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Life-Cycle Analysis and Reliability of Bridge Systems

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Thank you very much.

iv. Abstract

In this work, a study of the different models that perform the elements of a bridge like parallel, series or series parallel, is carried out. The aim is to assess the importance of these models along the lifetime of a structural system. In addition, the effects of aggressive environments which contains such as chlorides may affect directly to the resistance of the bridge and thus, lead it to collapse.

Since we are dealing with a real case study, a probabilistic analysis must be done, where the uncertainties of the random variables are accounted for the problem. For these reasons, an exhaustive analysis, in terms of reliability index and probability of failure is carried out.

First, we present a brief introduction to the life-cycle of structures and some definitions that will help us to follow the work and its results. Also, inspection methods used to control and detect the damage on the structures are presented.

Secondly, a deep explanation of reliability in civil engineering is given. Herein, the probability of failure and the reliability index are described in detail. In addition, stochastic models which use probabilistic inputs to study the reliability of the structural system are introduced. Moreover, some optimization concepts and decision making in civil engineering are superficially exposed.

Thirdly, concepts of Life-Cycle Management (LLM) in order to lead the structural system into a better performance are described. In this chapter, the reliability-based, risk-based performance and the maintenance for bridges are briefly detailed.

Fourthly, due to the fact that any construction is related with expenditure, Life-Cycle Cost (LLC) is needed to be introduced.

Fifthly, a case study is carried out. Herein, the analysis of the performance of the models series, parallel and series-parallel along the lifetime of the bridge affected by chloride concentration is performed. Thus, a discussion of the results obtained is done.

Finally, conclusions and future works are exposed on. Within this last section we can find many results regarding the behavior of series and parallel models. How they are related to the correlation between inputs and the extent of the chloride attack along the lifetime of the bridges. In addition some ideas to develop in next future are introduced.

0. Introduction

The economic growth and social development of most countries are well dependent on the durable and reliable performance of the structures and infrastructures. Currently, civil engineering is focusing in a smarter point of view that will make structures more efficient in terms of lifetime and cost. For that reason, a service life prediction of deteriorating structures must be studied in order to have an optimal design of the bridge.

The functionality in the design of the each element in the bridge, modeled as series, parallel or series parallel, will affect to the performance of the structural system along its lifetime. In addition, the presence of chlorides in the environment quite often coming from salts used in winter to melt the snow on the bridge will affect to the reinforcement of the structure. Corrosion is an important issue that must be assess so that the bridge will not fall down a reliability target that ensure the serviceability and safety of the bridge.

The aim of this work is divided into two parts. First of all, gather all the information and research regarding the life-cycle of bridges in civil engineering. Within this part, all the concepts related with reliability in civil engineering are presented. Secondly, a case study based on a possible scenario where two big cities need a connection through bridges is carried out. On this practical case, the analysis of performance of the models series, parallel and series-parallel is carried out along lifetime affected by chloride concentration.

0.1. Motivation

The growth of society and civilizations has always been related with transportation that still enables countries bring people and goods from one point to another. Then, maintain our actual bridges safety in terms of serviceability is required.

According to the American Road & Transportation Builders Association (ARTBA), more than 600,000 are nowadays in service within the United States by 2013. In addition, more than 61,000 are identified as structurally deficient, what means more than a 10% overall the bridges in the country.

Moreover, the Federal Highway Administration (FHWA) in 2013 estimated that 71 billion dollars is needed to address the current deficient bridges. This is an amount of expenditures that really concerns the government to apply maintenance to the actual structural systems.

Finally, this matter is of interest for my advisors, Dr. Joan Ramon Casas and Dr. Dan M. Frangopol, whose main research interest are in the application of probabilistic concepts, structural reliability, probability-based design, life-cycle performance maintenance and management of structures and infrastructures under uncertainty among others.

0.2. Outline of This Document

This document is organized as follows. After this introduction about the aim of the work, an overview of Life-Cycle of structures is carried out in chapter 1, including some aspect concerning the actual civil engineering objectives and what is understood by life-cycle of bridges. In chapter 2, we present the concepts of probability of failure and reliability index, stochastic models to face a real problem where uncertainties are accounted and a brief introduction to optimization and decision making on civil engineering. In chapter 3 the Life-Cycle Management (LCM) and Life-Cycle Cost (LCC) are explained, respectively. The case study is performed in chapter 4 where different scenarios are taken into account. Finally, main conclusions of this work can be found in chapter 5, including a provision for future possible works.

1. Overview on Life-Cycle of Structures

1.1. Introduction to Life-Cycle of Structures

The economic growth and social development of most countries are well dependent on the durable and reliable performance of the structures and infrastructures [Frangopol, 2011]. Structural systems are meant to satisfy the needs of society such as transportation (bridges, subways, railways), water storage (dams, rafts), or coastal defense (dikes, breakwaters), among others. Currently, civil engineering is focusing in a smarter point of view that will make structures more efficient in terms of lifetime and cost. In other words, the more time it is spent in the design stage, will conclude with a better structural system. Analyzing, studying and optimizing crucial phenomena like environmental effects, the time interval of inspections, the reach of the techniques used to detect damage, among others, will give an extra value in terms of reliability. Therefore, optimization is gaining more weight in the field of civil engineering so as the future structures will life more with less expenditure.

In this work, the life-cycle of structures is carried out in bridges. Therefore, the extrapolation of the studies to other structures mentioned above is out of the scope of this document.

Natural hazards like earthquakes or hurricanes, ageing or increased structural performance demand may significantly affect the vulnerability of constructed facilities [Tsompanakis, 2010; Esteva et al., 2010; Casciati and Faravelli, 2010]. These aspects can inflict detrimental effects on the performance of structural systems during their life-cycles. Moreover, an important aspect to take into account is the environment where the structure is located. Structures are generally under progressive environmental deterioration, which limits their service life.

The general concept of life-cycle of structures refers to the integral cycle subdivided into different time steps that any structural system presents. The following picture illustrates the stages of a bridge and its management from the initial of service until the bridge must be replaced:

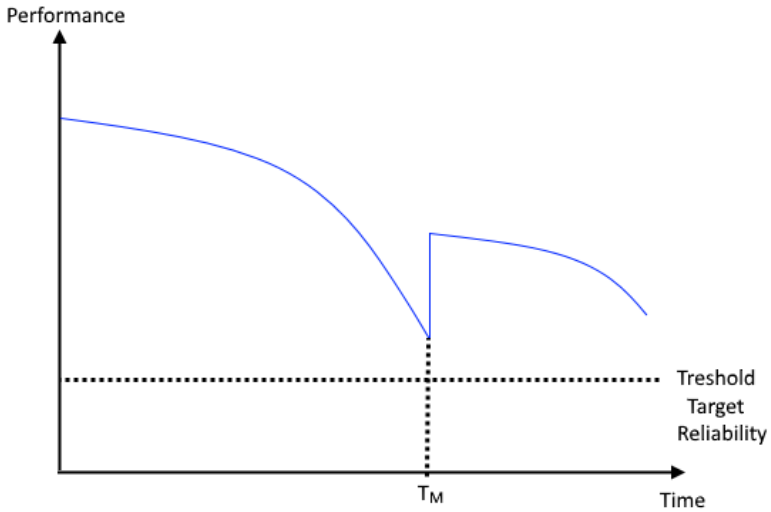

Figure 1: Life-cycle of a bridge [Frangopol, 2011]

Regarding several studies on bridges, fatigue and corrosion have been considered as predominant deterioration mechanisms [Fisher, 1984; Chaker, 1992; Zayed et al., 2002; Schijve, 2003; Chung et al., 2006; NCHRP, 2006]. For that reason, a service life prediction of deteriorating structures must be studied in order to have an optimal design of the bridge. The aim is to have both the optimal lifetime with the optimum expected total cost.
Maximizing the service life and minimizing the life-cycle cost implies a maintenance plan directly linked to the cost of repair and thus, the final total cost. A correct decision in terms of inspections and repair actions must be made.
The Figure 2 shows the optimum cost with the optimum number of inspections. We see how the cost of failure decreases with the number of inspections, while the cost of repairs increases though.

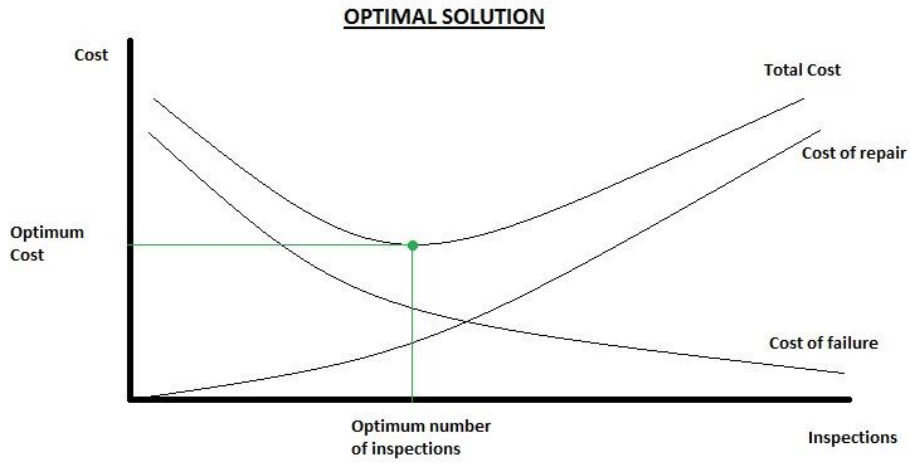

Figure 2: Optimal solution for a bridge management [Frangopol, Yin and Estes, 1997]

1.2. Inspection Methods

Inspections methods are needed to have a complete control of the deteriorating of the structural state. Bridges serviceability is highly dependent on the frequency and quality of the maintenance programs and the methods used to evaluate the damage. Two common techniques in the field of bridges are described herein.

1.2.1. Visual Inspection Method

Visual inspections are basically used to estimate roughly the structural condition of the bridge by a specialist. One of the main disadvantages is that the actual safety level is not explicitly accounted for and that discrete stochastic transitions between condition states fail to account for previous structural performance and prohibit more accurate continuous modelling approaches [Frangopol and Liu, 2007].
For that reason, successful visual inspections do depend on considering all possible damage scenarios, which is not easy to ensure even for an experienced worker.

One the one hand, an advantage is the cost of the method. Visual inspections are carried out when the maintenance plan is required and then, the cost of this technique is less than more complex ones. On the other hand, it has a lack of continuity in front of the other methods.

Thus, a human error or a bad ability of the inspector will not succeed in terms of service life of the bridge. In fact, more than 50% of bridges are being classified incorrectly via visual inspections [Catbas et al., 2008].

1.2.2. Structural Health Monitoring

Structural Health Monitoring (SHM) is a non-destructive testing evaluation that provides great results. In addition, it presents a continuous supervision of the state of the bridge that is being monitored. This gives live and current response data (in time over time) from different sensors located in a smart way on the bridge. There exists no doubt that this method provides better control rather than visual inspections. Also, SHM is meant to reduce uncertainties on the behavior of the bridge and improve the design of future structures.
However the cost of this technique is much more expansive and might be questionable in terms of expected final cost.

2. Reliability in Civil Engineering

The main objective of any structural design is to ensure safety and economy of the structure operating under a given environment. For this purpose, designers always check whether the capacity of the structure exceeds the demand:

$$\text{Capacity (R)} > \text{Demand (Q)}$$

Where capacity is the resistance of the structure and demand is the applied load. As long as this condition is satisfied, the safety of the structure is ensured for the intended purpose for which the structure is built.

However design parameters like geometry, material property, boundary conditions and loads cannot be taken as deterministic elements but random in nature. The uncertainties behind the design of a structure must be considered in order to have a probabilistic analysis of the structure. Therefore, stochastics methods are needed to face this phenomenon of uncertainty.

In the following pictures, a probability density function (PDF) of the demand and the capacity of the structure is represented.

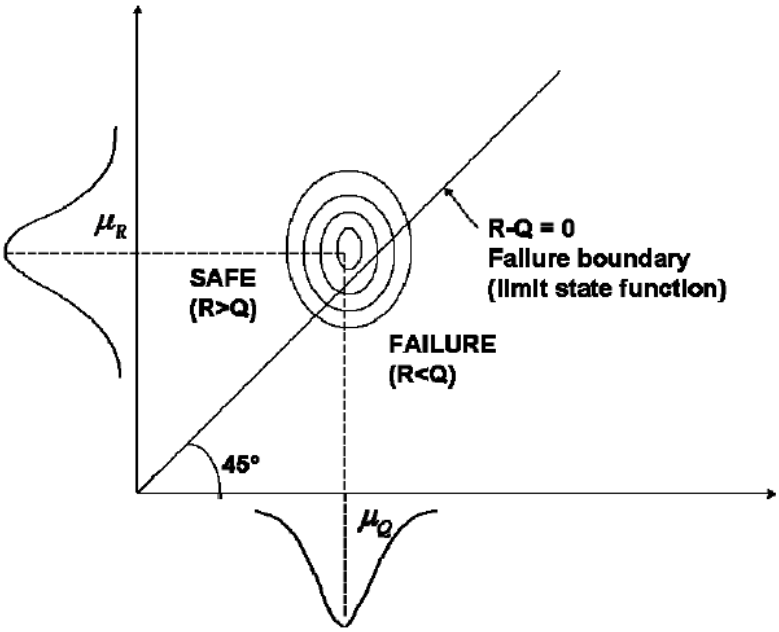

Figure 3: Safe domain and failure domain in 2D space. Source: nptel.ac.in

Here we see how the there is a line, which indicates that the resistance is equal to applied loads, separating between two regions. On the left-upper side the safety is assumed since R>Q and any point belonging to this area will be considered safe. On the right-bottom side the failure appears due to R<Q and then the structure cannot resist the efforts applied.

It can be extended to a 3D plot in which the volume represents the PDF and the safety and failure area is identified through the limit state equation.

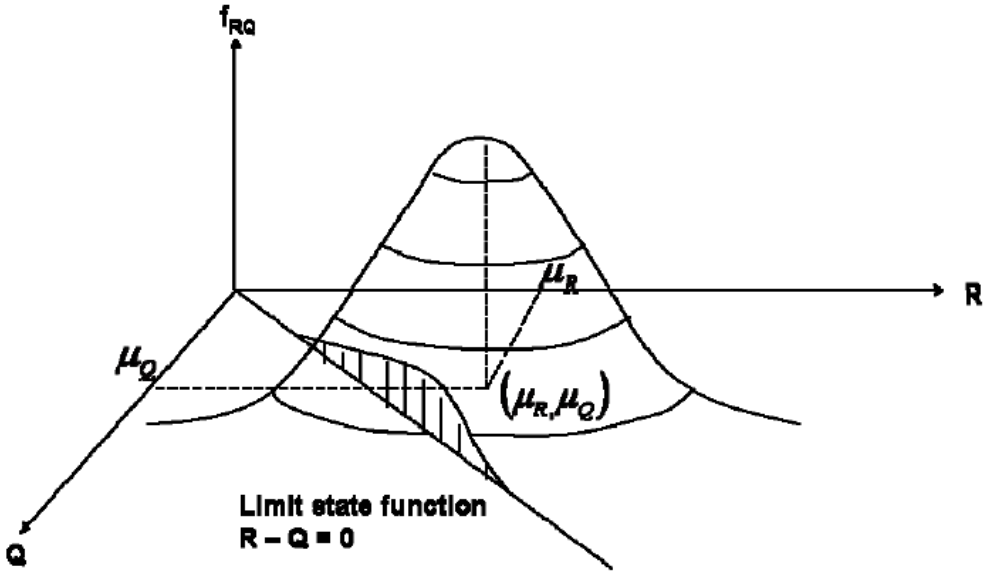

Figure 4: 3D representation of a possible joint density function f_{RQ}. Source: nptel.ac.in

Some techniques are used in order to study and know about the serviceability, reliability and cost of the bridge during his life-cycle [Saydam, Frangopol, and Dong, 2013]. Herein both the reliability based structural and the risk-based performance are introduced. Certain parameters related to the risk on civil engineering such as Probability of Failure (PoF) and Reliability Index (β) are described. From the point of view of safety, β can be understood as a safety indicator and thus, PoF is the inverse.

A detailed explanation, showing different graphs, is done within the scope of this chapter.

Considering these uncertainties, reliability models to solve stochastic problems must be applied to solve the design problem. Some of them are introduced herein such as Monte Carlos Simulation (MCS), First Order Reliability Method (FORM) or Second Order Reliability Method (SORM). They are also well-known as Structural Reliability Methods pretty extended in the field of civil engineering design problems.

Regardless the main objective described as the structural capacity, optimization design is gaining more weight time by time so as the future structures will life more with less expenditure. In addition, since the structural system will get damage due to aging, corrosion, hazards, among others, an optimization maintenance plan must be carried out throughout the lifetime of the structure. For that reason, techniques like genetic algorithms are required in order to find the optimum maintenance. The objective is to know how many inspections or/and repairs must be taken and the optimal time to do it.

Life-cycle cost analysis, obviously, is directly linked to the optimization solution. The more money it is saved at the beginning, the more money it can be spent on the maintenance plan so as to give maximum lifetime for the structural system.

2.1. Risk-Based Performance

2.1.1. Probability of Failure (P_f)

Design of structures is clearly related with the probability of failure. Although in the design process, many safety factors are taken into account in order to decrease the risk of failure, the probability of failure cannot remain 0%. There not exists risk zero when dealing with structures due to the fact that uncertainties in material, loads, resistance or natural hazards are constantly present during the lifetime. In the risk-based structural design, the overall probability of failure (PoF), denoted as P_f plays an important role. The behavior along time is represented in Figure 5.

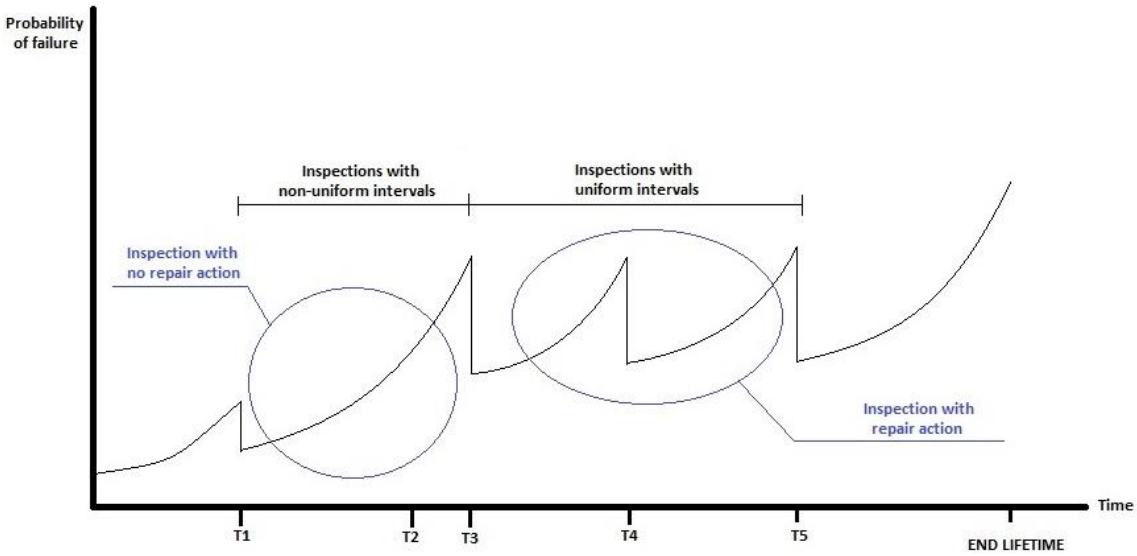

Figure 5: Probability of failure along the lifetime of a bridge [Frangopol, Lin and Estes, 1997]

The intervals of inspections might be uniform or non-uniform depending on the maintenance plan regarding to the optimum solution, which is detailed within this work. As we see, there exist some inspections that required a repair action and some do not. The probability of failure decreases when a repair is done, as expected. Obviously this is due to an improvement of an element in the structure that affects to the global behavior (see 3.3).

