LINEAR PHASE ADAPTIVE LINE ENHANCER FOR IMPROVING
THE PERFORMANCE OF PHASE SYNCHRONIZERS


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Abstract:
This paper deals with the problem of symbol recovery under low SNR conditions and nonlinear channel effects. In order to improve the performance of phase synchronizers the use of some linear ALE methods is proposed: classic ALE, CMA, and ML filtering. Comparisons among them are made from different signal processing aspects such as capacity of noise removal and speed properties.

A linear regression is shown to be a good quick form for phase and frequency acquisition. Finally two types of ALE structures are suggested.

I - INTRODUCTION

The main goal of this paper is the inclusion of advanced DSP methods in classical communication structures. In our study we specially consider open loop diagrams to achieve good solutions for burst PSK modulation.

As it is well known, in communication systems good estimation of amplitude, phase and synchronise are required.

An easy method for symbol recovery without demodulation process can be implemented with a bank of matched filters to each possible waveform. Only a good symbol synchronization is required.

Unfortunately, this structure is not flexible and cannot adapt itself to the variant characteristics of the incoming signal. Therefore a Doppler deviation, for instance, forces the system to work in a different frequency which is not the nominal one, so the performance becomes worse.

Thus, this paper suggests the use of a linear regression method as frequency and phase estimator. In order to improve it, we deal with an ALE, CMA and ML filtering as a previous process.

II - THE LINEAR REGRESSION METHOD.

A linear regression is an easy and quick form for obtaining the phase and frequency of an unmodulated carry signals from its phase samples.

The next picture shows the block diagram of a frequency and phase estimator based on linear regression.

![Figure 1: Block diagram of frequency and phase estimator based on linear regression.]

If an arctangent phase detector is used, (fig 2), the phase unwrapping reconstructs a segment of the phase straight line.

![Figure 2: Block diagram of phase detector and phase unwrapping.]

Then, a linear regression from the noisy samples \( \hat{\phi}(n) \) is performed:

If \( \hat{\phi}(n) \) represents the least square linear, then

\[
E(\hat{\phi}(n) - \phi(n))^2 \rightarrow \min
\]

where

\[
\hat{\phi}(n) = \hat{\omega}_0 n + \hat{\theta}_0
\]

Minimizing the cost function (1) we achieve the optimal solution for \( \hat{\omega}_0 \) and \( \hat{\theta}_0 \), that is

\[
\hat{\omega}_0 = \frac{E(\phi(n)) - E(n) E(\phi(n))}{E(n^2) - E(n) E(n)}
\]

\[
\hat{\theta}_0 = \frac{E(\phi(n)) - E(n) E(n)}{E(n^2) - E(n) E(n)}
\]

Graphically \( \hat{\omega}_0 \) and \( \hat{\theta}_0 \) can be seen as the slope and the origin ordinate of the straight line that matches the sample set \( \phi(n) \).

The sample window length becomes a tradeoff between the ability of the system to adapt itself to frequency shifts and the noise sensitivity.

This work was granted by the CICYT National Research Plan on Information and Communication Technologies of Spain.
The structure presents a threshold level in noise and a maximum estimated frequency since a nonlinear operation (the phase unwrapping) is employed; a high level of noise masks the $2\pi$ crossings of the sawtooth phase signal.

We can improve the properties of the linear estimator placing a previous equalizer that removes both noise and modulation from the signal.

An adaptive equalizer can adjust its behavior to the statistics of the incoming signal.

III - ADAPTIVE SYSTEMS

- Introduction

In the next sections we'll suppose that symbol synchronization is known.

Furthermore only digital angular modulation is employed, due to its immunity in front of noise, channel nonlinearities, etc. For this reason the use of linear phase adaptive systems is desired (information "travels" on the phase).

![Adaptive System Diagram](image)

Figure 3

An adaptive system obtains from the incoming signal a specific information (output signal), thanks to an additional or collateral external information.

We can consider two types of collateral information:

- Independent information from the input and output signals.
  For example, the modulus of the output signal (CMA algorithm), the nominal carry frequency (ML filtering), etc.

- Dependent information.
  For example, a reference signal (ALE).

The type of collateral information and its use determine the different methods of adaptive filtering.

- The Adaptive Line Enhancer (ALE)

The ALE [1], which basic scheme can be seen in figure 4, was initially designed as a degenerate form of a noise canceller.

![Classic ALE Structure](image)

Figure 4. Classic ALE structure

The ALE uses the LMS adaptive algorithm to recursively adjust the weights of the filter.

So

\[ W(n) = W(n-1) + 2 \mu e(n) X(n) \]

with

\[ e(n) = x(n) - y(n) \]

The collateral information employed is an appropriate delayed replica of the incoming signal.

The delay $\Delta$ is chosen to obtain a high level of correlation at the input of the comparator.

Therefore if

\[ x(n) = A \sin (\omega_0 n T + \theta_0) + N_0(n) \]

with

\[ \omega_0 = 2\pi / \tau_0 \]

then $\Delta$ is forced to be

\[ \Delta = m \tau_0 \] with $m = 1, 2, \ldots$

Thus the error signal $e(n)$ has several uncorrelated components, basically noise, and it's used to properly adjust the frequency response of the filter.

ALE reaches a high level of noise cancelling, even in very low SNR cases, as shown below.

![Signal Graphs](image)

Figure 5. (A) Ideal signal
(B) Noisy signal
(C) Output signal of the filter
SNR= 0 dB

For a fast convergence a weight vector

\[ W(0) = 1/LF \cdot \left( 1, \cos(\omega_0), \ldots, \cos(\omega_0(LF-1)) \right) \]

is used, so initially the algorithm performs a DFT.

