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SIMULTANEOUS FREQUENCY AND PHASE ESTIMATION OF DIGITAL MODULATED SIGNALS IN BURST MODE TRANSMISSION

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ABSTRACT

Simultaneous frequency and phase acquisition and tracking methods in very short data records, are reviewed and studied from a ML-estimation point of view.

The frequency estimate of Tretter and Kay [1] is extended to a multiple parameter estimate. The resulting algorithm uses ML estimates for magnitude and phase prior to the linear regression procedure. The procedure provides good performance even under narrowband interferences and coloured background noise.

The application of interest is the estimation of doppler frequency and channel leakage in burst communication systems.

I. INTRODUCTION

When block accuracy is required in frequency and phase estimation at very low convergence rate, close-loop systems are not adequate to be used. Open-loop techniques with optimum performance under short data-records are appropriate in full digital receivers.

MPSK burst transmission of digital data and speech signal "consists" on emission and reception of signal slots. Each slot contains some symbol periods ($T_B = NT_s$ where T_B is the slot duration and T_s the symbol duration, N uses to be between 16 and 64).

The slot duration is shorter than the time between two consecutive bursts. So, there must be a very well accurate and controlled timing system. Thus the time uncertainty Δt will be a negligibly small fraction of the symbol time T_s. Other parameters of signal, the carrier and the frequency uncertainty have to be reestimated for each successive burst (slot). Viterbi and Viterbi in [1] described a class of nonlinear algorithms to estimate the unknown phase of a carrier which is fully modulated by m-ary PSK modulation. With the assumption that the frequency uncertainty Δf is insignificant, this is

$$2\pi \, \Delta f \, N \, T_s << 2\pi \qquad (1)$$

the phase estimator obtained results only moderately less efficient than the optimum linear technique for estimating the phase of an unmodulated carrier. It will be seen here as the so-called FFT-Processor.

In an other line, Kay in [2] reported a new frequency estimator for a single complex sinusoid in complex white Gaussian noise, the estimator is computationally very efficient and it attains the Cramer-Rao bound, at moderately high signal-to-noise ratios.

In next sections a maximum-likelihood multiparameter estimate is presented. In a first stage an optimum filtering over the received signal is introduced in the Viterbi algorithm. ML magnitude and phase estimates are obtained prior to the linear regression procedure in the phase plane.

It concentrates the potential phase estimation on a frequency range nearby the frequency where the sinusoid to be detected is expected to be.

In a second stage the extension of Tretter and Kay ideas in frequency detection to the case of digital modulated carriers, in presence of narrow band interferences and coloured noise, is given; and, also the problem of simultaneous magnitude, phase and frequency estimation in digital modulated signals is studied.

The signal environment and first and second stage with the introduction of Kay frequency detection will be reported in II. In III different results for the system performance are shown. Finally conclusions and references are given in IV and V respectively.

II. SIMULTANEOUS PARAMETER ESTIMATION
II.1. Signal environment

The received signal in its analog or continuous form is shown in (2) within a symbol interval.

$$x_r(t) = A_{ck} \cos\left[(\omega_N + \omega_d)t + \theta + \theta_k\right] + n(t) \qquad (2)$$

A_{ck} is the magnitude of the symbol labeled with index k, and θ_k is the modulation phase in the same symbol. ω_d denotes the frequency error due to doppler effects and ω_N is known as nominal frequency.

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The discrete signal (3) appears when sampling the analog signal. It is considered f_N. T = 0,2 where T is the sampling period.

$$x(n)=\sum_{k=1}^{N} A_{ok}\cos\left[(\omega_N+\omega_d)n+\theta+\theta_k\right]\Pi\left(n-k\frac{T_s}{T}\right)+n(n) \quad (3)$$

ω_N and ω_d in (3) are normalized to the sampling frequency. Our objective in this work is to find an estimate for the following set of parameters: A_o, f_d, θ.

