Lecture / Ponencia XX

Uncertainty Analysis in the IS-LM Model

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Abstract

We introduce a model for uncertainty in the IS-LM linear macroeconomic model with exogenous parameters. An uncertainty profile $\mathcal{U}$ is a short and macroscopic description of an stressed situation. We use $\mathcal{U}$ to define a strategic game where two agents, the angel and the daemon, act selfishly and have different goals. Those Nash equilibria provide the stable strategies in stressed situations, giving a natural estimation of risk. We apply this analysis to a linear version of the IS-LM model and analyse the structure of the Nash equilibria in some particular games.

1. Introduction

Knight (1921) made a distinction between *risk* and *uncertainty* (see also Akerlof, Schiller (2009), Chapter 11): "*Risk* refers to something that can be measured by mathematical probabilities. In contrast, *uncertainty* refers to something that cannot be measured because there are no objective standards to express probabilities." We explore a way to transform uncertainty into risk by modelling stressed uncertain situations by an strategic situation among two agents and use game theory to analyse those models. We propose a simplified way to model uncertain situations through an *uncertainty profile* $\mathcal{U}$. A profile $\mathcal{U}$ gives a short and macroscopic description of the potential stress of an economic system, together with the a description of an strategic situation where two agents, the angel $\mathfrak{a}$ and the daemon $\mathfrak{d}$ have opposite goals. Uncertainty profiles were introduced in Gabarro *et al.* (2014) to analyse Web orchestrations under stress in uncertain situations.
We are interested here in extending the approach to the *InvestmentSavings-LiquidityMoney (IS-LM)*, introduced in Hicks (1937) (see also Baldani *et al.* (2007)) providing a way to express, in equilibrium, the national income and the interest rate as a a function of several exogenous parameters. Our motivation comes from the fact that one of the limitations in the IS-LM model is the lack of risk. Hicks (1980-1981) wrote that "there is no sense in liquidity, unless expectations are uncertain".

You can imagine $\mathfrak{a}$ as the government and $\mathfrak{d}$ as the people, the unions or the crude reality, those agents can act by stressing some of the exogenous parameters of the system in a positive or negative way. This point of view is far from being unique, in southern Europe many citizens believe just the opposite. Thus by considering different uncertainty profiles we can model different situations. This give us a possible algorithmic interpretation of the Keynesian animal spirits[1] in macroeconomics: animal spirits ($\mathfrak{a}$ and $\mathfrak{d}$) are players in a strategic game.
We consider a linear approximation of the IS-LM model, present the global framework of analysis and analyse the Nash equilibria of the resulting $\mathfrak{a}$-$\mathfrak{d}$ for some simple uncertainty profiles. In the studied cases, both $\mathfrak{a}$ and $\mathfrak{d}$ have limited capability to act over the exogenous parameters. The paper is structured as follows. In Section 2 we present the IS-LM model. In Section 3 we model the behaviour of the *whole set* of exogenous parameters under stress. In 4 we introduce uncertainty profiles and the associated $\mathfrak{a}$-$\mathfrak{d}$ games tailored to the linear IS-LM model and analyse the Nash equilibria of some cases. In particular We prove that adding uncertainty to any fiscal policies generates strategic situations in which there is always a a dominant strategy equilibria. Finally, in Section 5 we raise some lines for future research.

2. The InvestmentSavings-LiquidityMoney (IS-LM)

We recall here the definition of the IS-LM model, which deals with equilibrium states and is given by two equations. The first equation, the IS (InvestmentSavings) line, describes the equilibrium points in the market of goods and services. The second equation, the LM (LiquidityMoney) line, represents the equilibrium points in the money market.
The IS line, $Y = C(Y - T) + I(r) + G$, gives an accounting identity where, Y is the national income, Y, r is the interest rate, the remaining parameters are the sum of the annual rates of spending: C by consumers, $I(r)$ by the investors (as a function of the interest rate), and G by government G.
The LM line, $M/P = L(r, Y)$, expresses the fact that the money supply M/P, where M is the money and P the price leve,l is given by the liquidity preference $L(r, Y)$ which is a function of the national income Y and the interest rate r (see Keynes (1936) pag 166).
An equilibrium point (Y, r) is a pair solving the system. This pair (Y, r) gives the point where both markets are on mutual equilibrium.
We consider a *linear approximation* of the IS-LM model taking $C(Y - T) = a + b(Y - T)$, $I(r) = c - dr$ and $L(r, Y) = eY - fr$. Those expressions leads to a set of *exogenous parameters* $\mathcal{E} = \{a, b, c, d, e, f, T, G, M, P\}$. The set of *endogenous variables* $\{Y, r\}$ can be expressed, in equilibrium, as a function of the parameters. The parameters and their constraints are the following:

[1]"Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as the result of animal spirits - a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities" (Keynes (1936) pages 161-162). Look also at Akerlof, Schiller (2009), Part One.

autonomous consumption	$0 < a$	interest sensitivity for real money	$0 < f$
marginal propensity to consume	$0 < b < 1$	taxes	$0 < T$
exogenous investment	$0 < c$	exogenous government spending	$0 < G$
interest sensitivity	$0 < d$	price index	$0 < P$
income sensitivity for real money	$0 < e$		

The IS *line* $Y = Y(r)$ is obtained from $Y = C(Y-T) + I(r) + G = a + b(Y - T) + c - dr + G$. The LM *line* $r = r(Y)$ is obtained from $M/P = eY - fr$. Thus $Y = (a + c + G - bT - dr)/(1-b)$ and $r = (eY - \frac{M}{P})/f$. Solving the linear system $Y = Y(r)$, $r = r(Y)$ we obtain the following *equilibrium point* (Y, r) where $g = (1-b)f + de$:

$$Y = \frac{f}{g}(a + c + G - bT) + \frac{d}{g}\frac{M}{P} \text{ and } r = \frac{e}{g}(a + c + G - bT) - \frac{(1-b)}{g}\frac{M}{P}$$

As (Y, r) depends on $\mathcal{E}$, when needed we write $(Y(\mathcal{E}), r(\mathcal{E}))$.

Example 2.1. Consider the following valuation or the set $\mathcal{E}$:

a	b	c	d	e	f	T	G	M	P
200	3/4	200	25	1	100	100	100	1000	2

We get, $Y = 1700 - 100r$, $r = Y/100 + 5$ and solving $Y = 1100$, $r = 6$. □

3. Stress model for the IS-LM model

We assume the existence of two adversary agents called the *angel* $\mathfrak{a}$ and the *daemon* $\mathfrak{d}$. Those agents act at the same time attempting to stress the model by altering the values of some of the exogenous parameters of the linear IS-LM model. A *stress model* for $\mathcal{E}$ is a tuple of pairs $\mathcal{S} = ((\delta_{\mathfrak{a}}(e), \delta_{\mathfrak{d}}(e)))_{e \in \mathcal{E}}$ describing the perturbation that can be applied to the parameter. The perturbation values are real numbers, so they can be either positive or negative.

Example 3.1. Let us give an example of a stress model $\mathcal{S}$:

agent	a	b	c, d, e, f	T	G	M	P
$\mathfrak{a}$	0	$+1/20$	0	0	$+50$	0	0
$\mathfrak{d}$	0	0	0	$+50$	-25	0	$+1$

The angel $\mathfrak{a}$ has the (potential) capability to act upon the parameters $\{b, G\}$. The marginal propensity to consume could be increased from $3/4$ to $4/5$. This is modelled by $\delta_{\mathfrak{a}}(b) = 1/20$. The government spending G could be increased by $\delta_{\mathfrak{a}}(G) = 50$. For any other $e \in \mathcal{E} \setminus \{b, G\}$, the agent $\mathfrak{a}$ has no possibility to act upon and therefore $\delta_{\mathfrak{a}}(e) = 0$. The daemon $\mathfrak{d}$ has the following (potential) capability to act upon some of the parameters in $\{P, T, G\}$. The price of money could increase by $\delta_{\mathfrak{d}}(P) = 1$. Taxes could increase $\delta_{\mathfrak{d}}(T) = 50$. The G spending could decrease by $\delta_{\mathfrak{d}}(G) = -25$. For any $e \in \mathcal{E} \setminus \{P, T, G\}$, $\delta_{\mathfrak{d}}(e) = 0$. □

