EVOLUTIONARY LATTICE MODEL FOR THE
COMPACTION OF DEFORMABLE GRANULAR
MATERIAL

AUTHORS
R. Pieralisi a, *, S. H. P. Cavalaro a, A. Aguado a

a Departamento de Ingeniería de la Construcción, E.T.S. Ingenieros de Caminos,
Canales y Puertos, Universidad Politécnica de Cataluña, BarcelonaTech, Jordi Girona
Salgado 1-3, Módulo C1, Despacho 202, 08034 Barcelona, Spain

CORRESPONDING AUTHOR
* Tel.: +34 934016507; fax: +34 934054135.
E-mail address: ricardo.pieralisi@upc.edu

ABSTRACT
This paper presents a simplified model for the simulation of the compaction of
deformable granular materials, such as pervious concrete. The strategy is to use an
evolutionary Lattice system with local instability to simulate the intensive internal
rearrangement expected during compaction. This is associated with a bar layout and
material properties that vary with the time steps. Furthermore, a biphasic particle
composed by a rigid inner core involved by a deformable exterior layer is considered.
This versatile approach allows a simplified representation of complex granular media
made of very deformable particles or of rigid particles covered by a flexible binder.
Numerical and experimental comparisons were conducted to evaluate the capacity of
the method to predict the behavior of different materials. The results indicate that the
model may provide an estimation of the geometrical disposal of the media and provides a prediction of the forces applied during the compaction, being suitable for a wide range of applications.

KEYWORDS
Deformable granular media; Biphasic particle; Compaction; Lattice Model; Pervious concrete

1. INTRODUCTION
Granular materials are widely employed in construction, ceramic, metallurgy, seeds and pharmaceutical industries, being largely available in nature (rock and soil formation). They are composed of grains that might be involved by a binder layer that will harden with time or temperature, providing cohesion to the system. In certain applications, before hardening occurs, the material is subjected to compaction pressure during production or processing. The final density obtained may present a large variation depending on the applied loads and may change completely the material properties, affecting the porosity, mechanical strength, permeability, acoustic and thermal conductivity. It is clear that the compaction is as important as other initial conditions (particle size distribution of the material, specific weight and surface area) when it comes to obtaining the required characteristics of the granular media.

In the literature, an increasing interest on the simulation of the compaction process of granular media is observed. In general, two types of computational methods are used to simulate this problem: the discrete-particle approach and the continuum approach. A well-known example of the former is the discrete or distinct element method (DEM), where the interactions between particles are usually classified as rigid or soft, depending
whether the particle deformation during a collision is explicitly incorporated in the model or not. Soft particle interaction was developed by Cundall and Strack [1-2]. In this method the particles interact with each other and with the boundary walls. The Cundall and Strack method simulate the contacts by virtual dashpots and springs that represent the interaction between particles. To calculate the force acting in the normal direction of each contact, the distance of overlap between the particles is assessed and multiplied by the stiffness constant of the spring and/or used in the equation of the dashpot. In the tangential direction, the procedure is analogous, applying the change of the relative tangential displacement of the surfaces of both particles instead of the overlap. Also in the method developed by Cundall and Strack, the analysis is performed with time steps that should be small enough to assure convergence and representativeness of the results, which means the disturbances do not propagate from any particle further than its immediate neighbors [2]. Even though it has been successfully used for the simulation of granularly media [3-8], this approach usually presents a high computational cost. In addition to that, some simplifications and new implementations have to be assumed for the simulation of biphasic deformable particles in DEM.

In the second approach, a finite element method (FEM) is used to simulate a continuous media whose behavior is described with the elastoplastic theory. This numerical solution was widely used to evaluate the compaction of powder metals and ceramics [9-12]. Although the computational cost is lower than with the DEM and deformation may be directly taken into account, the assessment of parameters such as the porosity or the particles position with the compaction pressure is not possible.

A recent alternative to simulate the compaction process more realistically is to integrate both methods [13-18]. In this case, the discrete element consideration from the DEM is
combined with a continuum representation of particles and contacts, which is typical of the FEM. The DEM/FEM simulation is more accurate than the previous ones and is considered promising since it accounts for aspects such as the deformation of particles and allows the estimation of the porosity. However, the high computational demand still limits its use.

Another approach developed and successfully applied by Katsman and Aharonov [19, 21] uses a modified lattice model to simulate the compaction process of rocks and soils. The authors establish a fixed distribution of bars whose properties and length are changed depending on the level of compaction. Despite the reduced computational time required to analyze these models, they do not consider the effect of the outer binder layer disposed around the grains. Moreover, the materials simulated present an initial porosity of up to 20%, meaning that the level of rearrangement of particles is small and the same lattice layout may be maintained throughout the analysis. On the contrary, in the case of particles with a binder layer, the initial porosity may vary from 20% to 50% when it is poured on the molds. Consequently, the level of initial disorder and of rearrangement experienced during the compaction process is much higher than in the case of rock masses. This means that a lattice model with fixed distribution of bars might not be representative of the material in intermediate compaction stages due to the high level of internal reorganization, which is characterized by a significant increase in the number of contacts.

The objective of this study is to propose a simplified approach for the simulation of the compaction of a deformable granular media without the drawbacks mentioned previously. The strategy is to use an evolutionary lattice system with local instability to allow the intensive rearrangement expected during compaction, associated with a bar layout and material properties that vary with the load steps. Furthermore, a biphasic
particle composed by a rigid inner core involved by a deformable exterior layer is considered. It allows a simplified representation of complex granular media made of very deformable particles or of a rigid inner core covered with a viscoelastic binder layer. This is especially interesting for the simulation of composites like pervious concrete - a material formed by aggregates involved with mortar binder that generates a microstructure greatly affected by the compaction process [22].

Initially the model is described and its application is evaluated. Then, a numerical comparison using the results by other authors with DEM/FEM simulation is presented. Finally, an experimental program is conducted to evaluate the compaction process of pervious concrete. The results from the tests are compared with those numerically estimated with the new model. The good fit obtained supports the use of the model proposed here, which is capable of providing a simplified straightforward representation of the compaction process.