II.2. Optimum filtering for complex sinusoids

The Viterbi receiver is reviewed as a base to obtain the optimum sinusoid detector. The receiver presented by Viterbi in [1] could be interpreted in the following form. First of all, the arriving signal shown in (3) is filtered by a Q-order pre-steered filter at the nominal frequency ω_N. The filter output signal y(n)

$$y(n)=\frac{1}{Q}e^{-jQn\omega_N}\sum_{i=0}^{Q-1} x[n(Q-1)+i]\exp\left[-ji\omega_N\right] \quad (4)$$

with a vectorial notation and ignoring the modulation factor $\exp[-j\omega_N n]$ results

$$y(n)=\underline{S}^H.\underline{X}(n) \quad (5)$$

The filter response $\underline{S}$ is the steering vector at the nominal frequency, and the input signal $\underline{X}(n)$ contains the Q input-samples included in each snapshot $\underline{X}(n)$. Given that synchronism is ensured, the filter length Q is assumed to be equal to the number of samples per symbol. Then, the number of independent measures contained in each burst is the number of symbols by burst N. The filter output is assumed to be a pure sinusoid (6)

$$y(n)=\left(A_o+a(n)\right)\exp j\left[\omega_d nQ+\theta+\theta_n+\varepsilon(n)\right] \quad (6)$$

being a(n) and $\varepsilon(n)$ the resulting noise in the phase and envelope estimation due to the output filtered noise n(n). Viterbi uses this framework to derive and estimate the phase, after a non-linearity which in phase terms, removes the modulation by just multiplying by the number of levels M the output signal phase in (6). The last step is to do an average of the above samples, that will provide, on the phase plane, an unbiased estimate of the phase leakage θ, independent of the ω_d term, whenever it is small compared with the symbol rate (1). The resulting structure is optimum to track phase evolution from burst to burst, always than (1) is ensured.

As an alternative to the drawback appeared by (1), an optimum multiparameter (A_o, ω_d and θ) detector is presented, supported by the framework of optimum detection of a sinusoid in coloured noise. The vectorial notation reflected in (5) is used. The components of $\underline{X}(n)$ are a complex sinusoid (phasor) with additive Gaussian noise n(n).

$$x_{n,i}=A_o\exp j\left[((n-1)Q+i)(\omega_N+\omega_d)+\theta_n+\theta\right]+n(n,i) \quad (7)$$

An approximation is done in this point with the assumption that the Q components of the data snapshot have a constant leakage phase due to the doppler frequency, equal to the average leakage.

$$x_{n,i}\cong A_o\exp\left[j\omega_N((n-1)Q+i)\right]\exp j\left(\omega_d(n-1)Q+\theta_n+\theta+\omega_d\frac{Q}{2}\right) \quad (8)$$

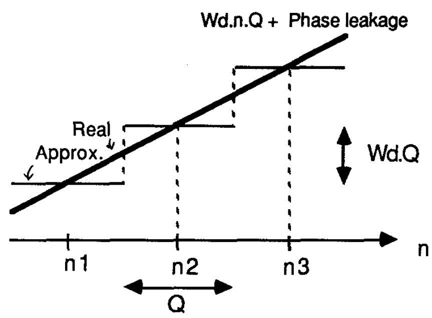

FIGURE 1. Doppler leakage approximation.

With this approximation the input signal vector $\underline{X}(n)$ is decomposed as

$$\underline{X}(n)=A_o(n)\,\underline{S}+\underline{N}(n) \quad (9)$$

where $\underline{N}$ is a coloured Gaussian noise snapshot and

$$\underline{A}_o(n)=A_o\exp\left[-j\omega_N(N-1)Q\right]\exp j\left[\omega_d(n-1)Q+\omega_d\frac{Q}{2}+\theta_n+\theta\right] \quad (10)$$

is a complex envelope in the snapshot $\underline{X}(n)$. When minimizing with a likelihood criteria

$$\underline{X}(n)-A_o(n)\underline{S} \quad (11)$$

an optimum envelope (12) is obtained

$$\hat{A}_o(n)=\frac{\underline{S}^H\,\underline{\underline{R}}^{-1}\,\underline{X}(n)}{\underline{S}^H\,\underline{\underline{R}}^{-1}\,\underline{S}} \quad (12)$$

$\underline{\underline{R}}$ is the autocorrelation signal matrix estimated using all the snapshots contained in a burst. With the N $A_o(n)$ resulting envelope symbols the different parameters are evaluated.

The amplitude measure $A_o(n)$ could be used to validate the phase and frequency estimates.

In the other hand, the phase estimate (13)

$$\measuredangle A_o(n)=\omega_d\,Q(n-1)+\theta_n+\theta+\omega_d\frac{Q}{2}+\varepsilon(n) \quad (13)$$

has to give a line, plus noise phase $\varepsilon(q)$. In order to use the phase matching procedure the modulation θ_n is removed multiplying the phase (13) by the level number M.

