gLAB upgrade with BeiDou Navigation System Signals

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ABSTRACT

Global navigation satellite system (GNSS) consist of a constellation of satellites orbiting the earth. Through continuously transmitting signals, it is able to provide real time 3D-position, velocity and time for the users on the earth. It has been used in many fields, as transportation, meteorology, disaster prevention and mitigation. GNSS is an indispensable part of our life. BeiDou navigation satellite system is developed by China and it is one of four GNSS in the world. The capability to send and receive short messages is a unique feature of BeiDou system. BeiDou involve three types of satellites: GEO, IGSO and MEO. It develops fast, and now it can provide region service and is expect to have full operation capability in 2020.

gLAB is a software tool for GNSS data processing. It was developed by the Research group of Astronomy and Geomatics (gAGE) at the Technical University of Catalonia under contract of the European Space Agency (ESA). The initial version of gLAB can fully process GPS data and a partial handing of GLONASS, Galileo, SBAS, GEO data, but it didn’t include BeiDou module. With fast development of BeiDou navigation satellite system, it is extremely essential to add the function to gLAB to process BeiDou data.

The purpose of this project was to develop a BeiDou data processing module in gLAB. With this upgrade, gLAB has the capability to read and process BeiDou signal. The performance of BeiDou navigation satellite system has been assessed in different conditions. As with GPS, we find that the NEU error is affected by different factors. As latitude, number of visible satellites, DOP, etc. The IONEX model correction is better than Klobuchar model. Compared with GPS performances, BeiDou performance is slightly worse than GPS, but it meets the specified requirement of BeiDou interface control document.
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摘要

全球导航卫星系统用来为全球用户提供实时位置、速度、时间和导航服务，现已应用于多个领域，如交通、气象、防灾减灾等，与我们的生活息息相关。其中北斗导航卫星系统是由中国开发的，为四大全球导航卫星系统之一。目前，北斗导航卫星系统已经可以提供亚太地区的定位、授时服务，预计2020年将可以提供全球服务。北斗由地球同步卫星、倾斜轨道地球同步卫星及中轨道卫星三种卫星组成，而其他全球导航卫星系统只包含中轨道卫星，且比其他全球导航卫星系统增加了接收和发送短消息的功能。

gLAB是加泰罗尼亚理工大学gAGE小组开发的一款软件，用于处理全球导航卫星系统数据。现有版本可以处理GPS数据，并可以读取并存储GLONASS，Galileo的数据。但不包括北斗模块，因此尚不能读取和处理北斗数据。但随着北斗导航系统的快速发展，对该部分数据的处理十分必要，因此本文在已有的gLAB程序基础上，进行了北斗数据处理模块的开发。

本文升级了gLAB，增加了北斗模块，使其能够读取、存储并处理北斗导航卫星系统的数据，实现标准动态定位，并达到了北斗界面控制文件（ICD）所提出的精度要求。通过比较在不同条件下，使用北斗信号计算的位置精度，可以发现影响位置精度的因素有很多，包括所处纬度，可见卫星数目，DOP值等。通过对比不同的电离层模型对北斗信号的修正作用，发现IONEX电离层模型修正效果比Klobuchar电离层模型。结果发现，在同一位置且使用相同的电离层模型的条件下，北斗精度略微低于GPS精度，但亦满足了北斗界面控制文件（ICD）文件所规定的精度。
Resum

El Sistema Global de Navegació per Satèl·lit (GNSS) consisteix en una constel·lacions de satèl·lits que orbiten la Terra. A través de senyals que transmeten contínuament, és capaç de proporcionar posicionament 3D en temps real, velocitat i temps per als usuaris sobre la Terra. S'ha utilitzat en molts camps, com el transport, la meteorologia, la prevenció i mitigació de desastres. GNSS és una part indispensable de la nostra vida. El Sistema de navegació per satèl·lit BeiDou està sent desenvolupat per la Xina i és un dels quatre GNSS en el món. La capacitat per enviar i rebre missatges curts és una característica única del sistema BeiDou. BeiDou implica tres tipus de satèl·lits: GEO, IGSO i MEO. S'està desenvolupant ràpidament, i actualment pot oferir servei regional. Està previst tenir la capacitat plena operació en 2020.

gLAB és una eina per al processament de dades GNSS. Va ser desenvolupat pel grup d'Investigació d'Astronomia i Geomàtica (gAGE) de la Universitat Politècnica de Catalunya sota contracte de Agència Espacial Europea (ESA). La versió inicial de gLAB permet fer un processat complert de les dades GPS i un tractament parcial de GLONASS, Galileo, SBAS, dades GEO, però no inclou cap mòdul per BeiDou. Amb el ràpid desenvolupament del sistema de navegació per satèl·lit BeiDou, és molt necessari afegir una nova funció a gLAB per processar dades de BeiDou.

El propòsit d'aquest projecte és desenvolupar un mòdul de processament de dades de BeiDou dins de gLAB. Amb aquesta actualització, gLAB té la capacitat de llegir i processar el senyal BeiDou. El rendiment del sistema de navegació per satèl·lit BeiDou s'ha avaluat en diferents condictions. Igual que amb el GPS, ens trobem amb que l'error NEU es veu afectat per diferents factors, com la latitud, el nombre de satèl·lits visibles, DOP, etc. La correcció del model IONEX és millor que la del model de Klobuchar. En comparació amb el rendiment de GPS, BeiDou és lleugerament pitjor que el GPS, però compleix amb els requisits específics del document de control d'interfície BeiDou.
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INTRODUCTION

Global satellite navigation systems provide real-time position, velocity, and time for users with global coverage. GNSS application has successfully extensive used in the area of civil aviation, ground transportation, seismic survey, and geodetic survey etc. They have made a significant contribution to the development of economy and the convenience of our life.

There are four Global satellite navigation systems in the world. BeiDou navigation satellite system is being developed by China. It is an important member of GNSS community. It has begun providing region navigation service since 2011 and expected to provide global navigation service in 2020.

The aim of this master degree final project is to upgrade gLAB with necessary functionality to make it to obtain user’s solution with BeiDou navigation satellite system signals.

The current version of gLAB can process GPS data with High Accuracy Positioning capability, at the centimeter level, and only a very limited data handling of Galileo and GLONASS capability. The Chinese Global Satellite Navigation System BeiDou signal can't be handled and processed. So now we need add a BeiDou data processing module.

The basic algorithms of GPS and BeiDou are similar, then they can share part of gLAB functions. Nevertheless there are several important differences between them. BeiDou has MEO, IGSO and GEO three kinds of orbits while GPS only has MEO. They use different time system, reference frame, so they use different values for some constant values. To upgrade it to read and process BeiDou data, we need add BeiDou parameters in data handing module. Implement algorithm for different part for BeiDou system. With the upgrade, it can read and process BeiDou data properly.

The first chapter is a brief introduction to all Global Navigation Satellite System. Structure of GNSS and development and constellations of each satellite systems are introduced.

In the second chapter, the development and current status of BeiDou Navigation Satellite System are discussed in detail. It will introduce space segment, ground stations, and user segment. BeiDou signals, the current state and the ionosphere model are also introduced in this chapter.

In the third chapter is a description of gLAB. It introduces gLAB’s structure and each part of the functions. It also shows the upgrade of gLAB for processing BeiDou data and explains in detail the functions implemented.

The forth chapter give simple concepts about time reference and coordinate frame, as well as the BeiDou time system and BeiDou coordinate frame. The fifth chapter is devoted to introduce satellite orbits elements and the computation of coordinates from broadcast message according to the BeiDou interface control document.
In the sixth chapter, the main modeling for BeiDou measurement is presented. The main algorithms and the effect of each model are discussed. Most of them are the same as in GPS except the reference to the TGD error.

The seventh chapter introduced several methods to solve the navigation equations. The Least Squares is the most common way to solve linear equations.

The eighth chapter shows the performance of solution with BeiDou data, and compares BeiDou and GPS in different cases.

The last part is a conclusion of this work. It summarizes work of the project and suggests future work.

At last, an appendix provides all data website used in the project and some useful information links.
Chapter 1 GNSS Introduction

1.1. Global Navigation Satellite System

A Global Navigation Satellite System is used to provide global continuous real time positioning, velocity and time service over global with a constellation of satellites.

They mainly consist of space segment, user segment and the ground control segment. As shown in Figure 1.1.

![Figure 1.1 GNSS architecture](image)

**Space segment:**

Space segment consist of satellite constellation. The main functions of the space segment are to generate and transmit code and carrier phase signals, and to store and broadcast the navigation message uploaded by the control segment. These transmissions are controlled by highly stable atomic clocks onboard the satellites.

The satellite constellation with enough satellites can ensure that users will have at least four satellites in view simultaneously from any point on the earth’s surface at any time.

**Ground control segment**

The ground control segment is responsible for the GNSS operation and control. It consists of the Master Control Station (MCS), Time Synchronization/Upload Stations (TS/US) and Monitor Stations (MS). The basic functions are:

- To control and maintain the status and configuration of the satellite constellation
- To predict ephemeris and satellite clock evolution
- To keep the corresponding GNSS time scale
- To update the navigation message for all the satellites.

**User segment:**

The user segment is composed of GNSS receivers. It is used to receive GNSS signals, determine pseudorange and solve the navigation equations in order to obtain the coordinates and provide very accurate time. It can be a smartphone, the equipment in a car or in a ship.

For different GNSS system, the user segment mostly has same requirement. Some receivers are compatible across different systems.

### 1.2. The current global navigation systems

Currently, there are four global navigation satellite systems in the world. They are GPS, GLONASS, Galileo and BeiDou. Also there are several regional navigation systems.

**GPS:** It is developed by United States. It is the first GNSS in the world, and it began works in 1983, and achieved full operational capability in 1995. Now it is the most Maturity global navigation system in the world.

![GPS satellite constellation](image)
The GPS satellites are arranged in six equally spaced orbital planes surrounding Earth, with four satellites in each plane. These orbital planes are separated by 60° right ascension of the ascending node (angle along the equator from a reference point to the orbit's intersection). This constellation structure can ensure that there are at least four satellites in view from any point on the earth. The satellites are placed in a Medium Earth Orbit (MEO) orbit, at an altitude of 20 200 km and an inclination of 55° relative to the equator. Orbits are nearly circular, with an eccentricity of less than 0.02, a semi-major axis of 26560 km and a nominal period of 11h58m2s, repeating the ground track each sidereal day.

**GLONASS:** It is developed by Russia Aerospace Defense Forces. It began provide service in 1983, having full operational in 2011.

![Figure 1.3 GLONASS satellite constellation](image)

The nominal GLONASS constellation consists of 24 MEO satellites deployed in three orbital planes with eight satellites equally spaced in each plane. The orbits are roughly circular, with an inclination of about 64.8°, and at an altitude of 19100 km with a nominal period of 11 hours, 15 minutes and 44 seconds, repeating the geometry every eight sidereal days. Due to funding problems, the number of satellites decreased from the 24 available in 1996 to only 6 in 2001. In August 2001, the Russian government committed to recover the constellation and to modernize the system. A total of 24 operational satellites plus 2 in maintenance were again available in December 2011, restoring the full constellation.

**Galileo:** It is developed by European Space Agency, it is quite new and still in developing.

The fully deployed Galileo system will consist of 30 satellites (27 operational + 3 active spares), positioned in three circular Medium Earth Orbit (MEO) planes at 23222 km altitude above the Earth. Ten satellites will occupy each of three orbital plane inclined at an angle of 56° with respect to the equator, with an orbital period 14h 4min 45s, repeating the geometry each 10 sidereal days. This constellation guarantees, under nominal operation, a minimum of six satellites in view from any
point on Earth’s surface at any time, with an elevation above the horizon of more than 10°.[1]

![Figure 1.4 Galileo satellite constellation](image)

**BeiDou**: The Chinese navigation satellite system, more details about BeiDou are in Chapter 2.
Chapter 2 BeiDou Navigation Satellite System

BeiDou Navigation Satellite System is called BeiDou System for short, with the abbreviation as BDS. It is developed by China. It began providing regional service from 2011 and is expected to provide global positioning with full operational capability in 2020. The BDS can provide highly reliable and precise positioning, navigation and timing (PNT) services as well as short-message with 120 Chinese characters communication for all users under all-weather, all-time, and worldwide conditions.

2.1. The development strategy of BeiDou

The development of BeiDou navigation satellite system follows “three step” strategy as it is shown in Figure 2.1. They are “the demonstration system”, “the regional system”, and “the global system”.[2]

![Figure 2.1 BeiDou satellite navigation system develop process](image)

From 2000 to 2012 is the stage of “Demonstration System”. It is the first stage of BeiDou system development. From the research to provides the radio determination satellite service (RDSS) and the short message services for users in China and the surrounding area. The satellite constellation consists of three geostationary (GEO) satellites located at 80°E, 110.5°E and 140°E, respectively.

The Regional System: From 2012 to 2020, the radio navigation satellite service (RNSS) will be increased from the basis of BeiDou Demonstration System to possess regional navigation ability. It is the current status of BeiDou system.
The Global System: After 2020, it will be in full operation status. The number of MEO satellites will increase to 27 and work in the Walker 24/3/1 constellation. The positioning accuracy will be superior to 10m and the timing accuracy to 20 ns.

The Table 2.1 shows the current BeiDou constellation satellite information. We can see 16 satellites have been sent and two of them were discarded. So there are 14 satellites in orbit providing service. It is developed very fast. It shows that after one GEO and one MEO satellite were launched respectively in 2007 and 2009, the implementation became speeded up, and five satellites (three GEO and two IGSO) were launched in 2010, three IGSO satellites in 2011, and six satellites (two GEO and four MEO) in 2012. The BDS satellites were designed to be compatible with the demonstration system.