Therefore, the probability of failure P_f is defined as the probability of occurrence of the event $R \leq S$ and can be evaluated by solving the following convolution integral [Lu, Luo and Conte, 1994]:

$$P_f = \int_{\{g(x) \leq 0\}} f_X(x)\, dx$$

Where, the random vector X contains all the uncertain basic variables such as the material properties, loads, and model uncertainties parameters; $f_X(x)$ represents the joint probability density function (PDF) of the basic random variables; $g(x)$ is the limit state function corresponding to the failure mode considered and defined such that the failure event corresponds to $g(x) \leq 0$. Notice that this integral is pretty complex to solve due to f_X cannot

be defined and the integral process becomes very difficult and expensive computationally. This is why reliability methods (see 2.3) are carried out to compute and solve the problem.

Assuming that the probability of failure must be studied during the whole life of the structure:

$$P_f = P\{R(t) < S\}$$

Where, the time t ranges from 0 to the end of lifetime. Notice that this approach requires a complex process of integration.
Also, the probability of failure is related to the reliability index, explained beneath this section.

$$P_f = \Phi(-\beta)$$

Where Φ is the standard normal cumulative distribution function (CDF).

The following Figure 6 illustrates the behavior of the PDF when the P_f is studied:

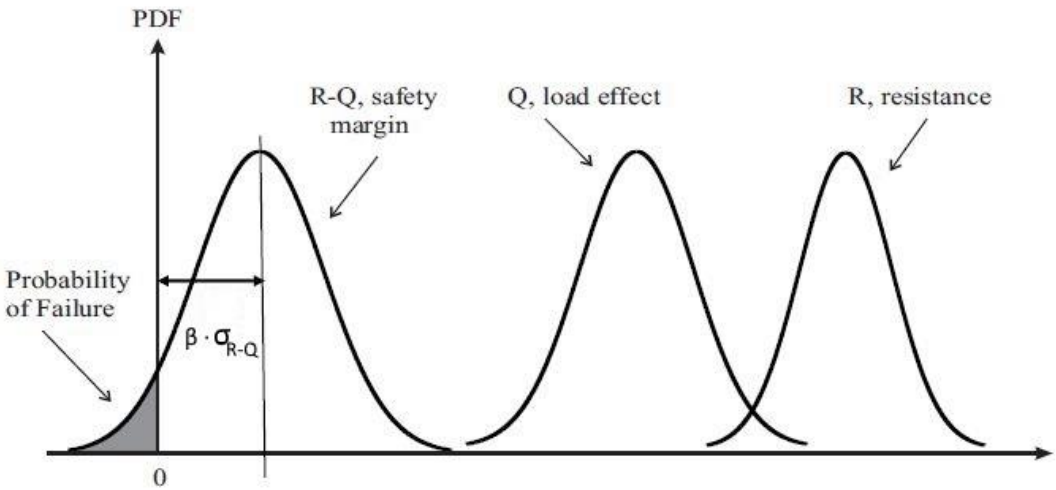

Figure 6: Probability of failure regarding load (Q) and resistance (R) [Nowak, 2007]

Focusing on the relationship between the Pf and β, the next table can be obtained to know how varies one parameter with the other:

P_f	β
10^{-1}	1.28
10^{-2}	2.33
10^{-3}	3.09
10^{-4}	3.71
10^{-5}	4.26
10^{-6}	4.75
10^{-7}	5.19
10^{-8}	5.62
10^{-9}	5.99

Table 1: Probability of failure and reliability index

2.1.2. Probability of Detection (PoD)

In order to not let the P_f increases during the serviceability time, it is crucial to detect the possible damage on the structure such as corrosion or cracking, among others. Then, knowing the probability of detection is important due to the fact that the sooner the damage is known, the less cost it will imply. Therefore, by the time it is repaired, the lifetime of the structure could either suffer slightly changes or being affected seriously.

In the case of fatigue inspection, the quality of an inspection type can be generally expressed by the probability of detection of a given crack size. The probability of detection has been widely used in the probabilistic inspection and repair planning of structures [Chung et al. 2006; Orcesi and Frangopol, 2011]. The probability of detection PoD is [Crawshaw and Chambers 1984]:

$$PoD = 1 - \Phi\left[\frac{\ln(a) - \lambda}{\beta}\right]$$

Where $\Phi(\cdot)$ is the standard normal cumulative distribution function (CDF); λ and β, respectively, are the location and scale parameters of the cumulative lognormal PoD curve. They are dependent on the quality of the inspection type; the parameter *a* is the crack size. Certainly, if $a < a_{min}$, then PoD remains zero since no damage can be detected at that time.

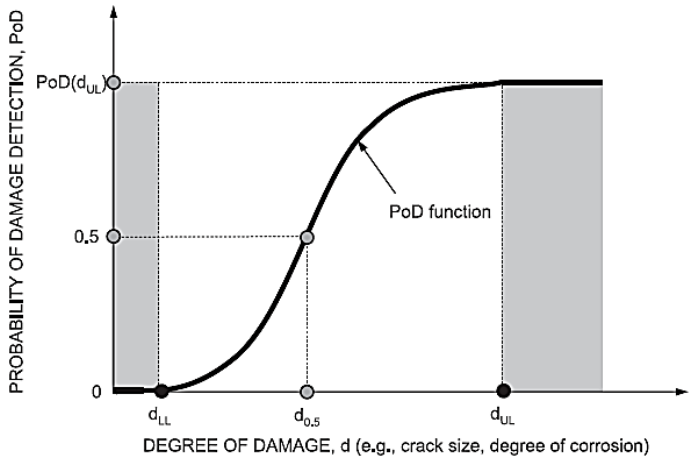

Figure 7: Probability of detection distribution [Frangopol, Kim and Soliman, 2013]

This Figure 7 represents that until the degree of damage reaches the lower limit d_{LL}, the damage cannot be detected by inspection method, whereas if it is larger than d_{UL}, there will be no increase in the PoD with a further increase in the degree of damage. Therefore it really depends on the technique used to detect it. A higher quality inspection method has lower values of d_{LL} and d_{UL}, and probability of detection associated with the upper limit d_{UL} almost 1,0.

2.2. Reliability-Based Performance

2.2.1. Reliability Index (β)

The reliability index β is a parameter used to describe the probability that a component or even a system does not fail.

$$\beta = -\Phi^{-1}(P_f)$$

Where Φ^{-1} is the inverse of the cumulative distribution function of the standard normal distribution function.

$$\beta = \frac{\mu_M}{\sigma_M}$$

Where μ_M are the mean (resistance minus load); σ_M is the deviation (resistance minus load) shown in Figure 6. It is interesting to point out that while lifetime increases, the reliability index decreases and so the mean does. Logically, the probability of failure increases time by time.

When designing structures, we may have a look at the reliability index or the probability of failure. On the one hand, if the reliability index β is too small, there are problems, even structural failures. On the other hand if it is too large, the structure is too expansive. Assuming this, a balanced between the cost and the reliability/safety must be agreed to maximize the lifetime and minimize the total expenses.

This index might be affected by many other parameters taken into account on the optimization analysis such as loads on the bridge, environmental effects, and probability of damage detection, among others. Consequently, the obtained results may vary according to these reliability indices.

The next figure shows the behavior along lifetime of a bridge and how it affects if a maintenance plan is made or not in the structure. Improvements in the reliability index are obtained when repairs are taken and thus, a higher β is performed throughout the life-cycle.

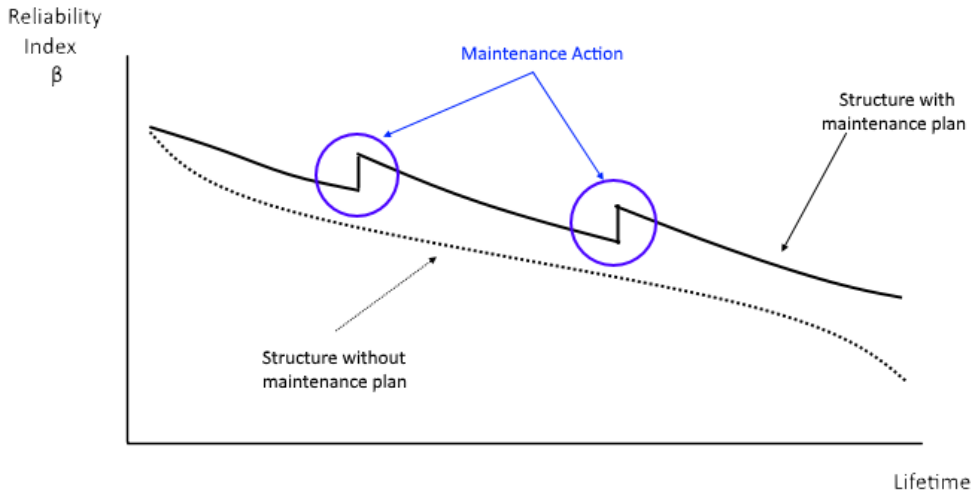

Figure 8: Reliability index vs Lifetime (with and without maintenance) [Frangopol, 2011]

Each single point represented on this curve, comes from a reliability analysis (Monte Carlo, FORM or SORM) with at a certain time t. The graphic obtained is similar to the Figure 3 where a safety region and a failure region are determined. Then by running all the possible cases for the estimated time studied, the whole curve is plotted. A more detailed explanation is done in its corresponding chapter regarding simulation (MC) and approximate methods (FORM/SORM).

2.3. Stochastics Models

It has been explained at the beginning of this work that uncertainties are present on the design of structures. Therefore, deterministic models will not be accurate enough since they do not take into account some randomness in materials, loads or resistance of structure, among others. The stochastic process models assume that the deterioration over time is represented by a collection of random variables [Frangopol, Kalle and Noortwijk, 2004]. Thus, a higher computational cost must be considered [Frangopol and Maute, 2003]. Within this chapter some well-known and widely used stochastics models are described.

2.4. Reliability Methods to Solve Stochastic Problems

2.4.1. Simulation Methods

A simulation model take the form of a set of assumptions about the operation of the system, expressed as mathematical or logical relations between the objects of interest in the system.
The simulation process involves executing or running the model through time, to generate representative samples of the measures of performance. In this respect, simulation may be seen as a sampling experiment on the real system, with the results being sample points.
To obtain the best estimate of the mean of the measure of performance, we average the sample results. Clearly, the more sample points we generate, the better our estimate will be. However, other factors, such as the starting conditions of the simulation, the length of the period being simulated, and the accuracy of the model itself, all have a bearing on how good our final estimate will be.

2.4.1.1. Monte Carlo Simulation (MCS)

It is a simulation technique in which statistical distribution functions are created by using a series of random numbers. This method feature generality, simplicity, and effectiveness on problems that are highly non-linear with respect to the uncertainty parameters to give the decision-maker a fair idea about the probabilities associated with various outcomes. As a result, several predictions of the behavior are obtained. Monte Carlo simulation yields a solution which is very close to the optimal, but not necessarily the exact solution. However, it should be noted that this technique yields a solution that converges to the optimal or correct solution as the number of simulated trials lead to infinity. Finally, engineers make their decision based on the risk they are willing to take to get the outcome the want or need.

This method uses the given density functions to create multiple sets of realizations of all random variables. For each set of realizations, a deterministic analysis of the researched limit state function g(x) is performed. Afterwards, the results are evaluated concerning failure or survival. In order to simplify the description of the analysis results, an indicator function must be specified [Julic et al., 2004]:

$$I\big(g(x)\big) = \begin{cases} 1, & for\ g(x) < 0 \\ 0, & for\ g(x) > 0 \end{cases}$$

Then an alternative of the probability of failure is obtained:

$$P_f = \int_{-\infty}^{+\infty} I\big(g(x)\big) \cdot f_X(x)dx$$

Or in case of a discrete simulation it can be simplified to a finite sum:

$$P_f = \frac{1}{n}\sum_{i=1}^{n} I[g(x_i) < 0]$$

Where *n* is describing the number of simulations and x_i the i-th set of generated realizations.

The most serious drawback is the computational costs, in particular when the reliability level is high, that is the failure probability low. In addition, the accuracy of the estimated results are proportional to $1/\sqrt{n}$. Therefore, an increase of accuracy by one order of magnitude demands an increase execution of discrete simulations by around two orders of magnitude. The main reason is the clustered generation of realizations of the random variables near their mean values. As the demanded failure probabilities in structural engineering are very small, an uneconomic number of simulations have to be performed intending to get good estimations.

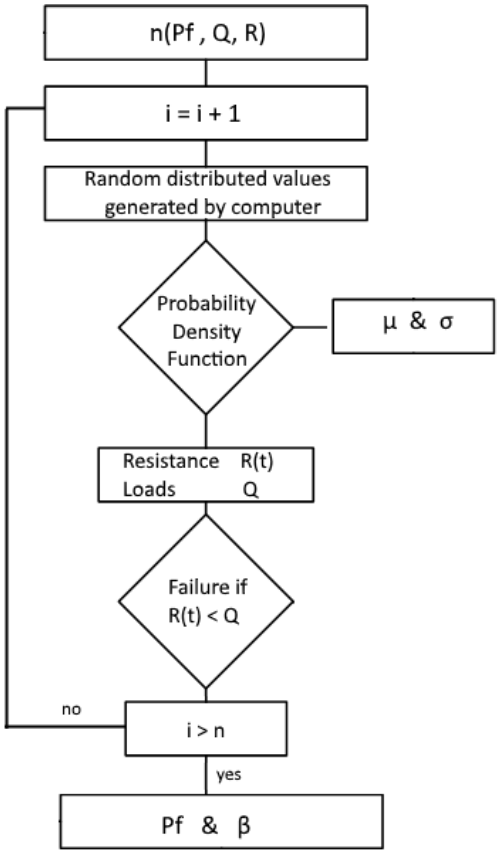

Figure 9: Monte Carlo Simulation flow chart. Source: brighton-webs.co.uk

Moreover, some variations are studied such as Importance Sampling, Stratified Sampling or Adaptive Sampling, in order to arrive faster to the results. More details can be found in some studies [Bucher, 1988; Schuëller, 1998], which are out of this work.

2.4.2. Approximation Methods

Analytical solutions converting the integration into an optimization problem are known as approximation methods. In order to simplify the calculation, the distribution function of the random variables and the limit state equations are transformed into a standardized space.

2.4.2.1. First Order Reliability Method (FORM)

The First Order Reliability Method (FORM) is an analytical approximation in which the reliability index is interpreted as the minimum distance from the origin to the limit state surface in a standardized normal space (u-space), and the most likely failure point (design point) is searched [Zhao, 1999]. Because the limit state function is approximated by a linear function in u-space at the design point, accuracy problems occur when the performance function is strongly nonlinear [Thorndahl and Willems, 2007].

The principle of First Order Reliability Method, denoted as FORM, is to find the probability of a failure of a component in a given system. This method searches the combinations of input values that are most likely to cause failure in a system. For doing so, FORM requires the failure space to be defined discretely. After the choice of outputs and definition of the failure space,

the next step in a FORM analysis is to locate and choose the input variables for the analysis. When the input variables are chosen, these must be associated with probability density functions, and possible correlation between the variables must be derived.
The point on the hyperplane where the failure probability is the highest is labelled the design point. In Figure 10 a theoretical example of a two-variable FORM analysis is shown, and as two variables are applied, the failure surface is a line. The failure surface (or limit state function) in this analysis is defined as:

$$g(x) = R - Q$$

Where greater values than zero means safe, otherwise failure occurs.

And the Probability of failure is denoted as:

$$P_f = \Phi(-\beta)$$

In the following picture is illustrated the search of the point with highest probability of failure.

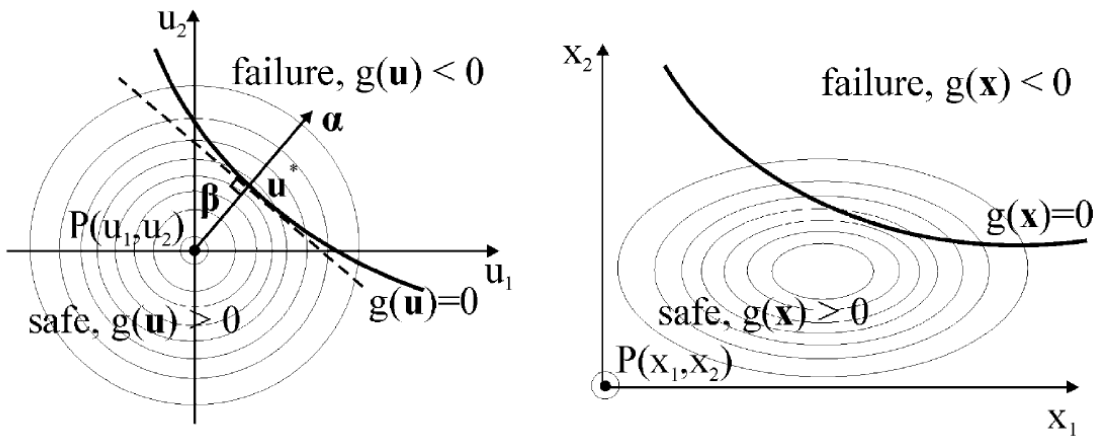

Figure 10: Left: failure-surface and linear approximation in a standard normal space. Right: failure surface in the real space. Source: statisticalengineering.com

Thus, β is the reliability index, which is the minimized distance perpendicular from the linearized failure surface (the point with the highest joint probability density) to the origin in a standard normal space. Represents the point with the largest failure probability, given the probability distributions of *x*, and is a measure of safety.

FORM is based on an iteration procedure in which new values of *u* are calculated until convergence of β and *x* (or *u*) is obtained. This technique employs a linear approximation of the limit state function at the Most Probable Point (MPP).

To sum up with the FORM analysis, the next steps must be followed:

1. Transform the real joint probability density into an equivalent multivariate standard normal density function.
2. Plot the joint probability density of demand and capacity which is now multivariate standard normal with zero means and identity covariance matrix.
3. Partition this probability space into safe and unsafe regions with some suitable *g*-function. Since a *g*-function is often defined in terms of probability of failure, this results in either circular reasoning or an iterative solution.
4. Solve for the point on the *g*-function closest to the origin, which is called the Most Probable Point (MPP).
5. β is defined as distance in standard deviation units from the center of the joint density to the MPP.
6. The transformed multivariate normal density is bisected by that line conveniently producing a univariate normal density which can then be used to assign probabilities.

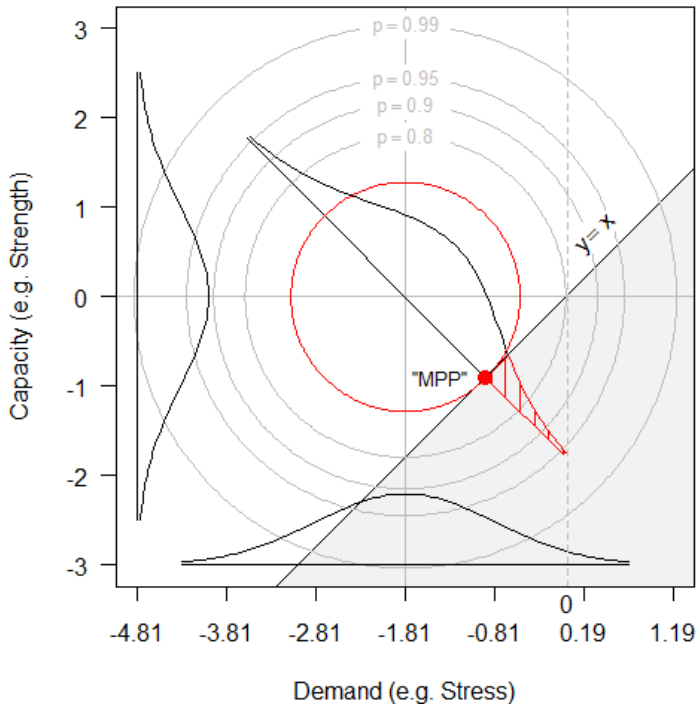

Figure 11: Bivariate normal density (Capacity vs Demand).
Source: statisticalengineering.com

An example of that bivariate normal density function is the Figure 11. It shows the 80%, 90%, 95% and 99% concentric circles of the bivariate normal density. The MPP is indicated by a red dot which is tangent to the limit state equation g(x).The shaded area indicates the failure region where Q > R.

2.4.2.2. Second Order Reliability Method (SORM)

The second-order reliability method (SORM) has been established as an attempt to improve the accuracy of FORM [Zhao, 1999]. This method can be demanding for large problems with a large number of basic variables or where the limit state function is complex. SORM is obtained by approximating the limit state surface in u-space at the design point by a second-order surface.