2. WORK PHILOSOPHY

The model to simulate the compaction of granular media was divided in two complementary stages. The sole purpose of the first stage is to create a particle distribution with the desired initial porosity. The algorithm applied organizes the particles in space; creating a system similarly to that obtained by pluviation with Hertzian contacts in DEM with limitations of no particle movement and with the desired initial porosity (other distribution laws could be implemented depending on the type of material simulated). The distribution generated is the input for the second stage, which is the focus and main contribution of the present study. Based in an adapted lattice method, it simulates the reorganization of the particles after an imposed compaction displacement or vibration. As mentioned before, the algorithm generated is versatile and
may be simply adapted for a wide range of applications. The program MATLAB 7.10.0 was chosen for the development since it is of common usage in the scientific community and includes a large math library that is capable of solving complex matricial problems. A detailed explanation of both stages is presented in the following sections.

Although the model described here was developed for 2D simulations of circular particles, it could also be generalized to 3D. For that, it would be necessary to modify the original algorithm, including an additional degree of freedom. Furthermore, the verifications performed in the lattice model would have to be adapted for a 3D condition. This would make the code more complex and would lead to additional computation time but would lead to a model more representative of the particle distribution.

2.1. Stage 1 –Random Fall

In the first stage, a series of steps are needed to ensure the global and local stability as well as a homogeneous distribution of the particles. The numerical approach used is based on a simple algorithm designed by Vold [23] for the packing of circular random particles. This approach was modified by several authors [24-26] that implemented new types of contact between particles.

Here, the algorithm was modified to represent the packing of circular particles that might be involved by a binder with special characteristics, considering several simplifications that would allow reaching the initial porosity. The interaction between particles is based in the Hertzian contact law without friction coefficient. The frictionless condition is assumed in this first stage to simplify the calculation process. This means that new particles are allowed to freely spin and slide over the existing ones.
Such interaction is a simplification of what would occur if a rigid particle without binder or if a particle with a dry binder layer was dropped over another. It is important to remark that the Hertzian contact law is only used in Stage 1 and that other laws could be used instead if necessary.

To find a stable place for each particle, three steps were conducted as illustrated in Fig. 1. First, an X coordinate is chosen to start the vertical falling process inside the container (see Fig. 1.a). In the studies about the disposition of granular media, Jullien [24] demonstrated that a segregation phenomenon might occur in case a large number of particles with different sizes are considered and the x coordinate of the initial falling point is fixed. Such condition favors a heap accumulation inside the container just under the coordinate x. Moreover, the large particles may easily spin over the bed formed by smaller particles, being more likely to accumulate close to the lateral wall of the container and to generate big voids or a non-homogeneous distribution.

Fig. 1. Steps to find a stable position for each particle in Stage 1 (free fall).

To mitigate this problem, two modifications are taken into account in the first step regarding the original algorithm. For once, the starting coordinates x are randomly chosen according with a probabilistic curve. To diminish even more the heap accumulation, the probabilistic curve is assessed every time a particle falls. It is assumed inversely proportional to the distance in y-axis between the dropping point and the height of the granular bed at each position in x. Consequently, points with bigger accumulation of particles have a smaller probability of being selected. Once the x coordinate is defined, the particle falls until the first contact is verified. If the contact occurs with the walls of the container, the particle are automatically fixed. On
the contrary, if it occurs with another particle (see Fig. 1.b), the third step is called. In the latter, the stability of the falling particle is verified considering the support provided by the surroundings.

Each contact point is only capable of generating compressive normal forces since, in this stage, the contact between particles is weak. To evaluate the stability, the balance of forces in x and in y and the balance of moments are used to estimate the contact reactions applied to the contact points of the falling particles, similarly to what is done in statics. If all reactions estimated are positive (indicating compressive forces), the particle is stable. Conversely, if any of the calculated reactions is negative (indicating tensile forces) or if the system of equations may not be solved, the particle is assumed unstable.

In case an unstable condition is observed, the falling particle is allowed to spin around the particle that received the last contact until additional contacts are observed and stability may be verified again, as shown in Fig. 1.c. During the spin, the center of the particle will always move with the same direction of a vector traced tangent to the last contact, pointing towards the bottom line of the recipient. This procedure is repeated until a stable condition is found or a contact with the walls of the recipient occurs.

After finding a stable condition for the falling particle, a small deformation may exist in the contact region due to the self-weight of the particle (see Fig. 2). This deformation is simulated as an overlap.

Fig. 2. Deformation of the particle represented as an overlap.

Because of the deformation, an increase of the contact length ($l$) is observed. The value of $l_c$ may be calculated through the Hertzian Contact Law [27]. This is represented
through Eq. 1 that depends on the stiffness of the materials in contact ($k_{pc}$) and of the force (F) due to the self-weight of the particle.

$$L = 2 \cdot k_{pc} \cdot \sqrt{F}$$  \hspace{1cm} (1)

With the stable contact established and the initial overlap estimated, the particle becomes fixed and cannot change position or be moved by others. The whole procedure is repeated with the next falling particle. As a result, the stability of the distribution is always assured. The recipient is filled until the fixed particles cross the line to which the compaction will be applied. All particles located above this line are then removed so a reference initial height of the sample is defined. In this moment, the initial porosity of the system is estimated and compared with the desired one.

The algorithm used for the Stage 1 tends to generate systems of particles with porosities that are below that found in a granular medium with a binder (cohesive material). This occurs because, in reality, particles are not dropped one by one until a stable condition is found. Rather than that, they are poured over the surface all together in big groups. This increases the level of internal disorder, hence the porosity. To correct this and assure the desired initial porosity, some of the particles are removed from the system.

A special procedure - independent of the stage 1- is used to select from which points the particles will be removed in order to minimize the modification of the grading curve of the granular medium. First, a diameter is selected according with the proportion of the grading curve used to introduce the particles in the system. Then, all particles with this diameter are listed and one is randomly selected. A verification is performed to evaluate whether the self-weight stability of the system will be maintained after it is removed. If stability is guaranteed, it is removed and another diameter is selected. If not, another one from the same diameter is randomly selected and the stability is verified again. This
is repeated until a suitable particle is found and removed. In case no particle in the list fulfills the stability criterion, another diameter from the grading curve is selected. In some cases, this will modify slightly the grading curve of the granular media. Anyhow, the authors have found that this procedure is more efficient than using an algorithm that considers the real interactions between all particles that are poured together. Another possible situation that will ask for a modification of the initial porosity happens in case non-circular particles are used in reality. In this situation, the porosity generated will be higher than that achieved with Stage 1, which considers only circular particles. In order to correct the porosity of the system, the same procedure described in the previous paragraph for the removal of particles is applied. For that, the equations that correlate the initial compactness of the sample with the sphericity of the particles presented in the studies of German [28] were used.