**Table 2.1 Satellite of the current BeiDou constellation**

<table>
<thead>
<tr>
<th>Satellite</th>
<th>PRN</th>
<th>Launch Date</th>
<th>Int. Sat. ID</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>C01</td>
<td>2010.01.17</td>
<td>2010-001A</td>
<td>140.0° E</td>
</tr>
<tr>
<td>G2</td>
<td>n/a</td>
<td>2009.04.15</td>
<td>2009-018A</td>
<td>Inactive; uncontrolled</td>
</tr>
<tr>
<td>G3</td>
<td>C03</td>
<td>2010.06.02</td>
<td>2010-024A</td>
<td>110.5° (moved from 84.0° to new position between Nov 7 and 22, 2012)</td>
</tr>
<tr>
<td>G4</td>
<td>C04</td>
<td>2010.11.01</td>
<td>2010-057A</td>
<td>160.0° E</td>
</tr>
<tr>
<td>G5</td>
<td>C05</td>
<td>2012.02.25</td>
<td>2012-008A</td>
<td>58.75° E</td>
</tr>
<tr>
<td>G6</td>
<td>C02</td>
<td>2012.10.25</td>
<td>2012-059A</td>
<td>80.0° E</td>
</tr>
<tr>
<td>I1</td>
<td>C06</td>
<td>2010.08.01</td>
<td>2010-036A</td>
<td>~120° E</td>
</tr>
<tr>
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<td>C07</td>
<td>2010.12.18</td>
<td>2010-068A</td>
<td>~120° E</td>
</tr>
<tr>
<td>I3</td>
<td>C08</td>
<td>2011.04.10</td>
<td>2011-013A</td>
<td>~120° E</td>
</tr>
<tr>
<td>I4</td>
<td>C09</td>
<td>2011.07.27</td>
<td>2011-038A</td>
<td>~95° E</td>
</tr>
<tr>
<td>I5</td>
<td>C10</td>
<td>2011.12.02</td>
<td>2011-073A</td>
<td>~95° E</td>
</tr>
<tr>
<td>M1</td>
<td>C30</td>
<td>2007.04.14</td>
<td>2007-011A</td>
<td>Discarded</td>
</tr>
<tr>
<td>M3</td>
<td>C11</td>
<td>2012.04.30</td>
<td>2012-018A</td>
<td>55°</td>
</tr>
<tr>
<td>M4</td>
<td>C12</td>
<td>2012.04.30</td>
<td>2012-018B</td>
<td>55°</td>
</tr>
<tr>
<td>M5</td>
<td>C13</td>
<td>2012.09.19</td>
<td>2012-050A</td>
<td>55°</td>
</tr>
<tr>
<td>M6</td>
<td>C14</td>
<td>2012.09.19</td>
<td>2012-050B</td>
<td>55°</td>
</tr>
</tbody>
</table>
2.2. Space segment

As other Global navigation systems, BDS consists of the space segment, the ground control segment and the user segment.

At the current status, BeiDou navigation satellite system has 14 satellites in orbits. They are five Geostationary Earth Orbit (GEO) satellites, four Medium Earth Orbit (MEO) satellites in two orbit planes and five Inclined Geosynchronous Satellite Orbit (IGSO) satellites in two orbit planes. As shown in Figure 2.2.

![Figure 2.2 The current status of BeiDou navigation satellite system](image)

The GEO satellites are operating in orbit with an altitude of 35,786 kilometers and positioned at 58.75°E, 80°E, 110.5°E, 140°E and 160°E respectively. The special purpose of GEO satellite is to provide the SMS function as BeiDou-1.

The IGSO satellites are in the orbit with an altitude of 35,786 kilometers and an inclination of 55° to the equatorial plane. The phase difference of right ascensions of ascending nodes of those orbital planes is 120°. The sub-satellite tracks for three of those IGSO satellites are coincided while the longitude of the intersection point is at 118°E. The sub-satellite tracks for the other two IGSO satellites are coincided while the longitude of the intersection point is at 95°E. The five IGSO satellites are intended to reduce the “urban canyon” effect, because satellite signals are degraded or lost in dense urban areas.[3]

The MEO satellites are operating in orbit with an altitude of 21,528 kilometers and an inclination of 55° to the equatorial plane. The current 4 MEO satellites are in the 7th and 8th phases of the first orbital plane, and in the 3rd and 4th phases of the second orbital plane respectively. The satellite recursion period is 13 rotations within 7 days.
The phase is selected from the Walker24/3/1 constellation, and the right ascension of ascending node of the satellites in the first orbital plane is 0°.

BeiDou system is scheduled to attain full operation capability in 2020. When fully deployed, the space constellation of BDS will consist of 5 GEO satellites, 27 MEO satellites and 3 IGSO satellites. The status of BeiDou with full operation capability is shown in Figure 2.3.

![Figure 2.3 The status of BeiDou satellite system with full operational capability](image)

### Compare with GPS Space Segment

- GPS only have MEO satellite, while BDS has GEO, IGSO and MEO satellites.
- For providing global navigation service, GPS need 24 MEO satellites, while BeiDou need 35 satellite, they are 5 GEO, 3 IGSO and 27 MEO satellites.
- The orbits of BeiDou MEO satellites are higher than GPS, so the satellite period is 12h53min while GPS satellite period is 11h58m2s.

### 2.3. Ground Control Segment

The ground control segment is design for the BDS operation and control. In June 2011, the system has completed the ground segment commissioning, including the test section of the user terminal development. It consists of the Master Control Station (MCS), Time Synchronization/Upload Stations (TS/US) and Monitor Stations (MS).

#### 2.3.1. Master Control Station (MCS)

The BeiDou Master Control station is located in Beijing, and it is the operation and control center of BDS with the main tasks including:
a) To collect observation data of NAV signals from each TS/US and MS, to process data, to generate satellite NAV messages;

b) To perform mission planning and scheduling, to conduct system operations, management and control;

c) To observe and calculate the satellite clock bias, to upload satellite NAV messages;

d) To monitor the satellite payload and analyze anomalies, etc.

2.3.2. Time Synchronization/Upload Stations (TS/US)

The main tasks of TS/US are time synchronizations of satellite clocks, the navigation message uploading and the data exchange with MCS. [4]

BeiDou is planning to establish three time Synchronization/Upload Stations in China, one is in Beijing, that will be the same location as the Master Control Station, and another two will be in Sanya and Kashgar. The Figure 2.4 shows the specific locations. It can cover the Chinese territory at the most extent. [5]

![Figure 2.4 TS/US station locations](image)
2.3.3. Monitor stations (MS)

There are 30 monitor stations for BDS. They are well distributed in China. The main tasks of MS are to continuously observe satellite navigation signals, to provide real-time data to the MCS, and provide measurements for orbit determination and wide-area differential information. The pseudorange is transmitted to master station in real time. [4]

There are some corporations between Russia’s Global Navigation Satellite System office and Chinese satellite navigation office to construction of monitoring stations in each other’s territory and share the monitor station to promote improved performance and integration of BeiDou and GLONASS. It can benefit both GLONASS and BeiDou, because BeiDou has a better performance at middle and low latitudes while GLONASS has a good performance at high latitudes. [6]

Also they have taken a test for the first BeiDou monitor station in Brazil RIOS station. It indicates that BeiDou will develop monitor stations around the world.[7]

2.4. User segment

The BeiDou user segment and applications are similar to other GNSSs for the open service. The particularity of BeiDou user segment is exchanging short messages between ground stations and users. It is very useful in emergency case; the holder will able to communicate with others even without cellphone signal in remote areas.

The BeiDou User Segment consists of BeiDou user terminals, like smartphones, vehicle navigator, which can receive BeiDou navigation signals, determine pseudorange (and other observables) and solve the navigation equations to obtain the user position, velocity and precise time (PVT). Also there are some terminal is portable navigator with short message communication function.

2.5. BeiDou signals

BeiDou satellites transmit right-hand circularly polarized signals centered on three radio frequencies in the L band, referred as: B1, B2 and B3. The frequency of B1 signal is 1561.098 MHz, the frequency of B2 signal is 1207.140 MHz, and the frequency of B3 signal is 1268.52 MHz.

All the signals are the sum of channel I and Q, which are in phase quadrature of each other. The ranging code and NAV message are modulated on carrier.

The signals are composed of the carrier frequency, ranging code and NAV message. It ranging signals are based on the CDMA technique as GPS and Galileo. The different navigation signals, structure and supported services are summarized in Table 2.2 The radio signal characteristics of the BeiDou Regional Satellite system.
Table 2.2 The radio signal characteristics of the BeiDou Regional Satellite system

<table>
<thead>
<tr>
<th>Band</th>
<th>Carrier frequency/MHz</th>
<th>PRN code</th>
<th>Chip rate/Mcps</th>
<th>Bandwidth/MHz</th>
<th>Modulation type</th>
<th>Service type</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1</td>
<td>1561.098</td>
<td>B1I</td>
<td>2.046</td>
<td>4.092</td>
<td>QPSK(2)</td>
<td>Open</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B1Q</td>
<td></td>
<td></td>
<td></td>
<td>Authorized</td>
</tr>
<tr>
<td>B2</td>
<td>1207.14</td>
<td>B2I</td>
<td>2.046</td>
<td>24</td>
<td>BPSK(2)</td>
<td>Open</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B2Q</td>
<td>10.23</td>
<td></td>
<td>BPSK(10)</td>
<td>Authorized</td>
</tr>
<tr>
<td>B3</td>
<td>1268.52</td>
<td>B3</td>
<td>10.23</td>
<td>24</td>
<td>QPSK(10)</td>
<td>authorized</td>
</tr>
</tbody>
</table>

As GPS, BeiDou system service has open service and authorized service:

◆ **Open Service**: The SPS (Standard Accuracy Signal Service) is an open service to all the users without charge. B1I and B2I are the public frequencies.

◆ **Authorized Service**: This service will ensure very reliable use, providing safer positioning, velocity and timing services, as well as system information, for authorized users. B1Q, B2Q, and B3 are only for authorized users.

2.6. BeiDou System Construction Principles

BeiDou system steady development means China is independent in positioning and navigation domains. Also BeiDou development boosts the growth of space. It is a reliable system, specially keeps the quality, safety, practical and benefits. During the process of construction and development, it stands by the following principles: [5]

A. **Open.** The construction, development and application of the system will be open. It will provide high quality service free of charge for all users in the world. China is ready to cooperate with other countries in developing satellite navigation projects, enhance the exchanges of the existing satellite navigation systems, and promote the development of satellite navigation technology and industry.

B. **Independent.** China constructs and operates the BeiDou Navigation Satellite System independently, and the system will be able to provide global service on its own.

C. **Compatibility.** China hope that compatibility and interoperability will be realized between BeiDou and other satellite navigation systems, in the frame of the International Committee on Global Navigation Satellite Systems (ICG) and the International Telecommunication Union (ITU), making all users sharing the development of satellite navigation.

D. **Progressive.** China will actively promote BeiDou Navigation Satellite System's construction and development, improve service quality continuously, and realize the seamless integration between each stage.
2.7. What can it do now?

Now BeiDou can provide regional area positioning service. The area is shown in Figure 2.5, from 55°S to 55°N, 70°E to 150°E. In this region, most areas can receive signals from 8 to 9 BDS satellites on average at the same time and achieve a standard positioning service (SPS) level with the following real-time accuracies:

- Horizontal positioning: 10 meters;
- Vertical positioning: 10 meters;
- Velocity: 0.2 meters per second and one-way timing, 20 nanoseconds.

Beyond the service area, in the 55S to 55N, 55E to 160E area, the accuracies can reach 20m in horizontal and vertical direction. In 55S to 55N, 55E to 180E, the accuracies can reach 30m in horizontal and vertical direction.

For the current regional navigation service of BDS, the main contribution to positioning is from the GEO and IGSO satellites. Due to the characteristics of the BDS constellation, the positioning service will be affected by the geometry of the satellites. The GEO satellites are static relative to the earth; so only two dimensional positioning can be achieved if using only GEO satellites. Moreover, in the northern and southern hemispheres, the elevation of GEOs is generally low. This leads to some signal blockage and decrease in the availability of the GEO satellite signals. [8]

After BeiDou with full operation capability in 2020, the positioning signals are primarily from MEO satellites, receivers in Asia/Pacific region can receive more signals than other area. It will produce higher accuracies. According to [8], It is
supposed to have 5% to 10% improvement in BDS positioning accuracy in Asia/Pacific area compared with the rest of the world. And at that time, the GEO satellite will be maintained as SBAS of BeiDou. It will improve sub-meter and meter level positioning accuracy in Asia/Pacific for dual-frequency and single-frequency receivers, respectively.

The future BeiDou is expected to support two different kind of general services: Radio Determination Satellite Service (RDSS) and Radio Navigation Satellite Service (RNSS).

In the RDSS, the user position is computed by a ground station using the round trip time of signals exchanged via GEO satellite. The RDSS Long term feature further includes:

- Short message communication
- Large volume message communication
- Information connection
- Extended coverage

RNSS is very similar to the service provided by GPS and Galileo and is designed to achieve similar performances.

### 2.8. BeiDou Ionosphere Model

BeiDou ionosphere model (BMI) is based on the Klobuchar model, although it has some modifications. As Klobuchar model, it has eight parameters, $a_0, a_1, a_2, a_3, b_0, b_1, b_2, b_3$. The difference between them is that BIM is formulated in terms of geographic while Klobuchar is formulated in terms of geomagnetic coordinates, another difference is BIM use the ionosphere height is 375km, compared with Klobuchar model use 350km.