Thus, SORM features an improved accuracy by using a quadratic approximation and a modified formula to obtain the probability of failure must be used:

$$P_f = \Phi(-\beta) \sum_{j=1}^{k} \left[\prod_{i=1}^{n-1} (1 - \beta \cdot \kappa_i) \right]^{-0.5}$$

Where the parameter:

$$\kappa_i = \frac{\partial^2 y_n}{\partial y_i^2}$$

It is the ith principal curvature of the limit state surface $g(y^*) = 0$. In SORM, the difficult time consuming portion is the computation of the matrix of second-order derivatives. Therefore in practical engineering cases, a first order reliability method is applied. It takes a lot of time finding the design point. Several iterations have to be calculated until the distance measure shows good convergence.

The next figure shows how the PDF of each point of the analysis of the reliability index changes. This is why simulation and approximations methods are run over time. Each blue point is analyzed to obtain the probability density function and then calculate the reliability index. Each PDF has associated a safety and failure area as seen in Figure 11. The points belonging to the green circle must be understood as a safe. At this point the structure has no significant damage and no maintenance is required. The orange ones represent a certain visible cracking and damage in the structure that certain maintenance should be applied such as preventive maintenance so that the reliability index does not go down. Finally, the red circle implies several damage on the structure that may it fall down and an essential maintenance must be done.

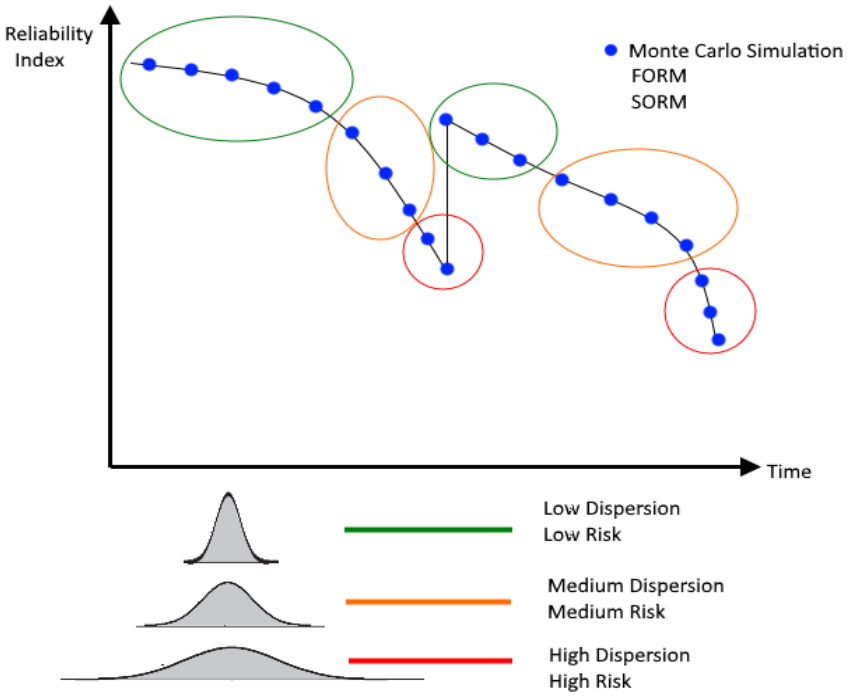

Figure 12: Life-cycle performance of the reliability index over time [Frangopol, 2011]

2.5. Optimization in Engineering

2.5.1. Genetic Algorithms (GA)

Optimization in engineering is gaining weight time by time since the expenditures are getting higher because of the amount of structures built. The aim is to optimize both the cost and lifetime. For this reason, genetic algorithms will allow to minimize the expected final cost and maximize the serviceability of the structure. Regarding the solution of this problem, a maintenance plan will be planned as a result of the optimization, in which the times when inspections/repairs must be done will determine the lifetime of the bridge.

A genetic algorithm (GA) is a search heuristic that mimics the process of natural selection. This heuristic (also sometimes called a metaheuristic) is routinely used to generate useful solutions to optimization and search problems.

The Figure 13 is a flow chart that shows this method in order to assess the optimization of the problem.

First of all, the inputs to be optimized such as the number of inspections and their interval time are introduced. Secondly, the random variables are simulated to know the effects on the maintenance on the service life, in other words, how to optimize the minimum cost and the maximum lifetime. Thirdly, iterations are carried out in the optimization problem until the solution converges.

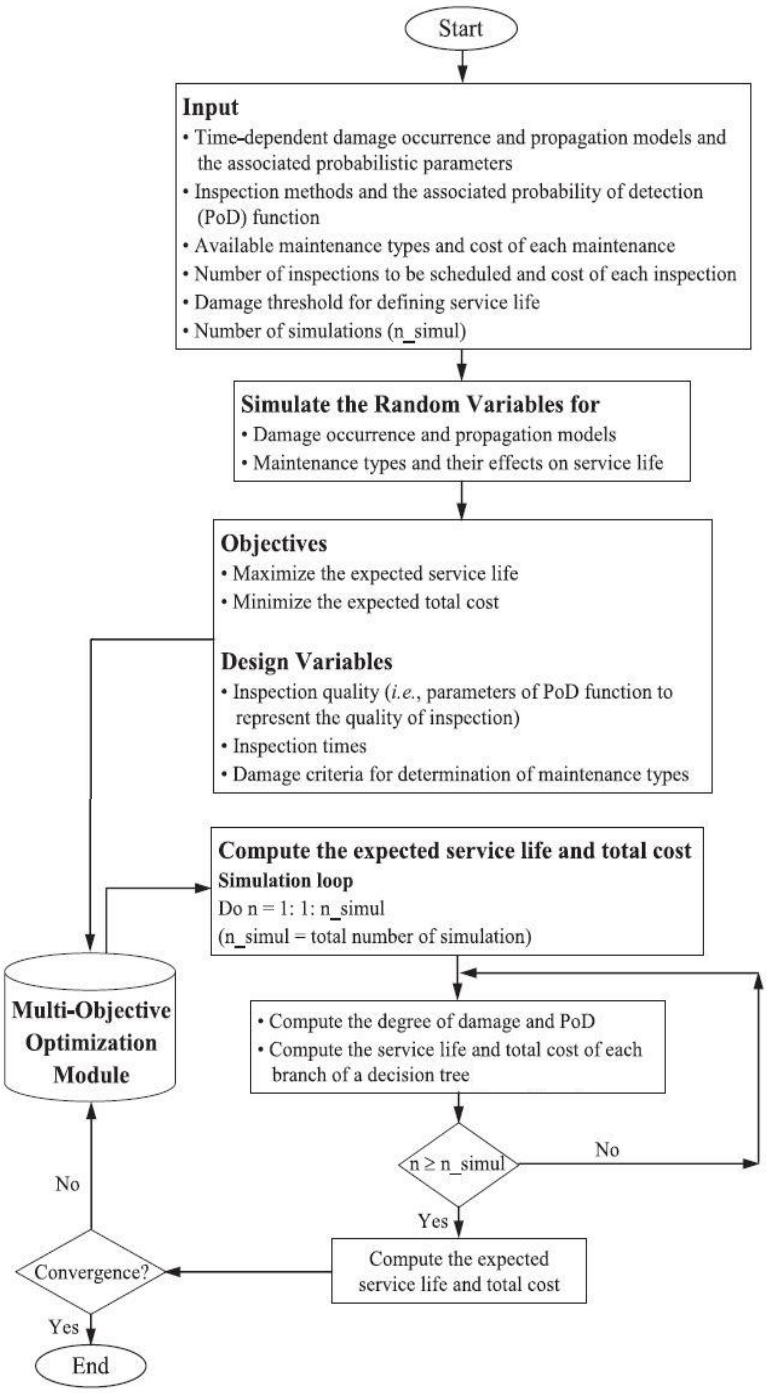

Figure 13: Flow chart of genetic algorithms [Furuta, Kameda, Fukuda and Frangopol, 2003]

2.5.2. Multi-Objective Genetic Algorithm (MOGA)

When dealing with more than one objective, which is the case of studying life-cycle of structures, a multi-objective genetic algorithm is required. The methodology is the same as it is for a unique objective.

Regarding life-cycle of bridges, the objectives to optimize are:

- Minimize Life-Cycle Cost
- Maximize target safety level (in terms of long serviceability)

Once the Multi-Objective Genetic Algorithm (MOGA) is carried out, a set of optimal solutions with a local Pareto front will be obtained. It is performed according to the following steps [Goldberg, 1989]:

- Step 1: Generation of initial populations
- Step 2: Crossover
- Step 3: Mutation
- Step 4: Evaluation of fitness function
- Step 5: Selection

The flowchart of the MOGA process is shown in Figure 13 where the optimization problem is carried out.

For instance, let's assume a given structure that we know the resistance (R) and loads (S) applied. Then, the following function describes the case to optimize:

$$f(t_1, t_2, t_3, R(t), S)$$

Defined for $\beta_{Lifetime}$ of 75 years (according to standards and guidelines). So the statement of the problem is:

$$\begin{cases} \text{Find } t_1, t_2, t_3 \\ \text{s.t.} \\ 1)\ \text{Maximizes Target Reliability } (\beta_{Lifetime}) \\ 2)\ \text{Minimizes Total Cost } (C_{Final}) \end{cases}$$

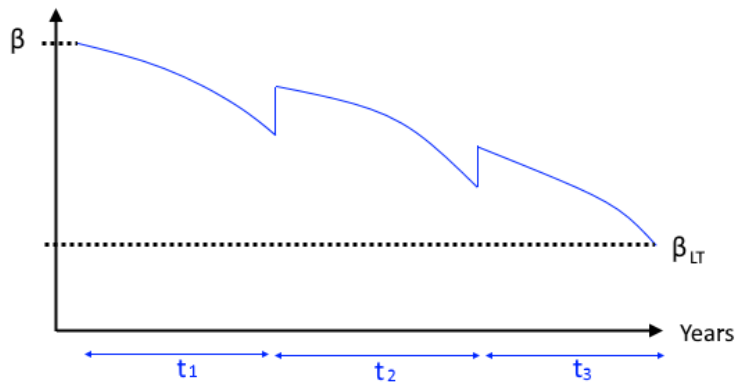

Figure 14: Optimization parameters (Genetic Algorithms)

After this analysis, we will obtain the best optimal solutions in a local Pareto front that will make easy to make a decision in the design of the structure, as it is illustrated in Figure 15.

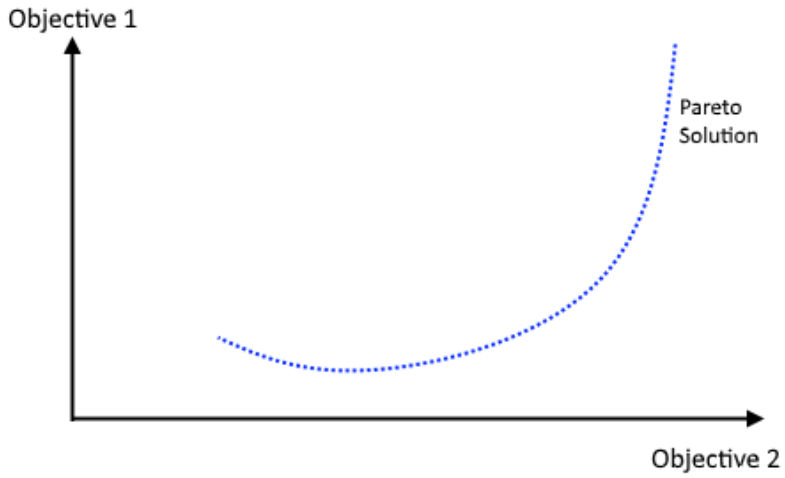

Figure 15: Pareto Solution from MOGA

2.6. Decision Making - Event Tree Model (ETM)

The types of maintenance actions are determined based on the degree of damage, where the service life, life-cycle cost consisting of inspection and maintenance costs, and maintenance delay are formulated using a decision tree model.

2.6.1. Tree Model

The Event Tree Analysis (ETA) is a forward logical modelling technique for both success and failure that explores responses through a single initiating event and lays a path for assessing probabilities of the outcomes and overall system analysis. Therefore this model provides a systematic method to evaluate the repair possibilities related to an uncertain environment of inspections. After each inspection, a decision either to repair or not has to be made and these decisions are affected by actions done in the past.

The following figure shows how to create this tree model assuming 1 as a repair action and 0 as a non-repair action. T_0 indicates when bridge is placed in service and T_1, T_2, …, T_i are the times when an inspection is carried out.

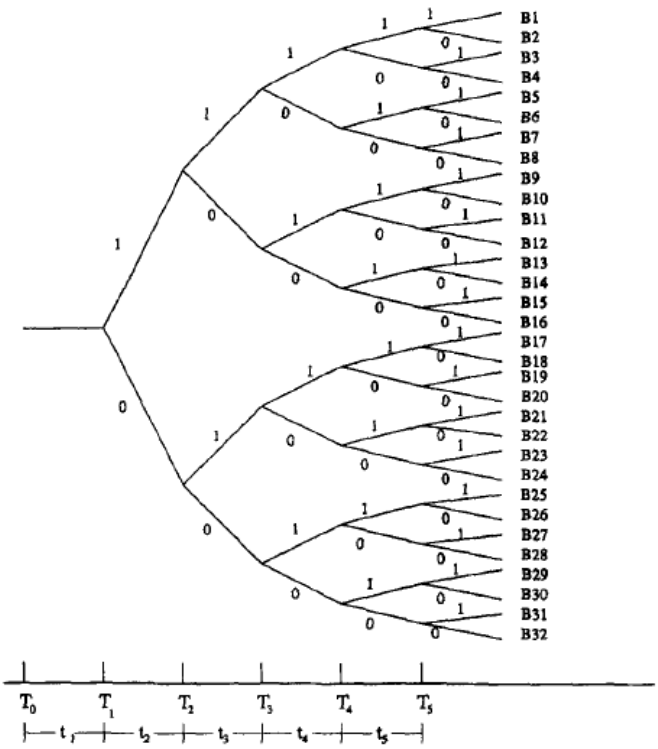

Figure 16: Tree model [Frangopol, Lin and Estes, 1997]

This tree model gives the probabilities associated with the events and the probabilities of failure before and after a decision is made.

There are two different methods to get the optimum number of inspections and repairs [Frangopol, D. M., K-Y. Lin and A.C. Estes, 1997], uniform interval inspection strategy or non-uniform interval inspection strategy.

2.6.1.1. Uniform Interval Inspection Strategy

This method is restricted to uniform time intervals considered in the analysis. Then, only the value of inspections *m* that gives the minimum expected total cost is optimized and designated as m_{opt}. This is the number of inspections to minimize the life-cycle cost of the structure and maximize the lifetime. This method is easier to manage and implies less computational cost.

2.6.1.2. Non-Uniform Interval Inspection Strategy

The objective of this method is to optimize both the number of inspections *m* and the time which inspections are carried out. It gives more efficiency by considering non-uniform time intervals.

$$min \quad Cost = \sum c_i$$

Subject to:

$$Interval = \sum_{i=1}^{m} t_i$$

In contrast with the uniform interval, this strategy gives more accurate results. Therefore, a cheaper solution, regarding the final expected cost, is obtained.

3. Life-Cycle Management

Successful management of deteriorating structures requires the reliable prediction of damage occurrence as well as the time-dependent damage propagation under uncertainty. The reliability of the performance prediction process can be significantly improved by integrating information gained from inspection and monitoring actions. This integration leads to a more accurate prediction of the time-dependent damage level and, eventually, to a better supported decision-making process. The probabilistic approach utilizes a probabilistic time-dependent damage criterion, inspection cost, and failure cost to find the optimum inspection times under uncertainty. New information resulting from inspection actions performed during the lifetime of the structure is used to update the damage propagation parameters as well as the optimization procedure. This process results in an enhanced management plan which can provide managers the ability to make real-time decisions based on inspection results.

The need to provide new approaches for effective fatigue life-cycle management (LCM) of such structures is growing. Effective LCM plans should provide information regarding fatigue critical locations, optimized inspections, monitoring, and repair times [Das, 2000; Frangopol, 2011]. Effective inspection and monitoring actions are crucial aspects in the LCM framework. They provide the useful means to:

a) Reduce uncertainties in the loading and resistance of the structure
b) Indicate the current condition of the structure
c) Detect the possible damaged locations within the structure

Since the results of these actions allow for decision making, the inspection and monitoring times should be optimized to ensure that damage will be detected before causing a significant drop in the structural performance.

Recent research has shown that irregular inspection schedules are more cost-effective than regular inspection plans [Kwon and Frangopol, 2011].

Some techniques are used in order to study and know about the serviceability, reliability and cost of the bridge during his life-cycle [Saydam, Frangopol, and Dong, 2013]. Herein both the reliability based structural and the risk-based performance are introduced.

Time-dependent reliability analysis is necessary to develop optimum strategies for the life cycle management of infrastructure systems [Pandeya, Yuana and van Noortwijkb, 2009].

In structural reliability estimation, the deterioration of structures is usually modelled through random variables (such as the deterioration rate) and the computation is based on the MCS, FORM and SORM, [Pandey and van Noortwijk, 2004], see section 2.3.

3.1. Reliability-Based Performance

The reliability index of a structural system is evaluated based on demand, such as loading effect, and capacity, such as resistance. Both demand and capacity vary during the system lifetime. Therefore, the reliability index varies with time [Kong and Frangopol, 2005]. The variation of reliability index with time is called the reliability index profile. An initial reliability index profile can be obtained by using time-dependent structural reliability methods [Chan and Melchers 1993; Mori and Ellingwood, 1993; Enright and Frangopol 1999]. Efforts to maintain the performance of a system above a minimum prescribed reliability level need inspection, maintenance, and rehabilitation.
This method uses parameters that reflect uncertainties in resistance or loads which must be taken into account in the analysis of the reliability of a structure. The probability of failure and the reliability index are obtained in 2.1.1 and 2.2.1.

The discussed reliability-based inspection scheduling methods plan the inspection based on the predicted reliability profile and the target reliability index; i.e., an inspection is performed when the reliability index reaches the threshold value [Soliman and Frangopol, 2014]. If updating is performed, yielding an updated reliability profile, the next inspection is scheduled when the updated profile reaches the threshold. However, the updating process in this manner does not modify the model parameters based on the inspection outcomes and, therefore, the updated reliability profile may not be representative of the actual damage propagation. Therefore, the results of the updated inspection scheme may be questionable, especially considering the fact that the detected damage level, in most cases, will be different than the predicted one at the inspection time [Zheng and Ellingwood, 1998].

Successful management of deteriorating structures requires the reliable prediction of damage occurrence as well as the time-dependent damage propagation under uncertainty. The reliability of the performance prediction process can be significantly improved by integrating information gained from inspection and monitoring actions. This integration leads to a more accurate prediction of the time-dependent damage level and, eventually, to a better supported decision-making process. The probabilistic approach utilizes a probabilistic time-dependent damage criterion, inspection cost, and failure cost to find the optimum inspection times under uncertainty. New information resulting from inspection actions performed during the lifetime of the structure is used to update the damage propagation parameters as well as the optimization procedure. This process results in an enhanced management plan which can provide managers the ability to make real-time decisions based on inspection results.

3.2. Risk-Based Performance

Risk is usually defined as the product of the failure probability with the consequence of failure. Hence, the highest risk items will be those with both a high failure probability and severe consequences. The probability of a bridge component to be in a specific condition state is time-variant. The risk associated with the failure of a component is the sum of the risks associated with the failure of this component in various condition states. Therefore, the total risk associated with component failure increases as the component deteriorates.

This method does take into account the outcome of a failure event in terms of economic losses, in contrast with the reliability-based performance. Hence, the methodology integrates the probabilities of structural failure and the events that cause the failure, with the consequences of the failure. Since the objective is to minimize the expected final cost, this indicator is essential in the study of life-cycle of structures. Thus, combining the reliability (or probability of failure) with economic consequences will enable to manage the structural system in a smart way. This approach is aimed to be useful in decision making regarding the design and maintenance of deteriorating structures. When an optimal maintenance plan is required, the efficiency of the resources must be carried out to minimize the impact on the society.

3.3. Maintenance

There are two main types of maintenance actions that could affect the reliability index profile: preventive maintenance and essential maintenance, as indicated [Frangopol and Das, 1999].