It is important to remark that the algorithm used in Stage 1 includes simplifications to decrease the computational effort required to generate a distribution of particles with the desired initial porosity. More realistic alternatives based on DEM may be found in the literature.

### 2.2. Stage 2 – Compaction

This stage is responsible for the simulation of the compaction process. It consists of calculating the geometric position of all the particles after a static compaction produced by a small vertical displacement of the top surface of the granular sample. Although this study considers only vertical compressive displacement to compact the granular bed, the same approach is valid for compaction or vibration in all directions.

Fig. 3 outlines the algorithm used for each load step in the compaction stage. It initiates with a modified Delaunay Triangulation (DT) method, where nodes and bars are created.
between adjacent particles. An interactive Lattice model analysis with incremental displacement is conducted (the same numerical solution is used to study the concrete fracture by other researchers [29-31]). Then, the balance of forces is verified and an interactive process is performed until reaching convergence. In the present study, the convergence is verified using the Euclidian norm of the out-of-balance forces. The iterations are stopped and the step is concluded if this norm is smaller than $10^{-4}$. Once this occurs, nodal displacement results obtained by the Lattice model are used to update the position of the centers of the particles in each time step. This procedure – including the DT to detect new contacts – is repeated at every load step. A detailed description of the different steps from Stage 2 is presented in the following sections.

Fig. 3. Algorithm of Stage 2 (Compaction).

2.2.1. Triangulation

Initially, the particles center coordinates and the radius are detected (Fig. 4.a). After that, a DT (Fig. 4b) is executed using the same algorithm applied by [32]. Then, a geometrical verification (Fig. 4.c) is conducted to ensure consistency in the interaction. The principal points of this geometrical verification is to avoid no potential particles contacts and for numerical efficiency. In this sense, it is considered that the interaction occurs when the distance between particles is below 1.5 times the sum of the radius. Notice that the resulting DT includes bars that may represent the contact between particles and a virtual contact between particles that do not interact with each other. The latter does not exist in reality but is necessary to obtain a well-conditioned matricial system. As described in section 3, special material properties are assigned to each type of contact to reproduce the interactions between particles and to avoid the virtual
contact interfering in the compaction results. Fig 4.d shows the final triangulation in a
certain time step, highlighting the real and the virtual contacts in blue and in green,
respectively.

Fig. 4. Triangulation process for a certain load step.

Even though the arrangement of bars depicted in Fig 4.d is globally stable, local
instabilities are found if groups of bars are analyzed separately. Consider, for instance,
the arrangement highlighted in Fig 4.e. It is evident that the relative movement between
all nodes present in the local system is restricted. On the other hand, in the arrangement
highlighted in Fig 4.f, the relative displacement between nodes is possible. This
freedom of displacement enhanced by the use of virtual bars with very small stiffness is
essential to reproduce the highly porous medium and the rearrangements of particles
throughout the time steps.

The resulting lattice layout works as a mesh with bar elements. In each load step, the
granular medium is re-meshed since new contacts might arise and old ones might be
eliminated. To illustrate it, Fig. 5.a represents the initial condition without compaction
and Fig. 5.b represents the situation after a compaction of 14.4 %. It is clear that new
contacts are detected and the internal instability is reduced throughout the steps. The
increase in the number of real contacts is not considered a computational problem since
the equations are solved by matricial methods and in this kind of solution the
computational cost is more related with the number of nodes, which remains constant
throughout the analysis.

Fig. 5. Triangulation at different steps.
2.2.2. Lattice Model Process (LMP)

Once the triangulation for all the particles is finished, the nodes and the bars are passed to the Lattice model process (LMP). In a 2D model, all the nodes are articulated and have two degrees of freedom, corresponding to the displacement in two orthogonal directions. Equilibrium conditions regarding the forces in the x direction (Eq. 2) and in the y direction (Eq. 3) are applied. According to the Principle of Virtual Work (PVW), the total virtual work of the forces acting on the node should be zero in each time step. The boundary conditions for this numerical problem are applied only at the nodes and defined by the particles in contact with the surfaces of the container, which might have their displacement fixed or might be used for the application of the compaction.

\[ \sum F_x = 0 \]  
\[ \sum F_y = 0 \]

To understand how the forces and displacements are calculated, consider the simplified example of the bar from Fig 6 that originally extends from node 1 to node 2. If resultant forces are applied in the nodes \( F_1^{(x)} \) and \( F_2^{(x)} \), respectively, the bar may deform and move assuming the new position defined by the nodes 1' and 2'. In this situation, the displacements \( u_1^{(x)} \) and \( u_2^{(x)} \) will be observed.

Fig. 6. Forces and displacements in the nodes of a bar.
Since equilibrium is maintained in the system, the forces should cancel each other. In this condition, the variation in the position of the nodes may be related with the forces through Eq. 4. In this equation, \( k^{(e)} \) represents the stiffness of the bar in a certain time step (for a detailed description on the contact laws used to define \( k \) see section 3).

\[
F_1^{(e)} = -F_2^{(e)} = k^{(e)} \cdot (u_2^{(e)} - u_1^{(e)})
\]  

(4)

In the matricial form, Eq. 4 could be represented through Eq. 5. In the latter, the decomposed stiffness \( K_{ij}^{(e)} \) should be calculated according to Eq. 6, considering the angle \( \alpha \) formed between the forces and the coordinate system selected.

\[
\begin{bmatrix}
K_{11}^{(e)} & K_{12}^{(e)} \\
K_{21}^{(e)} & K_{22}^{(e)}
\end{bmatrix}
\begin{bmatrix}
u_1^{(e)} \\
u_2^{(e)}
\end{bmatrix}
=
\begin{bmatrix}
F_1^{(e)} \\
F_2^{(e)}
\end{bmatrix}
\]  

(5)

\[
K_{ij}^{(e)} = k^{(e)} \cdot \begin{bmatrix}
\cos^2 \alpha & \sin \alpha \cdot \cos \alpha \\
\sin \alpha \cdot \cos \alpha & \sin^2 \alpha
\end{bmatrix}
\]  

(6)

In a more complex lattice model composed by several bars, Eq. 5 may be generalized to Eq. 7. Notice that it has the form \( K \cdot u = F \), in which \( K \) is the stiffness matrix, \( u \) is the displacement vector and \( F \) is the forces vector.