Flowing is the algorithm for BeiDou ionosphere model:

The user computes the vertical ionosphere delay correction $I_z'(t)$ with the 8 parameters and Klobuchar model as follows:

$$
I_z'(t) = \begin{cases} 
5 \times 10^9 + A_2 \cos \left[ \frac{2\pi(t-50400)}{A_4} \right] , & \text{when } |t - 50400| < A_4/4 \\
5 \times 10^9 , & \text{when } |t - 50400| \geq A_4/4 
\end{cases}
$$  \hspace{1cm} (2.1)

$I_z'(t)$ is the vertical ionospheric delay in seconds for B1I, $t$ is the local time (range 0–86400 sec) for the place under the intersection point (M) of ionosphere and the direction from receiver to satellite.

$A_2$ is the amplitude of Klobuchar cosine curve in the day time computed from the $\alpha_n$.

$$
A_2 = \left\{ \begin{array}{ll}
\sum_{n=0}^{3} \alpha_n |\phi_m|^n , & \text{when } A_2 \geq 0 \\
5 \times 10^9 , & \text{when } A_2 < 0 
\end{array} \right.
$$  \hspace{1cm} (2.2)
$A_4$ is the period of cosine curve in seconds. It is computed from the $\beta_n$.

$$A_4 = \begin{cases} 172800 & , \text{when } A_4 \geq 172800 \\ \sum_{n=0}^{3} \beta_n |\phi_m|^n & , \text{when } 172800 > A_2 \geq 72000 \\ 72000 & , \text{when } A_4 < 72000 \end{cases} \quad (2.3)$$

Where, $\phi_m$ is the geographic latitude of earth projection of the ionosphere intersection point in semi-circles ($\pi$). The geographic latitude $\phi_m$ and longitude $\lambda_m$ of the intersection point M are computed as:

$$\phi_m = \arcsin(\sin\phi_u \cdot \cos\psi + \cos\phi_u \cdot \sin\psi \cdot \cos A) \quad (2.4)$$

$$\lambda_m = \lambda_u + \arcsin\left(\frac{\sin\psi \cdot \sin A}{\cos \phi_m}\right) \quad (2.5)$$

Here, $\phi_u$ is the user’s geographic latitude in radians. $\lambda_u$ is the user’s geographic longitude in radians. $A$ is the satellite azimuth from the user location in radians. $\psi$ is the earth’s central angle in radians between the user location and ionospheric intersection point. It is computed as:

$$\psi = \frac{\pi}{2} - E - \arcsin\left(\frac{R}{R+h} \cdot \cos E\right) \quad (2.6)$$

Here, $R$ is the mean radius of the earth, the value is 6378 km. $E$ is the satellite elevation from the user’s location in radians. $h$ is the height of ionosphere, the value is 375 km.

$I_z'(t)$ can be converted to the ionospheric delay along the B1I propagation path $I_{B1I}(t)$ through the equation as follows and the unit is seconds.

$$I_{B1I}(t) = \frac{1}{\sqrt{1 - \left(\frac{R}{R+h} \cdot \cos E\right)^2}} I_z'(t) \quad (2.7)$$

For B2I,

$$I_{B2I}(t) = k(f) \cdot I_{B1I}(t) \quad (2.8)$$

Here

$$k(f) = \frac{f_2^2}{f_z^2} = \left(\frac{1561.098}{1207.140}\right)^2 \approx 1.6724 \quad (2.9)$$
Where, $f_1$ refers to the nominal carrier frequency of B1I, $f_2$ refers to the nominal carrier frequency of B2I, and the unit is MHz.

Note: When user adopts the ionospheric delay model in the south hemisphere, the ionospheric correction accuracy is slightly worse than that in the north.

### 2.9. Ionosphere-free combination

As in GPS, for dual-frequency (B1I and B2I) users, we can use the ionosphere-free combination.

\[
PR = \frac{PR_{B2I} - k(f)PR_{B1I}}{1 - k(f)} - \frac{c(T_{GD2} - k(f)T_{GD1})}{1 - k(f)}
\]  

(2.10)

PR: pseudorange corrected for ionospheric effects;

$PR_{B1I}$: Pseudorange measured on B1I (corrected by the satellite clock correction parameters and $T_{GD1}$);

$PR_{B2I}$: Pseudorange measured on B2I (corrected by the satellite clock correction parameters and $T_{GD2}$);

$T_{GD1}$: Equipment group delay differential on B1I;

$T_{GD2}$: Equipment group delay differential on B2I;

$k(f)$: The ratio of frequency square, here $\frac{f_1^2}{f_2^2} = 1.672$;

c: The light speed, and its value is $2.99792458 \times 10^8$ m/s.

There is a lot of research about comparing Klobuchar model and BIM, in[9], we can see BIM has a better performance than Klobuchar model in China territory even Asian area. But due to uneven distribute of the monitor stations of BDS, for global user, Klobuchar model is better. For BIM, it has better accuracy in northern hemisphere than in southern hemisphere. Mid-latitude regions can get the best correction and it has a decreasing trend to the low latitude and high latitude regions. Also it depend on a local time, a constant delay of 5ns is applied in the night.
Chapter 3 gLAB Introduction

3.1. Introduction of gLAB

The GNSS-LAB tool suite (gLAB) was developed by Research group of Astronomy and Geometrics (gAGE) at the Technical University of Catalonia (UPC) under ESA Education Office contract N. P1081434. It is an advanced, interactive, educational, multipurpose software for processing and analyzing GNSS data and can get high accuracy positioning (for GPS data, get centimeter accuracy). It can be used for professionals GNSS data processing, education and self-learning for students.

The previous version of gLAB is able to perform precise modeling of GPS observables (pseudorange and carrier phase) at centimeter level, allowing both standard point positioning (SPP) and Precise Point Position (PPP) service. Additionally, it is able to handle Galileo and GLONASS data. But it can’t handle and process BeiDou data. This project is to upgrade gLAB for handing and processing BeiDou data.

gLAB is a flexible software package. It is able to deal with a variety of standard formats like RINEX 3.00, SP3, ANTEX and SINEX files, among others. Moreover, it is able to run under Linux and Windows environment. It is programmed in ANSI C and Python languages and contains three main software modules:

- **Data Processing Core** (DPC)
- **Graphic User Interface** (GUI)
- **Data Analysis Tool** (DAT)

Table 3.1 For running gLAB under different system, use different command

<table>
<thead>
<tr>
<th>Modules/ System</th>
<th>Windows</th>
<th>Linux</th>
</tr>
</thead>
<tbody>
<tr>
<td>DPC</td>
<td>gLAB.exe</td>
<td>gLAB_linux</td>
</tr>
<tr>
<td>GUI</td>
<td>gLAB_GUI.exe</td>
<td>gLAB_GUI.py</td>
</tr>
<tr>
<td>DAT</td>
<td>graph.exe</td>
<td>graph.py</td>
</tr>
</tbody>
</table>

The DPC the core of gLAB; it implements all the data processing algorithms and can be executed both in command line and with the GUI. The GUI connects DPC and DAT consists in different graphic panels for a user friendly managing of the SW and the tool configuration. They provide all the options to configure the model and navigation. The DAT provides a plotting tool for the data analysis and results visualizing in a user friendly environment. Both the DPC and DAT modules can run independently to processing GNSS data and plotting for analysis.
3.2. Data Processing Core (DPC)

DPC consist DATAHANDLING, PREPROCESS, INPUT, MODEL, FILTER, OUTPUT six modules. Except DATAHANDLING module, others are shown in the GUI interface. The following section introduces all these modules in detail and shows the upgrade of gLAB when introduce the DPC structure.

• DATAHANDLING: It defines all the structures and enumerators in the program and has functions to access data. This module does not appear in the GUI interface, because it does not have any configuration options.

In this part, the upgrade adds all necessary options about BeiDou. It includes all the BeiDou measurements, signal frequencies, and BeiDou time (BDT) week. Also some constant parameter which has different value in BeiDou system, like gravitational constant $\mu$, the earth’s angular velocity $\omega$.

For the measurement, one thing need to mention is that inconsistent observation types for BeiDou B1 observations due to inconsistent definitions in recent RINEX versions, C2I in versions RINEX 3.01 vs. C1I in RINEX 3.02. so in gLAB, for easy to deal with these measurement. we make C2I =C1I, L2I =L1I, D2I =D1I, S2I =S1I in gLAB. [10]

• INPUT: It can be understand as a "driver" between the input data file and the rest part of the program. This module implements all the input reading capabilities and stores it in structures which have defined in the DATAHANDLING module.

BeiDou data is only available in Rinex 3, so when “select system” is BDS, gLAB just read and process BeiDou data, skipping other systems. The next step is to check the version of Rinex file, if it is Rinex 2 or even lower version, which means there is no BeiDou data in this file, just quit out.

The main change in this part is when read data from navigation file and observation file, we need change the condition. Because in the previous version of gLAB cannot handle and processing BeiDou data, so the condition was always “when it is BeiDou, just skip.” But now we need to change the condition and defined when we need process BeiDou data, how to read and store the data.

It is difficult to define the condition properly. When read the observation header, for some system, the measurement types occupy more than one line. So we need have a special defined for this case.

BeiDou navigation information structure is similar with GPS; the first 5 rows are exactly the same as GPS navigation message. In the last three rows has slight difference:
In the sixth row, the second column and the forth column, in GPS navigation message is “code on L2 channel” and “L2 P data flag”, respectively. In BeiDou navigation message these two columns is spare. BeiDou ICD document only defined B1I signal. The third column is “BDT week” instead of “GPS week”. The difference between BDS week and GPS week is 1356 week.

In the seventh row, the forth column in BeiDou navigation message is “TGD2” while in GPS navigation message is “IODC Issue of Data, Clock”.

In the eighth row, the second column in BeiDou navigation message is “IODC Issue of Data, Clock” while in GPS is “Fit interval”.

• **PREPROCESS:** This module process the data before the MODEL. It includes cycle-slip detection, inconsistent pseudorange carrier-phase jump detection for Septentrio receivers, epoch decimation, carrier phase pre-alignment, and pseudorange smoothing.

In this project, we only deal with pseudorange measurement, so we only use “epoch decimation” and “pseudorange smoothing” this two function. They are the same for BeiDou and GPS. So there is no change in this module.

• **MODEL:** This module has all the functions to fully model the receiver measurements. As said, it implements several kinds of models, like ionosphere model, troposphere model, which can be activated or deactivated independent by choose different options or different comments.

Most models are the same for BeiDou and GPS. In the previous version of gLAB, some model is only for GPS system, so we need change the condition and add BeiDou system.

Another difference is when calculate the position of satellite, because BeiDou has three types orbit, the MEOs and IGSOs use the same way to calculate satellite position as GPS, but for the GEO satellites need do a rotation based on the way of calculation of MEO and IGSO.

The third difference is ionosphere model, the BeiDou ionosphere model use ionosphere layer height as 375km instead of GPS model as 350km. And the constant parameters $\Omega_e$ and $\mu$ are slightly different in BeiDou system and in GPS, and when choose BeiDou system, it will use correspond the constant parameters for BeiDou system.

The last difference is the clock offset reference, it has explained in the clock offset and instrument delay section.
• **FILTER:** This module implements an Extended Kalman Filter fully configurable, and obtains the estimations of the required parameters.

This part is the same for BeiDou system and GPS.

• **OUTPUT:** This module outputs the data obtained from the FILTER. Output is to display all the results. It can show input content, model result, prefit value, output result. Depends on choosing different options, it can show different contents as we need.

At last, the integrated module is gLAB. It is not a separate module; it is not shown in the GUI. But it includes the main function to connect all the other modules.

The DPC is able to work both with command-line parameters and a configuration file.

This project is only for processing standard position processing. The main idea is make gLAB read and process BeiDou data independently. When choose BDS, gLAB will not read and processing other systems data, while we process GPS data, it will read and store other system data.

### 3.3. Graphic User Interface (GUI)

The GUI consists in different graphic panels for a user friendly managing of the software and the tool configuration. The GUI is an interface between the other two components, the DPC and the DAT. It will allow the user changing different parameters through choose different options, and execute the other two programs with the proper arguments.

In previous version, it only includes the GPS option, and the upgrade adds BeiDou option to the GUI. As shown in Figure 3.1. And two tabs can be found:
Figure 3.1 gLAB preprocess interface with BeiDou option

- **Positioning**: This tabs interfaces with the **DPC** tool, and allows selecting all the different processing options.

- **Analysis**: This tabs interfaces with the **DAT** tool, and allows selecting all the different plotting options.

  Tooltips is one of the highlights of gLAB. When the user hover the mouse over an option, the relative information is automatically displayed. It helps the users to understand what is mean and result of the options.

### 3.4. Data Analysis Tool (DAT)

The DAT is an advanced plotting utility to graph different combinations of columns taking into account several user-defined conditions. Each graphic window can contain several plots with the same scale. It has following options:

- **Graphic general options**: The General Options are specified only once per graphic, and will affect the entire graphic window. It includes show help, defined title, and label of axis, the minimum and maximum value for each axis.
• Plot dependent options: The Plot Dependent Options are specific to each plot. One new plot is considered from the point that a '-f' or '--file' is found. All the options coming after this parameter belong to that specific plot. The user can set which column in the input file is for each axis, also can use the mathematic function when defined the axis. And can defined different colors, styles for the plot, also can set condition and setting axis.

With BeiDou signals, gLAB works well. It will show the test result in Chapter 8.
Chapter 4 Time System and Reference Frame

The time system and reference frame are playing significant roles in a GNSS to get proper positioning result. The characteristics of time and reference frame have direct effects on time synchronization, satellite orbit determination and prediction. The measurements obtained and used in GNSS are also relevant to them.

In this chapter, it will briefly introduce the main kinds of time reference and coordinate frames. The time system of BeiDou is called BeiDou navigation satellite system Time (BDT). The space coordinates are referred to the China Geodetic Coordinate System 2000 (CGCS2000), which is consistent with ITRS on the centimeter level.