3.3.1. Preventive Maintenance

Preventive maintenance actions such as painting, saline treatment, and minor repairs are undertaken when the reliability index is above its target value (minimum acceptable reliability level). In general, preventive maintenance delays the deterioration process and, therefore, reduces the rate of reliability index deterioration over a period of time.

3.3.2. Essential Maintenance

Essential maintenance actions such as major repairs and replacement of damaged members are normally undertaken when the reliability of the structure has fallen in the vicinity of the target value. The purpose of essential maintenance is to substantially improve the reliability and condition. In general, deteriorating structural systems experience multiple maintenance interventions during their lifetime.

The reliability index (see 2.2.1) profile associated with maintenance including the effects of all maintenance actions over a given time horizon is obtained by superposition as follows:

$$\beta_j(t) = \beta_{j,0}(t) + \sum_{i=1}^{n_j} \Delta\beta_{j,i}(t)$$

Where n_j is the number of maintenance actions associated with reliability index profile j, $\beta_{j,0}(t)$ is the reliability index profile without maintenance, and $\beta_{j,i}(t)$ is the additional reliability index profile associated with the i-th maintenance action. Then the reliability index is being updated after actions made on the structure.

It is important to point out that the maintenance cost is often considered as fixed and independent of both the state of the structure and the effect of a maintenance action on

reliability and/or condition of the structure. However, the cost of maintenance depends not only on the type of maintenance action, but also on the reliability and/ or condition state of the structure before and after its application.

The scenario associated with preventive maintenance is more economical than the one associated with essential maintenance beyond a certain time horizon [Kong and Frangopol, 2003].

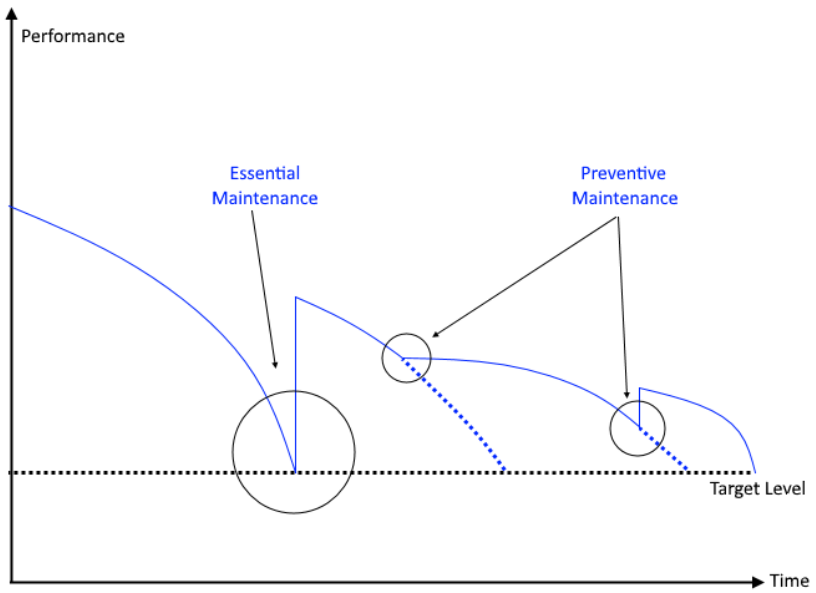

Figure 17: Type of maintenance on bridges [Frangopol et al., 2001]

3.4. Life-Cycle Cost Analysis

So far, initial construction cost has been considered as the most dominant economic factor. There has been no total insight that accounts for the maintenance cost. Recently, Life-Cycle Cost (LCC) has been paid attention as a powerful and useful tool to achieve a rational maintenance program [Frangopol and Furuta, 2001]. Since it is important to have a maintenance program, there is a need to develop a cost-effective decision-support system for the maintenance of existing infrastructures. Life-Cycle Cost is a useful concept in reducing the overall cost and achieving an appropriate allocation of resources [Frangopol and Furuta, 2001]. In general, LCC optimization consists in minimizing the expected total cost which includes the initial cost involving design and construction, routine or preventive maintenance cost, inspection, repair and failure cost [Furuta, Kameda, Fukuda and Frangopol, 2003]. Nevertheless, the optimal strategy obtained by LCC optimization might be slightly different according to the safety level and required service life.

In the following figure, it is shown the different costs mentioned and the relationship with the number of inspections on the lifetime of a bridge.

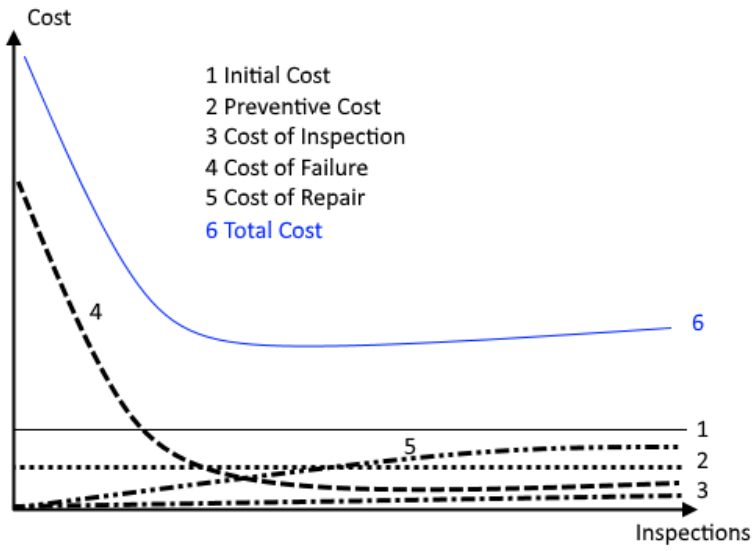

Figure 18: Cost vs number of inspections [Frangopol, Lin and Estes, 1997]

Regarding the economic and safety plan from the viewpoints of total economy, it implies a strategy with the minimum LCC. However, there still remain several problems to calculate the LCC, such as prediction of deterioration or the effects of repair, among others.
One problem associated with life-cycle cost estimation, as currently implemented, is the fact that the cost of repair and retrofit of the bridges associated with natural hazards, particularly for future earthquakes, is not taken into consideration in the estimation [Chang and Shinozuka, 1996]. While previous studies have examined risk deriving from parameter uncertainty [Flanagan et al. 1987], risk from natural hazards has been relatively neglected in the development of life-cycle cost frameworks.

Developing reliable LCC evaluation models is essential for establishing cost-effective management systems for deteriorating structures. In general construction, inspection, maintenance, and failure costs are essential for the LCC analysis of deteriorating structures [Chang and Shinozuka, 1996; Ang and De Leon, 1997; Frangopol et al. 1997, 2001; Ang et al. 1998a, b; Frangopol and Das, 1999; Maunsell, 1999; Das, 2000; Frangopol and Kong, 2001].
Many uncertainties are usually associated with inspection and maintenance expected total cost. The main purpose of life-cycle cost analysis is finding the optimal maintenance scenario for an individual or a group of similar deteriorating structures over a specified time horizon.
It is interesting to notice that interaction between the effect of maintenance intervention on system reliability and corresponding cost should be considered in life-cycle cost analysis of deteriorating structures. The main advantages of using cost functions are the flexibility and expandability required for general purpose reliability-based structure management systems.

According to the optimum maintenance plan and decisions made, the life-cycle cost at time horizon changes significantly depending on the characteristics of cost functions. Variation of intervention costs according to time-varying reliability of systems has significant effect on optimum life-cycle cost solution [Kong and Frangopol, 2004]. Life-cycle cost analysis of deteriorating structures has to consider not only time-varying resistance and loading affecting the reliability of these structures but also maintenance interventions applied during their lifetime [Kong and Frangopol, 2004]. There is a clear relationship between maintenance intervention cost and the effect of the intervention on system reliability. However, this relationship is not included in modern structural management systems since costs of interventions are prescribed as fixed values independent on their effects on system reliability.
There are two important additional issues in life-cycle cost analysis:

- The loss of the service time during inspection/maintenance activities
- The failure cost

In general, the loss of the service time is converted into the user cost. Failure cost is an important parameter for the optimization of the life-cycle analysis of inspection and/or maintenance scenarios.

One of the major deficiencies of the classical management methods of civil infrastructure systems is the cost evaluation procedure. Inspection, maintenance, and rehabilitation costs invested during the lifetime of a system should be closely related to the reliability index profile of the system and interaction between cost and reliability should be considered. For instance, the cost of maintenance depends not only on the maintenance type but also on the reliability level when the maintenance is applied, the improvement of reliability due to the applied maintenance, and the effective duration of the maintenance action. Conversely, the reliability index profile is also influenced not only by the maintenance cost spent but also by the time when the maintenance is applied.

4. Case Study: Analysis of the Performance of the Models Series, Parallel and Series-Parallel along Lifetime of Bridges Affected by Chloride Attack

4.1. Case Study I

4.1.1. Objective

The objective of this case study is not to design a bridge with its different structural components such as a deck, girders or piles among others, but to study the reliability of the structural system. What is supposed to analyze is the behavior of the design models: series, parallel or a combination. Once the problem is exposed, the reliability index and the probability of failure over time are obtained in a probabilistic analysis, where the uncertainties take part of the problem.

The next sections describe the environment of this scenario. The structural analysis of the bridge (strength, strains, and stresses) is out of the scope of this case study.

First of all, different scenarios are exposed in order to study the different models: series, parallel or series-parallel. They are intended to show a possible real case where going from one city to another is only possible through bridges that allow people crossing.

Secondly, the equivalent of theses bridges are performed as simply-supported beams. All the mechanical and material parameters are described herein. The elements are designed to flexure and the reinforcement is calculated with the American Concrete Standard (ACI-318-08).

Thirdly, the corrosion by chloride attack is taken into account in the problem. For that, a Monte-Carlo simulation is carried out to study the time initiation and how it affects to the reinforcement of the concrete. The analysis overtime in the remaining size of the rebar and the nominal strength moment are numerically obtained with Matlab.

Fourthly, the reliability analysis of the bridge throughout the life time is calculated. Therefore, both the reliability index and probability of failure are accounted for this section.

Finally, the results and plots are exposed so that conclusions and future works are explained in further chapters.

4.1.2. Scenario of the Problem

First of all, in order to model the different systems, a hypothetical scenario must be presented.

It is supposed that there exist 2 different cities, A on the left and B on the right side. Both are separated by a big river which does not allow the citizens crossing from one point to another, unless one or more bridges are connecting these cities.

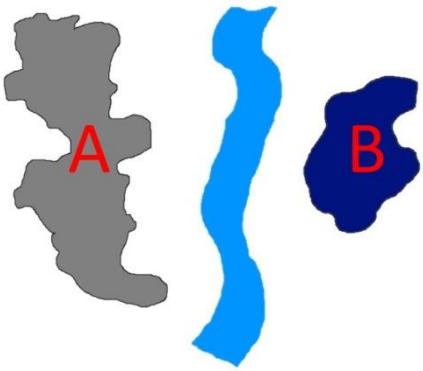

Figure 19: Scenario with 2 cities to connect

The different possibilities to go from the *A city* to the *B city*, which can be modelled as mentioned before, are the following:

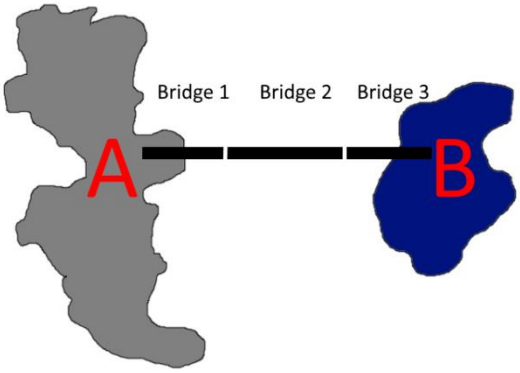

Figure 20: Scenario simulating series model

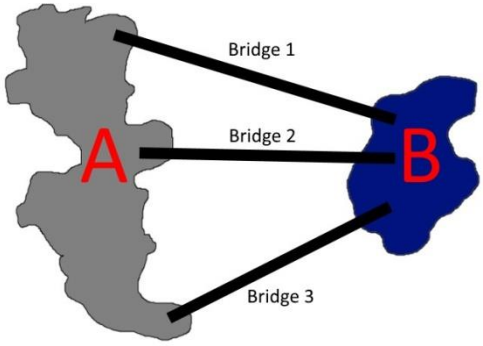

Figure 21: Scenario simulating parallel model

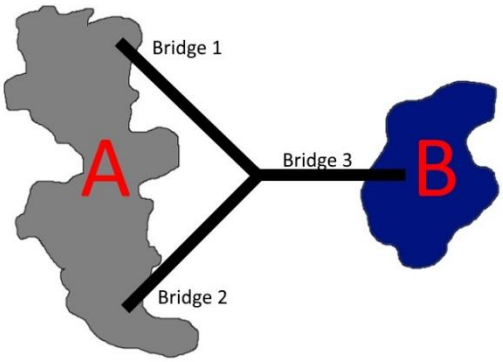

Figure 22: Scenario simulating series-parallel model

The first figure represents a series model where the connection between the two cities is uniquely through 3 bridges (B1, B2 and B3). From now on, this is called scenario number 1.

The second figure represents a parallel model where the connection between the two cities can be done, at least, by one of the three bridges (B1, B2 or B3). From now on, this is called scenario number 2.

The third figure represents a series-parallel model where the connection between the two cities has to be done by one of either B1 or B2 and afterwards B3. From now on, this is called scenario number 3.

As explained at the beginning of this section, the analysis of internal efforts, stresses or strains are not the aim of this work, then those bridges are represented as simply-supported beams. Loads and self-weight are taken into account to create a failure mode where the bridge remains safe or unsafe and will affect directly to the study of reliability and probability of failure of the entire system, which will be developed within this chapter.

4.1.3. Design of the Beam

The beam that simulates the bridge that allows crossing from the *Grey city* to the *Blue city* is designed according to the American Concrete Institute Code. The Standard [ACI, 2008] is called *Building Code Requirements for Structural Concrete (ACI 318-08)* and deemed to satisfy ISO 19338:2007(E).

4.1.3.1. Structural Demand

The Standard aims to design the beam according to the Ultimate Limit State where the reinforcement has to be determined so that the structural capacity (resistance) is greater than the loads applied (demand), as explained in chapter 2 :

$$Resistance > Demand$$

The concrete design theory for the Ultimate Limit State is defined as:

a) Flexure (bending moment)
b) Shear (shear strength)
c) Compression (axial strength)

In case of a simply-supported beam subjected to live-loads and self-weight, the greatest effort to supply is the bending moment (flexure). Thus, the beam is designed to resist the effort such that:

$$Mn > Mu$$

Where Mn is the nominal moment, in other words, the admissible bending moment given a cross-section; Mu is the ultimate moment or the highest bending moment applied on the beam by the designing loads.

The steps to design each beam according to the ACI 318-08 (SI units) are the following:

4.1.3.2. Design the Midspan Section of a Beam

The design bending moment at this section is located in the middle of the beam and takes the value of:

$$M_{u,max} = \frac{qL^2}{8}$$

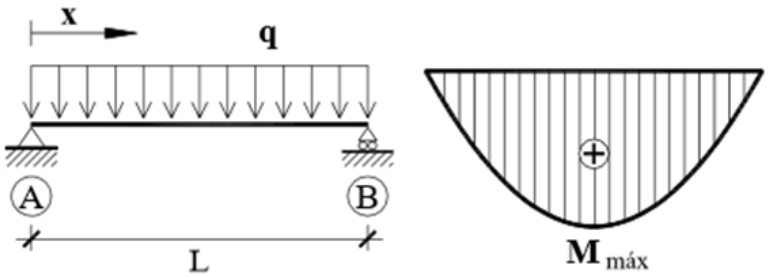

Figure 23: Distribution of the bending moment along a simply-supported beam

Where q is the total load applied on the beam, understood as the sum of the self-weight and the traffic loads; L is the length of the beam or span. According to the ACI 318-08, the minimum load for design is HS 15-44. In this work, both the HS-15-44 and HS-20-44 are used which are the highest possible truck loads. The next picture illustrates how these loads are distributed:

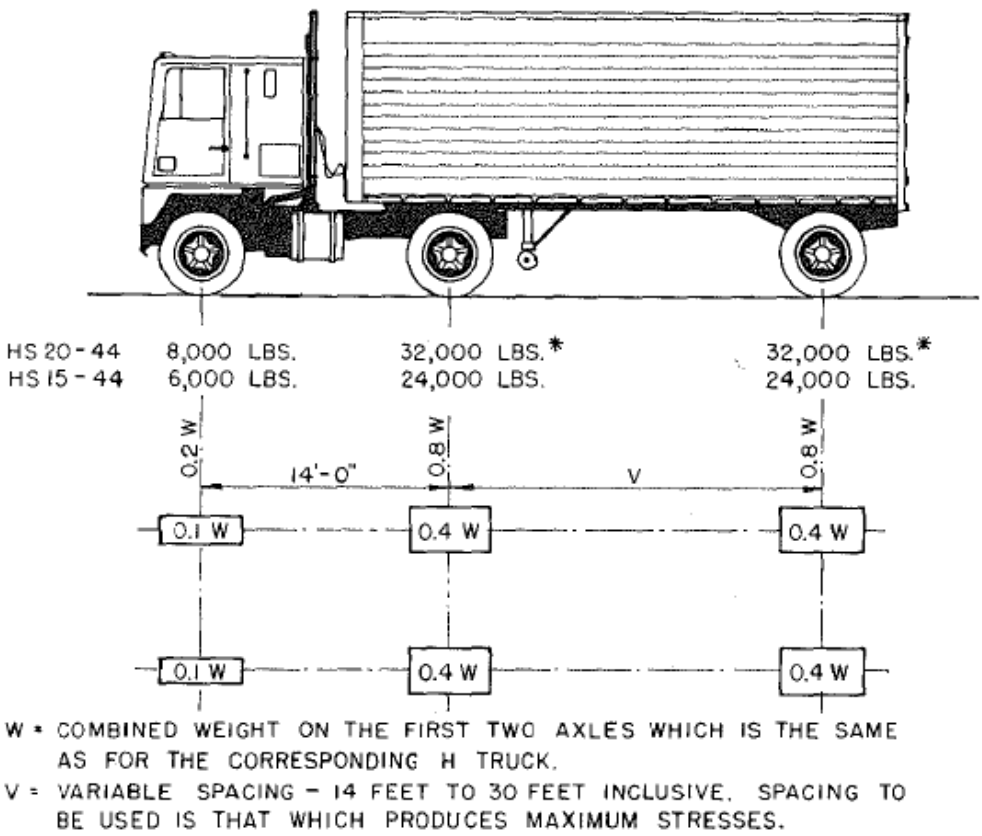

Figure 24: Traffic Loads according to the AASHTO Standard [AASHTO, 2014]

4.1.3.2.1. Structural Capacity

Because this is a rectangular cross-section in positive bending moment, we initially assume a moment arm, *jd*, equal to 0.95*d* (wide compression zone). Although we will use a double layer of reinforcement, there is no big importance in between both, so *d* can be taken as h-63.5mm.