\[
\begin{bmatrix}
K_{11}^{(e)} & K_{12}^{(e)} & \cdots & K_{1n}^{(e)} \\
K_{21}^{(e)} & K_{22}^{(e)} & \cdots & K_{2n}^{(e)} \\
\vdots & \ddots & \ddots & \vdots \\
K_{n1}^{(e)} & K_{n2}^{(e)} & \cdots & K_{nn}^{(e)}
\end{bmatrix}
\begin{bmatrix}
u_1^{(e)} \\
u_2^{(e)} \\
\vdots \\
u_n^{(e)}
\end{bmatrix}
=
\begin{bmatrix}
F_1^{(e)} \\
F_2^{(e)} \\
\vdots \\
F_n^{(e)}
\end{bmatrix}
\]  

(7)

At each step, an updated stiffness matrix (\( K \)) for all elements and a vector with the applied forces (\( F \)) are assembled. Once the boundary conditions are imposed, the direct
stiffness method is applied to determine the displacement vector (u) of the nodes. For that, similarly to what is done in FEM, the system $K\cdot u = F$ must be solved. In the present study, the solution of Eq. 7 is iteratively approximated through the Newton-Raphson Method. Convergence criteria based on the displacement and the forces are considered. Maximum relative errors of $10^{-4}$ should be simultaneously fulfilled to stop the iteration.

All equilibrium equations are applied and the stability of the system is assured in the load steps. This might seem contradictory given that local instabilities might occur during the compaction process. To simulate such instabilities, the lattice mesh is redefined at each step and virtual bars are included or eliminated.

In the compaction process of a granular material characterized by an inner rigid core and an external deformable layer (such as a binder), the main interactions between particles are deformation of the contact region and rotation of one particle over the others. Although deformations of the contacts may occur by the action of forces in normal and tangential directions, in a uniaxial compaction process the normal forces are the main responsible for the interaction between particles. Indeed, due to the soft nature of the binder, the normal forces generated tend to be several times bigger than the tangential forces. Consequently, the former governs the contact between particles. This allows a simplified consideration of the lattice bars in the present study, which only have axial stiffness.

The rotation of the particles is not explicitly considered in the modified lattice model proposed. This movement is indirectly represented by the possible rotation of the bars around the nodes. Such approximation is only feasible for granular materials in which the rigid connections between particles have not been established so that interactions are governed mainly by normal forces and large displacements may occur. This is the
typical situation found in the compaction of many granular medium without binder or
with binder in the non-hardened state.

Even though it goes beyond the scope of the present study, it is important to remark that
the model proposed could produce unreasonable results once rigid connections are
established between particles. In this situation, other models are required since the
influence of tangential forces associated with the rotation of particles should be taken
into account to achieve accurate results.

2.2.3. Verifications

Several verifications are applied to evaluate the correctness of the boundary conditions
in each time step. If all verifications are satisfied, the step is concluded and another
incremental displacement is applied. If not, the boundary conditions are modified, the
force and displacement matrices are changed and the code calls the LMP function to
process the step once again. This is repeated until all verifications are satisfied.

Two types of modifications in the boundary conditions are possible, as shown in the
Fig. 7. The first of them occurs when the calculated reactions of particles in contact with
the walls are tensile forces (Fig. 7.a1) that exceed the tensile strength of this contact. In
this case, the change imposed is to exclude the support, as illustrated in Fig. 7.a2. The
second type of modification occurs when a particle passes throw the wall of the
container (Fig. 7.b1). The correction here is to introduce a new support on the node of
the particle and to fix its position in contact with the wall, as demonstrated by Fig. 7.b2.

Fig. 7. Verification of boundary conditions (a and b) and of overlap condition (c).
Besides the modifications in the boundary conditions, three additional modifications might be necessary at each step. The first of them occurs when an unrealistic overlapping of the inner core of particles occurs after the LMP (Fig. 7.c1) concludes the analysis. In this case, the code does not send the model back to the LMP to recalculate the step again. Instead, it introduces in the next step a displacement in the overlapping particles equivalent to the overlap distance (Fig. 7.c2). The second additional modification concerns the change of the material properties of the bar. The distance between the centers of the particles is used to define the type of contact and its properties according with the criterion presented in section 3. The last additional modification only takes place in the initial load steps when the model presents a global instability and the stiffness matrix may not be inverted. If this happens, the triangulation function is called again and the minimum distance between particles is increased to achieve a bigger number of bars in the DT. If the increase in the minimum distance is performed several times in the same step, the analysis is stopped. Based on the experience of the authors, the number of repetitions observed will depend on the porosity of the medium. For the models analyzed here, the initial time steps might require 2 to 3 repetitions. As the analysis proceeds, the number of contact increases and porosity reduces, so that no repetition of the DT is needed within the same step.

Although the successive DT at each time step allows identifying new contacts, it might lead to convergence and stability problems. This is especially evident in the first stages of compaction of systems with high initial porosity due to the significant movement between particles. To mitigate these problems, it is necessary to apply sufficiently small time steps.

2.2.4. Time steps
The compaction of a granular material is usually a dynamic process given the typical rate of load application and since the value of the forces depends on the physical properties of the interactions between particles, which are affected by the velocity of their relative displacement. Instead of using a purely dynamic analysis, some simplifications are assumed in the alternative approach proposed in this study. The relative velocities between particles is used as an input parameter to estimate the stiffness of the interaction in each time step, according with the formulation presented in section 3.2. Once all bars of the Lattice layout have their stiffness calculated, the loads are applied as small vertical displacements by steps. This vertical displacement is related to the velocity of the compaction process applied in reality. In other words, the displacement is the velocity of the compaction multiplied by the time step. With all the loads and boundary conditions applied, the lattice model is statically solved for each time step. The variation in the position of the particles is used to assess the velocity of their relative displacements and the new stiffness of each contact. For example considering two particles \(i\) and \(j\) and the relative distance between them as \(\Delta u\), the initial relative distance between both particles at the last step is \(\Delta u^{n-1}_{i} - j\) and the relative distance between both particles at the considered step is \(\Delta u^{n}_{i} - j\). The modulus of the difference between the relative difference \(\Delta u^{n-1}_{i} - j\) and \(\Delta u^{n}_{i} - j\) divided by the time step \(\Delta t\) is the velocity of their relative displacements \(v_{i-j}\), expressed by Eq. 8. After that, a new time step is initiated.

\[
v_{i-j} = \frac{|\Delta u^{n-1}_{i} - j - \Delta u^{n}_{i} - j|}{\Delta t}
\]

It is evident that the approach proposed combines a static consideration of the balance of forces within every time step but includes the dynamic effects through the redefinition of the stiffness of the interactions between particles and through the
adaptive DT. Therefore, it may be considered a quasi-static analysis that intends to simulate a dynamic process. It is important to remark that this represents a simplification made with the intent of reducing the time required for the analysis of such dynamic phenomenon. Other models should be used if a more precise representation of the interaction between particles is needed.