4.1. Time Reference

Based on different periodic processes associated with earth rotation, celestial mechanics or transitions between energetic levels in atomic oscillators, there are several time references are in operation currently. Summarize of these time reference is shown in table 4.1.

<table>
<thead>
<tr>
<th>Periodic</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth rotation</td>
<td>Universal Time (UT0, UT1, UT2)</td>
</tr>
<tr>
<td></td>
<td>Sidereal Time</td>
</tr>
<tr>
<td>Earth revolution</td>
<td>Terrestrial Dynamic Time (TDT)</td>
</tr>
<tr>
<td></td>
<td>Barycentric Dynamical Time</td>
</tr>
<tr>
<td>Atomic oscillators</td>
<td>International Atomic Time (IAT)</td>
</tr>
<tr>
<td></td>
<td>Universal Coordinate Time (UTC)</td>
</tr>
<tr>
<td></td>
<td>GNSS Reference Time</td>
</tr>
</tbody>
</table>

Universal and sidereal times associate with earth daily rotation. Universal time (solar time) uses the sun as a reference while Sidereal time uses a direction out of the Solar system (Aries point). This lead to the fact that in a year both times differ in a lap (24h), which supposes 3 minutes 56.4 seconds per day.

\[1 \text{ mean sidereal day} = 1 \text{ mean solar day} - 3^{m}56^{s}.4 \quad (4.1)\]

Universal times UT0, UT1, UT2 are not completely uniform in contrast of atomic times. Due to this, Universal Coordinated Time (UTC) was introduced, which is an atomic time that keeps itself at least at close as 0.9s to UTC 1, By means of the systematic introduction of the certain number of seconds called leap seconds. This
causes that, along time, the difference between UTC and IAT varies in integer leaps of 1 second.

![Figure 4.1 UT1-UTC and the leap second adjustments](image)

For different time system, they have following relations:

\[ IAT = UTC + 1s \times n \]  \hspace{1cm} (4.2)

\[ UTC = UT1 - dUT1 \]  \hspace{1cm} (4.3)

Where \( n \) is the number of leap seconds introduced for a given epoch. (E.g. January 1st 1999, \( n=32 \), January 1st 2006, \( n=33 \), January 1st 2009, \( n=34 \), July 1st 2012, \( n=35 \); see Figure 4.1.

### 4.2. Julian Day

In order to facilitate calculations in long time intervals, Julian date is used (invented by Julio Scaliger), having as a reference epoch the 1st of January of the year 4713 before Christ, and starting from there, days are being counted in a correlative way. The Julian Day (JD) commences at 12h of the corresponding civil day. The current reference standard epoch for the scientific community is

\[ J2000.00 = 1d.5 January 2000 = JD2451545.0 \]  \hspace{1cm} (4.4)
Because the number of Julian data is very big, and difficult to manage. So the modified Julian date is used, which is obtained by subtracting 2400000.5 day from the Julian day.

The following relation allows us to calculate Julian Date (JD) from civil date. (YYMMDDUT)

\[
JD = \text{int}[365.25 \times y] + \text{int}[30.6001 \times (m + 1)] + DD + UT(h)/24 + 1720981.5
\]

When \( y = YY - 1 \),

\[
m = MM + 12, MM \leq 2
\]

When \( y = YY \),

\[
m = MM, MM > 2
\]

4.3. BeiDou Time System

The time system of BeiDou is called BeiDou navigation satellite system Time (BDT). It is a continuous timekeeping system, with its length of second being a SI second. BDT adopts international system of units (SI) seconds, as the basic unit for continuous accumulation.

The origin epoch of BeiDou time is 00:00 UTC (midnight) of 1st of January of 2006 (GPS week-BDT week=1356). And it is synchronized to UTC within 100ns. BDT is counted with week and seconds of week (SOW). The BDT week starts at midnight Saturday/Sunday. At the first epoch, the difference IAT-BDT was 33s. And at the first epoch of GPS, IAT-GPST is 19s, so

\[
GPST - BDT = 14s
\]

When calculate the position with BeiDou signals, we need to take it into account.

In BeiDou Navigation System, there is a time and frequency system in the master control station to supply time and frequency signals. The satellite signals are based on the satellite clocks. All the satellites are loaded with high-quality rubidium clocks, which are made by Spectra Time Company, Switzerland, and Chinese companies.[4]

The BeiDou System provides time service in three modes which are:

1. RDSS one-way time service. One-way time service means all the corrections used in timing are calculated by users themselves on the basis of the NAV data. All of the time signals are emitted from the master station and transferred by the GEO satellite to the users. Currently, with respect to BDT, the uncertainty of the one-way time service is about 100ns and with system development, it is expected to reach 50ns.
(2) RDSS two-way time service. The ‘two-way time service’ means that the user responds with a signal back to the master station, and the round-trip time delay is measured by the station and then sent to the user. Currently, with BDT, the uncertainty of the RDSS two-way time service is about 20 ns (one sigma). With further development of the system, it is expected to reach 10 ns.

(3) RNSS one-way. The RNSS service emits all the signals from the navigation satellites.

At present, the performance of BDT is as follows [4]:

- Time (frequency) accuracy: $< 2 \times 10^{-14}$
- Time (frequency) stability: $< 1 \times 10^{-14} / 1 \text{ day}$
  - $< 6 \times 10^{-15} / 5 \text{ days}$
  - $< 5 \times 10^{-15} / 10 \text{ days}$
  - $< 6 \times 10^{-15} / 30 \text{ days}$

### 4.4. Reference Frames

To obtain precise positioning, satellite coordinates and user receivers must be expressed in a well-defined reference system. Following is the introduction of Conventional Inertial System (CIS), Conventional Terrestrial System (CTS) and China Geodetic Coordinate 2000 (CGCS2000).

#### 4.4.1. Conventional Celestial Reference System (CRS)

![Figure 4.2 Conventional Celestial Reference System](image)

It is also called Conventional inertial system (CIS). The origin point of the CIS is in the earth mass center. X axis points in the direction of the mean equinox in J2000.0 epoch, Z axis is orthogonal to the plane defined by the mean equator in J2000.0 epoch (fundamental plane) and Y axis is orthogonal to the former ones, so the system is directly (right handed) oriented. The practical implementation is called inertial Reference Frame and it is determined from a fundamental set of stars.
4.4.2. Conventional Terrestrial System (CTS)

It also called Earth Centered Earth Fixed System (ECEF). Its origin point is in the earth mass center. Z axis is identical to the direction of the earth rotation axis defined by conventional international origin, X axis is defined as the intersection of the orthogonal plane to Z axis and Greenwich mean meridian, and Y axis is orthogonal to both of them, making the system directly oriented.

4.5. CGCS 2000

BeiDou reference system is the China Geodetic Coordinate System 2000 (CGCS 2000). It is developed by China in 2000. It is referred to ITRF97 (International Terrestrial Reference Frame 1997) at the epoch of 1 January 2000(2000.0). The adoption of the new system will allow closer integration with international coordinate frames, global navigation satellite systems, scientific applications and routine spatial data management.

Figure 4.3 Conventional Terrestrial Reference System

Figure 4.4 CGCS 2000
CGCS 2000 coordinate system is right hand earth centered earth fixed coordinate system. The Z-axis is the same as IRP in IERS reference system. Its X-axis directs from the origin to the intersection between Greenwich meridian and the equator (at the epoch of 2000.0). Its Y-axis completes a right-handed, Earth-Centered Earth-Fixed (ECEF) orthogonal coordinate [11]. See figure 4.4.

CGCS 2000 is defined as Table 4.1.

**Table 4.1** The definition and parameters of CGCS2000

<table>
<thead>
<tr>
<th>The origin of coordinates</th>
<th>The earth mass center (including oceans and atmosphere);</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit of length</td>
<td>meter SI, consistent with TCG (Geocentric Coordinate Time);</td>
</tr>
<tr>
<td>The initial orientation</td>
<td>given by the BIH orientation at 1984</td>
</tr>
<tr>
<td>The time evolution in orientation</td>
<td>no residual global orientation with regard to the crust</td>
</tr>
<tr>
<td>Semi-major axis</td>
<td>(a = 6,378,137.0 \text{meters})</td>
</tr>
<tr>
<td>Flattening</td>
<td>(f = 1/298.257222101)</td>
</tr>
<tr>
<td>Earth's gravitational constant</td>
<td>(GM = 3.9860044181432 \times 10^{14} m/s)</td>
</tr>
<tr>
<td>Angular velocity of the Earth</td>
<td>(\omega = 7.292115 \times 10^{-5}) rad/s \times 10^{-5})</td>
</tr>
<tr>
<td>The coefficient of the zonal spherical harmonics of the second order</td>
<td>(J2 = 0.001 082 629 832 258);</td>
</tr>
</tbody>
</table>
Chapter 5 Orbit and clocks of BeiDou satellites

In order to get the receiver position, we need to calculate the satellite coordinate first. So the orbit and clock information are required. This information is included in the satellite navigation message. The following part will introduce detail of BeiDou navigation message and the method of calculate the satellite coordinates from navigation information.

5.1. Keplerian elements (two-body problem)

If it is only considered the attractive force between two masses, the motion of mass \(m_2\) relative to another mass \(m_1\) is defined by the differential equation.

\[
\frac{d^2 r}{dt^2} + \frac{\mu}{r^3} r = 0
\] (5.1)

Where \(r\) is their relative position vector, \(\mu = G(m_1 + m_2)\) and \(G\) is the universal gravitational constant. In the case of motion of an artificial earth satellite, its mass can be neglected with respect to the mass of the earth.

The integration of this equation leads us to the Keplerian orbit of the satellite as:

\[
r(t) = r(t; a, e, i, \Omega, \omega, \tau)
\] (5.2)

Defined by the following six orbital parameters as shown in Figure 5.1:

![Figure 5.1 GNSS satellite orbital elements](image)

- \(i\) inclination
- \(\Omega\) argument of perigee
- \(\Omega\) arg. ascending node (Aries)
- \(\lambda\) arg. ascending node (Greenwich)
- \(V\) true anomaly
- \(\Theta\) sidereal time
- \(\Upsilon\) vernal equinox
- \(G\) Greenwich meridian
• **Ω**: **Right ascension of ascending node** is the geocentric angle between the ascending node direction and the Aries point. The node line is the intersection with the equatorial plane and the orbital plane. Its intersection with the unit sphere defines two points: the ascending node, through which the satellite crosses to the region of positive Z, and the descending one.

• **i**: **Inclination of the orbital plane**: is the angle between the orbital plane and equator.

• **ω**: **Argument of perigee**: it is the angle between node direction and perigee, measured in the orbital plane. The perigee is the point of closest approach of the satellite with respect to the center of mass of the earth. The most distant position is the apogee. Both are in the semi-major axis direction.

• **a**: **Semi-major axis of orbital ellipse**: it is the semi-major axis of the ellipse defining the orbit.

• **e**: **Numerical eccentricity of the orbit**: it is the eccentricity of the orbital ellipse.

• **$T_0$**: **Perigee passing time**: it is the time of the satellite passage through the closest approach with the earth (perigee), satellite orbit position can be obtained at a moment $t$ using $\tau(t) = t - T_0$ or any of the three following anomalies

  - $v(t)$: **True anomaly**: it is the geocentric angle between perigee direction and satellite direction

  - $E(t)$: **Eccentric anomaly**: it is the angle, measured from the center of the orbit, between the perigee and the direction of the intersection point of the normal line to the major axis passing through the satellite with the circle of radius $a$.

  - $M(t)$: **Mean anomaly**: it is a mathematical abstraction.

The three anomalies are related by the formulas:

\[
M(t) = n(t) - T_0 \tag{5.3}
\]

\[
E(t) = M(t) + e \sin E(t) \tag{5.4}
\]

\[
V(T) = 2\arctan \left[ \frac{1-e}{1+e} \tan \frac{E(t)}{2} \right] \tag{5.5}
\]

\[
n = \frac{2\pi}{P} = \sqrt{\frac{\mu}{a^3}} \tag{5.6}
\]
When \( n \) denotes the mean angular velocity of the satellite, or mean motion, with revolution period \( P \). Replacing \( a = 21528 \text{ km} \) (nominal value for BeiDou MEO satellites) in the last of the above equations, we can obtain orbital period of BeiDou MEO satellite is 12 h 53 m.

### 5.2. Perturbed motion

The two-body problem considered in the previous section is only a first approximation to the real case. In practical, an additional set of accelerations \( k \) or disturbing terms must be added to the equation (5.1), therefore, our previous differential equation becomes one equation.

\[
\ddot{r} = -\frac{\mu}{r^3} r + k
\]  

(5.7)

These perturbations are mainly due to:

1. Non-sphericity of the earth and non-homogeneous mass distribution;
2. The presence of other celestials bodies, foremost, the sun and the moon;
3. Tidal effect;

Although the gravitational effects appears very small, but in a long period of time, their accumulated effect can produce significant changes in GNSS satellite orbits. So we need take into account the effect to orbital elements from all these perturbations osculation is changing with time, thus:

\[
r(t) = r(t; a(t), e(t), i(t), \Omega(t), \omega(t), \tau)
\]  

(5.8)
5.3. BeiDou Navigation messages

BeiDou navigation messages are formatted in D1 and D2 based on their rate and structure. D1 navigation message contains basic NAV information (fundamental NAV information of the broadcasting satellites, almanac information for all satellites as well as the time offsets from other systems) and it is broadcast by MEO and IGSO satellites. It is modulated with 1 kbps secondary code and its rate is 50 bps. D2 NAV message contains basic NAV and augmentation service information (the BDS integrity, differential and ionospheric grid information) and it is broadcast by GEO satellites. Its rate is 500 bps, ten times of D1 rate.[12]

5.3.1. D1 NAV Message Frame Structure

The NAV message in format D1 is structured in the superframe, frame and subframe. Every superframe has 36000 bits and lasts 12 minutes. Every superframe is composed of 24 frames (24 pages). Every frame has 1500 bits and lasts 30 seconds. Every frame is composed of 5 subframes. Every subframe has 300 bits and lasts 6 seconds. Every subframe is composed of 10 words. Every word has 30 bits and lasts 0.6 seconds.