The problem considered in (1) is also solved by means of a phase unwrapping algorithm, that increases the maximum ω_d permitted to $2\omega_d QM < 2\pi$. This means that the phase discontinuity must be separated by two or more symbols.

$$\varphi(n) = M\omega_d\, Q(n-1) + M\left[\theta + Q\frac{\omega_d}{2}\right] + M\varepsilon(n) \qquad (14)$$

After the θ_n modulation has been removed and the phase has been unwrapped the phase matching is done with the best fit of parameters α and β to (14)

$$\varphi(n) = \alpha(n-1) + \beta \qquad (15)$$

II.3. Optimum parameter detection

In the receiver just reported before, the main problem arises when computing phase and frequency estimates from the filter output, $A_0(n)$. Due to the process of removing phase modulation the noise phase $\varepsilon(n)$ is multiplied by M. Also there is the unwrapping phase, a highly non linear operation that promotes a threshold effect similar to the classical one associated with frequency modulation.

Even for low SNRs, it would be reasonable to match, in a deterministic sense, to the received signal y(n), after phase unwrapping

$$y(n) = A_0(n) = \left[A_0 + a(n)\right] \exp j(\alpha(n-1) + \beta) \qquad (16)$$

a complex sinusoid with a phase leakage β and a frequency α with amplitude $A_0(n)$.

If the objective is the mean square error (MSE) it will be formulated as

$$\xi = \sum_{n=0}^{N-1} \left| y(n) - \hat{A}_0\, e^{j\hat{\alpha}(n-1) + \hat{\beta}} \right|^2 \qquad (17)$$

Taking derivatives with respect $\hat{A}_0$, $\hat{\alpha}$ and $\hat{\beta}$ the respective optimum estimates are

$$\hat{A}_0 = \frac{1}{N}\sum_{n=0}^{N-1} A_0(n)\ \cos(\varepsilon(n)) \qquad (18)$$

$$\hat{\beta} = tg^{-1}\frac{Real\ [*]}{Imag\ [*]} \quad \text{where} \quad * = \sum_{n=0}^{N-1} y(n)\ e^{-j\hat{\alpha}(n-1)} \qquad (19)$$

$\hat{\alpha}$ optimum is just the value that maximizes the periodogram of y(n), $|Y(w)|^2$.

As conclusions of the precedent estimations it can be said that

-. Wheneve· the phase noise $\varepsilon(n)$ is small enough to assume $\cos(\varepsilon(n))=1$, the amplitude estimate (18) becomes the same to that one that could be directly estimated from (12).
-. The magnitude $A_0(n)$ acts as a quality window in the phase estimate (19).
-. The optimum way to determine the frequency leakage α will be a DFT of the filter output samples y(n) or measurements available.

Considering the frequency estimate $\hat{\alpha}$, although is an optimum result, computationally is poorly efficient because of the high number of operations required to compute the DFT, even with only 17 symbols by burst. In this sense, as an alternative, the method proposed by Kay in [2] can be used.

Subtracting each two consecutive sample phases, a finite phase difference vector $\underline{\Delta}$ is computed. The likelihood function to minimize $\underline{\Delta} - \alpha\underline{1}$ gives a resulting estimate in the same way that the optimum filtering described in II.2.

Kay's approach is very easy to compute, pays an increment of 7 dB in the SNR threshold with respect the MLE approach. Above 6 dB of SNR, when the approximation of Gaussian noise phase is valid, both estimates achieve the CRB given by (20)

$$VAR(\omega_d) = \frac{6}{SNR.N\ (N^2-1)} \qquad (20)$$

The equivalence between the FFT (high resolution periodogram) and the weighted phase average of Kay is ensured by the fact that if two estimators attain the CRB they must be the same.

As an alternative to the optimum computation of phase and frequency, it would be possible to use a linear regression algorithm to jointly estimate frequency and phase, when comparing (14) and (15). The resulting estimates would be accurate whenever the signal to noise ratio (SNR) is higher enough to justify the approximation involved in (18). From the authors experience, it seems to be that when linear regression is used in the phase plane, similar limits for α frequency detection are got, nevertheless higher values of SNR are necessary to get a phase estimate β similar to the optimum one.