### 3. MATERIAL PROPERTIES

This section focuses on the material properties for the bars that simulate the real and the virtual contact between particles. As shown in Fig. 8.a, the rigid inner core is involved by the binder and, for simplification proposes, the particle is assumed circular in a 2D analysis. Fig. 8 presents a schematic view of the study particle and the possible contact types. The total particle radius and the inner core radius are represented by \( R_p \) and \( R_{ic} \), respectively. This leads to three possible contacts defined by the distance \( d \) between the centers of the particles as illustrated in Fig. 8.b, 8.c and 8.d. Note that the stiffness is recalculated every load step since contacts might be generated or eliminated. The methodology used to obtain the contact length and the stiffness of each bar is outlined in the following sections.

![Fig. 8. Study particle (a) and situation of inner core contact (b), binder contact (c) and no contact (d).](image-url)

#### 3.1. Inner core contact simulation

The first case illustrated in Fig. 8.b occurs when the two inner cores are in contact, i.e. the distance between the center of the particles is equal or less than the sum of the radius of their cores (\( d \leq R_{ic,i} + R_{ic,j} \)). To simulate this interaction it is necessary to...
define the effective interaction area of particles in contact, which is used to calculate the stiffness of a bar. In the literature, some authors like Lu et al. [33] and Chidiac [34] defined that the interaction between two adjacent particles may be represented through Fig. 9.

According to this definition, the effective area \( A_{ef,ic} \) may be approximated as a circle with radius \( R \) equal the weight average of the diameter of the inner cores, expressed by Eq. 9. Usually, the elastic modulus of this contact is very high in comparison with that assumed for the contact of the binders. Therefore, in such applications, the interaction between inner cores may be simulated through a spring with high stiffness. The stiffness of this contact is calculated by multiplying the effective area by a surficial stiffness \( k_s \) that depends only on the properties of the material from the inner core. This is represented in Eq. 10, which takes into account that the sizes of the particles will affect the stiffness of the contact.

\[
A_{ef,ic} = R^2 \cdot \frac{R_{ic1}^2 + R_{ic2}^2}{R_{ic1} + R_{ic2}}
\] \hspace{1cm} (9)

\[
k_c = k_s \cdot A_{ef,ic}
\] \hspace{1cm} (10)

3.2. Binder contact simulation

Fig. 8.c represents a case in which the mortar layer of different particles overlap without putting in contact the aggregates. Here the distance of the center of the particles is less than the sum of the radius of the particles and more than the summed radius from the
rigid inner core (\( R_{ic,i} + R_{ic,j} < d < R_{p,i} + R_{p,j} \)). The binder is assumed as a soft layer susceptible to deformations. When an overlapping occurs, it is necessary to assess the binder deformation and to recalculate the stiffness of the contact.

3.2.1. Binder redistribution due to overlap

The contact between particles with a binder outer layer may produce a redistribution phenomenon. Depending on its rheology, the binder could relocate around the contact point as the overlap increases. This leads to the formation of a lateral increment of the contact length, as depicted in the Fig. 10. This increment should be taken into account when the normal forces between particles are estimated.

Fig. 10. Binder redistribution after overlap

To assess the final contact area after the overlap, a simple geometric calculation is performed. It is assumed that the area of binder located originally in the overlap region \( A_1 \) will equal the area that redistributes in region \( A_2 \) of Fig. 10. Therefore, once the area of \( A_1 \) is estimated for a certain overlap, the area of \( A_2 \) is also known. Then, the position of the points \( P_2 \) and \( P_3 \) may be determined, thus providing the final contact length \( (l_c) \).

To simplify this calculation, it is considered that the angle \( \omega_j \) that represents the increment is equal for both particles in contact. This consideration implies a negligible error as long as the thickness of the binder layer is small in comparison with the diameter of the inner core and the sizes of particles do not vary by one order of magnitude. Even though this is the common case with the materials analyzed here, if such conditions are not fulfilled, a more precise calculation should be used to estimate the angle \( \omega_j \) for each particle.
3.2.2. Stiffness relationship

A numerical simplification is also used to consider the binder-binder stiffness behavior. Fig. 11 presents in red this contact, expressed by the relation between the distance (d) and the equivalent stiffness (k_{eq}) estimated with the Kelvin-Voigt rheology model (f(E_n, \gamma_n, t)). This rheology model simulates the viscoelastic behavior of the binder by considering a parallel combination of the linear elastic spring and the linear viscous dashpot.

Fig. 11. Kelvin-Voigt rheology model for binder interaction.

It is assumed that no bending exists in this type of parallel arrangement. Consequently, the strain experienced by the spring is the same as that experienced by the dashpot. For each step, the strain (\varepsilon) is calculated with Eq. 11 that relates the overlap (\delta) and the sum of the radius of the particles (R_{p,i} + R_{p,j}). The total stress (\sigma) applied in the Kelvin-Voigt model due to the overlap is defined by Eq. 12, where E_n represents the elastic modulus of the spring, \gamma_n represents the viscosity of the dashpot, \xi represents the load time and \dot{\xi} is the relative velocity between the particles in interaction. In this sense, when a dashpot is used to simulate an interaction, the total stress is an approximation considering a dynamic solution. Parameters E_n and \gamma_n should be assessed in specific tests with the binder used.

\[
\varepsilon(t) = \frac{\delta}{(R_{p,i} + R_{p,j})} \tag{11}
\]

\[
\sigma = E_n \cdot \varepsilon(t) + \gamma_n \cdot \dot{\xi}(t) \tag{12}
\]
The stiffness in terms of forces and displacement for each bar is calculated through Eq. 13, considering the stress ($\sigma$), the strain ($\varepsilon$), the effective contact area ($A_{ef,b}$) and the distance between the center of the particles ($d$). This rheology model may be adjusted depending on experimental analysis. It is important to remark that the same model or a different one could be used to represent the behavior under tensile stress. If a change occurs in the verification function (section 2.2.3), the modified matrix has to be recalculated.

\[ k_b = \frac{A_{ef,b} \cdot \sigma}{d \cdot \varepsilon(t)} = \frac{(\frac{d}{2})^2 \cdot \pi \cdot \sigma}{d \cdot \varepsilon(t)} \]  

(13)

Notice that, in Eq. 13, the contact area ($A_{ef,b}$) is estimated assuming a circle with the diameter equal to the contact length ($l_c$) obtained according with section 3.2.1. In other words, the stiffness is related with the square of the contact length. However, in a purely 2D model it would seem more reasonable to use directly the contact length instead.