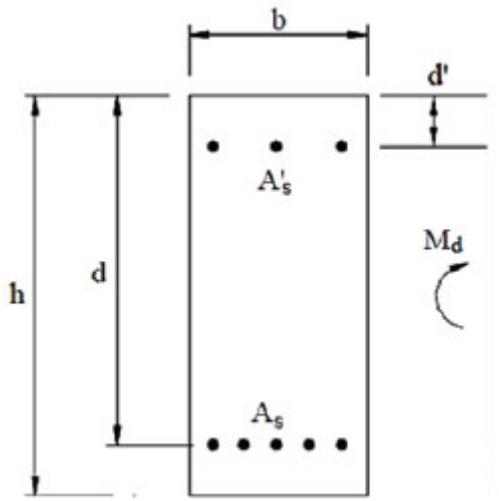

Figure 25: Sample of a rectangular cross-section and parameters

Assuming that this will be a tension-controlled section (ϕ=0.9), we will use the next equation to get the first estimate for the required area tension of steel:

$$A_s \geq \frac{M_u}{\phi f_y (jd)}$$

This area of steel obtained, must be greater or equal to the minimum reinforcement:

$$A_{s,min} = \frac{1.4 \cdot b_w \cdot d}{f_y}$$

Also, the minimum reinforcement for the compression zone, on top of the beam must be:

$$A'_{s,min,comp} = \frac{1}{2} A_s$$

Now, an iteration to improve the value of A_s is required. To determine the depth of the compression stress block, a, we must determine the effective width of the compression zone. Referring to ACI Code Section 8.12.2, the limits for the effective width of the compression flange is:

$$b_e \leq \frac{beam\ spam\ length}{4}$$

Then we will use a compression zone width:

$$a = \frac{A_s \cdot f_y}{0.85 \cdot f_c' \cdot b}$$

Finally, recalculating the value of the reinforcement area:

$$A_s \geq \frac{M_u}{\phi f_y \left(d - \frac{a}{2}\right)}$$

Also, dividing into the area of a single reinforcement bar:

$$\#_{rebars} = \frac{A_s}{A_{s,rebar}}$$

4.1.3.2.2. Strength Check

Once the structural demand and structural capacity are calculated, the final check is to confirm the strength of the section including the reduction factor, ϕ:

$$\phi M_n = \phi A_s \cdot f_y \cdot \left(d - \frac{a}{2}\right)$$

$$M_u = \frac{PL^2}{8}$$

Then, we check that:

$$\phi M_n \geq M_u$$

4.1.4. Deterioration Due to Corrosion

Under normal circumstances the reinforcing steel would not corrode. However, when concrete and steel join each other, a protective passivation layer of gamma iron oxide is formed because of a chemical reaction between oxygen and highly alkaline environment concrete [Estes, 1997]. This high pH environment, if left alone, will keep this protective film stable [Takewaka and Mastumoto, 1988]. Corrosion might be more aggressive as the same time that the chloride concentration does.

The more common cause of increased acidity is through chloride ion penetration, most commonly the result of road salts to prevent freezing on the roadway during the winter months. The increased acidity breaks down the passivation layer and corrosion is initiated as the reinforcement is exposed to oxygen and moisture which have penetrated the microscopic cracks present in all concrete [Lin, 1995]. Since practical experience [Thoft-Christensen et al. 1997] has shown chloride penetration to be a larger problem than carbonation, this study will focus on chloride penetration.

4.1.4.1. Corrosion Formulation

The chloride concentration at the reinforcement level must reach a minimum concentration before the corrosion process will start. This creates a two-step process. The first step is the penetration of chlorides from the concrete surface over time until a sufficient chloride concentration C_{cr} has built up at the reinforcing steel. The rate of chloride penetration into concrete as a function of time t and space x has been shown to follow Fick's second law of diffusion [Takewaka and Mastumoto, 1988]:

$$\frac{\partial C_{x,t}}{\partial t} = \frac{D_c \cdot \partial^2 C_{x,t}}{\partial x^2}$$

Where $C_{x,t}$ is the chloride concentration at distance x from the surface at time t, and D_c is the chloride diffusion coefficient. Assuming that the concentration of the chlorides at the surface is constant, the solution to the equation is:

$$C_{x,t} = C_0 \cdot \left[1 - erf\left(\frac{x}{2\sqrt{D_c \cdot t}}\right)\right]$$

Where C_0 is the equilibrium chloride concentration on the concrete surface as a percent weight of the cement and *erf* is the error function.

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-x^2}dx = \frac{2}{\sqrt{\pi}}\left(\frac{1}{0!}\frac{x}{1} - \frac{1}{1!}\frac{x^3}{3} + \frac{1}{2!}\frac{x^5}{5} - \frac{1}{3!}\frac{x^7}{7} + \cdots\right)$$

This function is also available in Matlab toolbox for users.

4.1.4.2. Corrosion Initiation Time

In a time dependent study, the variable of interest is the corrosion initiation time T_I which is the amount of time between the application of the surface chloride and the onset of corrosion (which occurs when the critical chloride concentration C_{cr} is reached) at some distance x from the surface. The corrosion initiation time $T_I$can be expressed as [Thoft-Christensen et al. 1997]:

$$T_I = \frac{d - D_I/2}{4D_c} \cdot \left(erf^{-1}\left(\frac{C_{cr} - C_0}{C_i - C_0}\right)\right)^{-2}$$

Where *d* is the concrete cover and D_I is the initial diameter of the reinforcement bar. C_i is the initial chloride concentration at the distance x which for this study will be assumed to be zero. For a specific distance from the surface x, the time t was iteratively varied until the chloride concentration at the point in question $C_{x,t}$ is equal to the threshold or critical chloride concentration C_{cr} that will initiate corrosion. C_0 is the Chloride concentration on the surface. The time t when this occurs is the corrosion initiation time T_I.

The second step in the process is the actual corrosion of the steel reinforcement. Once corrosion has started, then the diameter of the reinforcement bars as a function of time $D_I(t)$ is modeled by:

$$D_I(t) = D_I - C_{corr} \cdot i_{corr}(t - T_I)$$

Where D_I is the initial diameter of the reinforcing bar, C_{corr} is a corrosion coefficient which for this work is estimated to be C_{corr} = 0.0203, and i_{corr} is a parameter related to the rate of corrosion. Therefore the area of the steel reinforcement $A(t)$ at any time t is quantified as:

$$A(t) = \begin{cases} nD_i^2 \frac{\pi}{4} & \text{for} \quad t \leq T_I \\ n\big(D(t)\big)^2 \frac{\pi}{4} & \text{for} \quad T_I < t < T_I + D_i/(0.0203 i_{corr}) \\ 0 & \text{for} \quad t \geq T_I + D_i/(0.0203 i_{corr}) \end{cases}$$

Where $\mathrm{T_I + D_I/(0.0203 i_{corr})}$ becomes an upper limit time beyond which the reinforcing bar is assumed to provide no strength to the structure. This corrosion process is highly uncertain and a reliability analysis requires the introduction of many new random variables. One option is to include all of these random variables into the existing limit state equations.
This would be very difficult since the corrosion of the steel is a function of the corrosion initiation time T_I which is itself computed from the inverse error function which is also a function of random variables. The limit state equation can be solved and the uncertainty in the corrosion process can be included if an approximate distribution for the area of steel $A(t)$ at any time can be found.

The random variables used for this time-based reliability analysis and their associated values and distributions are taken from [Thoft-Christensen et al., 1997] and are shown the next table:

Random Variable	Units	Description	Values [μ; σ]	Source
$\mathrm{C_0}$	%	Chloride concentration on the surface	$\mathrm{N}\ [1.08; 0.072]$	Estes, 1997
$\mathrm{D_c}$	mm2/s	Diffusion coefficient	$\mathrm{N}\ [35; 2.49]10^{-5}$	
x	mm	Distance to reinforcement	$\mathrm{N}\left[\mathrm{d} - \frac{\mathrm{D}}{2}; 0\right]$	
C_{cr}	%	Critical chloride concentration	$\mathrm{N}\ [0.4; 0.05]$	
$\mathrm{D_I}$	mm	Initial diameter of bar	$\mathrm{N}\ [\mathrm{D}; 0]$	
i_{corr}	mm/year	Corrosion Parameter	$\mathrm{N}\ [78.9; 9.18]10^{-9}$	

Table 2: Random variables and the values

A Monte-Carlo simulation with a sampling of $\mathrm{np} = 10^6$ is carried out in order to know the behavior of the random variables. Also a plot of a histogram is needed to analyze the skewness of the data once the simulation is run. Finally, the initiation time (T_I) with its mean and standard deviation is obtained.

4.1.5. Reliability Analysis

The main purpose of this work is study the reliability of the system taking into account the uncertainties of the random variables so that a realistic case can be carried out. When dealing with this problem, the following inputs must be defined:

a) Probabilistic inputs: here we find the n random variables with its mean and standard deviation.
b) Correlation matrix: the correlation between inputs is used to describe statistical relationships involving dependence of the *n* variables.
c) Limit state equation: the criterion that defines if the structural system is safe or unsafe is determined by this equation that fulfills when $g(x) = 0$.
d) Analysis type: depending on the structural system, a component analysis for each part of the bridge or a system analysis might be carried out.
e) Solution method: as explained in section Stochastics Models2.3, there are several methods such as First Order Reliability Method (FORM), Second Order Reliability Method (SORM), Monte-Carlo Simulation or Genetic Algorithms. A personal decision must be made according to the computational cost and the needs.
f) Loop over time: the aim is to study the behavior of the structural system along the lifetime, which for this case is 75 years.
g) Reliability Index: as explained in section2.2.1, one of the main purposes of this work is to know at any time this index and how it varies along the time.
h) Probability of failure: as explained in section 2.1.1__, one of the main purposes of this work is to know at any time this index and how it varies along the time. This is directly related with the reliability index.

4.1.5.1. Probabilistic Inputs

These parameters are essential to proceed with a probabilistic analysis of the problem. Herein all the inputs with its mean and standard deviation are introduced. It is possible to separate mechanical inputs from the material, loads, corrosion or timing.

Point out that the initiation time will depend on the case to stud because variables related to the cross-section have influence on that.

The next table shows the information related to them:

Random Variable	Units	Values [μ; σ]	Type
C_0	%	N [1.08; 0.072]	Environmental
D_c	mm2/s	N $[35; 2.49]10^{-5}$	
C_{cr}	%	N [0.4; 0.05]	
i_{corr}	mm/year	N $[78.9; 9.18]10^{-9}$	
f_y	N/mm2	N [386; 42.5]	Material
f_c'	N/mm2	N [19; 3.45]	
ρ_s	N/mm3	N $[7.85; 0.785]10^{-5}$	
ρ_c	N/mm3	N $[2.4; 0.240]10^{-5}$	
$P_{HS-15-44}$	N	N $[2.4; 0.08]10^{5}$	Load
$P_{HS-25-44}$	N	N $[3.2; 0.106]10^{5}$	
T_i	years	N $[\mu(T_i); \sigma(T_i)]$	Timing

Table 3: Probabilistic inputs parameters

Random Variable	Description	Type	Source
C_0	Chloride concentration on the surface	Environmental	Estes, 1997
D_c	Diffusion coefficient		Estes, 1997
C_{cr}	Critical chloride concentration		Estes, 1997
i_{corr}	Corrosion Parameter		Estes, 1997
f_y	Yield stress of steel reinforcement	Material	Nowak, 1995
f_c'	28 day yield strength of concrete		Nowak et al., 1994
ρ_s	Specific weight of steel reinforcement		Estes, 1997
ρ_c	Specific weight of concrete		Estes, 1997
$P_{HS-15-44}$	Load tracks according to AASHTO	Load	AASHTO, 2014
$P_{HS-25-44}$	Load tracks according to AASHTO		AASTHO, 2014
T_i	Initiation Time	Timing	Thoft-Christensen et al., 1997

Table 4: Description of the probabilistic inputs parameters

4.1.5.2. Correlation Matrix

The correlation matrix of n random variables X_1, …, X_n is the $n \times n$ matrix whose i, j entry is corr(X_i, X_j). If the measures of correlation used are product-moment coefficients, the correlation matrix is the same as the covariance matrix of the standardized random variables $X_i / \sigma(X_i)$ for i = 1, …, n. This applies to both the matrix of population correlations (in which case σ is the population standard deviation), and to the matrix of sample correlations (in which case σ denotes the sample standard deviation). Consequently, each is necessarily a positive-semidefinite matrix.

$$\rho_{ij} = \begin{pmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1j} \\ \rho_{21} & \rho_{22} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{i1} & \cdots & \cdots & \rho_{ij} \end{pmatrix}$$

Where the diagonal elements $i = j$ are always equivalent to 1, and the rest of the elements depend on the degree of correlation such that:

- $\rho = 1$, if there is perfect direct (increasing) correlation
- $\rho = 0$, if the variables are completely independent
- $\rho = -1$, if there is perfect indirect (decreasing) correlation
- $-1 < \rho < 1$, indicates the degree of linear dependence between variables

The correlation matrix is symmetric because the correlation between X_i and X_j is the same as the correlation between X_j and X_i.

The next figure illustrates the degree of correlation and how the data behaves according to that:

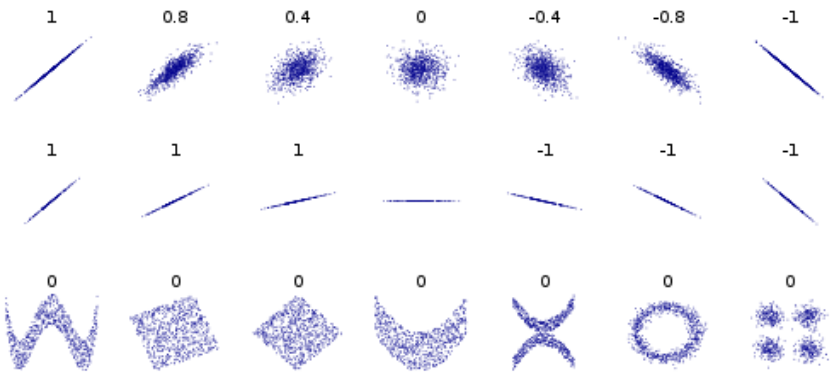

Figure 26: Degree of correlation

4.1.5.3. Limit State Equation

The limit state equations describe the behavior of the structure being investigated. This involves combining random variables with the load rating equations already used. This limit state equation is always in the form of $Capacity - Demand$, where a positive result means safety and a negative response implies the failure of the component. Therefore, a result equal to zero indicates a point on the failure surface.

The limit state equation used for this work is the following:

$$g(X) = M_{Capacity} - M_{Demand} = M_n - M_u = 0$$

Recalling the equations:

$$M_{Capacity} = M_n = \phi A_s \cdot f_y \cdot \left(d - \frac{a}{2}\right)$$

$$M_{Demand} = M_u = \frac{PL^2}{8}$$

4.1.5.4. Analysis Type

A general system can be modeled as any combination of series and parallel system. The reliability of a series system and a parallel system can be solved separately. The approach for a complex system will be to sequentially break the system down into simpler equivalent subsystems.

The next figure shows how all the components of a system are simplified step by step until a single system reliability index can be calculated.

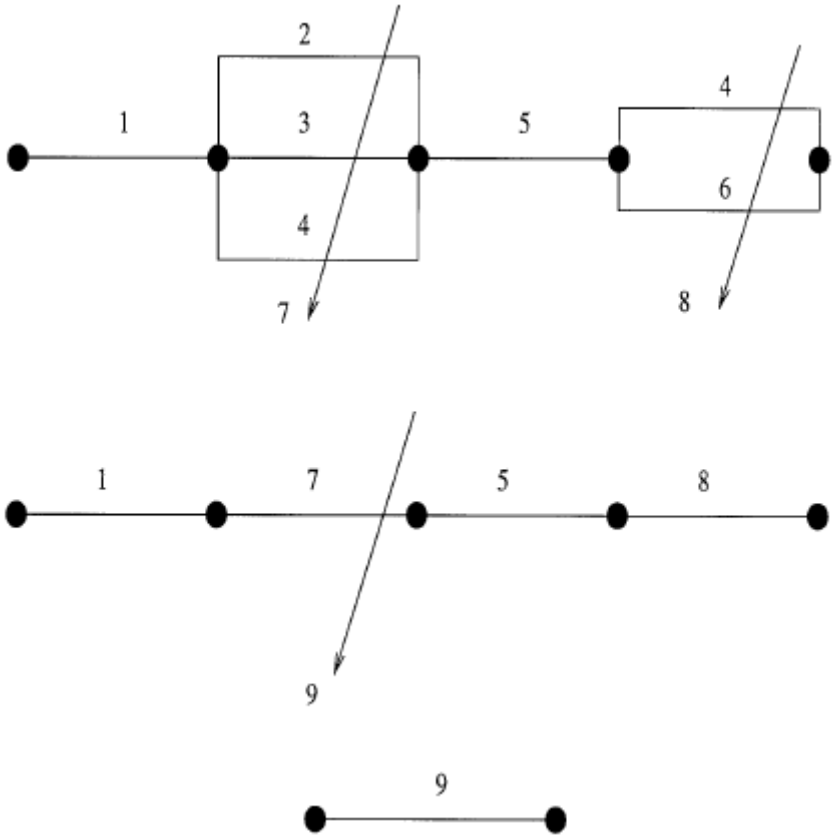

Figure 27: Reduction of a system [Estes, 1997]

Since the reliability of a system is dependent on the correlation of the individual members of the system, the equivalent correlation of the equivalent components must be included. Regardless the complexity of the problem, we can obtain either a component or a system reliability analysis.

4.1.5.5. Solution Method

The most common solution methods are explained widely in section 2.3. There exist the First Order Reliability Method (FORM), the Second Order Reliability Method (SORM), Monte-Carlo Simulation and Genetic Algorithms (GA).

FORM is the one used in this work to solve the reliability analysis of the structural system, due to the nature of the limit state equations and the computational cost. For this work, the convergence is assumed for 10^{-8} or a maximum number of iterations equals to 20.

4.1.5.6. Loop Over Time

The aim is to know how the corrosion affects to the reliability and probability of failure on the serviceability lifetime of the bridge. Therefore, the design lifetime assigned for this structural system is 75 years. All the results will be calculated for this period.

4.1.5.7. Reliability Index and Probability of Failure

At this point, the reliability analysis is carried out and all the results throughout the lifetime of the bridge are obtained. As a reminder of what the probability of failure is, as explained in section 2.1.1:

The probability of failure P_f is defined as the probability of occurrence of the event $R \leq S$ and can be evaluated by the following integral:

$$P_f = \int_{\{g(x) \leq 0\}} f_X(x)\, dx$$

Where $g(x)$ is the limit state function corresponding to the failure mode considered and defined such that the failure event corresponds to $g(x) \leq 0$, as defined in section 2.4.2

Due to the probability of failure are directly related, thus $\beta = \Phi^{-1}(P_f)$

The software that allows computing numerically this process is RELSYS, which is able to run from a Matlab function.

Herein, the bases of RELSYS and the different components of this program are detailed:

4.1.5.8. RELSYS

RELSYS (Reliability of Systems) is a FORTRAN 77 program written by Allen C. Estes and Dan M. Frangopol, which calculates the reliability of any structure which can be modeled as a combination of series and parallel structures. The program uses the average of the Ditlevsen bounds for series systems, numerical integration of the multi-normal distribution for two and three-member systems, the Hohenbichler approximation for larger parallel systems, and correlation coefficients based on equivalent alpha vectors to calculate the system reliability index. The algorithm reduces the various series and parallel systems to equivalent single components, gradually reducing the entire system to a single simplified component. The program is divided into four parts:

a) **relsys.f**: The program shell which reads the input file and calls the subroutines which will calculate the system reliability. The purpose of this shell is to create an environment which allows the user to repeatedly calculate the reliability of the structure and to define the problem to be solved. Specifically for a highway bridge, the shell defines the cost data, the deterioration functions, the number of lifetime inspections, and the minimum acceptable reliability. It contains the aspects of the problem that do not involve the reliability calculations;

b) **subrel.f**: Main program for reliability calculation. It acts as the switchboard 152 which calls the necessary subroutines to calculate reliability;

c) **subrel1.f**: Contains all subroutines for calculating the reliability of individual components

d) **subrel2.f:** Contains all subroutines for calculating the system reliability.

In general, RELSYS provides an excellent approximation for most problems with relatively little computational effort. There are certain limitations to this approach. The FORM methods are approximations and have errors associated with them, especially with problems that are highly non-linear. Sometimes a local minimum is identified in the minimum distance optimization process and the global minimum distance to the failure surface is missed which causes the reliability to be over-estimated. This FORM approach is a level 2 reliability method, which indicates that only the mean and standard deviation are considered. Other distribution parameters such as skewness are ignored. Creating an equivalent normal distribution from a non-normal distribution can create errors, especially when that assumption is carried through the entire system analysis. However, we are not dealing with non-linear inputs and the computational cost is very low.

4.1.6. Bridges in the Case Study

In this section, the different bridges that take part of the problem in order to study its behavior regarding the reliability index and probability if failure are widely explained herein.

