SIGNAL PROCESSING V: Theories and Applications
L. Torres, E. Masgrau, and M.A. Lagunas (eds.)

SIMULTANEOUS PARAMETERS ESTIMATION OF DIGITAL MODULATED SIGNALS

M. Cabrera, M. A. Lagunas

Dept. of Signal Theory and Communications
ETSIT-UPC
P.O. Box 30.002-Barcelona, Spain

A maximum likelihood estimate for frequency carrier, phase leakage and amplitude of digital PSK modulated signals is presented. The three optimum parameters obtained are analyzed and compared with suboptimum realizations in the synchronization process. In order to achieve an efficient system a final structure is closed where each method is selected in order to get not only appropriated performance as well as a relatively low computational load.

1. INTRODUCTION

When digital phase and frequency modulated signals are transmitted in a satellite communications environment, the principal characteristics of the communications system have to be reviewed. When the signal is received in Time Division Multiple Access (TDMA) mode, loop systems as Phase locked loops and adaptive estimation algorithms are not adequate to be used because of the short slots of signal received periodically. The convergency of the algorithms to estimate parameters of synchronization could not be got inside each slot of signal arriving at the system.

Here, different kind of estimation methods will be analyzed, as in an optimum sense as in a suboptimum mode in the parameters estimation part. First of all the environment and kind of signal will be studied. A presentation of the general system will be done with two different stages. In the first one, the signal is filtered and down converted to a low pass band signal. In the second stage the synchronization is implemented by mean of estimating parameters of the carrier signal as amplitude, residual frequency and phase. The complete system can be summarized by the blocks diagram of figure 1.

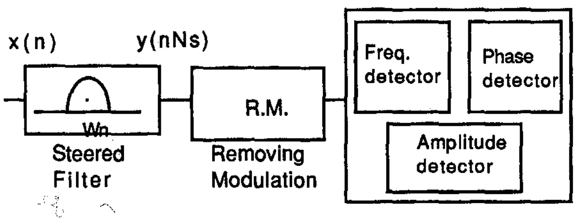


Figure 1. Basic scheme of the filtering/estimation process.

The three parameter detectors can be implemented jointly or separately inside the second stage.

Regardless our objective is the simultaneous estimation of the magnitude, phase and frequency, we may conclude that frequency represents the main difficulty and, for this reason, we will discuss initially the design of the filter of figure 1, in terms of frequency estimation only.

After modulation of the signal has been removed, the estimation stage is done, selecting a function objective to be minimized. The signal is modeled as a single tone with frequency, phase and magnitude to be estimated. The parameters are obtained in every burst or slot of signal, minimacing the Mean Square Error (MSE) between the real signal and its model, and they result optimum in a maximum likelihood sense, which allow to deal with modulated carriers and also provides narrowband interference rejection.

Phase and magnitude process are used in the synchronization of the system here presented, but optimum frequency obtained results high computationally inefficient. Because of this other kind of frequency estimation [2] with considerable lower computational load, is used here and it is compared with optimum methods to show that with Eb/No (bit energy to noise spectral density ratio) above 0 dB, both process give as result the same performance.

The experiments have been done over PSK-4 and PSK-8 modulated signals in coloured noise, and they confirm the statements on robustness and Eb/No threshold effects done previously.

Taking as base the optimum estimation method, other kind of suboptimal estimation for frequency and phase have been also proved. In a final system, each parameter detector, will have to be selected, and the particular solution will usually depend on a tradeoff between low computational load and optimum performance required in the system.

2. SIGNAL ENVIRONMENT

The type of signal used in this work is described. The kind of modulation is PSK. The information of the symbols is contained in the phase, added to the phase of the carrier signal. For N Symbols by burst, the received form will be as is shown in (1), where p(.) is the resulting pulse in the receiver without intersymbol interference.

$$x_r(t) = A.\sum_{s=1}^{N} \cos[(\omega_d + \omega_N)t + \theta + \theta_s]p(t - sT_s) \quad (1)$$

T_s is the symbol period, θ_s is the modulation phase and θ the leakage phase. It is assumed that a previous translation from the real carrier frequency ω_c to the nominal frequency ω_N has been done about the signal. There is a residual unknown frequency ω_d. This uncertainty has to be estimated to get appropriated accuracy.