Let us define the transformation of the exogenous parametres under stress model $\mathcal{S}$. when $\mathfrak{a}$ has decided to exert stress capabilities on $a \subseteq \mathcal{E}$ and $\mathfrak{d}$ has decided to exert stress capabilities on $d \subseteq \mathcal{E}$. For any $e \in \mathcal{E}$, the stressed version of e under (a, d) denoted as stress$_{\mathcal{S}}(e)[a, d]$ is defined as follows:

$$\text{stress}_{\mathcal{S}}(e)[a, d] = \begin{cases} e & e \notin a \cup d \\ e + \delta_{\mathfrak{a}}(e) & e \in a \setminus d \\ e + \delta_{\mathfrak{d}}(e) & e \in d \setminus a \\ e + \delta_{\mathfrak{a}}(e) + \delta_{\mathfrak{d}}(e) & e \in a \cap d \end{cases}$$

Finally the equilibrium point is $(Y(\text{stress}_{\mathcal{S}}(e)[a, d]), r(\text{stress}_{\mathcal{S}}(e)[a, d])$. When $\mathcal{S}$ is clear from the context, and we want to emphasize the role of the choice of parameters (a, d) we note $(Y(a, d), r(a, d))$.

Example 3.2. We continue with Example 2.1 and $\mathcal{S}$ given in Example 3.1 for $(a, d) = (\{b\}, \{P, G\})$. Writing the new values as $e' = \text{stress}(e)[\{b\}, \{P, G\}]$ we note stress$_{\mathcal{S}}(\mathcal{E})[\{b\}, \{P, G\}] = \{a', b', c', d', e', f', T', G', M', P'\}$ and:

agent	choice	a	b	c	d	e	f	T	G	M	P
		200	$3/4$	200	25	1	100	100	100	1000	2
$\mathfrak{a}$ $\mathfrak{d}$	$a = \{b\}$ $d = \{P, G\}$		$+1/20$						-25		+1
		a'	b'	c'	d'	e'	f'	T'	G'	M'	P'
		200	$3/4 + 1/20 = 4/5$	200	25	1	100	100	$100 - 25 = 75$	1000	2+1=3

Consider for instance the computation of b'. As $b \in a \setminus d = \{b\} \setminus \{P, G\} = \{b\}$ we have $b' = b + \delta_{\mathfrak{a}}(b) = 3/4 + 1/20 = 4/5$. The obtaiined equilibrium point is $Y(\{b\}, \{P, G\}) = 28700/27 \approx 1062.96$, $r(\{b\}, \{P, G\}) = 197/27 \approx 7.29$. $\square$

4. Uncertainty profiles and $\mathfrak{a}$-$\mathfrak{d}$ games in the IS-LM model

An *a priori* (global and macroscopic) view of the IS-LM model $\mathcal{E}$ in a stressed environment $\mathcal{S}$ with uncertainty is modelled by an *uncertainty profile* $\mathcal{U}$ introduced in Gabarro *et al.* (2014). In $\mathcal{U} = \langle \mathcal{E}, \mathcal{S}, \mathcal{A}, \mathcal{D}, b_{\mathcal{A}}, b_{\mathcal{D}}, u_{\mathcal{A}}, u_{\mathcal{D}} \rangle$, $\mathcal{A}$ and $\mathcal{D}$ are subsets of $\mathcal{E}$ which may be stressed. The analyser has the perception that when an angelic parameter in $\mathcal{A}$ is perturbed this is unlikely to have a serious impact (not malicious behaviour). In contrast when a daemonic parameter in $\mathcal{D}$ is perturbed this may well have catastrophic implications (malicious behaviour). The spread of the stress is modelled by $b_{\mathcal{A}}$ and $b_{\mathcal{D}}$. The effects are measured by a utility functions $u_{\mathcal{A}}$ and $u_{\mathcal{D}}$. Given $\mathcal{U}$ a strategic situation arises when the IS-LM model is subjected to combined angel and daemon actions.

An *uncertainty profile* is a tuple $\mathcal{U} = \langle \mathcal{E}, \mathcal{S}, \mathcal{A}, \mathcal{D}, b_{\mathcal{A}}, b_{\mathcal{D}}, u_{\mathcal{A}}, u_{\mathcal{D}} \rangle$, where $\mathcal{S}$ is a stress model for $\mathcal{E}$, $\mathcal{A} \cup \mathcal{D} \subseteq \mathcal{E}$ and $b_{\mathcal{A}} \leq \#\mathcal{A}$ and $b_{\mathcal{D}} \leq \#\mathcal{D}$.