4.1.6.1. Design Inputs

Definition	Parameter	Bridge 1	Bridge 2	Bridge 3	Units
Length	L	25000	20000	20000	mm
Height	h	1000	800	700	mm
Base	b	800	600	400	mm
Effective depth	d	936.5	736.5	636.5	mm
Diameter Rebar	ϕ	25.4	25.4	25.4	mm
Truck Load	HS-15-44 HS-20-44	240000 (HS-15-44)	320000 (HS-20-44)	240000 (HS-15-44)	N

Table 5: Design inputs

Note that the diameter of the reinforcement bars is according the US market stock for reinforced concrete. In this work the size #8 was taken:

Imperial Bar Size	Soft Metric Size	Mass per Unit Length (kg/m)	Nominal Diameter (mm)	Nominal Area (mm2)
#3	#10	0.561	9.525	71
#4	#13	0.996	12.7	129
#5	#16	1.556	15.875	200
#6	#19	2.24	19.05	284
#7	#22	3.049	22.225	387
#8	**#25**	**3.982**	**25.4**	**509**
#9	#29	5.071	28.65	645
#10	#32	6.418	32.26	819
#11	#36	7.924	35.81	1006
#14	#43	11.41	43	1452
#18	#57	20.284	57.33	2581

Table 6: Rebar size on stock (USA), source: Harris Supply Solutions

Also, the parameters of the strength related with the steel and concrete are the ones available in the American market.

$$f_y = 50\ ksi = 345MPa$$

$$f_c' = 3ksi = 20.7Mpa$$

Once the input parameters are defined, we proceed to calculate the different outputs of interest for the problem.

4.1.6.2. Design Outputs

Definition	Parameter	Bridge 1	Bridge 2	Bridge 3	Units
Nominal Bending Moment	M_N	$3.77 \cdot 10^9$	$2.31 \cdot 10^9$	$1.61 \cdot 10^9$	N/mm^2
Ultimate Bending Moment	M_U	$2.25 \cdot 10^9$	$1.37 \cdot 10^9$	$0.94 \cdot 10^9$	N/mm^2
Reinforcement	As	$8.61 \cdot 10^3$	$7.09 \cdot 10^3$	$6.08 \cdot 10^3$	mm^2

Number of Rebars	$\#_{Rebars}$	17	14	12	-
Minimum Reinforcement Compression	As'	$4.36 \cdot 10^3$	$3.52 \cdot 10^3$	$2.94 \cdot 10^3$	mm^2
Number of Rebars	$\#_{Rebars,comp}$	9	7	6	-
Initiation Time (mean)	$T_I\ (\mu)$	50.21	30.82	22.89	years
Initiation Time (standard dev.)	$T_I\ (\sigma)$	12.41	7.61	5.62	years

Table 7: Design outputs obtained

4.1.6.3. Evolution of the Diameter Overtime

First of all, is interesting to know how the diameter of the rebar is affected by the corrosion on the environment. This aspect is crucial since the reinforcement plays an important part in flexure. When the rebars are completely corroded, the concrete will have to resist all the loads and the beam is prone to fail.

The next figure illustrates the evolution of the diameter of the rebars overtime for different bridges:

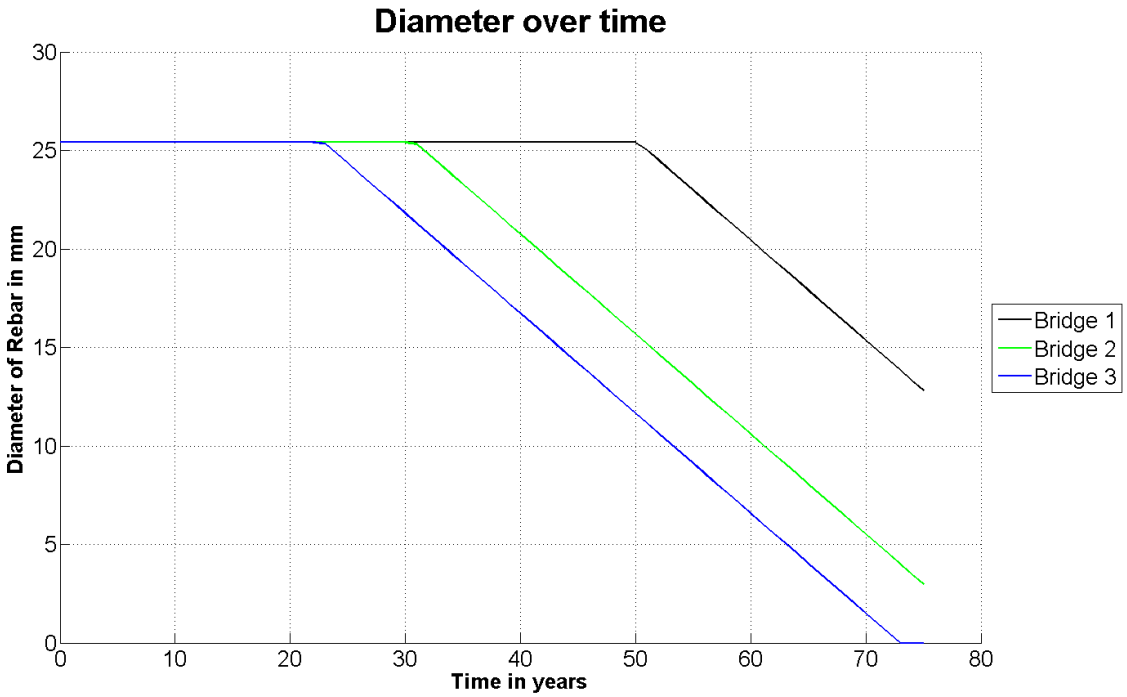

Figure 28: Evolution of the diameter of rebars for the 3 bridges

We see how the chart presents a clear trend. The diameter remains exactly the same until at a certain time, denoted as initiation time of the corrosion (T_I), when the size of the rebar starts to decrease overtime. This is due to the fact that, the concrete has reached the critical chloride

concentration C_{cr} and the corrosion starts to be aggressive. This will lead to cracking on the concrete's surface and time by time severe damage will be visible.

Clear differences are shown on the evolution of the different reinforcement for the three bridges. On the one hand, the Bridge 1, which has the thickest cross-section, implies no suffering the effects of corrosion in the reinforce steel for the first 50 years. On the other hand, the Bridge 3, which has the thinnest cross-section, needs only 22 years to notice some damage corrosion on its reinforcement. In addition, by the end of lifetime of the bridge, there will be a lack on the reinforce steel since all the rebar will be corroded by the time of 75 years. It is important to point out that this is a critical situation due to the concrete will have severe damage and no rebar, which will lead, at a certain degree, to a high probability of failure.

The Bridge 2 will be between both bridges since the design parameters are no larger than Bridge 1 and no lower than Bridge 3.

Finally, the evolution overtime of the corrosion seems to be linear and the diameter never gets to zero value for Bridge 1 and 2 but not in Bridge 3.

4.1.6.4. Evolution of the Capacity and Demand Overtime

Secondly, it is interesting to know how the overall resistance, capacity minus demand, behaves overtime.
The next figure illustrates the evolution of nominal bending moment $(\mathrm{M_N})$ and the ultimate bending moment $(\mathrm{M_U})$ overtime:

Figure 29: Evolution of the resistance over time

We find some similarities relating the plot of the evolution of the diameter. Again, all the bridges follow a trend. First of all, we see how the resistance remains constant until at a certain point, which is exactly the initial time of corrosion (T_I) when the resistance starts decrease. By that time, the diameter of the rebars starts to corrode (as seen in Figure 28) and the contribution of the steel, to the overall resistance, is lower. Afterwards, the failure might occur when the ultimate bending moment (M_U) is greater than (M_N).

Notice that the bridge with thickest cross-section (Bridge 1) takes longer time to decrease the resistance, while the one with the thinnest cross-section (Bridge 3) will start to suffer fall on its capacity almost half the time earlier.

Furthermore, the Bridge 2 will be between both bridges since the design parameters are no larger than Bridge 1 and no lower than Bridge 3.

Finally, the curve presented by the 3 bridges is slightly curved, but could be approximated by a straight line.

4.1.6.5. Reliability Analysis

Thirdly, we want to know how the reliability of the different bridge evolves over time. The next figure illustrates for each bridge how the reliability index changes for the lifetime of 75 years.

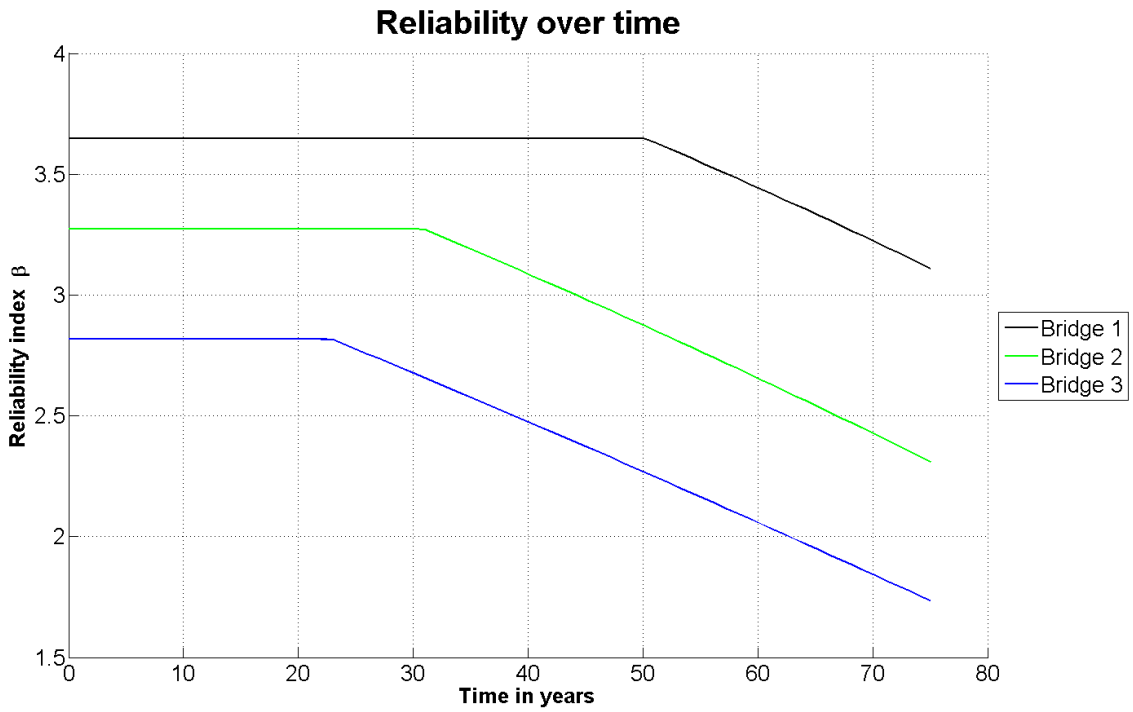

Figure 30: Evolution of the reliability overtime

In this graph we see the same trend like the plots described above this section. When the initiation corrosion time is reached, the reliability of the bridge starts to fall down. The results are expected due to the Bridge 3 (the weakest) ends up to a very low index. Also, it has the

earliest initiation time. For this reason, it will suffer more time the effects of corrosion in the structure leading to a higher degradation.

In addition, is important to highlight that the Bridge 1 and 2 would be under normal values in terms of reliability (between 3 and 5), from the start of serviceability until the end of the lifetime. In contrast, the Bridge 3, which has a lower reliability index at the beginning will end up to a close value of 1.5 which indicates is near the critical zone. This bridge will suffer important damages by the end of the lifetime.

Finally, the curves obtained seem to be linear when the reliability of the bridge is affected by the corrosion.

4.1.6.6. Probability of Failure

Fourthly, and related to the reliability index, the probability of failure for the different bridges is carried out.

Figure 31: Evolution of the probability of failure overtime

On this plot, there are several aspects to highlight regarding the probability of failure over the lifetime of the bridge, 75 years.

First of all, Bridges 1 and 2 present a probability of failure very low that can even be neglected in comparison with the Bridge 3. The former follows almost a straight distribution along the serviceability of the bridge. On the other hand, the latter starts with a low probability of failure but ends up with a close value to 0.45, which is not at all negligible. The reason why they behave completely different is associated with the reliability index. The probability of failure does not present a linear distribution, but a normal, as seen in the next table:

$\boldsymbol{\beta}$	$\boldsymbol{\Phi(-\beta)}$
0	0.5
0.5	0.3085
1	0.1587
1.5	$0.0668 \cdot 10^{-1}$
2	$0.227 \cdot 10^{-1}$
2.5	$0.621 \cdot 10^{-2}$
3	$0.135 \cdot 10^{-2}$
3.5	$0.232 \cdot 10^{-3}$
4	$0.316 \cdot 10^{-3}$
4.5	$0.339 \cdot 10^{-3}$

Table 8: Reliability index (left) vs probability of failure (right)

Thus, the closer to $\beta = 0$, the more change is shown in the probability of failure. For instance, the change in the PoF between $\beta = 4$ and $\beta = 4.5$ ranges from $0.316 \cdot 10^{-3}$ to $0.339 \cdot 10^{-3}$. But the difference between $\beta = 0.5$ and $\beta = 1$ ranges from 0.3085 to 0.1587 which is almost three times.

For this reason, the Bridge 3, which has the lower reliability indices overtime, will have important increments on the probability of failure very quickly. Then by far, is the bridge that will probably determine the behavior of the structural system afterwards in next sections.

Finally, the curves have an exponential trend which is more noticeable in Bridge 3 rather than in Bridge 1 and 2.

4.1.7. Structural Systems in the Case Study

In this section, a deep study of the probable structural systems, that allows crossing from one point to the other, is developed.

Moreover, the term of correlation between parameters of the bridges is introduced. The point is to compare a perfect correlated and an uncorrelated model. The aim is to know how much affects to the entire structural system and if it is a parameter that will determine how to perform the bridges in series, parallel or a combination.

4.1.7.1. Random Variables

In this part of the work we are dealing with 3 different bridges at the same time, so we need to identify the random variables for each bridge and are the following:

Random Variable	Units	Values [μ; σ]
$f_{y,1}$	N/mm^2	N [386 ; 42.5]
$f'_{c,1}$	N/mm^2	N [19 ; 3.45]
$\rho_{s,1}$	N/mm^3	N $[7.85 \cdot 10^{-5}\,; 0.785 \cdot 10^{-5}]$
$\rho_{c,1}$	N/mm^3	N $[2.4 \cdot 10^{-5}\,; 0.24 \cdot 10^{-5}]$
$P_{AASHTO,1}$	N	N $[2.4\,; 0.08]10^5$
$T_{i,1}$	$years$	N [30.826 ; 7.617]
$i_{corr,1}$	$mm/year$	N [2.5 ; 0.289]
$f_{y,2}$	N/mm^2	N [386 ; 42.5]
$f'_{c,2}$	N/mm^2	N [19 ; 3.45]
$\rho_{s,2}$	N/mm^3	N $[7.85 \cdot 10^{-5}\,; 0.785 \cdot 10^{-5}]$
$\rho_{c,2}$	N/mm^3	N $[2.4 \cdot 10^{-5}\,; 0.24 \cdot 10^{-5}]$
$P_{AASHTO,2}$	N	N $[3.2\,; 0.106]10^5$
$T_{i,2}$	$years$	N [50.21; 12.41]
$i_{corr,2}$	$mm/year$	N [2.5 ; 0.289]
$f_{y,3}$	N/mm^2	N [386 ; 42.5]
$f'_{c,3}$	N/mm^2	N [19 ; 3.45]
$\rho_{s,3}$	N/mm^3	N $[7.85 \cdot 10^{-5}\,; 0.785 \cdot 10^{-5}]$
$\rho_{c,3}$	N/mm^3	N $[2.4 \cdot 10^{-5}\,; 0.24 \cdot 10^{-5}]$
$P_{AASHTO,3}$	N	N $[2.4\,; 0.08]10^5$
$T_{i,3}$	$years$	N [22.89; 5.65]
$i_{corr,3}$	$mm/year$	N [2.5 ; 0.289]

Table 9: Random variables for bridges 1, 2 and 3

4.1.7.2. Correlation

The meaning of the correlation is explained in section 4.1.5.2. In the next cases, the different models such as series, parallel or series-parallel will be analyzed for perfect correlated or uncorrelated.

Therefore, the correlation matrix for each case is:

$$\rho_{ij,uncorrelated} =$$

	$f_{y,1}$	$f'_{c,1}$	$\rho_{s,1}$	$\rho_{c,1}$	$P_{AASHTO,1}$	$T_{i,1}$	$i_{corr,1}$	Parameters Bridge 2							Parameters Bridge 3						
$f_{y,1}$	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$f'_{c,1}$	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\rho_{s,1}$	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\rho_{c,1}$	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$P_{AASHTO,1}$	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$T_{i,1}$	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$i_{corr,1}$	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Parameters Bridge 2	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
Parameters Bridge 3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

$$\rho_{ij,correlated} =$$

		$f_{y,1}$	$f'_{c,1}$	$\rho_{s,1}$	$\rho_{c,1}$	$P_{AASHTO,1}$	$T_{i,1}$	$i_{corr,1}$	Parameters Bridge 2							Parameters Bridge 3						
	$f_{y,1}$	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0
	$f'_{c,1}$	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0
	$\rho_{s,1}$	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0
	$\rho_{c,1}$	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0
	$P_{AASHTO,1}$	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0
	$T_{i,1}$	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0
	$i_{corr,1}$	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1
Parameters Bridge 2		1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0
		0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0
		0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0
		0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0
		0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0
		0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0
		0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1
Parameters Bridge 3		1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0
		0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0
		0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0
		0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0
		0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0
		0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0
		0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1

4.1.7.3. Series Model

In this first model, the case 1 described in section 0 is carried out in order to study both the reliability and probability of failure of the structural system. This model is usually applied to a deck or piles of a bridge.

4.1.7.3.1. <u>Advantages</u>

If the bridges are robust, the structural system will be able to bear the loads and will give a high-order of reliability

4.1.7.3.2. <u>Drawbacks</u>

If there is (at least) one weak bridge, this element will dominate the reliability of the system and therefore, higher values of probability of failure will appear.

4.1.7.4. Parallel Model

In the second model, the case 2 described in section 0 is carried out in order to study both the reliability and probability of failure of the structural system. This model is usually applied to the girders of a bridge.

4.1.7.4.1. Advantages

This model usually gives a high-order of reliability to the structural. This is due to more than one element is required so that the system fails.

4.1.7.4.2. Drawbacks

Sometimes, the elements of a bridge cannot be modeled as a parallel system such as piles or a unique deck. Thus, a series design must be applied.

4.1.7.5. Series-Parallel Model

In this third model, the case 3 described in section 0 is carried out in order to study both the reliability and probability of failure of the structural system. This model is usually applied to whole structural system of a bridge, where both series (deck) and parallel (girders) make up the bridge.

4.1.7.5.1. Advantages

The point that mixes series and parallel elements will tend to increase the reliability of the system since the weak and strong elements of the bridge are combined.

