (57.96)
0.95	-8.36 (8.38)	11.21 (11.20)	31.21 (31.19)	51.21 (51.19)

Figure 3 shows a circuit that can be used as a perfect integration demodulator when $p(t)$ is a full rectangular pulse. Its power transfer function may be written:

$$|H(f)|^2 = \left[\Pi \left(\frac{f-f_c}{b} \right) + \Pi \left(\frac{f+f_c}{b} \right) \right] / 4 \sin^2 \pi T f \quad (21)$$

where $f_c = (f_{c1} + f_{c2})/2$ is the central frequency, and $b = f_{c2} - f_{c1}$ is the bandwidth of the final bandpass filter. Table I presents numerical results from our formulation, showing the small differences with respect to Wolf's values (in brackets) if P_T is not very different from 0.5, and the increasing differences when P_T goes to zero or 1.

If a single RC (leaky) integrator is used, we will have:

$$|H(f)|^2 = \left[\Pi \left(\frac{f-f_c}{b} \right) + \Pi \left(\frac{f+f_c}{b} \right) \right] / [1 + (f/f_3)^2] \quad (22)$$

where f_3 is the cutoff frequency of the RC filter. Table II shows numerical values corresponding to a typical f_3 equal to 150 Hz.

If $P_T = 0.5$ and we put $\text{sinc}^2 Tf \approx 1$ in $[f_{c1}, f_{c2}]$, we will obtain Johnson's formula for $P(n_0)$ [6] [7].

A double integrator is shown in Figure 4. Typical parameters allow us to write:

$$|H(f)|^2 \approx \left[\Pi \left(\frac{f-f_c}{b} \right) + \Pi \left(\frac{f+f_c}{b} \right) \right] \frac{f_1^2}{f^2 [1 + (f/f_2)^2]} \quad (23)$$

TABLE II
SIGNAL-TO-CHANNEL NOISE POWER RATIOS (IN dB), RC INTEGRATION

P_e/P_T	10^{-1}	10^{-3}	10^{-5}	10^{-7}
0.05	11.57	34.78	54.82	74.82
0.20	8.73	29.80	49.81	69.81
0.35	6.28	26.63	46.63	66.63
0.50	3.98	23.98	43.98	63.98
0.65	1.51	21.30	41.30	61.30
0.80	-1.69	17.98	37.97	57.97
0.95	-8.36	11.21	31.21	51.21

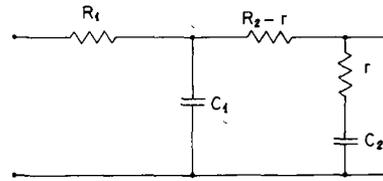


Fig. 4: Double integrator.

TABLE III
SIGNAL-TO-CHANNEL NOISE POWER RATIOS (IN dB), DOUBLE INTEGRATION

P_e/P_T	10^{-1}	10^{-3}	10^{-5}	10^{-7}
0.05	11.83	35.40	55.45	75.45
0.20	8.80	29.90	49.91	69.91
0.35	6.30	26.65	46.65	66.65
0.50	3.98	23.98	43.98	63.98
0.65	1.50	21.30	41.30	61.30
0.80	-1.70	17.97	37.97	57.97
0.95	-8.37	11.20	31.20	51.20

where

$$f_1 = 1/2\pi R_1 C_1 (1 + C_2/C_1) \quad (24)$$

$$f_2 = 1/2\pi R_2 C_2 [1 - R_2 C_2 (1 + R_1/R_2)^2 / 4R_1 C_1]. \quad (25)$$

Table III presents numerical results corresponding to a typical f_2 equal to 1 kHz. If $P_T = 0.5$ and we approximate $\text{sinc}^2 Tf$ by unity, our formulation will give an already known result for $P(n_0)$ [2]:

$$P(n_0) \approx 8P_e V^2 f_1^2 T^2 \{ 1/f_{c1} - 1/f_{c2} - [\tan^{-1}(f_{c2}/f_2) - \tan^{-1}(f_{c1}/f_2)] / f_2 \}. \quad (26)$$

In the case of delta-sigma modulation, the results obtained will not be directly comparable with the previous ones, since