Every word consists of NAV message data and parity bits. In the first word of every subframe, the first 15 bits are not encoded and the following 11 bits are encoded in BCH (15, 11, 1) for error correction. So there is only one group of BCH code contained and there are altogether 26 information bits in the word. For all the other 9 words in the subframe both BCH (15, 11, 1) encoding for error control and interleaving are involved. Each of the 9 words of 30 bits contains two blocks of BCH codes and there are altogether 22 information bits in it.

![Figure 5.3 Frame structure of NAV message in format D1](image-url)
5.3.2. D1 NAV Message Detailed Structure

The main information contents of NAV message in format D1 are basic NAV information, including fundamental NAV information of the broadcasting satellites (seconds of week, week number, user range accuracy index, autonomous satellite health flag, ionospheric delay model parameters, satellite ephemeris parameters and their age, satellite clock correction parameters and their age and equipment group delay differential), almanac and BDT offsets from other systems (UTC and other navigation satellite systems). It takes 12 minutes to transmit the whole NAV message.

The D1 frame structure and information contents are shown in Figure 5.4. The fundamental NAV information of the broadcasting satellite is in subframes 1, 2 and 3. The information contents in subframes 4 and 5 are subcommutated 24 times each via 24 pages. Pages 1~24 of subframe 4 and pages 1~10 of subframe 5 shall be used to broadcast almanac and time offsets from other systems. Pages 11~24 of subframe 5 are reserved.

![Figure 5.4 Frame structure and information contents of NAV message in format D1](image)

5.3.3. D2 NAV Message Frame Structure

The NAV message in format D2 is structured with superframe, frame and subframe. Every superframe is 180000 bits long, lasting 6 minutes. Every superframe is composed of 120 frames each with 1500 bits and lasting 3 seconds. Every frame is composed of 5 subframes, each with 300 bits and lasting 0.6 second. Every subframe is composed of 10 words, each with 30 bits and lasting 0.06 second. Every word is composed of NAV message data and parity bits. The first 15 bits in word 1 of every subframe is not encoded, and the last 11 bits is encoded in BCH(15,11,1) for error correction. For the other 9 words of the subframe both BCH(15,11,1) encoding and interleaving are involved. Thus there are 22 information bits and 8 parity bits in each word.
5.3.4. **D2 NAV Message Detailed structure**

Information in format D2 includes: the basic NAV information of the broadcasting satellite, almanac, and time offset from other systems, integrity and differential correction information of BDS and ionospheric grid information as shown in Figure 5.6. The subframe 1 shall be subcommutated 10 times via 10 pages. The subframe 2, subframe 3 and subframe 4 shall be subcommutated 6 times each via 6 pages. The subframe 5 shall be subcommutated 120 times via 120 pages.

---

**Figure 5.5** Frame structure of NAV message in format D1

**Figure 5.6** Frame structure and information contents of NAV message in format D2
5.4. Computation of BeiDou satellite coordinates

5.4.1. Ephemeris Parameters

The ephemeris parameters describe the satellite orbit during the curve fit interval, including 15 orbit parameters and an ephemeris reference time. All the parameters needed to calculate the orbital elements are transmitted at every observation epoch, and be renewed every hour [13]. They cannot be used out of 3 hours, because the extrapolating error grows exponentially beyond this period. However, due to the uplink station of BeiDou are limited to the china territory, the age-of-data for the non-geostationary satellites depends on the satellite location.[14]

The definitions of ephemeris parameters are listed in Table 5.1.

Table 5.1 Ephemeris Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{oe}$</td>
<td>Ephemerides reference epoch in seconds within the week</td>
</tr>
<tr>
<td>$\sqrt{A}$</td>
<td>Square root of semi-major axis</td>
</tr>
<tr>
<td>$e$</td>
<td>Eccentricity</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Argument of perigee</td>
</tr>
<tr>
<td>$\Delta n$</td>
<td>Mean motion difference from computed value</td>
</tr>
<tr>
<td>$M_0$</td>
<td>Mean anomaly at reference epoch</td>
</tr>
<tr>
<td>$\Omega_0$</td>
<td>Longitude of ascending node of orbital of plane computed according to reference time</td>
</tr>
<tr>
<td>$\dot{\Omega}$</td>
<td>Rate of right ascension</td>
</tr>
<tr>
<td>$i_0$</td>
<td>Inclination angle at reference time</td>
</tr>
<tr>
<td>$IDOT$</td>
<td>Rate of inclination angle</td>
</tr>
<tr>
<td>$c_{uc}$</td>
<td>Amplitude of cosine harmonic correction term to the argument of latitude</td>
</tr>
<tr>
<td>$c_{us}$</td>
<td>Amplitude of sine harmonic correction term to the argument of latitude</td>
</tr>
<tr>
<td>$c_{rc}$</td>
<td>Amplitude of cosine harmonic correction term to the orbit radius</td>
</tr>
<tr>
<td>$c_{rs}$</td>
<td>Amplitude of sine harmonic correction term to the orbit radius</td>
</tr>
<tr>
<td>$c_{ic}$</td>
<td>Amplitude of cosine harmonic correction term to the angle of inclination</td>
</tr>
<tr>
<td>$c_{is}$</td>
<td>Amplitude of sine harmonic correction term to the angle of inclination</td>
</tr>
<tr>
<td>$a_0$</td>
<td>Satellite clock offset</td>
</tr>
<tr>
<td>$a_1$</td>
<td>Satellite clock drift</td>
</tr>
<tr>
<td>$a_2$</td>
<td>Satellite clock drift rate</td>
</tr>
</tbody>
</table>
5.4.2. Compute BeiDou satellite coordinates process

The user receiver shall compute the satellite position in coordinate system CGCS2000 according to the received ephemeris parameters.

Due to BeiDou, GPS, Galileo use different reference frame, the values for the earth gravitational coefficient and the earth rotation rate are different. It is mandatory to use the constellation specific values to ensure full accuracy of the broadcast ephemerides in real time positioning application. Table 2.1 shows the respective value of gravitational coefficient and the rotation for BeiDou, GPS, and Galileo.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>BeiDou</th>
<th>GPS</th>
<th>Galileo</th>
</tr>
</thead>
<tbody>
<tr>
<td>The earth gravitational coefficient $\mu$ ($10^{14} m^3/s^2$)</td>
<td>3.986004418</td>
<td>3.986005</td>
<td>3.986004418</td>
</tr>
<tr>
<td>Earth rotation rate $\dot{\Omega}_e (10^{-5} \text{rad/s})$</td>
<td>7.2921150</td>
<td>7.2921151476</td>
<td>7.2921151476</td>
</tr>
</tbody>
</table>

Firstly, we defined the following constant parameter in the reference frame CGCS2000.

- The earth’s universal gravitational constant of CGCS2000:
  \[ \mu = 3.986004418 \times 10^{14} m^3/s^2 \]

- The earth’s rotation rate of CGCS2000:
  \[ \dot{\Omega}_e = 7.2921150 \times 10^{-5} \text{rad/s} \]

The following is algorithm to compute satellite coordinates.

a. Compute the mean motion of the satellite:
   \[ n_0 = \sqrt{\frac{\mu}{A^3}} \quad (5.9) \]

b. Compute the time $t_k$ from the ephemeris reference epoch:
   \[ t_k = t - t_{oe} \quad (5.10) \]

Here, $t$ is the time of signal transmission in BDT. $t_k$ is the total time difference between $t$ and ephemeris reference time $t_{oe}$ after taking account of beginning or end of a week crossovers. So:
If $t_k > 302400$ sec, subtract 6048000 sec from $t_k$; if $t_k < -302400$ sec, add 6048000 sec to $t_k$.

Note: 6048000 sec is one week.

c. Corrected mean motion

\[ n = n_0 + \Delta n \]  \hspace{1cm} (5.11)

d. Calculate the mean anomaly for $t_k$

\[ M_k = M_0 + nt_k \]  \hspace{1cm} (5.12)

e. Solving (iteratively) Kepler equation for the eccentricity anomaly $E_k$:

\[ M_k = E_k - e \sin E_k \]  \hspace{1cm} (5.13)

f. Calculate real anomaly $v_k$:

\[ \sin v_k = \frac{\sqrt{1-e^2} \sin E_k}{1-e \cos E_k} \]  \hspace{1cm} (5.14)
\[ \cos v_k = \frac{\cos E_k - e}{1-e \cos E_k} \]  \hspace{1cm} (5.15)

So:

\[ v_k = \arctan\left(\frac{\sqrt{1-e^2} \sin E_k}{\cos E_k - e}\right) \]  \hspace{1cm} (5.16)

g. Calculate the argument of latitude $\nu_k$ from the argument of perigee $w$, real anomaly $v_k$ and corrections and corrections $c_{uc}$ and $c_{us}$:

\[ u_k = w + v_k + c_{uc} \cos 2(w + v_k) + c_{us} \sin 2(w + v_k) \]  \hspace{1cm} (5.17)

h. Calculate of the radial distance $r_k$, considering corrections $c_{rc}$ and $c_{rs}$:

\[ r_k = a(1 - e \cos E_k) + c_{rc} \cos 2(w + v_k) + c_{rs} \sin 2(w + v_k) \]  \hspace{1cm} (5.18)

i. Calculate inclination $i_k$ of the orbital plane from the inclination $i_0$ at the reference time $t_{oe}$ and corrections $c_{ic}$ and $c_{is}$:

\[ i_k = i_0 + IDOT \cdot t_k + c_{ic} \cos 2(w + v_k) + c_{is} \sin 2(w + v_k) \]  \hspace{1cm} (5.19)

j. Compute satellite position in orbit plane:

\[
\begin{align*}
x_k &= r_k \cos u_k \\
y_k &= r_k \sin u_k
\end{align*}
\]  \hspace{1cm} (5.20)
k. Corrected longitude of ascending node in CGCS2000:

\[ \Omega_k = \Omega_0 + (\dot{\Omega} - \dot{\Omega}_e)t_k - \dot{\Omega}_e t_{oe} \]  \hspace{1cm} (5.21)

i1. Calculation of MEO/IGSO coordinates in CGCS2000 frame:

\[
\begin{align*}
X_k &= x_k \cos \Omega_k - y_k \cos i_k \sin \Omega_k \\
Y_k &= x_k \sin \Omega_k + y_k \cos i_k \cos \Omega_k \\
Z_k &= y_k \sin i_k
\end{align*}
\]  \hspace{1cm} (5.22)

i2. Calculation of GEO satellites

We need to do a rotation based on the same formula of MEO in the inertial system, the computation is as follows:

- Corrected longitude of ascending node in inertial coordinate system:

\[ \Omega_k = \Omega_0 + \dot{\Omega}t_k - \dot{\Omega}_e t_{oe} \]  \hspace{1cm} (5.23)

- Calculation of GEO satellite coordinates in user-defined inertial system:

\[
\begin{align*}
X_{GK} &= x_k \cos \Omega_k - y_k \cos i_k \sin \Omega_k \\
Y_{GK} &= x_k \sin \Omega_k + y_k \cos i_k \cos \Omega_k \\
Z_{GK} &= y_k \sin i_k
\end{align*}
\]  \hspace{1cm} (5.24)

- Calculation of GEO satellite coordinates in CGCS2000 frame, by applying three rotations (about \( u_k, i_k, \lambda_k \)):

\[
\begin{bmatrix}
X_k \\
Y_k \\
Z_k
\end{bmatrix}
= R_x(\dot{\Omega}_e t_k) R_x(-5^\circ) \begin{bmatrix}
X_{GK} \\
Y_{GK}
\end{bmatrix}
\]  \hspace{1cm} (5.25)

Here:

\[ R_x(\varphi) = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \varphi & \sin \varphi \\
0 & -\sin \varphi & -\cos \varphi
\end{pmatrix} \]  \hspace{1cm} (5.26)

\[ R_z(\varphi) = \begin{pmatrix}
\cos \varphi & \sin \varphi & 0 \\
-\sin \varphi & -\cos \varphi & 0 \\
0 & 0 & 1
\end{pmatrix} \]  \hspace{1cm} (5.27)

Combine the equation 5.25, 5.26, 5.27, and then the GEO coordinate can be obtained as following:
\[
\begin{align*}
X_k &= X_{GK} \cos(\hat{\Omega}_et_k) + Y_{GK} \sin(\hat{\Omega}_et_k) \cos(-5^\circ) + Z_{GK} \sin(\hat{\Omega}_et_k) \ast \sin(-5^\circ) \\
Y_k &= X_{GK} (-\sin(\hat{\Omega}_et_k)) + Y_{GK} \cos(\hat{\Omega}_et_k) \cos(-5^\circ) + Z_{GK} \cos(\hat{\Omega}_et_k) \sin(-5^\circ) \\
Z_k &= Y_{GK} \ast (-\sin(-5^\circ)) + Z_{GK} \ast \cos(-5^\circ)
\end{align*}
\]

Finally, we get the position of satellite, for MEO satellite, the computation is the same as GPS, except use different constant parameter \( \hat{\Omega}_e \) and \( \mu \). But for GEO satellite, it need do a rotation of \( R_z(\hat{\Omega}_et_k)R_x(-5^\circ) \) after use the same scheme as MEO satellites in user-defined inertial system.
Chapter 6 Signal modelling

The pseudorange between the satellite and the receiver is obtained through the correlation of the modulated code in the received signal from the satellite with the replica generated in the receiver \( p = c \Delta T \), this measurement is affected by a series of terms which are added to the geometric distance. In Error! Reference source not found., it shows a scheme with the different contributions.