The $\mu$ parameter controls the stability, convergence speed and filter capacity of the ALE.
In the classic ALE scheme it can be shown the reference is noisy because it’s obtained from the incoming signal. An improved scheme extracts the reference from the filter output.

![Improved ALE scheme](image)

**Figure 6.** Improved ALE scheme.

When the input signal is a modulate signal the performance of the ALE becomes worse because no $\Delta$ produces a high level of correlation. Then the CMA algorithm must be used.

- **Constant Modulus Algorithm (CMA)**

The objective of this adaptive filtering [2] [3] is to restore the incoming signal to a form which, on the average, has a constant instantaneous modulus.

The collateral information used is the knowledge about the constant modulus of the output signal.

The error function to minimize is

$$\zeta = E \left( \| y(n) - x(n) \|^2 \right)$$

Thus, using a gradient search algorithm

$$W(n+1) = W(n) - \mu \left( \| y(n) \|^2 - 1 \right) y(n) x^*(n)$$

This CMA was initially introduced as a method of correcting the multipath effects:

If $x(n) = A \exp(j\omega_0 n T + \theta_0) + \eta(n)$ then $y(n)$ defined as $y(n) = x(n) + \alpha x(n-\Delta)$ has not constant modulus.

In the next figure the correcting process is shown

![Correcting process](image)

**Figure 7.** (A) Ideal BPSK signal (B) Multipath corrupted signal (C) Output signal of CMA.

With unmodulated signals the CMA performance becomes worse than ALE, since ALE works with a reference signal, which supplies speed and noise removal capability.

- **Maximum Likehood (ML) Filtering**

The ML filter uses the knowledge of the nominal frequency of the carry signal as collateral information.

The filter is designed under the assumption on minimizing the output power.

$$\min (P_Y) = W^H R W$$

under the constrain equation

$$S^H W = 1$$

where

$$S = (1, \exp(-j\omega_0 T), \exp(-j\omega_0 (N-1) T))^T$$

and

$$R = \text{Autocorrelation matrix of X data}$$

The function to be minimized is

$$\zeta = P_Y - \lambda (S^H W \cdot 1)$$

being $\lambda$ a Lagrange multiplier.

Taking the gradient $\nabla \zeta = 0 = R W_{\text{opt}} + \lambda S$

Since $S^H W_{\text{opt}} = 1$

then

$$W_{\text{opt}} = R^{-1} S (S^H R^{-1} S)^{-1}$$

If, as usually, the $R$ matrix is unknown an adaptive algorithm can be used.

Thus, if $W(n+1) = W(n) - \mu \nabla \zeta$

then

$$W(n+1) = P (W(n) - \mu y(n) x(n)) + F$$

where

$$P = I - S (S^H S)^{-1} S^H$$ (Projection vector)

$$F = S (S^H S)^{-1}$$ (Shift vector)

The filter works respecting a single tone steered at $\omega_0$ and removing the rest components as possible.

In the particular case of considering white noise then

$$R^{-1} = \text{constant } I$$

Then

$$W_{\text{opt}} = R^{-1} S (S^H R^{-1} S)^{-1} = S/L$$

where

$$L = \text{DFT length}$$

This is equivalent to perform a DFT of the input signal steered at the frequency $\omega_0$.

$$\text{DFT}(x(n)) = \sum x(n) \exp(-j\omega_0 n m T))$$

with $m=1$

The DFT filtering provides information about the amplitude and phase of the signal.
Let's assume a MPSK transmission.

Then

\[ x(n) = A \exp(j \omega_0 n T + \phi_0 + \theta(n)) + n_o(n) \]

We can estimate the phase of each incoming symbol taking only one sample per symbol of the DFT argument output.

The modulation must be removed from the set of DFT samples. To do it, the phase samples are multiplied by M. Thus the modulation phase term becomes \(2\pi m\) jumps and they are removed by the phase unwrapping system.

Symbol synchronization can be achieved making a \(L\) length DFT over the whole data string (being \(L\) = number of samples per symbol). When the DFT window is centered just on a symbol the modulus is maximum. In other cases it clearly decreases.

Therefore adjusting the sampling symbol instants (\(L\) samples separated) to the modulus of the DFT output the symbol synchronization can be achieved.

The DFT results in an attractive useful method in burst transmission mode thanks to its speed and simplicity.

**IV - CONCLUSIONS**

The analysis of the previous methods suggests the implementation of two different structures:

- **Structure based on ALE filtering**

\[ x(n) \xrightarrow{ALE} \text{Phase Detector} \xrightarrow{Phase Unwrap} \hat{\theta}_0 \]

**Figure 8**

The adaptive characteristics adjusts the behavior of the filtering according to the variations of signal parameters (noise, frequency). No information about nominal frequency is required, so a wide frequency range can be tracked.

The convergence time makes it not fast enough for burst transmission mode cases. However an initial DFT vector supplies a high speed improvement.

Under modulated signals ALE only provides a first approximation for frequency estimation.

- **Structure based on ML filtering (DFT)**

\[ x(n) \xrightarrow{DFT} \text{Phase Detector} \xrightarrow{Phase Unwrap} \hat{\theta}_0 \]

**Figure 9**

The quick processing time is very useful in burst mode thanks to the open loop process employed.

Furthermore the structure can be used both in modulated and unmodulated cases.

This system requires the knowledge of the nominal carry frequency, so only Doppler deviations can be tracked.

**REFERENCES**