The signal is sampled with a sampling period T normalized to be one. Taking in consideration the modulation process, the sampling is done to set $f_N + f_d$ around 0.25 normalized value at the sampling frequency. N_{ss} will be the number of samples by symbol. In every burst there are N symbols to process the signal (2).

$$x(i) = \sum_{s=1}^{Q} A.\cos[(\omega_d + \omega_N)n + \theta + \theta_s]p(n - s.N_{ss}) + n(i) \quad (2)$$

With this kind of signal, we have N symbols and $N.N_{ss}$ samples in every burst.

3. FILTERING STAGE

In the first stage of the receiver the signal is filtered to obtain a sample by symbol for each group of N_{ss} samples. It is assumed than accurate timing synchronization is got in this stage. With a vector notation at the output filter it will be

$$y(n) = \underline{A}^H.\underline{X}(n) \quad (3)$$

where $\underline{A}^H$ is the filter response and $\underline{X}(n)$ is the signal vector corresponding to a single symbol. Taking as the filter order Q, the same as the number of samples by symbol N_{ss}, there will be N samples of signal y(n) at the filter output.

Inside each symbol, the signal can be considered as a single sinusoid without any phase change. Receiving the signal with the steering vector at the nominal frequency ω_N, at the output of the filter, there will be the signal y(n), where the frequency will be just the unknown doppler frequency.

$$y(n) = \frac{1}{Q}\sum_{i=0}^{Q} x(n-i)\exp(j\omega_N \frac{i}{Q}) \quad (4)$$

In these conditions, the filter response $\underline{A}(n)$ is the steering vector $\underline{S}$

$$\underline{A} = \underline{S} = [1, \exp(-j\omega_N), \ldots\ldots.(-j\omega_N(Q-1)] \quad (5)$$

Analysing y(n), analytically will be as (6)

$$y(n) = A.\exp[j\varphi(n)] + n(n) = A(n).\exp[j\psi(n)] \quad (6)$$

The instantaneous magnitude A(n) is the signal amplitude with noise and $\psi(n)$ is the instantaneous phase. $\varepsilon(n)$ is the output noise contained in $\psi(n)$.

$$\psi(n) = \omega_d.Q(n-1) + \theta_n + \theta + \omega_d\frac{Q-1}{2} + \varepsilon(n) \quad (7)$$

An approximation is assumed in (7). The phase of each sample is the central phase of the measure snapshot. Really in the case of no interferences and only additive Gaussian noise (7) will be the exact phase at the filter output.

Being our objective to estimate A and $\varphi(n)$ from a single snapshot $\underline{X}(n)$ there is an alternative to filter the signal x(n). It is found, deriving the maximum likelihood estimate for parameters A and ψ. The filter response is then obtained as a result of minimacing the difference between the signal vector and a sinusoid model for it, and it results as a data dependent vector. But in ideal conditions, we mean, without other sinusoid interferences, white noise, stationarity and signal autocorrelation matrix without estimation errors, the maximum likelihood filter results just as the filter presented in (5), the FFT processing of the current snapshot [1]. In other words, for the above mentioned assumption the optimum filtering is not longer data dependent and can be implemented as a DFT processor of length Q.

4 ESTIMATION STAGE

The available signal samples in this stage, are shown in (6) and (7). There are N_{ss} samples of y(n), a sample by symbol, and the objective is to estimate the residual frequency carrier ω_d, the phase leakage θ and also the magnitude A. The modulation phase θ_n is removed from the signal. In order to remove the modulation, we need to multiply the measured phase by the number of modulation levels M. This will remove from $\varphi(n)$ the contribution of modulation steps between symbols but it will introduce 2π steps in the corresponding phase. To remove the 2π steps existing in the new phase, a phase unwrapping algorithm can be used when it is necessary. Considering the above explained operations, the new available signal to process parameters will be as it is shown in (8)

$$y(n)=A_n{}^H.\exp j(M\omega_d n + M\theta + M\frac{n-1}{2}\theta\omega_d + \varepsilon'(n)) =$$

$$A_n{}'\cdot \exp j\varphi(n) \qquad (8)$$

Being the Mean Square Error (MSE) the selected objective, the problem is formulated in (9)

$$\xi = \sum_{n=1}^{N} |y(n) - A'(n).\exp(j\psi'(n)|^2) \quad (9)$$

where $\psi'(n)$ is equal to $\beta + \alpha(n-1)$. Identification of phase θ and frequency ω_d with the parameters α and β is trivial. It should be noted that the above objective is not an optimum criteria if the phase noise is not gaussian; but, even in the non gaussian case the associated performance is recognized. Due to the way y(n) is formed, if there are no wrong phase step correction in the unwrapping, the associated noise to the phase remains gaussian, when the Eb/No there is high enough, above 2 or 3 dB and the MSE estimation for magnitude A, frequency ω_d and phase θ remain optimum.