![Figure 6.1 Pseudorange components](image)

The modeling of pseudorange measures, between a receiver \( s \) and a satellite \( j \), must take into account the following terms

\[
P^i_j = \rho^i_j + c(dt_i - dt^j) + re_l^i_j + T^i_j + \alpha_1 I^i_j + K^i_j + M^i_j + \varepsilon^i_j \tag{6.1}
\]

6.1. Geometric range

The geometric range between the Euclidean distance between the satellite and receiver antenna phase center at the emission and reception time, respectively.

\[
\rho^{sat}_{rcv} = \| r^{sat} - r^{rcv} \| = \sqrt{(x^{sat} - x^{rcv})^2 + (y^{sat} - y^{rcv})^2 + (z^{sat} - z^{rcv})^2} \tag{6.2}
\]
6.1.1. Computation of satellite emission time

The satellite emission time can be computed from receive reception time using one of the next algorithms.

6.1.1.1. Algorithm using pseudorange

The emission epoch can be directly obtained from the reception epoch; taking into account that pseudorange $P$ is a direct measurement of the time difference between both epochs, it can be measured in the corresponding clock.

$$ P = c(t_{sta[reception]} - t_{sat[emission]}) $$

(6.3)

So the signal emission epoch can be:

$$ t_{sat[emission]} = t_{sta[reception]} - P/c $$

(6.4)

In order to calculate satellite coordinates, it must be used emission epoch, which is measured in BDT. This time scale may be obtained correcting $t_{sta}$ value with satellite clock offset $dt_{sat} = t_{sat} - T$, that can be acquired from the navigation message. Finally we get:

$$ T[emission] = t_{sat[emission]} - dt_{sat} = t_{sta[reception]} - P/c - dt_{sat} $$

(6.5)

This expression relate the emission epoch $T[emission]$ in the BDT scale with observation epochs ($t_{sta}$) recorded by the receiver, referring to receiver internal clock. It has the advantage of providing the signal emission epoch directly, without iterative calculation, although it does need pseudorange measurement in order to relate both instants. The accuracy in determination of $T[emission]$ is very high, and essentially depends on $dt_{sat}$ error, for BeiDou it is less than 50ns.

6.1.1.2. Purely geometric algorithm

The former algorithm provides signal emission epoch tied to satellite clock $t_{sat}$. On the other hand, the following algorithm ties epoch to receiver clock $t_{sta}$

$$ t_{sta[emission]} = t_{sta[reception]} - \Delta t $$

(6.6)

Where $\Delta t$ is calculated by iteration as following:

1. Calculate the position $r_{sat}$ of the satellite at signal reception epoch $t_{sta}$.

2. Calculate the geometric distance between satellite coordinates obtained previously and receiver position, and from it, calculate the signal propagation time between both points:
\[ \Delta t = \frac{||r_{\text{sat}} - r_{\text{olu}}||}{c} \]  \hspace{1cm} (6.7)

3. Calculate the satellite position at the instant:
\[ t = t_{\text{sta}} - \Delta t \Rightarrow r_{\text{sat}} \]  \hspace{1cm} (6.8)

4. Compare the new position \( r_{\text{sat}} \) with the former position. If they differ more than a certain threshold value, reiterate the process starting from step 2.

Finally, emission epoch at BDT scale is
\[ T[\text{emission}] = t_{\text{sta}}[\text{emission}] - dt_{\text{sat}} = t_{\text{sta}}[\text{reception}] - P/c - dt_{\text{sat}} \]  \hspace{1cm} (6.9)

Here, \( dt_{\text{sta}} \) is receiver clock offset referred to BDT, which can be obtained from navigation solution.

6.1.2. Satellite coordinates

Once the satellite emission time is known, the satellite coordinates can be computed using the algorithm of section 5.4.2.

6.2. Clock offset

The BeiDou clock model is essentially the same as GPS and describes the satellite clock offset as the sum of a second-order polynomial in time and a periodic relativistic correction depending on the eccentric anomaly. But there are two points needed to note:

The first point is due to on BeiDou satellite it use BDT, but for most of the receivers, it use GPS time (GPST), there is 14 seconds difference between BDT and GPST (see the section 4.3).
\[ BDT = GPST + 14 \text{ seconds} \]  \hspace{1cm} (6.10)

The second point is BeiDou clock reference is different from other GNSS constellations. In GPS, clock offsets in both broadcast and precise ephemeris products provided with respect an ionosphere-free dual frequency combination of conventional reference signals, which is itself considered to be free of group delays. In contrast to this, the BeiDou clock offset provided in the broadcast navigation message are referred to a single-frequency B3 signal. They correspond to the clock synchronism errors referring to BeiDou time scale.

The system time computation is as follows:
The user is able to compute BDT at time of signal transmission as:

\[ t = t_{sv} - \Delta t_{sv} \]  \hspace{1cm} (6.11)

Here, \( t \) is BDT in seconds at time of signal transmission;
\( t_{sv} \) is the effective satellite ranging code phase time in seconds at time of signal transmission;
\( \Delta t_{sv} \) is the offset of satellite ranging code phase time in seconds and can be calculated from values \( a_0, a_1, a_2 \) and \( t_0 \) which are transmitted in the navigation message:

\[ \Delta t_{sv} = a_0 + a_1 \left( t - t_0 \right) + a_2 \left( t - t_0 \right)^2 + \Delta t_r \]  \hspace{1cm} (6.12)

- \( a_1 \): clock drift
- \( a_2 \): clock drift rate
- \( t_0 \): time of clock

Then, \( t \) can be replaced by \( t_{sv} \) regardless of its sensitivity. \( \Delta t_r \) is the correction term to relativistic effect with value of

\[ \Delta t_r = F \cdot e \cdot \sqrt{A} \cdot \sin E_k \]  \hspace{1cm} (6.13)

Here, \( e \) is the orbit eccentricity, which is given in ephemeris of the broadcasting satellite;
\( \sqrt{A} \) is the square root of semi-major axis of satellite orbit, which is given in ephemeris of the broadcasting satellite;
\( E_k \) is eccentric anomaly of satellite orbit, which is given in ephemeris of the broadcasting satellite;

\[ F = -2\mu^{1/2} / c^2 \]  \hspace{1cm} (6.14)

\( \mu = 3.986004418 \times 10^{14} \text{ m}^3 / \text{s}^2 \), is the value of earth’s universal gravitational constant;
\( c \) is the light speed, the value is \( 2.99792458 \times 10^8 \text{ m/s} \).

The B1I user should make a further correction as follows:

\[ (\Delta t_{sv})_{B1I} = \Delta t_{sv} - T_{GD1} \]  \hspace{1cm} (6.15)

The B2I user should make a further correction as follows:
\[(\Delta t_{SV})_{B2I} = \Delta t_{SV} - T_{GD2}\] (6.16)

### 6.3. Relativistic correction

The rate of advance of two identical clocks, placed one in the satellite and the other on the terrestrial surface, will differ due to the difference of the gravitational potential (general relativity) and to the relative speed between them (special relativity). This difference can be break into:

- A constant component is that only depends on the nominal value of the semi-major axis of the satellite orbit, which is adjusted by modifying the clock oscillating frequency of the satellite:

\[
\frac{f_0 - f_0}{f_0} = \frac{1}{2} \left( \frac{v}{c} \right)^2 + \frac{\Delta U}{c^2} = -4.464 \cdot 10^{-10}
\] (6.17)

- A periodical component due to the orbit eccentricity, which must be adjusted by the user receiver

\[rel = 2 \frac{\sqrt{e}}{c} \sin(E) = 2 \frac{v \cdot v}{c}\] (6.18)

### 6.4. Tropospheric delay

At the frequency which the BeiDou signal is emitted, the troposphere behaves like a non-dispersive media, being its effect independent of the frequency. The tropospheric delay can be modeled in an approximate way use the following expression:

\[T_i = (d_{dry} + d_{wet})m(elev)\] (6.19)

Where \(d_{dry}\) is vertical delay due to the dry component of the troposphere (basically formed by oxygen and nitrogen in hydrostatic equilibrium), \(d_{wet}\) is vertical delay due to the wet component, which mainly is water vapor of the atmosphere, the expression is as following:

\[d_{dry} = 2.3 \exp(-0.116 \cdot 10^{-3} \cdot H)(m)\] (6.20)

H: height over the sea level, in meters.

\[d_{wet} = 0.1(m)\] (6.21)

\(m(elev)\) is the slant factor in order to project the vertical delay in the direction of the satellite observation.
Here, \( m(elev) \) is the elevation referring to the local horizon of the receiver.

### 6.5. Ionospheric delay

The ionosphere is the zone of the terrestrial atmosphere that extends itself from about 75km until more than 2000km high. Due to the interaction with free electrons, electromagnetic signals which go through it are affected in relation to the propagation in a vacuum that is expressed by

\[
\delta_{\text{ion}} = \int (n - 1) ds
\]  \( (6.23) \)

Where the integral itself through the ray trajectory and \( n = \frac{c}{v} \) is the refraction index. As ionosphere is a dispersive media, its refraction index depends on the frequency and affects in a different way, phase and code, delaying in code and advancing in carrier phase. This dependence on the signal frequency allows us to adjust its effect using two different frequencies. For receiver with only one frequency, we need use ionospheric prediction model to eliminate the ionosphere delay.

BeiDou ionosphere model is developed based on the Klobuchar model. Its parameters are transmitted in the navigation message. Apart from being a very simple ionospheric model in which one assumes that all electrons are concentrated in a thin layer placed at 375km high over the surface, it is able to reduce the ionospheric effect between 50% and 60%. But due to lack of BeiDou ionosphere data, in this project, we use Klobuchar ionospheric model and IONEX ionospheric model.

The refraction indexes of the ionosphere for phase velocity \( v_f \) and group velocity \( v_g \), of the BeiDou signal are given, in the first order of approximation by:

\[
n_f \approx 1 - a_f \cdot N
\]  \( (6.24) \)

\[
n_g \approx 1 + a_f \cdot N
\]  \( (6.25) \)

- \( N \) is the electron density of the ionosphere \( (e^-/m^3) \)
- \( a_f = 40.3/f^2 \ (m^2/e^-) \)
- \( f \) is the signal frequency(Hz).

With all this, the first approximation of the ionospheric delay (in meters) is:

\[
\delta_{\text{ion}} = a_f \cdot I
\]  \( (6.26) \)
Here, $I$ is the electron number per area unity in the direction of the observation or STEC (slant total electron content):

$$ I = \int N_e ds $$ \hfill (6.27)

For carrier phase measurements, the ionosphere delay is $-\delta_{ion}$, and for pseudorange measurements is $+\delta_{ion}$. It means phase measurement suffer an advancement when crossing ionosphere while pseudorange measurements suffer a delay.

### 6.6. Multipath

The interference by multipath is generated when a signal arrives, by different ways, at the antenna. Its principal cause is the antenna closeness to the reflecting structures and it is important when the signal comes from the satellite with low elevation. This error is different for different frequencies. It affects both in the phase measurements and the code measurements. In the case of the code, it can reach a theoretical value of 1.5 times the wavelength, this means that, for C/A code, it reaches up to 450m, although upper values more than 15m are difficult to observe. Typically, it is less than 2 to 3 meters. In the case of the phase, its theoretical maximum value is a quarter of the wavelength. The value is 4.7cm, 6.3cm, 5.9cm for B1, B2, and B3, respectively.

![Figure 6.2 Difference of the optical path between the direct signal and the reflected signal](image)

This error can be minimized by improving the antenna quality and antenna location. That is to say, making them reject signal coming from certain directions, and moving the antenna away from reflecting objects can avoid multipath error to some extent.

### 6.7. Instrumental delay (TGD1, TGD2)

Instrumental delay is corresponding to all possible sources delay of all kinds of antenna, cables, different kind of filters, and all other electronica delay used in
receivers and satellites. The delay is common for all satellites, so it is assumed to be zero and is included in the receiver clock estimate.

Due to BeiDou choose different clock reference than GPS constellations. In GPS, the satellite clocks are referred to the ionosphere-free combination of codes and instrument delay can be cancelled in a combination of two frequency signals. But the BeiDou satellite clock offsets provide in broadcast navigation message are referred to a single frequency B3 signal, so different code bias need to applied in both in B1 and B2 single frequency navigation as well as B1/B2 or B1/B3 dual frequency navigation.[14]

\[
P_{B1} = \rho + c(\delta t^{rcv} - \delta t^{sat}) + T + I_{B1} + DCB_{B1-B3} \tag{6.28}
\]

\[
P_{B2} = \rho + c(\delta t^{rcv} - \delta t^{sat}) + T + I_{B2} + DCB_{B2-B3} \tag{6.29}
\]

\[
P_{B3} = \rho + c(\delta t^{rec} - \delta t^{sat}) + T + I_{B3} \tag{6.30}
\]

Here \(\rho\) denotes the geometric range, \(\delta t^{sat}\) and \(\delta t^{rcv}\) are the satellite and receiver clock offsets and \(T\) and \(I\) describe the tropospheric and ionospheric path delays. When processing with dual frequency combination satellite, it can’t be canceled as in GPS. We need apply DCB as following:

For B1/B2 combination

\[
DCB_{IF(B1,B2)} = \frac{f_{B1}^2}{f_{B1}^2-f_{B2}^2}DCB_{B1-B3} - \frac{f_{B2}^2}{f_{B1}^2-f_{B2}^2}DCB_{B2-B3} \tag{6.31}
\]

In contrast, for B1/B3 dual frequency users just need apply a scale version with the B1/B3 code bias.

\[
DCB_{IF(B1,B3)} = \frac{f_{B1}^2}{f_{B1}^2-f_{B3}^2}DCB_{B1-B3} \tag{6.32}
\]

For B1, and B2 single frequency, the instrument delay is \(DCB_{B1-B3}\) and \(DCB_{B2-B3}\) respectively. \(DCB_{B1-B3}\) is TGD1 and \(DCB_{B2-B3}\) is TGD2, which is broadcast in the navigation message.