To understand why this was not assumed in the present study, consider a simplified example in which two spherical particles in a 3D medium are put in contact. Consider also a second set of spherical particles that are also put in contact but have half the diameter of the previous ones. If analogous conditions are assumed, it is evident that the contact area of the first set will be bigger than that of the second set. Since more area means more points to transmit forces, the total force transmitted in the interaction between particles will increase with the area. Consequently, the total force will be related with the square of the contact length, increasing with the square of the diameter of the particle. Now consider the same example in a 2D medium. In this case, the capacity to generate forces will be linearly related with the contact length and, hence, with the diameter of the particle.
This produces a contradiction once a model conceived purely to work in a 2D condition might not be representative of a 3D condition if different sizes of particles are simulated together. Such incongruence might be resolved by modifying the way the stiffness is calculated in 2D. In order to obtain representative results that could be used to validate the method proposed, the stiffness should be calculated from the area of contact as shown in Eq. 13 since this reflects the situation found in practice.

The rheology models used to simulate the contact between particles have a time- and load-dependent behavior. Consequently, the results obtained with the method become highly dependent on the compaction procedure applied. Therefore, to simulate the compaction process properly, it is necessary to define a loading procedure that approaches the expected in practice. Moreover, it is also necessary to define sufficiently small time steps that assure the convergence and the consistency of the steps. The maximum time step possible will depend on the size of the model, the complexity of the interactions between particles and the rheology of the contact.

3.3. No contact simulation (virtual contact)

The last case (Fig. 8.d) denotes a no contact situation that happens if the distance between particles $i$ and $j$ is bigger than or equal to the sum of their radius ($d \geq R_{p,i} + R_{p,j}$). The bars generated in this situation are only virtual and do not represent contact between the inner cores or the binders. In fact, they only contribute to the global stability of the model. Without this consideration, many analyses would not converge. In Fig. 4f, bars that represent real contact are indicated in blue. With only this consideration, the lattice layout obtained is statically unstable. In fact, a mechanism is formed and it is not possible to solve the equations for the balance of forces within each time step. With the virtual contact consideration (green bars), the lattice layout become
stable and can be solved numerically. To minimize the interference of these elements in the results, the virtual bars should have a stiffness several times lower than the bars that represent real contacts. The low stiffness consideration in the virtual contacts leads to axial forces several times lower than the others types of contact. As a result the bars that represent virtual contacts may present high deformation and may be related to the big relative displacement observed during compaction and to the rotation of one particles over the others. Also, during the compaction process tensile stress may surge and, for simplification, it is assumed that the bars that denote contacts subjected to tensile stress present no stiffness, being modeled as a ‘no contact’ material.

4. RESULTS

In order to evaluate the applicability of the model proposed, several analyses were conducted. The simulation considers a rigid inner core involved by a dry binder layer. Results were obtained from the average of 50 simulation with similar characteristics. Such a high number of simulation were required to assure a representative assessment since the randomness of the aggregate grading and distribution produce small variation in the results.

A random law was used to generate the particle with different sizes. According to this, the diameter of particles ranged from 2.5 mm to 10 mm (10 % of 10 mm, 30 % of 7.5 mm, 40 % of 5 mm and 20 % of 2.5 mm). The mix consisted of 54% by volume of inner core and 19.5% by volume of binder, which led to an initial volumetric porosity of 26.5%. The thickness of the binder layer is considered proportional to the diameter of the inner core.

A sample of 60 x 60 mm supported laterally and at the bottom by the walls of a container was subjected to a compaction of 10 mm (compaction degree of 16.67 %) applied at a rate of 100 mm/min to all particles located at the upper boundary. The
elastic modulus used for the spring and the viscosity of the dashpot in compression were 2x10^{-4} \text{ MPa}, and 1.74x10^{-3} \text{ MPa.s}, respectively.

Some parameters related to the compaction process should be defined beforehand for a better understanding of the results. The compaction degree \( (\mathcal{P}) \) may be represented by the ratio between the initial density and the density after a certain compaction, as shown in Eq. 14. In a bi-dimensional model, the density of the material is calculated through the division of its mass \( (M) \) and the volume occupied, which depends on the base length \( (b) \) of the recipient and the height of the sample \( (h) \). During the compaction, the variation of height relative to the initial condition is expressed by \( \Delta h \).

\[
\text{Compaction Degree} = \frac{\rho_i}{\rho_f} = \frac{\frac{M}{b \cdot (h - \Delta h)}}{\frac{M}{b \cdot h}} = 1 - \frac{\Delta h}{h}
\]  

(14)

The compactness and the density of the sample were calculated through Eq. 15 and 16, respectively. In these equations, the only parameter that changes along the course of the compaction simulation is the total compacted thickness \( (\Delta h) \). The area occupied by the inner core \( (A_{ic}) \), the area occupied by the binder \( (A_b) \), the density of the inner core \( (\rho_{ic}) \), the density of the binder \( (\rho_b) \), the sample initial length \( (b) \) and the sample initial height \( (h) \) are constant.

\[
\text{Compactness} = \frac{A_{ic} + A_b}{b \cdot (h - \Delta h)}
\]  

(15)

\[
\text{Density} = \frac{1}{b \cdot (h - \Delta h)} \cdot (A_{ic} \cdot \rho_{ic} + A_b \cdot \rho_b)
\]  

(16)

4.1 Types of Contact
An assessment of the different types of contact observed throughout the compaction process is performed. According with Pöschel and Schwager [35], the increase of the compaction degree should lead to an increase in the contact between particles. Consequently, an increase of the contact between inner cores and in the contact between the binders and a decrease of ‘no contact’ should happen.