4.1.7.5.2. Drawbacks

If the weak element is performed in the series part, this will lead to lower values of reliability

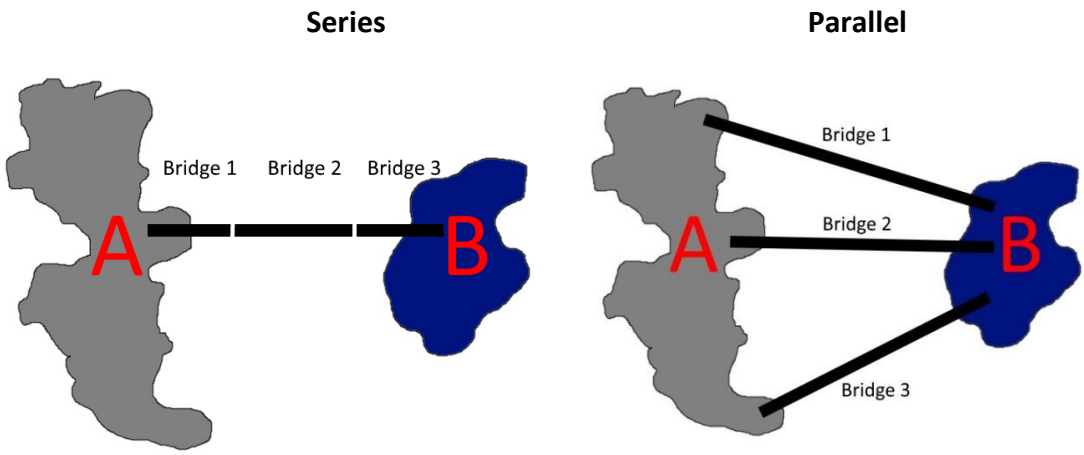

Figure 32: series scenario (left) and parallel scenario (right)

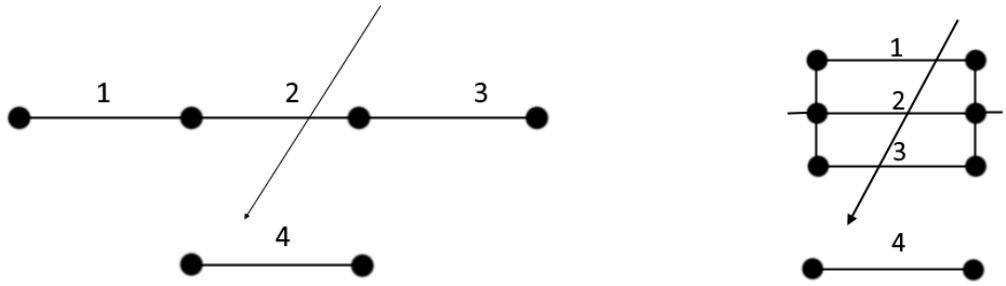

Figure 33: Scheme of series model reduction (left) and scheme of parallel model reduction (right)

Series-parallel

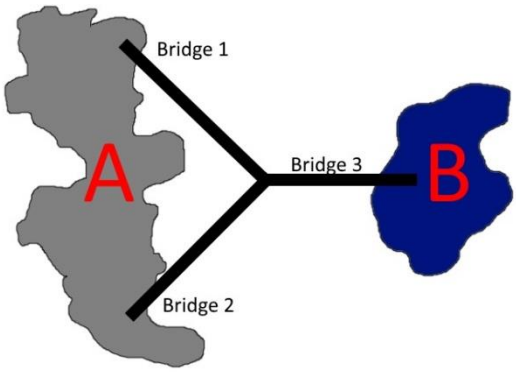

Figure 34: Scheme of series model

The scheme of the structural system of the different bridges may change according to the next configuration:

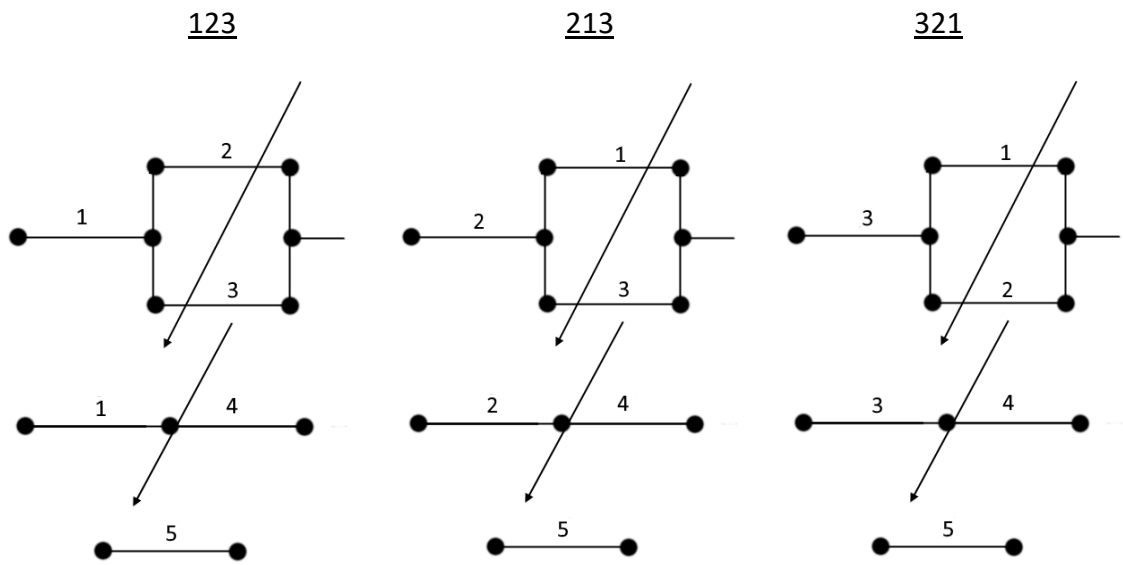

Figure 35: Scheme of series-parallel model reduction

After all the different models are presented, to cross from one city to another, the reliability analysis and the probability of failure for each case are carried out.

The indices used to identify each case are:

Definition	Legend
Series	S
Parallel	P
Series-Parallel	SP
Correlated	C
Uncorrelated	U
Bridge 1	1
Bridge 2	2
Bridge 3	3

Table 10: Identification of the legend for plots in this work

4.1.7.6. Reliability Analysis

The results of the reliability analysis are shown in the next figure:

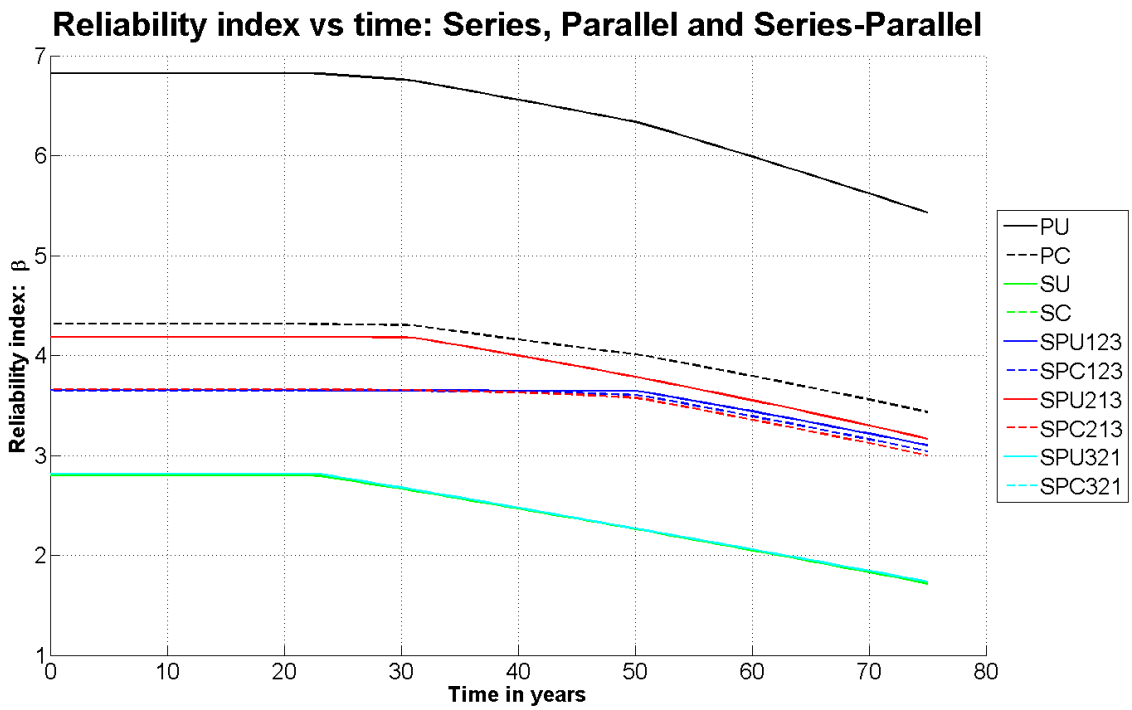

Figure 36: Evolution of the reliability index overtime for the different models

This figure clearly shows 3 different groups of behavior for the models plotted.

First of all, the parallel uncorrelated model is by far the one with a higher order of reliability index. This is expected since the parallel models, as explained in section 4.1.7.4, tend to have this performance. Also, the fact that do not exist correlation between bridges increases the chances to success in terms of serviceability.

Secondly, there is group in the middle. On the one hand there are no remarkable differences between correlated and uncorrelated models regarding the series-parallel models. The results are expected since there is a part of the system which is modeled in series and the reliability index is supposed to be lower than the pure parallel one. On the other hand we find the correlated parallel. It is important to point out the big effects of correlation in parallel models which is around 1.5 times lower. The correlation always tends to decrease the reliability of the systems.

Thirdly we see the group with the lowest reliability index. Those curves are the series model and series-parallel, where the series element is the weakest bridge among the three designed. This behavior is expected because series models will dominate the entire structural system driving to the model to lower values of reliability indices.

4.1.7.7. Probability of Failure

The results of the probability of failure analyzed are shown in the next figure:

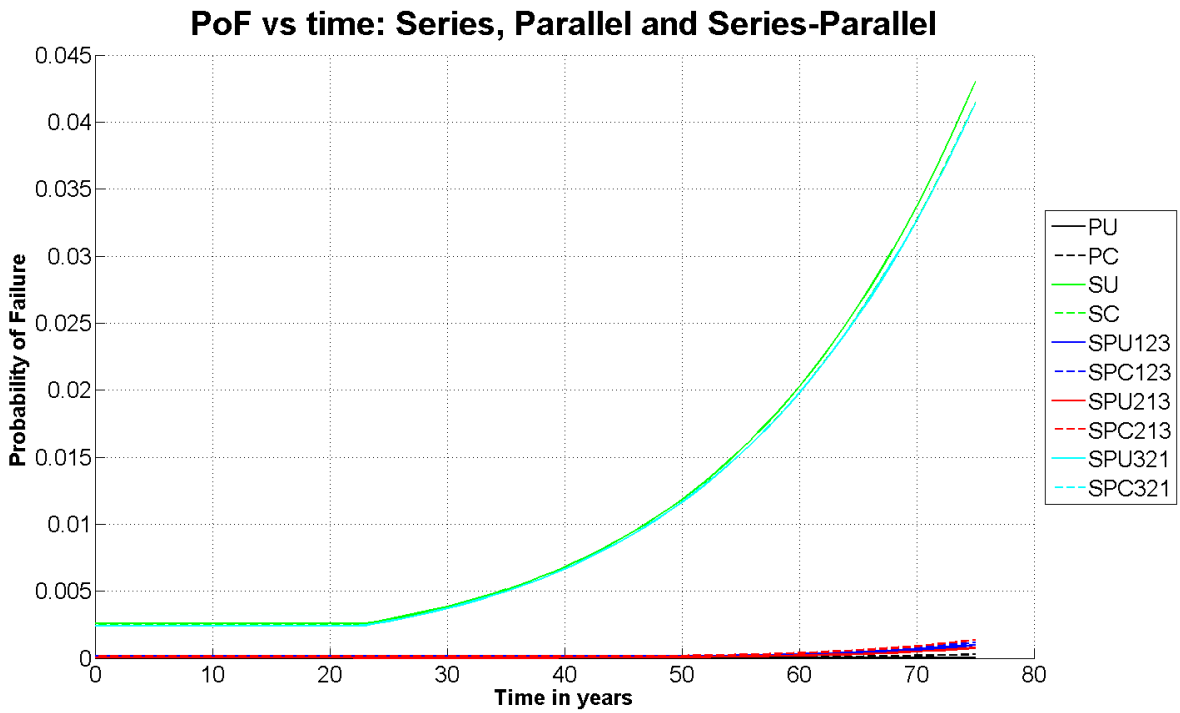

Figure 37: Evolution of the probability of failure overtime for the different models

The obtained results plotted above give important information on the behavior of the different models. First of all, 2 groups of curves are clearly separated:

4.1.7.7.1. Exponential Curves

In this group, we basically identify how the series and the correlation affect to the performance of the structural system.

On the one hand the, the green lines (continue and dotted) refer to the series model, both correlated and uncorrelated. The fact that high probability of failure is associated to this case is because, once a bridge fail, the global system will fall down as well. In addition, if there is correlation between inputs of the bridge, this will increase, even ore, the probability of failure. However there is not important different between correlation in this case.

On the other hand, the blue lines (continue and dotted) refer to the series-parallel model, both correlated and uncorrelated. The reason why they are very similar to the case explained above, it is because the weak bridge is modeled in series with respect to the whole system, and as it is explained in sections above, the domination of the weakest elements is crucial in those models. Therefore, it is expected to see that both results are pretty similar.

4.1.7.7.2. Plain Curves

These curves with a lower probability of failure are related with parallel systems, where all the elements must fail in order to lead the system into collapse. For that reason, the higher number of elements in parallel, the lower probability of failure will have the system.

In addition, we found that the series-parallel models, that follow the same trend, are those whose weak bridge is in the parallel group instead of in the series. Therefore, the weak element will not be dominant over the entire structural system as it happened in the case before.

4.1.7.8. Reliability Index vs Probability of Failure

It is remarkable to highlight the importance of studying both the reliability index and the probability of failure. As shown in the figures above, the reliability index clarifies some aspects that were not really easy to notice in the probability of failure and the other way around. Since they follow different distribution, each one shows with more detail some different aspects. For instances, the plot related with the probability of failure, two different groups are clearly separated but not in the plot regarding the reliability index. The fact that the lines related with reliability index are not too distant, it does not mean that there are slightly variations on the probability of failure as shown above. In terms of probability of failure might not be big differences but not in the reliability index.

4.2. Case Study II

4.2.1. Objective

The aim of this second case study is to know what happens if the initial chloride concentration in the atmosphere is higher than the one applied to the case study I. The reasons why this may happen are the followings:

- Excessive use in of salts during winter (abnormal high rates of chlorides)
- Raise in the sea level because of global warming or environmental issues
- The bridge is located next to an industry and there is an emergency situation of an unexpected escape of gases which contain high level of chlorides
- Spill of any liquid with a high concentration of chlorides

Therefore, it is interesting to know which will be the final reliability index of the structural system. For this study, the initial chloride concentration (T_I) ranges from x1 to x10 times.

The methodology used is the same done in the case I and the design parameters and random variables are taken from this first approach, as well. Thus, only the recalculation of initiation time, reliability index and probability of failure are carried out.

Since the bases of this second case are founded on the explained in the first section, no further details are described herein.

After computing the all the results of this second scenario, where the concentration of chlorides ranges from the initial rate until ten times, one of the cases is analyzed separately to comment the trend obtained. Then, all the results from the different models (series, parallel or series-parallel) are plotted together in order to see differences on the behavior.

4.2.2. Sample: Parallel Uncorrelated Model

4.2.2.1. Reliability Analysis

In this section, the increase on the chloride concentration for the parallel uncorrelated model is described. The next figure shows the evolution of the reliability index along the lifetime of the structural system (Bridge 1, 2 and 3).

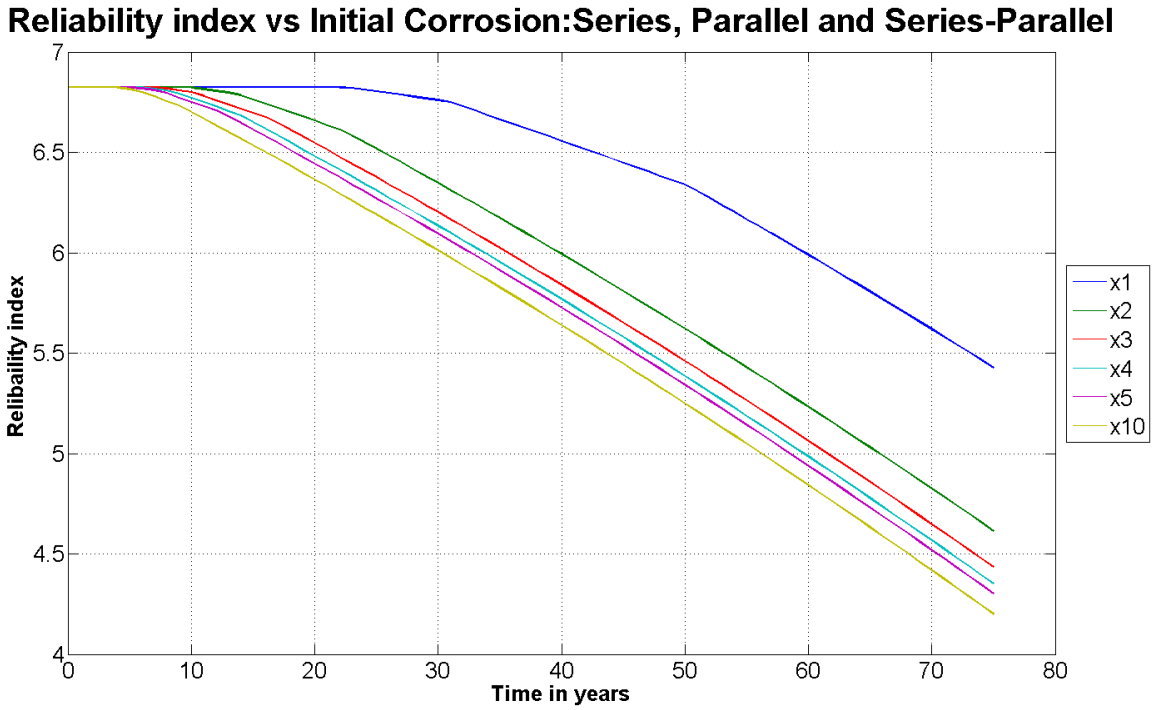

Figure 38: Reliability index over time for different rates of initial chloride concentration

The results shown on this graph give some remarkable information for the different rates.

First of all we see how the turning point of the reliability index is different for each line. This is because the initiation time changes for every chloride concentration on the structure. The higher is the initial concentration, the sooner it starts to corrode and thus decrease the reliability index.

Secondly, there are 2 big differences between the initial chloride concentration and the rest. On the one hand, the blue line (x1) seems to be far from the rest of concentration rates (x2, x3, x4, x5 and x10). On the other hand the rest of colored lines are slightly separated, not remarkable differences.

The reason why there exist these big differences between chloride concentrations in terms of the reliability index is the following. When the critical chloride concentration is reached, the reinforcement starts to corrode. Once that level is achieved by high rates there will not imply significant changes. Then, the initial chloride concentration, in this case, is close to the critical concentration.

Therefore a different increment of chlorides must be done to see a gradually change in the plot before, closer to the initial concentration.

4.2.2.2. Probability of Failure

The next figure shows the probability of failure, according to the reliability indices explained above.

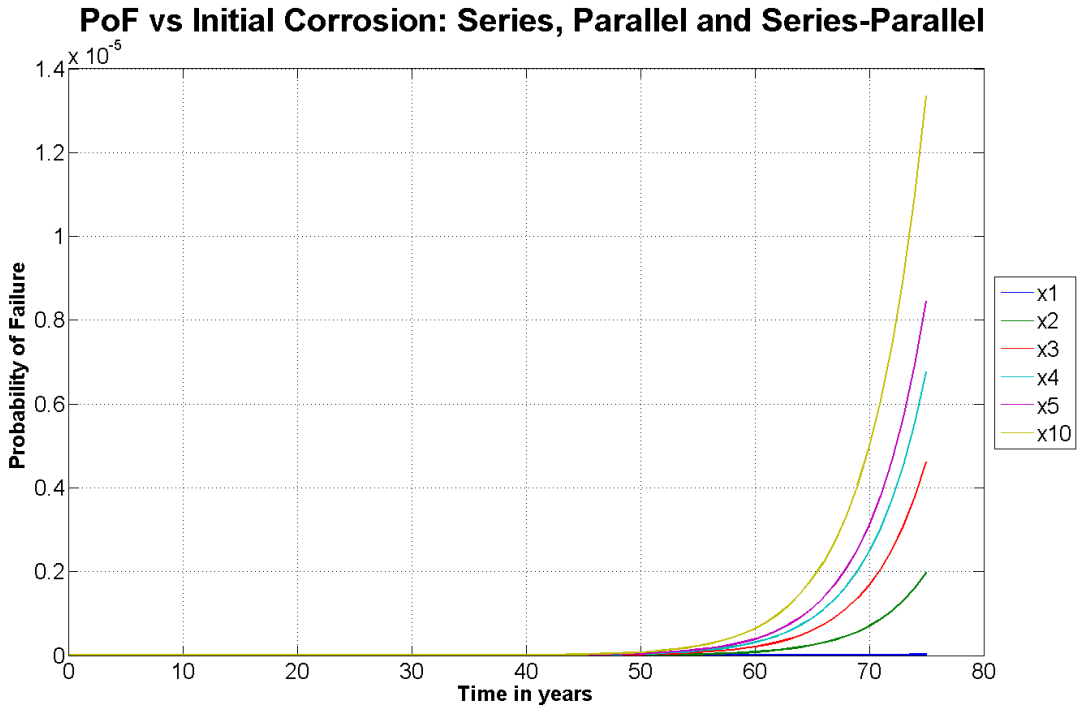

Figure 39: Probability of failure over time for different rates of initial chloride concentration

From the analysis of the probability of failure some information is extracted. Despite of the fact that in terms of reliability index there are no big differences (the ultimate β ranges from 5.5 – 4.5), once again we see the importance of doing both analysis.