III. RESULTS

The final multiparameter detector proposed here is:

-. Use ML filter to obtain y(n). With Q=5 robustness in filter length is got and hardware complexity is maintained under reasonable limits.
-. Remove modulation from the N independent output samples multiplying the phase by the number of modulation levels.
-. Compute frequency leakage estimate using the weighted phase estimate of S. Kay. (Unwrapping phase is not necessary with this method).
-. Compute magnitude estimate by averaging the magnitude of the filter output samples.
-. Compute phase estimate using the optimum phase detector, after removing the frequency leakage, and the measured magnitude.

In figure 2, for a 4-PSK signal received there is the frequency error (2-a) and the phase error (2-b), versus Eb/No from 0 to 6 dB, with $\theta=0$, $f_d=0$, N=17 and Q=5. There have been done 10 randomized trials per Eb/No value. Errors are shown in percentage with respect to nominal or upper bound values.

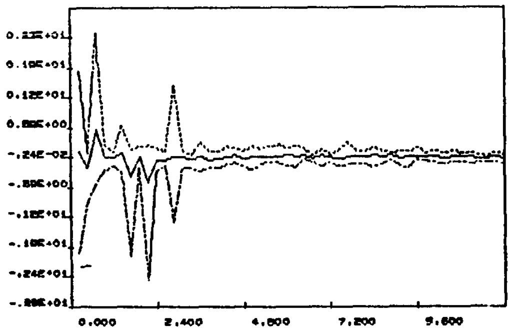

FIGURE 2-a) Frequency error

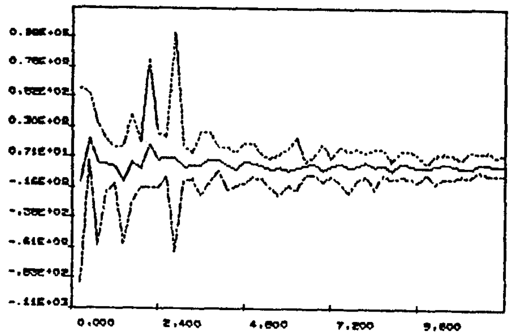

FIGURE 2-b) Phase error

FIGURE 2. Parameter error versus Eb/No for a 4-PSK signal with $\theta=0$, $f_d=0$, a) Frequency error, b) Phase error.

Same kind of results are reported in Figure 3 with worse reception conditions. In these proves $\theta=0{,}32\pi$ rad., $f_d=0{,}01$, the rest of dates are the same as in figure 2.

IV. CONCLUSIONS

The experiments carried out by the authors let us conclude that when f_d is greater than 0,01 (5% of the carrier frequency) the performance of the detector is destroyed. In order to guarantee the correct quality of the receiver the Eb/No has to be above 6 dB. This value is 2,4 dB below the required one for uncoded PSK of four values, to achieve a BER of 10^{-4}. Lower values of f_d will allow further reduction in the required Eb/No.

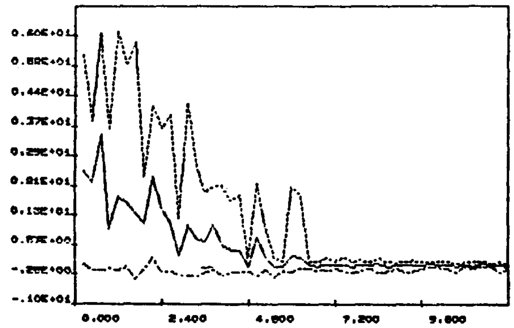

FIGURE 3-a) Frequency error

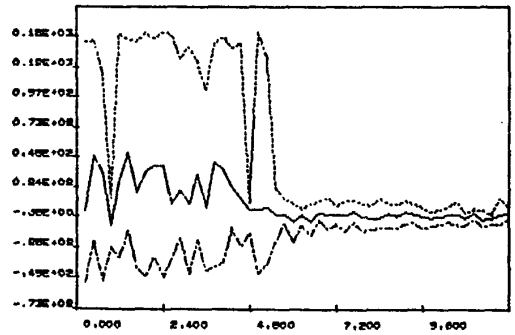

FIGURE 3-b) Phase error

FIGURE 3. Parameter error versus Eb/No for a 4-PSK signal with $\theta=0{,}32\pi$ rad., $f_d=0{,}01$ a) Frequency error, b) Phase error.

VI. REFERENCES

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