In SP3 file, the clock is calculated by using B1/B2 combination, so refers the clock offset to B1/B2 combination, with the reference \(DCB_{IF(B1,B2)}\), instead of B3 with the reference \(DCB_{B1-B3}\). So when use SP3 file to compute the position, for B1 signal, the DCB should be \(TGD1 - DCB_{IF(B1,B2)}\). [14]

There are other time delay terms only for carrier phase measurement or for Precise Point Positioning. The wind-up effect is only affecting the carrier phase measurement and the correction is required only for high accuracy positioning. Antenna phase center correction and the earth deformation effect modeling are also only for PPP. This
project is only for use pseudorange measurement to obtain SPP solution, so they are out of the scope.
Chapter 7 Solving Navigation Equations

The aim of chapter is to determine the position $\vec{s}$ and the offset $\tau$ of the clock of a receiver from the pseudorange $P_j$, with at least 4 satellites, and positions $\vec{s}_j$ and the offset $\tau_j$ of the clocks of these satellites.

![Figure 7.1 Geometric concept of GNSS positioning](image)

7.1. Code-Based Positioning

From the pseudorange between satellite and receiver for $n \geq 4$ satellites:

$$P_j = \rho_j + c(\tau_j - \rho_j - \delta_j) + \epsilon_{P_j}$$

(7.1)

In this equation, the unknown parameter is the receiver position $\vec{s} = (x, y, z)$ and the receiver clock offsets $\tau$.

Then we have the following measurement equation system with four unknowns $(x, y, z, \tau)$:

$$P_j + c\tau_j - \delta_j \approx \sqrt{(x - x_j)^2 + (y - y_j)^2 + (z - z_j)^2} + c\tau_j, j = 1, 2, ..., n (n \geq 4)$$

(7.2)

Where the multipath terms have been rejected and noise in general, and it has been called

$$\delta = \epsilon_{P_j} + T_j + \alpha_j T_j + TGD_j$$

(7.3)
It is a nonlinear system, and in general over dimensioned, its usual resolution technique consists of linearizing the distance \( \rho \) in the neighborhood of a point \((x_0, y_0, z_0)\) corresponding to an approximate position of the receiver.

Then, linearizing

\[
\rho^j(x, y, z) = \sqrt{(x - x^j)^2 + (y - y^j)^2 + (z - z^j)^2}
\]

\[ (7.4) \]

in the point \( \vec{r}_0 = (x_0, y_0, z_0) \), can obtain:

\[
\rho^j = \rho_0^j + \frac{x_0 - x^j}{\rho_0^j} dx + \frac{y_0 - y^j}{\rho_0^j} dy + \frac{z_0 - z^j}{\rho_0^j} dz
\]

\[ (7.5) \]

with \( dx = x - x_0, dy = y - y_0, dz = z - z_0 \)

Resulting into the linear equation system:

\[
P^j = \rho_0^j + \frac{x_0 - x^j}{\rho_0^j} dx + \frac{y_0 - y^j}{\rho_0^j} dy + \frac{z_0 - z^j}{\rho_0^j} dz + c(dt - dt^j) + \delta^i, j = 1, ..., n
\]

\[ (7.6) \]

The system of navigation equations can be expressed as a matrix:

\[
\begin{bmatrix}
P^j - \rho_0^j + cdt^j - \delta^j \\
n^j - \rho_0^n + cdt^n - \delta^n
\end{bmatrix} = \begin{bmatrix}
\frac{x_0 - x^1}{\rho_0^1} & \frac{y_0 - y^1}{\rho_0^1} & \frac{z_0 - z^1}{\rho_0^1} \\
\vdots & \vdots & \vdots \\
\frac{x_0 - x^n}{\rho_0^n} & \frac{y_0 - y^n}{\rho_0^n} & \frac{z_0 - z^n}{\rho_0^n}
\end{bmatrix} \begin{bmatrix}
dx \\
dy \\
dz \\
cdt
\end{bmatrix}
\]

\[ (7.7) \]

In general, overdimensioned systems will be obtained (for \( n > 4 \)) and will have to be solved using the least mean square technique or Kalman filter.

Note that the differences \((dx, dy, dz)\) are being estimated between the true position\((x, y, z)\) and the approximate one \((x_0, y_0, z_0)\) where a linearization has been made. This value can be refined, iterating with successive corrections obtained for the same epoch, to the point of reducing the error underneath a threshold.
7.2. Solving navigation equation:

7.2.1. Least Squares solution

Least Squares is one common way to solve the linear overdimensioned system.

\[ \hat{Y} = G \hat{X} \]  

Minimizes the residual

\[ \min \| Y - \hat{Y} \|^2 = \min [\sum_{i=1}^{n} (y_i - \hat{y}_i)^2] \]

Obtain the solution

\[ \hat{X} = (A^t A)^{-1} A^t Y \]  

7.2.2. Least Squares with weights solution

If \( W \) is a weight matrix for the observations vector \( Y \), then the least mean square solution with weight matrix \( W \) is:

\[ \hat{X} = (A^t W A)^{-1} A^t W Y \]  

The weight matrix \( W \) is usually expressed as below

\[ W = \begin{pmatrix} \frac{1}{\sigma_{y_1}^2} & \cdots & \frac{1}{\sigma_{y_n}^2} \end{pmatrix} \]  

Where \( \sigma_{y_i}^2 \) is the noise variance of the observations \( Y = (y_1, \ldots, y_n)^t \).

If \( P_Y \) is the covariance matrix of the observation vector \( Y \), for \( W = P_Y^{-1} \) the least variance solution is obtained for \( X \), being:

\[ P_{\hat{X}} = (A^t W A)^{-1} \]  

7.2.3. Kalman filter

If the \( \hat{X}(n-1) \) is the obtained estimation for the \( n \)-th epoch, a prediction of the vector \( X(n) \) will be done for the following epoch \( \hat{X}^-(n) \), according to the first model of Gauss-Markov

\[ \hat{X}^-(n) = \Phi(n-1) \hat{X}(n-1) \]
\[ P_{\hat{x}(n)}^- = \Phi(n - 1)P_{\hat{x}(n-1)}\Phi(n - 1)^T + Q(n - 1) \] (7.15)

With these predictions \( \hat{x}^-(n) \), one can extend the observation equation \( Y(n) = A(n)X(n) \), as if they were new observations, obtaining the system.

\[
\begin{bmatrix}
Y(n) \\
\hat{x}^-(n)
\end{bmatrix} = (A(n))X(n) \tag{7.16}
\]

\[ W = \begin{pmatrix}
P_Y(n) \\ P_{\hat{x}(n)}^-
\end{pmatrix}^{-1} \tag{7.17}
\]

It is solved in the usual way by the least mean squares with weight matrix \( W \), and this algorithm can be summarized in the following scheme.

**Figure 7.2 Kalman filter diagram**

Note: the formulation here presented is algebraically equivalent to the classical formulation defined in the following scheme.
7.3. Bancroft method

Bancroft method is used for the direct calculation of the receiver position and the satellite offset. It allows us to obtain a direct solution of the receiver position and the clock offset, without requesting any knowledge kind “a priori” for receiver. Thus, this method can provide and initial value \((x_0, y_0, z_0)\) for the navigation equations seen before.

7.3.1. Raising and resolution

Developing equation

\[
P^j \approx \sqrt{(x-x^j)^2 + (y-y^j)^2 + (z-z^j)^2 + cd t}
\]  

(7.18)

Then obtains

\[
\left[ x^j^2 + y^j^2 + z^j^2 - P^j \right] - 2[x^j x + y^j y + z^j z - P^j cd t] + [x^2 + y^2 + z^2 + (cd t)^2] = 0
\]

(7.19)

Then, calling \(r = [x, y, z]^T\) and considering the inner product of Lorentz can be expressed in a more compact way as

\[
\frac{1}{2} \left[ \begin{array}{c} \langle r^j | \rho_j \rangle - \langle r^j | \rho_j \rangle \end{array} \right] + \frac{1}{2} \left[ \begin{array}{c} \langle r | cd t \rangle + \langle r | cd t \rangle \end{array} \right] = 0
\]

(7.20)

The former equation can be raised for every satellite.
Let us suppose that one disposes of four measurements $P^j$, and let us consider the following matrix which contains the available information of satellite coordinates and pseudorange (every row corresponds to a satellite):

$$B = \begin{pmatrix} x^1 & y^1 & z^1 & P^1 \\ x^2 & y^2 & z^2 & P^2 \\ x^3 & y^3 & z^3 & P^3 \\ x^4 & y^4 & z^4 & P^4 \end{pmatrix} \quad (7.21)$$

Then calling

$$\Lambda = \frac{1}{2} \left\langle \frac{r}{\text{cdt}}, \frac{r}{\text{cdt}} \right\rangle$$

$$1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix}$$

Being

$$a_j = \frac{1}{2} \left\langle \frac{r^j}{P^j}, \frac{r^j}{P^j} \right\rangle$$

The four equations for pseudorange can be expressed as:

$$a - BM \left[ \frac{r}{\text{cdt}} \right] + \Lambda 1 = 0$$

Being

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Then we can get:

$$\left[ \frac{r}{\text{cdt}} \right] = MB^{-1} (\Lambda 1 + a) \quad (7.22)$$

Then taking into account that the equality is carried out $\langle Mg, Mh \rangle = \langle g, h \rangle$, and that

$$\Lambda = \frac{1}{2} \left\langle \frac{r}{\text{cdt}}, \frac{r}{\text{cdt}} \right\rangle$$
From the former expression, we can obtain:

\[
\langle B^{-1}1, B^{-1}1 \rangle \Lambda^2 + 2[\langle B^{-1}1, B^{-1}a \rangle - 1] \Lambda + \langle B^{-1}a, B^{-1}a \rangle = 0
\]

The previous expression is a quadratic equation in \( \Lambda \) and provides two solutions; one is the searched solution \( \begin{bmatrix} r \\ cdt \end{bmatrix} \), another one is far from earth.

### 7.3.2. Generalization to the case of n-observations

If more than four observations are done, the matrix B is not square. However, multiplying by \( B^t \), we can obtain (least mean square solution):

\[
B^t a - B^t B M \begin{bmatrix} r \\ cdt \end{bmatrix} + \Lambda B^t 1 = 0
\]

Where:

\[
\begin{bmatrix} r \\ cdt \end{bmatrix} = M (B^t B)^{-1} B^t (\Lambda + a)
\]

And then:

\[
\langle (B^t B)^{-1} B^t 1, (B^t B)^{-1} B^t 1 \rangle \Lambda^2 + 2[\langle (B^t B)^{-1} B^t 1, (B^t B)^{-1} B^t a \rangle - 1] \Lambda
\]

\[
+ \langle (B^t B)^{-1} B^t a, (B^t B)^{-1} B^t a \rangle = 0
\]
Chapter 8 Performance of BeiDou system

In this chapter, we will present the solution of BeiDou system calculated by gLAB. The following aspect will be analyzed:

1. Different stations. Compare NEU errors and DOP values which are obtained by using broadcast file and IONEX\(^1\) model. Also compare NEU errors and DOP values which are obtained by using SP3\(^2\) file and IONEX model at different location.

2. Different ionosphere model. Compare NEU errors at the same station by using broadcast file with IONEX model and Klobuchar model, respectively.

3. Orbit comparison. Compare difference between the satellite orbit which are calculated from broadcast file and the satellite orbit provided by SP3 file.

4. BeiDou and GPS. We compared BeiDou and GPS from the above three aspect.

8.1. Comparison of Different Stations Performance

How to choose the location?

Due to BeiDou system is quite new, the service area is limited. There are not many public stations available for BeiDou signal. According to the receivers' availability and the BeiDou service area, we choose the following four stations, JFNG station is in Wuhan, China; the GMSD station is in Japan; the XMIS station is in the Christmas Island and the NNOR station is in Australian.

Station information

Table 8.1 shows the general information of the stations. Except GMSD station, other three stations have the same receiver type and antenna type.

---

\(^1\) See IONEX file format in [http://gage.es/sites/default/files/gLAB/HTML/IONEX_v1_0.html](http://gage.es/sites/default/files/gLAB/HTML/IONEX_v1_0.html).

\(^2\) See SP3 file format in [http://gage.es/sites/default/files/gLAB/HTML/SP3_Version_C.html](http://gage.es/sites/default/files/gLAB/HTML/SP3_Version_C.html).
Table 8.1 Station information

<table>
<thead>
<tr>
<th>Station</th>
<th>JFNG (China)</th>
<th>GMSD (Japan)</th>
<th>XMIS (Australia)</th>
<th>NNOR (Australia)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Receiver type</td>
<td>TRIMBLE NETR9</td>
<td>TRIMBLE NETR9</td>
<td>TRIMBLE NETR9</td>
<td>SEPT POLARX4</td>
</tr>
<tr>
<td>Antennae type</td>
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<td>TRM59800.00</td>
<td>TRM59800.00</td>
<td>SEPCHOKE_MC</td>
</tr>
<tr>
<td>Latitude (degree)</td>
<td>30.52</td>
<td>30.56</td>
<td>10.45</td>
<td>-31.05</td>
</tr>
<tr>
<td>Longitude (degree)</td>
<td>114.49</td>
<td>131.02</td>
<td>105.69</td>
<td>116.19</td>
</tr>
<tr>
<td>Height (meters)</td>
<td>71.30</td>
<td>142.65</td>
<td>261.58</td>
<td>234.98</td>
</tr>
</tbody>
</table>

**Comment:** They provide observation data from BeiDou Experimental Tracking Stations (BETS) for Chinese researchers. BETS is a continuous tracking network for BeiDou and GPS satellites established worldwide for scientific and engineering applications by the GNSS Research Centre of Wuhan University with cooperation partners since March 2011 and currently comprises fifteen stations. The data is not open. For us, we can only use the data from MGEX website.