Fig. 12.a presents an area chart with the average proportion of the different types of contact depending on the compaction degree. As expected, in the initial step (compaction degree equal 0 %) almost 30 % of the bars were simulating contact between binders and the remaining 70% comprised bars with ‘no contact’. However, in the final step (compaction degree equal 16.7 %) of this simulation, the ‘no contact’ decreased to 25 %, whereas the contact between binders increased to 72 %. The remaining 3 % were simulating inner core contact.

Fig. 12. Types of contact depending on the compaction degree.

Fig. 12.b and 12.c show the types of contact between particles for the compaction degrees of 1.67% and 16.67%, respectively. Again, it is evident that the increase in the compaction leads to the reduction on the number of green bars that represent a ‘no contact’. On the contrary, the increase in the number of blue and red bars is observed.

4.2. Support reactions

Another result to be considered is the support reaction on the walls and base of the container. When a pressure is applied on the top of a granular material, the reaction on the walls depends on the height and compactness of the sample. On the other hand, the reaction on the base of the container equals the applied forces. In fact, the reaction in the
base (vertical) increases with the compaction pressures, whereas the wall reaction (horizontal) should increase with less intensity.

Fig. 13 presents two figures that illustrate this situation. The first of them (Fig. 13.a) shows the ratio between the horizontal and the vertical reactions for different compaction degrees of all simulations. It is observed that the adjusted curve presents a negative slope, indicating that the average ratio goes from 0.69 to 0.57, approximately. This means that the sum of horizontal reaction is between 35% and 50% smaller than the sum of vertical reaction. It is worth mentioning that the ratio calculated between the horizontal and the vertical reactions is analogous to the coefficient of lateral pressure at rest ($K_0$) of a soil. Even though the material compacted may present special characteristics, the values obtained here are close to the typical $K_0$ considered for loose granular soil.

Fig. 13. Relation between the horizontal and vertical pressure with the compaction.

Fig. 13.b correlates the vertical and horizontal reaction pressure with the compaction degree for one of the simulations performed. This figure helps to understand the behavior observed in Fig. 13.a. It shows that both pressures increase with the compaction degree. However, the rate of increase is bigger in the vertical direction due to a better connection between particles that favors the transmission of vertical loads.

5. NUMERICAL COMPARISON

Considering the lack of studies on the simulation of the compaction of granular media formed by particle involved by a binder, the results from Frenning [18] are used to
validate the model proposed here. Even though binder is not present, the simulations
performed by the author using a coupled Finite/Discrete Element Model assume the
contact of deformable 3D particles. In theory, this deformation could be represented by
the binder behavior considered in the model proposed here.
To calibrate the properties of the interaction, first a simulation of the contact of two
particles is performed just like the described by Frenning [18] (see Fig. 14.a). The two
particles are spheres with diameters equal to 1 mm, density of 1.5 g/cm$^3$, Young’s
modulus of 250 kPa and zero Poisson’s ratio. They were enclosed in a rectangular
container that restrained the lateral motion while a relative displacement of 0.01 mm/ms
is applied (small enough for the particles never get out of equilibrium). Time steps of
$10^{-5}$ s were used to assure the convergence of the iterations. Fig. 14.b shows the force
applied and the total relative displacement. It is important to remark that all simulations
with the model developed here were performed in equivalent 2D configuration.

Fig. 14. Representation (a) and behavior (b) of the contact between two particles

The results from the simplified model fit well the obtained from the literature and shows
a correlation coefficient (R) of 0.974. For that, the damper without coefficient and
elastic modulus of 0.08 MPa were used as input parameters for the Kelvin-Voigt model.
Once the contact parameters were calibrated, the analysis of a granular media was
performed. Such analysis was originally conducted by Frenning [18] for a granular
media composed by spherical particles randomly disposed in a cubic container of 17.3
mm of side. The grading of the particles considered by the author is summarized in Tab.
1.
Tab. 1. Grading and properties of the particles simulated by Frenning [18]

Fig. 15 presents the results of the compaction simulation for the granular media. A very good fit is obtained between the simplified model and the model by Frenning [18]. The comparison of the results for equivalent displacements shows a correlation coefficient (R) of 0.99. Notice that the results of Frenning [18] were validated with the results from Hassanpour [36]. The latter used a DEM with the contact law based on the model of Herzt for initial contact. When the force exceeds the yield value, the contact law was changed to the model of Thornton and Ning [37]. Therefore, the new approach developed here seems to reproduce satisfactorily the results from other more complex models.

Fig. 15. Relation between density and compressive pressure.

6. EXPERIMENTAL COMPARISON

The experimental comparison is performed with a special type of concrete known as pervious concrete, which is usually obtained by eliminating the finer fraction of sand from the composition and by reducing the water-to-cement ratio until a very dry paste is achieved. As a result, the aggregates become involved by the cement paste that has a sufficiently high viscosity not to flow (see Fig. 16.a). The material obtained after compaction is porous with interconnected macro voids that provide good thermal and acoustic insulation, as well as, very high permeability (see Fig. 16.b).

Fig. 16. Isolated particle (a), pervious concrete in hardened state (b) and test setup (c, d and e).
To evaluate the compaction process of pervious concrete, the testing equipment shown in Fig. 16.c, 16.d and 16.e was used. It is composed by an external support frame that holds a piston that moves vertically. A recipient is placed just below the piston with the material to be tested. The piston is positioned over the sample and the load is manually increase by placing known weights at a plate located on the opposite side of the piston. After a weight is placed, enough time is waited for the displacement to stabilize before additional load is applied. This is necessary given that the cement paste surrounding the aggregates has a viscoelastic behavior that induces a time dependent response. During the whole procedure, 4 displacement sensors measure the movement of the piston, which is used to assess the porosity and the compactness of the sample throughout the test.

Total loads of up to 80 kg were reached, leading to compactions of 6.5%. Higher load values were not applied for safety reasons and due to difficulties to keep the piston aligned with the supports. In the experimental program, two typical compositions of pervious concrete were tested with different cement contents and the same water-to-cement ratio (w/c) of 0.35. This way, the rheology of the cement paste is maintained while the thickness of the layer surrounding the aggregate is modified. Consequently, if the model proposed is working properly, the same input parameter should allow the simulation of the compaction curve of both compositions.