TABLE IV
SIGNAL-TO-CHANNEL NOISE POWER RATIOS (IN dB),
DELTA-SIGMA MODULATION

$\frac{P_e}{P_T}$	10^{-1}	10^{-3}	10^{-5}	10^{-7}
0.05	9.88	31.54	51.56	71.56
0.20	8.36	29.28	49.29	69.29
0.35	6.20	26.53	46.53	66.53
0.50	3.98	23.98	43.98	63.98
0.65	1.53	21.33	41.33	61.33
0.80	-1.65	18.02	38.01	58.01
0.95	-8.32	11.25	31.25	51.25

$P_T' \neq P_T$ in a general case. The receiver has:

$$|H(f)|^2 = \Pi\left(\frac{f-f_c}{b}\right) + \Pi\left(\frac{f+f_c}{b}\right). \quad (27)$$

Some signal-to-channel noise ratio values are included in Table IV.

$P_T' = 0.5$ and $\text{sinc}^2 Tf \approx 1$ in $[f_{c1}, f_{c2}]$ will lead to Johnson's formula for $P(n_0)$ [5] [6].

The case of linear delta-sigma modulation can be solved by calculating the corresponding $I(mT)$ in the time domain. Sine-integral functions appear in the resulting expression, which is difficult to manipulate and to interpret. Making $\text{sinc}^2 Tf \approx 1$, we obtain:

$$\begin{aligned} I(mT) &\approx 2V^2T^2 \int_{f_{c1}}^{f_{c2}} \cos 2\pi mT df \\ &= 2V^2T^2 [f_{c2} \text{sinc } 2mTf_{c2} - f_{c1} \text{sinc } 2mTf_{c1}] \quad (28) \end{aligned}$$

and the resulting $P(s_0)/P(n_0)$ can be expressed with the help of (11). The same approximate method can be applied in other cases, but the integrals $I(mT)$ have to be calculated numerically.

V. SUMMARY AND CONCLUSIONS

The general formulation of the signal-to-channel noise power ratio in (direct) LDM systems has been presented and compared with earlier results. The main conclusions are:

1) A slight modification of Wolf's work allows us to extend it to all LDM systems;

2) Wolf's formula is accurate enough in practical systems having P_T not very different from 0.5, but it fails when P_T goes to zero or P_T goes to 1, and especially when P_e is small or when T is large.

Some numerical results are comparatively presented.

Additional comments are:

3) The method can be easily extended to correlated channel errors (e.g., to the case of burst errors accordingly to the Gilbert model [7], considered by Wolf), the only modification being the alteration of $E(n_i n_{i+m})$ values (not very problematic in the case of the Gilbert model);

4) By properly redefining signal and noise terms, the method can be extended to coded LDM systems and to other practical DM systems (adaptive delta modulation, ADM, and, particularly, digitally syllabic companded delta modulation, DSCDM). We are obtaining results along these lines at the present time.

APPENDIX I

From the model, it is easy to obtain [1]:

$$E(b_i b_{i+m}) = (1 - 2P_T)^{|m|} \quad (I.1)$$

$$E(n_i n_{i+m}) = \begin{cases} 4P_e, & \text{if } m = 0 \\ 4P_e^2 (1 - 2P_T)^{|m|}, & \text{if } m \neq 0 \end{cases} \quad (I.2)$$

where P_e is the error probability. Then, rearranging the power expressions, we can write:

$$\begin{aligned} &\sum_{m=-\infty}^{\infty} E(b_i b_{i+m}) \exp(j2\pi mTf) \\ &= \sum_{m=-\infty}^{\infty} (1 - 2P_T)^{|m|} \exp(j2\pi mTf) = \\ &= \frac{P_T(1 - P_T)}{P_T^2 + (1 - 2P_T) \sin^2 \pi Tf} \quad (I.3) \end{aligned}$$

$$\begin{aligned} &\sum_{m=-\infty}^{\infty} E(n_i n_{i+m}) \exp(j2\pi mTf) \\ &= 4P_e \left[1 + P_e \sum_{m=-\infty}^{\infty} (1 - 2P_T)^{|m|} \exp(j2\pi mTf) \right] \\ &= 4P_e \left[1 + P_e \frac{P_T(1 - 2P_T) - (1 - 2P_T) \sin^2 \pi Tf}{P_T^2 + (1 - 2P_T) \sin^2 \pi Tf} \right] \quad (I.4) \end{aligned}$$

where the prime sign in the summation indicates exclusion of the zero index term. The ratio of the integrals of these functions multiplied by $S_{pD}(f) |H(f)|^2$ will be the signal-to-channel noise ratio, formula (13) in the main text.

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