8.1.1. **NEU error with broadcast orbits and clocks and IONEX ionospheric model for BeiDou**

The current BeiDou satellites still do not broadcast the Klobuchar coefficients of the ionospheric model. For this reason, we will use the IONEX file with Global Ionospheric Maps in this assessment.

The following figures are the results with broadcast orbits and clocks file and IONEX ionospheric model for BeiDou system in different stations
The XIMS station has the largest error, and the JFNG station has the best solution. It is because the XIMS station is the nearest one to the equator, has the worst ionosphere condition. The solution can be affected by the ionosphere activity. According to [15], the maximum ionosphere value is at the latitude N15° and S15°. Other three stations are more or less at the same distance to the equator, so they have similar performance. JFNG station is in middle of China. All the parameters of BeiDou should fit best in China area. At all of these stations, the errors are smaller than 10 meters, so they met the requirement from BeiDou official documents.

Also, the NEU error is affected by the number of visible satellite, with more satellites in view, the error will be smaller. For example, we can see in JFNG station, the error in north and east direction become smaller from 10000 seconds to 30000 seconds with the satellite number increasing. The up errors also go down in this period, but it is always larger than north and east error and it is not as stable as other two directions error. This is because in north and east directions, users can see many satellites, and can receive signals from different directions. So it can compensate the errors in these directions, and get better solution. But in up direction, half of satellites
are at the other side of planet and their signal have blocked by the earth. So it has larger error. Additional, due to the characteristics of the BDS constellation, the positioning service will be affected by the geometry of the satellites. The GEO satellites are static relative to the earth, so only two dimensional positioning can be achieved using only GEO satellites.

8.1.2. DOP with broadcast orbits and clocks file and IONEX ionospheric model

DOP represents an approximate ratio factor between the precision in the measurements ($\sigma$) and that in positioning. This ratio is computed only from the satellite–receiver geometry. It is meaning the additional multiplicative effect of navigation satellite geometry on positional measurement precision. It does not depend on the anything that cannot be predicted in advance.

Figure 8.2 The DOP with broadcast orbits and clocks file and IONEX ionospheric model at different stations
(Up left: JFNG; Up right: GMSD; Down left: station; Down right: NNOR)

From the Figure 8.2, the XMIS station has the smallest value of DOP. There are two reasons. The first one is because it has the most visible satellites. At most time, the number is more than 10. The second reason is for current constellation of BeiDou system; the main contribution to positioning comes from the GEO and IGSO satellites. The 5 GEO satellites positioned at 58.75°E, 80°E, 110.5°E, 140°E and 160°E respec-
The longitude of XMIS station is 105.69°E; it has lower elevation to one of the GEO satellites positioned 110.5°E than other station, so it is one of the main reason for the lower DOP value in this station.

The GMSD station has the biggest DOP value. In Figure 8.1, the JFNG station and the GMSD station has the most similar shape plots, because they are the nearest station from each other. They have similar latitude, and the ionosphere effect is similar. But the solution accuracy is relative to the DOP value. Due to GMSD big DOP value, the solution is worse than in JFNG.

8.1.3. NEU error with SP3 file and IONEX model in different stations for BeiDou

![Figure 8.3](image)

Figure 8.3 The NEU error with precise SP3 orbits and clocks file and IONEX ionospheric model at different stations
(Up left: JFNG; Up right: GMSD; Down left: station; Down right: NNOR)

Precise SP3 orbits and clocks file provide precise orbit and clock of the satellite, so compare with Figure 8.1 and Figure 8.3, we can see the solution with SP3 file is better than the solution with broadcast. Due to publicly unavailable antenna phase center offsets for the BeiDou satellites, we use the value (+600.0 mm, 0.0 mm, +1100.0 mm) according to [16].
8.1.4. DOP with precise SP3 orbits and clocks file and IONEX ionospheric model

In figure 8.4, we can see that in the XMIS and NNOR station, the DOP values are lower than other two stations. This is because they have more satellites in visible than other two stations.

Figure 8.4 DOP with precise SP3 orbits and clocks file and IONEX ionospheric model at different stations
(Up left: JFNG; Up right: GMSD; Down left: station; Down right: NNOR)

8.2. The same location with broadcast file and different ionosphere model

To compare different ionosphere model correction, from Figure 8.5 to Figure 8.8, we can find the solutions with IONEX ionospheric model are a little bit better than the solution with Klobuchar model. According to ionosphere data figure in [17], DOY 201 is the day with best ionosphere condition day in the year, which is clearly visible when comparing with DOY 208, which has a worse ionosphere condition. In the former day, we can see the results are more accurate while in the latter day larger error peak appeared. The results of DOY 208 are shown from figure 8.9 to figure 8.12.
**DOY 201**

**At JFNG station**

![Graph: NEU error with broadcast file at JFNG station with different ionosphere models in day 201](image)

*Figure 8.5* The NEU error with broadcast file at JFNG station with different ionosphere models in day 201 (left: IONEX; right: Klobuchar)

**At GMSD station**

![Graph: NEU error with broadcast file at JFNG station with different ionosphere models in day 201](image)

*Figure 8.6* The NEU error with broadcast file at JFNG station with different ionosphere models in day 201 (left: IONEX; right: Klobuchar)
At XMIS station

Figure 8.7 The NEU error with broadcast file at XMIS station with different ionosphere models in day 201 (left: IONEX; right: Klobuchar)

At NNOR station

Figure 8.8 The NEU error with broadcast file at NNOR station with different ionosphere models in day 201 (left: IONEX; right: Klobuchar)
DOY 208

To compare different ionosphere models’ correction to BeiDou signals, here we choose DOY 208 which with worse ionosphere condition and compare two ionosphere models’ correction.

At JFNG station

Figure 8.9 The NEU error with broadcast file at JFNG station with different ionosphere models in day 208 (left: IONEX; right: Klobuchar)

At GMSD station

Figure 8.10 The NEU error with broadcast file at GMSD station with different ionosphere models in day 208 (left: IONEX; right: Klobuchar)
At XMIS station

Figure 8.11 The NEU error with broadcast file at XMIS station with different ionosphere models in day 208 (left: IONEX; right: Klobuchar)

At NNOR station

Figure 8.12 The NEU error with broadcast file at NNOR station with different ionosphere models in day 208 (left: IONEX; right: Klobuchar)

8.3. Comparison of BeiDou solution and GPS solution at same condition

From the following results, we can see the GPS solution is always better than BeiDou solution. Because GPS is in full operation status and it has more satellite than BeiDou. We can see from the following figures, in most time, the GPS has better geometry than BeiDou. However, the BeiDou system NEU error is smaller than 10 meters at most time; it has met the requirement of the BeiDou ICD.

Also we can see, the GPS solution at XMIS station has the worst result, it is because the ionosphere effect as explained in section 8.1.
At JFNG station

Figure 8.13 The NEU error with broadcast file and IONEX model at JFNG station (Left: BDS; Right: GPS)

At GMSD station

Figure 8.14 The NEU error with broadcast file and IONEX model at GMSD station (Left: BDS; Right: GPS)
At XIMS station

**Figure 8.15** The NEU error with broadcast file and IONEX model at XIMS station
(Left: BDS; Right: GPS)

At NNOR station

**Figure 8.16** The NEU error with broadcast file and IONEX model at NNOR station
(Left: BDS; Right: GPS)
8.4. GPS solution in different station

Station JFNG

Figure 8.17 NEU error and DOP of GPS with IONEX model At JFNG station
(Up Left: NEU errors with broadcast file; Up Right: NEU errors with SP3 file
Down Left: DOP with broadcast file; Down Right: DOP with SP3 file)
Station GMSD

Figure 8.18 NEU error and DOP of GPS with IONEX model At GMSD station
(Up Left: NEU errors with broadcast file; Up Right: NEU errors with SP3 file
Down Left: DOP with broadcast file; Down Right: DOP with SP3 file)
Figure 8.19 NEU error and DOP of GPS with IONEX model At XMIS station
(Up Left: NEU errors with broadcast file; Up Right: NEU errors with SP3 file
Down Left: DOP with broadcast file; Down Right: DOP with SP3 file)
Station NNOR

Comparing the result with broadcast orbits and clocks file and precise SP3 orbits and clocks file for GPS system, we can find that the result with SP3 is slightly better than the result with broadcast file, but not as apparent as BeiDou. It is indicated GPS broadcast orbits and clocks is more accurate than BeiDou broadcast orbits.

GPS has the worst solution at XMIS station, even the DOP is similar as other stations; it is due to the bad ionosphere condition at this latitude. It also happen to BeiDou.

8.5. Comparison of the error of broadcast orbit and the SP3 precise orbit

In this part we will compare the difference between the broadcast orbit and SP3 precise orbit for BeiDou system and GPS, respectively. Satellite orbits refer to radial, along-track, cross-track directions as shown in Figure 8.21.
8.5.1. GPS: Difference between the broadcast orbit and SP3 precise orbit

From Figure 8.22 we can see the difference between the broadcast orbit and SP3 orbit is less 5 meters for GPS, most of them are concentration between -3 to 3 meters. The cross-track error is the smallest one, at most time, it is less than 1 meter, and the along-track has the largest error.

Figure 8.22 Difference between the broadcast orbit and SP3 precise orbit for GPS
8.5.2. BeiDou: Difference between the broadcast orbit and SP3 precise orbit

From Figure 8.23 we can see for most of the satellite the difference is less than 10 meters. Only one or two satellites have a bigger value, which is between 30 and 40 meters. Through checking, we find only C01 has a big value of orbit difference. For comparison with GPS, we get the result without satellite C01. Figure 8.24 shows the result without C01; we can see the value of orbit difference for BeiDou is between -10 to 8 meters, which is larger than the orbit difference for GPS. Also, we can see the difference is convergent to a stable value between -4 to 2 meters at the end of the plot.
Figure 8.25, Figure 8.26 and Figure 8.27 show the BDS GEO, IGSO and MEO satellite difference respectively. The MEO has the smallest value of orbit difference while the GEO has the largest value. This is because currently the BeiDou monitor station is distributed uneven in the Asia/Pacific region and BeiDou satellite constellation geometry is weak, the accuracy of orbit determination of the GEO satellites is affected in some directions, especially in the along-track direction.

Figure 8.25 Difference between the broadcast orbit and SP3 precise orbit for BDS GEO Satellites without C01

Figure 8.26 Difference between the broadcast orbit and SP3 precise orbit for BDS IGSO
For all satellites, the error in cross-track direction always is the smallest one; the value is between -2 meters to 2 meters. And at the along track direction, the orbit difference has the largest error. This is because of the observation geometric. The orbit difference for MEO is between -1 meter and 1 meter, for IGSO is from -2 meters to 2 meters and for GEO is from -4 meters to 2 meters.
Chapter 9 Conclusion

In this project, gLAB has been upgraded with a BeiDou signal processing module. The main task is to use single frequency pseudorange measurements to compute standard point positioning.

During the upgrade work, some differences between BeiDou system and GPS were considered. Firstly, BeiDou system consists of satellites with three kinds of orbits, so different orbit's algorithms are needed to obtain satellite position with GEO and MEO/IGSO. The MEO and IGSO use the same algorithm as GPS, while GEO needs to use a different algorithm. Secondly, a 14-seconds difference between BDT and GPST would be taken into account when calculating the clock difference between satellite clocks and receiver clocks. Thirdly, BeiDou defined B3 as reference of clock offset, while other GNSS use ionosphere free combination as a reference of clock offset. But in SP3, the clock is calculated by using combination of B1/B2. So when use SP3 data, we need take it into account.

The upgraded gLAB works well with BeiDou signals, through the result analysis, the solution is able to achieve the BeiDou ICD specific requirements. When comparing different stations’ solution, the accuracy is related to the receiver position, DOP, the visible satellites number, etc. Due to BeiDou is developed by China, all the parameters fit best in the middle part of the China, so it has the best performance in JFNG station. The ionosphere condition near equator is worse than in other areas, so the NEU error in XMIS station is larger than other station. At the same condition, by using SP3 orbit and clock files can get better performance than with broadcast orbits and clocks. Also, it was found that IONEX ionosphere model has better correction than Klobuchar ionosphere model. GPS orbits quality is a slightly better than BDS. In BDS, the MEO has the smallest orbit error while the GEO exhibit the largest orbit error. The BeiDou positioning performance is a little worse than the GPS one. But in the future, it will get improved with more MEO satellites and more even distribution of monitoring stations.
Appendix A: Sources of Data Files Used in the Exercises and other useful link

Public server for downloading RINEX3 navigation and observation files

Mixed navigation messages (including BeiDou):
Or ftp://igs.bkg.bund.de/MGEX/BRDC_v3/

For example:
Or ftp://igs.bkg.bund.de/MGEX/BRDC_v3/2014/

SP3 Data
ftp://cddis.gsfc.nasa.gov/pub/gps/products/mgex/

For example:

IONEX files
ftp://cddis.gsfc.nasa.gov/gnss/products/ionex/

For example:

GPS navigation file
ftp://cddis.gsfc.nasa.gov/pub/gps/data/daily/2014/208/14n/

GPS ANTEX file
ftp://igscb.jpl.nasa.gov/igscb/station/general/pcv_archive/

Get the GPS calendar
http://adn.agi.com/GNSSWeb/

Information about ANTEX for all GNSS
http://igs.org/mgex/status-GPS
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