There is a remarkable variation among the different chloride concentrations, that ranges from 0 to 1.4 ($x10^{-5}$). However it is not critical since $x10^{-5}$ might be negligible, but in another case, that would be a serious increase in terms of failure.

Furthermore, the higher is the chloride concentration, the higher is the slope of the curve plotted.

4.2.3. Series, Parallel and Series-Parallel Systems

After this first example of increase in the initial chloride concentration, a deepest study with the rest of structural systems is carried out.

4.2.3.1. Reliability Analysis

The first figure represents all the possible models, series, parallel and series-parallel with all the combinations between bridges.

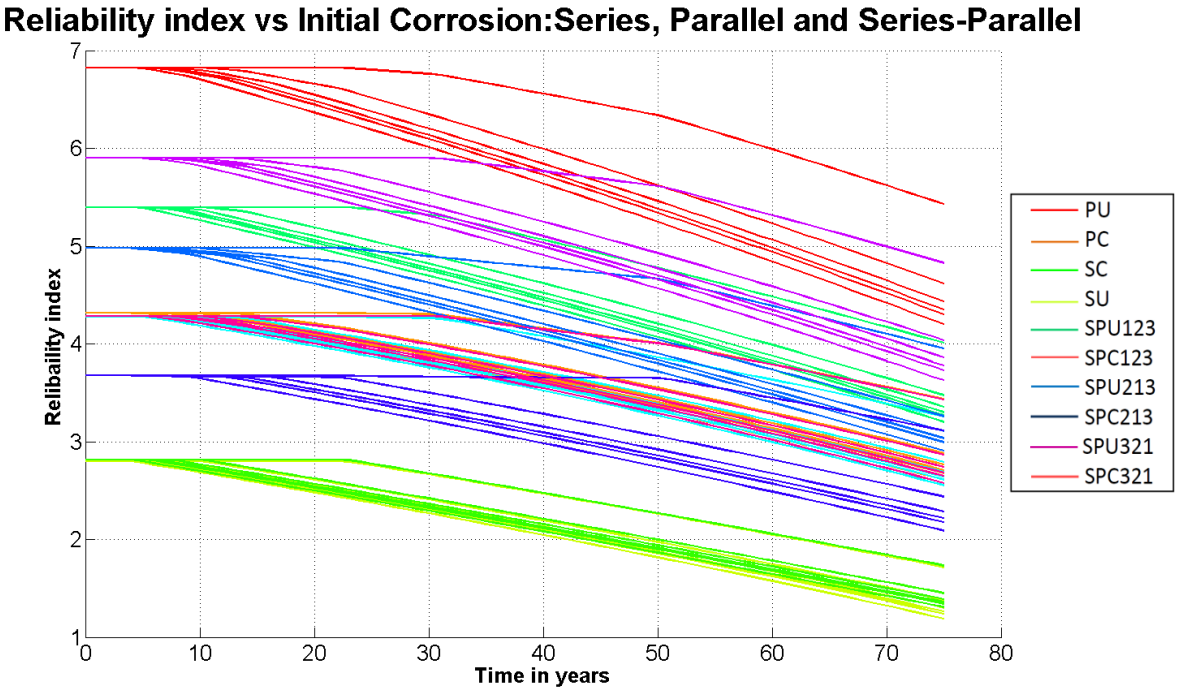

Figure 40: Reliability index for different models and chloride concentrations over time

From the results of this plot we can highlight the following information regarding the evolution of the reliabilit index.

First of all, we see how the trend is clearly similar for all the models, where the initial chloride concentration is less aggressive rather than the rest.

Secondly, three groups might be analysed slightly separated as happened in section 4.1.7.6. The Parallel uncorrelated model is by far the one that presents higher reliability index for all the cloride rates at anytime. This is due to the fact that the structural system needs the failure of all the components to collapse. In conrast, the series correlated and uncorrelated present the lowest reliability index at any time and they do not even present bg differences. Series models always are know for lower reliability indices since only one element is needed in order to fall down the structure. On the other hand we find combination of series-parallel model, in which the lowest are the ones where the series model is the weakest. Also, in the same group, we find the parallel correlated. The correlation between elements always lead to decrease the reliability of the system, as explained in section 4.1.7.2.

4.2.3.2. Probability of Failure

In this second figure, the probability of failure for the different models is obtained.

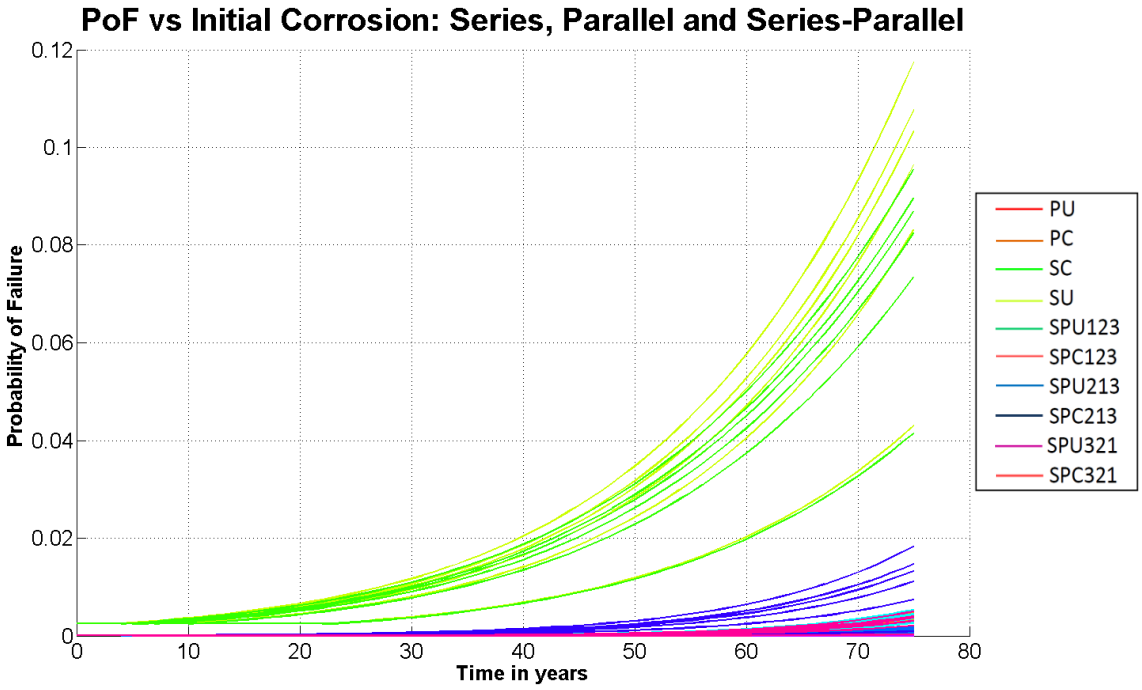

Figure 41: Probability of failure for different models and chloride concentrations over time

We can group the lines in 3 different groups that present different trend over time.

Firstly we find the group of lowest probability of failure where we found all the parallel and series-parallel models. Nevertheless we find some series-parallel with a higher rate but always below 0.02.

Secondly, in the middle we see the series correlated and series uncorrelated regarding the initial chloride concentration. They present a final probability of failure near 0.04 which is not negligible at all.

Thirdly, the group of highest probability of failure and presents the highest slope in the graph. All of them take part of the series correlated and series uncorrelated. There is a remarkable increase in the probability of failure almost reaching 0.12 which is an important value that may cause severe damage in the structure. In addition, the ones belonging to series uncorrelated have the highest indices. The explanation to this is because when there is no correlation between elements, the focus of the source that makes fail the structural system may come from different points. However, when correlation exists in the bridges, the focus must come from the same source which is slightly more improbable.

Finally, it is important to point out again that the change from 3 to 1.5 in terms of reliability means big variations in terms of probability of failure.

4.3. Case Study III

4.3.1. Objective

So far, it has been studied the behavior of the different structural systems: series, parallel and series-parallel with the initial chloride concentration and increasing that ratio until 10 times.

In this chapter, it is interesting to know the final reliability index to ensure the serviceability by the end of the lifetime of the structural system, for the different chloride concentration. Therefore, a detailed study on the final points obtained in the section before will be analyzed.

4.3.2. Reliability Analysis

In the next figure it is presented the values of the reliability index at the end of lifetime for different chloride concentrations.

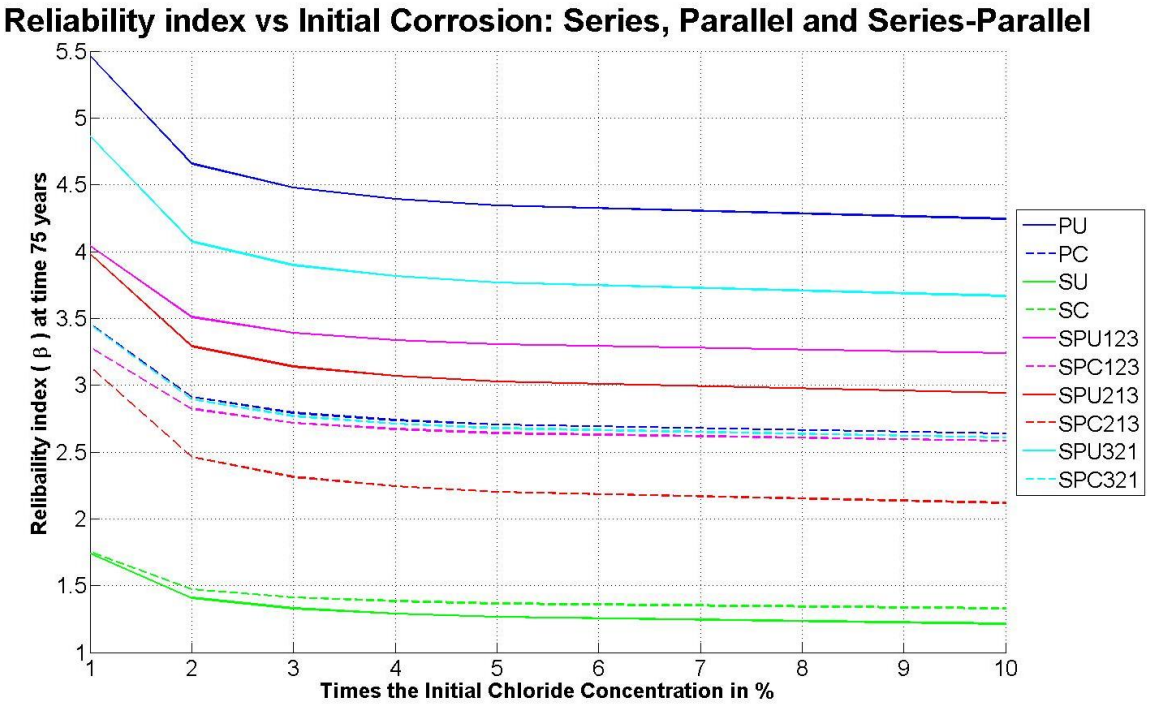

Figure 42: final reliability index for different chloride concentration at the end of lifetime

The graph above shows the variation of the reliability index while the chloride concentration increases. We see how, once again, the series models present a low reliability index and the parallel correlated the highest. In the middle, as usual, we find the series-parallel models and the parallel correlated.

Also, is interesting to notice that the reliability index only decreases from 1-1.5 at the end of life time for chloride concentrations from x1 to x10, which is not an extreme change.

Finally, the slope presented by the curves tends to stabilize gradually as long as the concentration increases.

4.3.3. Probability of Failure

In the next figure, the probability of failure for the case III is obtained.

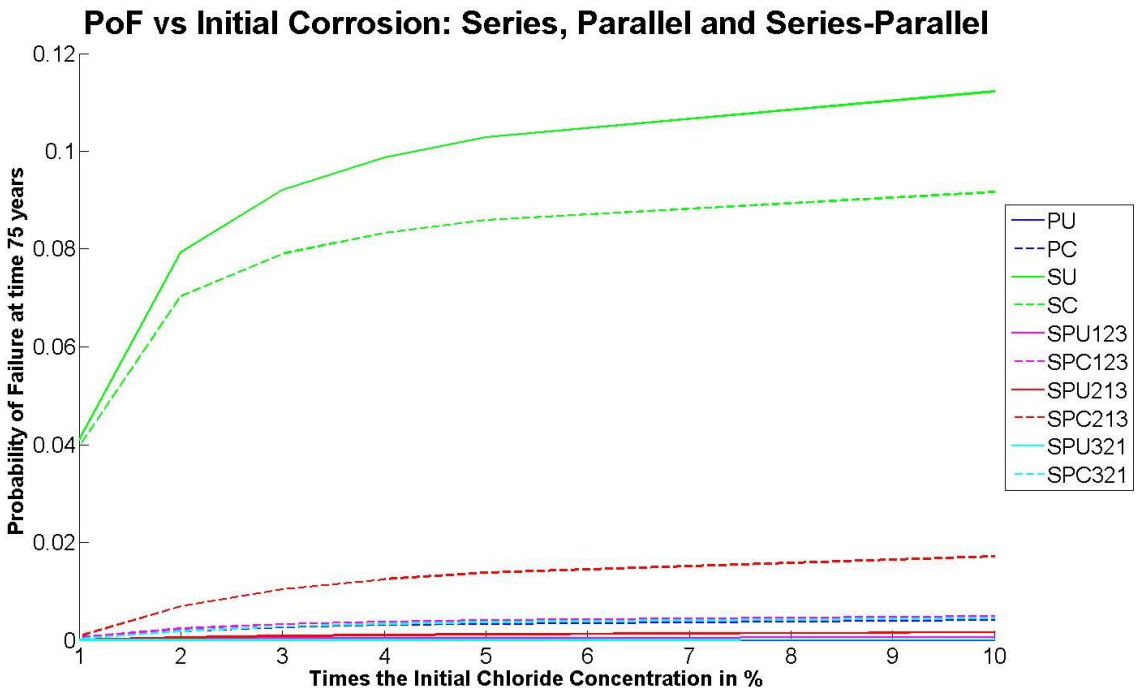

Figure 43: final probability of failure for different chloride concentration at the end of lifetime

The figure above shows some remarkable aspects regarding the results of the probability of failure for the different models and chloride concentration at the end of lifetime. We identify two big groups: the curves with low probability of failure and the curves with highest.

On the one hand, the series model, as expected, present the highest probability of failure, and the uncorrelated one has the highest among them. This behavior is the same as explained in 4.3.2.

On the other hand, the rest of parallel and series-parallel models are plotted at the bottom of the graph belonging to low values of probability of failure, as usual and explained in section 4.1.7.7.

Finally, the slope of all the curves tends to remain constant at for high values of chloride concentration.

5. Conclusions and Recommendations

After the case study where an analysis of the performance of the models series, parallel and series-parallel along lifetime affected by chloride concentration has been carried out, the most remarkable aspects from the results obtained are introduced herein.

5.1. Series

It has been shown that the series models are very dominant in the behavior of the general structural system. The fact that only one element is need to lead the structure into failure, it comes to be a determinant factor. Low values of reliability indices will be obtained when dealing with risky elements, in terms of high probability of failure. Therefore, a special attention must be taken when designing these elements due to its importance in the whole system.

Moreover, when dealing with correlation between the different bridges that make up the system, we have seen that is not very relevant for the evolution of the reliability index. It is true that there exist a slightly difference, where the uncorrelated models seem to be the worst case, since the focus of the damage has not to come from the same source and make it much more vulnerable the model. However we do not identify it as a factor that will drive the model to collapse.

Finally and regarding the increase of the chloride concentration, there are no significant differences as far as behavior. The higher is the concentration, the faster will affect to the model.

5.2. Parallel

In the parallel models, an increase in the reliability index is always obtained. The fact that the n number of elements that perform the structural system must fail to collapse will decrease the probability of failure of the whole. For this reason is recommended to perform as much number of elements as possible to increase the reliability of the structure.

Secondly, it is very important to highlight the importance of the correlation of this model. As obtained in the results, there is a big difference with the uncorrelated and perfect correlated. This is due to the same factor described for series above. If we add to the difficulty to fail all the parallel elements the adversity that the same origin of failure comes from the same source, it will bring the system to high values of reliability and negligible values of the probability of failure.

Finally, the corrosion due to the chloride concentration will have effect as in the series model, but they still keep high values of reliability of the model and low values of probability of failure.

5.3. Series-Parallel

The conclusions about the series-parallel models are as expected, in between the other models. Basically, the final performance of the system depends on the configuration of the elements. When an element is designed in the series part, there are two possibilities. If it is a weak point, the reliability of the whole will be low, whereas is a strong one it will give high values of reliability to the final reduction of the system. Thus, a wise design of the elements has to be done if low values of probability of failures want to be achieved.

In addition, the correlation will be more decisive if the series elements have more weight in the model. Needless to say that anyhow it is not as important as it is in the pure parallel case.

5.4. Increasing Chloride Concentration

After studying different chloride concentrations that ranges from x1 times the initial until x10 times, some conclusions can be extracted. There is an important change for low values of the chloride concentration and not very significant for high rates. The reason is that the closer you are to critical chloride concentration the faster you will see changes. However, for instance, differences between x8 and x10 will not be remarkable since both are almost in the critical threshold.

Also, the changes induced by corrosion will follow the configuration of the structural system (series, parallel or series-parallel). In other words, the trend that a series systems present in the reliability index along lifetime is the same. It may be faster depending on the rate of chlorides, but will not change the typical curve plot.

Finally it is interesting to point out that, if we need to fix a threshold of reliability index or probability of failure, maintenance actions will take place later or sooner depending on the initial chloride concentration.

5.5. Final Reliability Index and Probability of Failure for Different Chloride Concentrations

As it is explained in 5.4, here it is easier to appreciate that as we increase the initial chloride concentration, the results tend to stabilize for a given value of reliability index. Thus, for a given structural system and boundary conditions (input design parameters and environmental effects) a time horizon reliability index could be assess as shown in Figure 42.

It is important to point that plotting the reliability index and probability of failure will give us complementary information but with different point of view of the structural system analysis.

5.6. Extrapolating the Results Obtained

Now that we have seen the behavior of each structural system (series, parallel and series-parallel) and the effects of correlation and corrosion of the reinforcement due to chlorides, the next extrapolations can be done:

When designing a series element, we have to make sure that it will have a high reliability index. If not, once the reduction of the system is done, the structure will be very close to the reliability index of that part of the bridge.

For instance, if a series element has a $\beta_{ele} = 1.5$, it does not matter that the $\beta_{sys} = 8$, because when reducing the system, the series part will be very dominant giving low values for the whole.

However, it does not affect as decisive as it is in parallel models, where we have some freedom to keep high values of the reliability index.

The next figure shows the typical scheme of the different parts of a bridge. Then we can see which elements must be well-designed so that the reduced system will have high values of β_{sys}.

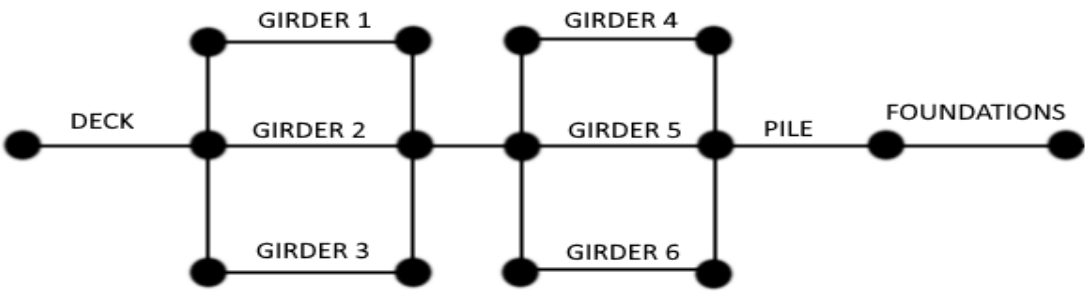

Figure 44: Scheme of the elements that make up the structural system of a bridge

5.7. Future Works

As future studies, it would be interesting to know what happen in case that a maintenance plan has to be performed to achieve a minimum target of reliability. Then a different behavior through lifetime should be obtained.

Also, how the important is the diameter of the reinforcement in the concrete. Study if the size affects to the chloride effects. The fact that, the steel could be allocated differently on the cross-section may or not help to face the corrosion problems.

Finally, try to find out if there is any input of the material that can give more reliability to the system.

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