The composition PC0.25 included 1400 kg of limestone aggregate (with specific density of 2.6 g/cm³) and 300 kg of cement (with specific density of 3.1 g/cm³), whereas the PC0.30 had 1400 kg of the same limestone aggregate and 350 kg of cement. The equivalent thickness of the cement paste layer in the compositions PC0.25 and PC0.30
are approximately 0.25 times and 0.30 times the radius of the aggregate, respectively. For each composition, a minimum of 8 samples were tested. The recipient used in the test was always filled to the same initial height and the weight of the sample was measured before starting the test. With those measurements, the density and the proportion of the components, it is possible to estimate the initial density, the initial porosity and the variation of the compaction degree during the procedure.

Fig. 17.a presents a typical curve that relates the displacement and the time for one of the tests. In this curve, two phases are systematically observed. When the weight is applied, a fast displacement occurs (see Fig. 17.b). After that, additional displacement is observed at much smaller rates, which decreases with time due to the viscous behavior of the contact between the cement paste layers. Fig. 17.c and 17.d show the curves that relates the compaction stress and the compaction strain measured during the tests of the pervious concrete PC0.25 and PC0.30, respectively. In both of them, a red line indicates the average result obtained. The maximum compaction reached in PC0.25 is approximately 3.5%, whereas in PC0.30 it was approximately 6.0%. This difference is attributed to the bigger cement paste content in PC0.30, which induces a thicker binder layer and higher deformability.

Fig. 17. Typical displacement-time curve (a and b), experimental and numerical curves for pervious concrete PC0.25 (c) and PC0.30 (d).

The average experimental results was used to validate the model developed here. The grading curve measured in the laboratory for the aggregate was used to define the proportion and the sizes of the particles introduced in the model. Based on that, the total
volume of aggregate was divided in 28% of particles with 5.0 mm, 40% of particles with 6.3 mm and 32% of the particles with 8.0 mm. The initial porosity assessed in the laboratory is taken as a reference for the definition of the initial particle distribution in the Stage 1, according with section 2.1.

The elastic modulus ($E_r$) and the viscosity ($\eta_r$) of the cement paste were assumed 50 MPa and 25 MPa$\cdot$s, respectively. The surficial stiffness ($k_z$) of the aggregate was set to 65 MPa/mm. The stiffness of the bars that represent the situation of no contact or virtual contact according with section 3.3 was set to approximately $2\times10^{-3}$ N/mm. The same parameters are used for the simulation of both pervious concrete tested, despite their differences in terms of the thickness of the cement past layer. The rate of displacement applied during the test was approximately the same as during the test. Time steps of $10^{-5}$ s were used in all analyses.

Fig. 17.c and Fig. 17.d show compaction curve (in blue) obtained in the simulations of the test with PC0.25 and PC0.30. Despite considering the same input parameters, a good agreement is obtained for the whole extent of the average experimental curves. This confirms that the model is capable of reproducing the compaction curves and the repercussions of changes in the composition of the pervious concrete.

A verification of the accuracy of the rearrangement of the particles in the numerical model during compaction was not possible in the present study due to the difficulty to follow the path of the particles during the experiment. However, such rearrangement may be related with other properties, like the evolution of the compaction degree or loss of porosity for a certain applied load. Even though these are macro scale properties, they are the most useful ones from the practical point of view. Notice that a good agreement is obtained between experimental and numerical results considering the same input parameters and despite the variation of the paste content. This suggests that the method
developed should be capable of representing the macro-scale behavior of the material expected in reality and possibly the rearrangement of the particles during the compaction process.

7. CONCLUSIONS

In this paper, a method for predicting the geometrical disposal of a granular media submitted to compaction was proposed, using a Lattice model. The method yields reasonable results and is suitable for a wide range of applications, potentially requiring less computation effort than some of the existing methods from the literature. The change of the element type of the bars and the modification of the boundary conditions along the course of the steps represent well the movements of the particles. Furthermore, the proposal of an evolutionary lattice system that is redefined in each time step and admits internal instability has proven to allow the intensive rearrangement of particles expected in a poorly compacted medium. Finally, this method presents a possibility to correlate the compaction pressure with the porosity, density and the compacted thickness. The results from the simulation of a deformable granular medium show a good agreement with those obtained in the literature with more complex models (DEM or in combined FEM/DEM). Likewise, the numerical results estimated with the model developed here agree with those assessed during the experimental compaction process of two compositions of pervious concrete. Once again, this suggests that this method is capable of reproducing the behavior of a deformable granular medium subjected to compaction. In fact, it provides an alternative approach and a useful tool to the prediction of aspects such as the optimum compositions of the materials or the possible repercussions of modifications in the compaction process. Despite the good results,
more studies are required to assess the input parameters related with the contact
between two particles by means of specific tests. This way it would be possible to
estimate the stiffness of the spring and the viscosity of the dashpot more accurately.

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REFERENCES

[1] Cundall PA, Strack ODL. BALL – A program to model granular medium using the
[4] He H, Guo Z, Stroeven P, Stroeven M, Sluys LJ. Characterization of the packing of
[6] Chen J. Discrete element method (DEM) analyses for hot-mix asphalt (HMA)


Fig. 1. Steps to find a stable position for each particle in Stage 1 (free fall)
Fig. 2. Deformation of the particle represented as an overlap.
Fig. 3. Algorithm of Stage 2 (Compaction).
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Fig. 8. Study particle (a) and situation of inner core contact (b), binder contact (c) and no contact (d).
Fig. 9. Effective interaction area of particles in contact.
Fig. 10. Binder redistribution after overlap
Fig. 11. Kelvin-Voigt rheology model for binder interaction.
Fig. 12. Types of contact depending on the compaction degree.

Label:
- Particle
- Inner core contact
- Binder contact
- No contact

(a) Curve % of contact along the compaction
(b) Compaction degree: 1.67 %
(c) Compaction degree: 16.67 %
Fig. 13. Relation between the horizontal and vertical pressure with the compaction.
Fig. 14. Representation (a) and behavior (b) of the contact between two particles
Fig. 15. Relation between density and compressive pressure.
Fig. 16. Isolated particle (a), pervious concrete in hardened state (b) and test setup (c, d and e).
Fig. 17. Typical displacement-time curve (a and b), experimental and numerical curves for pervious concrete PC0.25 (c) and PC0.30 (d).
Tab. 1. Grading and properties of the particles simulated by Frenning [17]

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<thead>
<tr>
<th>Diameter (mm)</th>
<th>Amount (%)</th>
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</tr>
<tr>
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