Taking derivatives of (9) with respect $A'$and φ'in terms of β and α, and setting to zero, the maximum likelihood estimations obtained are the following.

Magnitude estimation:

$$A=\frac{1}{N}\sum_{i=1}^{N} A(i).\cos\varepsilon'(n) \qquad (10)$$

the classical estimate for magnitude (11) results the optimum estimate MSE whenever $\varepsilon'(n)$ is small enough to assume the second term of the sum as one for all the measurements done

$$A \cong \frac{1}{N}\sum_{i=1}^{N} |y(n)| \qquad (11)$$

This estimate can be used to validate frequency and phase estimates, depending on the A level.

Concerning the phase derivative, and taking first the case of derivative with respect the phase leakage β (12) is obtained to compute the optimum phase.

The obtained optimum phase is just the same presented by Viterbi in [3] also as the optimum, but for the case of no doppler frequency, α=0. It is just the phase obtained, in the Fourier transform of the signal evaluated at α frequency.

$$\text{Phase: } \theta = \frac{1}{M}\,\mathrm{tg}^{-1}\frac{\mathrm{real}(\sum_{m=0}^{N-1} y(mT_s)\exp(-j\alpha(m-1)))}{\mathrm{imag}(\sum_{m=0}^{N-1} y(mT_s)\exp(-j\alpha(m-1)))} \qquad (12)$$

The frequency estimate is obtained from the derivative of (9) with respect to parameter α. After some algebra it can be shown that the optimum α just sets out the condition to find an extrema of the periodogram of y(.) or the square magnitude of its Fourier Transform. Thus, the optimum way to determine the frequency leakage α, will be a DFT of the filter output samples or measurements available.

Three optimum solutions for magnitude, frequency and phase estimates have been given. They represent the maximum likelihood estimates with the only drawback for the frequency computes of its computational load. With the DFT method it would require a lot of samples to get a BER of the order of 10^{-4}.

The precedent method give the optimum ω_d estimation but with a considerable computation cost. As an alternative for frequency estimation, Kay's approach [2] minimizes an objective function between a finite consecutive samples phase differences vector ($\underline{\Delta}$) and $\alpha\underline{1}$. The computational load of the method is very low. It is synchronous with the symbol period, and gets the Cramer-Rao bound when Eb/No is above 6 dB, possible threshold for its use.

The resulting algorithm, yet preserving the mentioned properties, becomes optimum even for signal contaminated with coloured noise. This is of capital importance taking in mind that many currently TDMA systems in communications will require to track amplitude, phase and frequency from symbol to symbol, with a maximum of 16 symbols in the burst and with no more than 4 samples per symbol.

The determination of amplitude estimates provides an useful quality index of the phase and frequency estimates. This is due to the role played by carrier, or sinusoid, magnitude in the threshold effect. Note that, in order to obtain gaussian noise in the phase estimates, the input signal to noise ratio is above 0 dB is required. Once this constraint holds, the frequency and phase estimates achieve the Cramer- Rao bound. At this point, it is clear that narrow band interferences, coloured noise or sinusoid modulation destroy this property and, as a consequence, the over-all system fails.

As a suboptimum alternative to the optimum phase and frequency estimations, linear regression can be used with the phase of the removed modulation signal, to obtain α and β estimates. In this case the algorithms work directly with the signal phase samples.

Summarizing the final structure for the receiver, the general process to synchronize the TDMA signal is shown in figure 2.

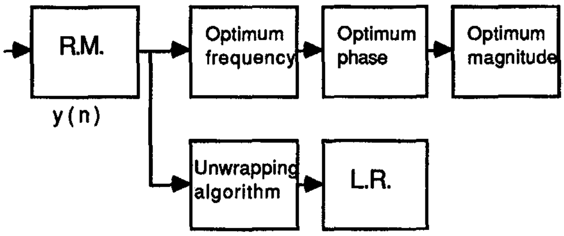


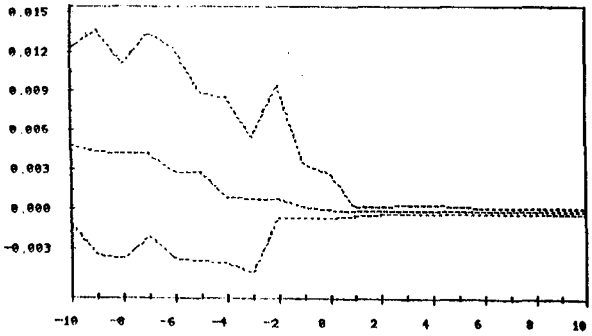
Figure 2. General Receiver

5. RESULTS AND COMMENTS

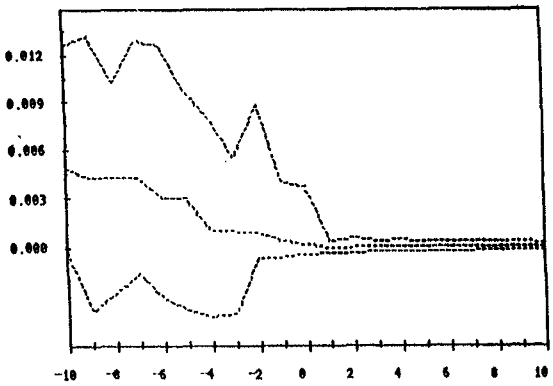
Simulations results were obtained over PSK-4 and PSK-8 modulated signals in coloured noise.

In the figure 3, for a 4-PSK signal with N=18 symbols, N_{ss}=5 samples/symbol, ω_d=.005, θ= 10º, the normalized errors are compute for Eb/No from -10 dB to 10 dB, with 50 randomized trials for each Eb/No value. For each signal, the maximum, average and minimum estimate errors are compute for parameters ω_d, θ and A.

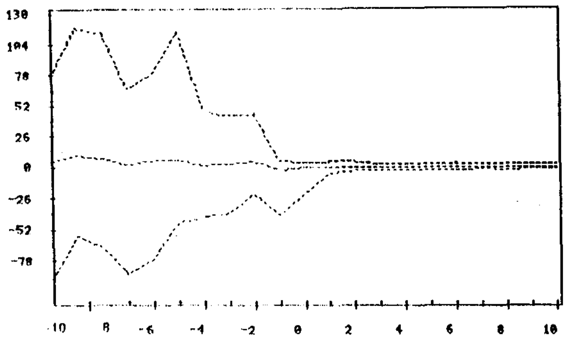
Thresholds of Eb/No for work, could be estimated from figure 3. In these conditions 1 dB for Eb/No is appropriated to get low values of errors. For other values of parameters to estimate, the appropriated thresholds can also be processed and they always result better than the required Eb/No in PSK transmission if f_d is greater than 0,01 (5% of the carrier frequency).



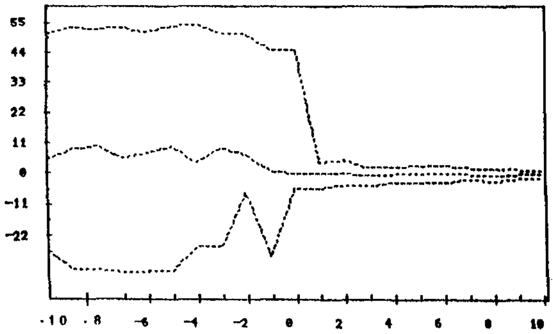
a) Linear regression frequency estimate



b) Kay frequency estimate



c) Linear regression phase estimate



d) ML phase estimate

Figure 3. 4PSK signal, ω_d=.005, θ=10º, 18 symbols and 5 samples/symbol. Values of Eb/No from -10 dB to + 10 dB.

6. REFERENCES

[1] M. A. Lagunas, M. Cabrera. "Open Loop Joint Parameter Estimation for Burst Communication Mode". ESA (European Spatial Agency) Report, ESA-ESTEC Noorwijk (Netherlands), February 1990.

[2] S. Kay, "Statistically/computationally Efficient Frequency Estimation". IEEE-ICASSP 88, paper E3.5, New York 1987.

[3] A. J. Viterbi, A. M. Viterbi, "Nonlinear Estimation of PSK Modulated Carrier Phase with Applications to Burst Digital Transmission". IEEE Tr. on IT, Vol IT_29 nº 4, July 1983.

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