A $\mathfrak{a}$-$\mathfrak{d}$ strategic game (the $\mathfrak{a}$-$\mathfrak{d}$ game) associated to the uncertainty profile $\mathcal{U}$ is given by $\Gamma(\mathcal{U}) = \langle \{\mathfrak{a}, \mathfrak{d}\}, A_{\mathfrak{a}}, A_{\mathfrak{d}}, u_{\mathfrak{a}}, u_{\mathfrak{d}} \rangle$ where $\{\mathfrak{a}, \mathfrak{d}\}$ are the *angel* and the *daemon* players. The player actions are $A_{\mathfrak{a}} = \{a \subseteq \mathcal{A} \mid \#a = b_{\mathcal{A}}\}$ and $A_{\mathfrak{d}} = \{d \subseteq \mathcal{D} \mid \#d = b_{\mathcal{D}}\}$. The utilitites are $u_{\mathfrak{a}} = u_{\mathcal{A}}$ and $u_{\mathfrak{d}} = u_{\mathcal{D}}$.

Example 4.1. Let us continue with Example 3.2. We assume that having high income is good and high interest rate is bad. Then we obtain the bimatrix game $\Gamma(\langle \mathcal{E}, \mathcal{S}, \{b, G\}, \{P, G, T\}, 1, 2, Y, r \rangle)$:

$\mathfrak{a}$ \ $\mathfrak{d}$	$\{P,G\}$	$\{P,T\}$	$\{T,G\}$
$\{b\}$	1062.96 , 7.29	1029.62 , 6.962	1233.33 , $22/3 \approx 7.33$
$\{G\}$	1066.66 , $22/3 \approx 7.33$	1041.66 , 7.08	1075 , 5.75

The PNE are $(\{G\},\{P,G\})$ $(\{b\},\{T,G\})$ with different values for Y and r. □

It is well know that any strategic game has a Nash equilibrium so from the $\mathfrak{a}$-$\mathfrak{d}$ game, the stable stressed situations are described by the strategies of the players in the Nash equilibria. In the following we analyze the properties of such equilibria for some particular cases.

A case of fiscal policy under uncertainty. Consider the case where $\mathfrak{a}$ and $\mathfrak{d}$ have the capability to act over G and T, that is $A_{\mathfrak{a}} = A_{\mathfrak{d}} = \{\{G\},\{T\}\}$ and $u_{\mathfrak{a}} = Y$ and $u_{\mathfrak{d}} = r$ (as in Example 4.1). We ask if the addition of uncertainty can generate a unstable situation where each agent is trying to catch the other and no PNE exists. Theorem 4.1 shows a negative answer.

Theorem 4.1. *Given $\mathcal{U} = \langle \mathcal{E}, \mathcal{S}, \{T,G\}, \{T,G\}, 1, 1, Y, r\rangle$ where the stress model $\mathcal{S}$ verifies is $\delta_{\mathfrak{a}}(x) = \delta_{\mathfrak{d}}(x) = 0$,for $x \notin \{T,G\}$, it holds that $\Gamma(\mathcal{U})$ has always a dominant strategy equilibrium.*

Proof Sketch. In $\Gamma(\mathcal{U})$ the sets of actions are $A_{\mathfrak{a}} = A_{\mathfrak{d}} = \{\{T\},\{G\}\}$ and $u_{\mathfrak{a}}(a,d) = Y(a,d)$, $u_{\mathfrak{d}}(a,d) = r(a,d)$. Defining $\mu_{T,T} = -b\big(\delta_{\mathfrak{a}}(T) + \delta_{\mathfrak{d}}(T)\big)$, $\mu_{T,G} = \delta_{\mathfrak{d}}(G) - b\delta_{\mathfrak{a}}(T)$, $\mu_{G,T} = \delta_{\mathfrak{a}}(G) - b\delta_{\mathfrak{d}}(T)$ and $\mu_{G,G} = \delta_{\mathfrak{a}}(G) + \delta_{\mathfrak{d}}(G)$ game $\Gamma(\mathcal{U})$ is :

$\mathfrak{a}$ \ $\mathfrak{d}$	$\{T\}$	$\{G\}$
$\{T\}$	$Y + (f/g)\mu_{T,T},\ r + (e/g)\mu_{T,T}$	$Y + (f/g)\mu_{T,G},\ r + (e/g)\mu_{T,G}$
$\{G\}$	$Y + (f/g)\mu_{G,T},\ r + (e/g)\mu_{G,T}$	$Y + (f/g)\mu_{G,G},\ r + (e/g)\mu_{G,G}$

it can be proved that $(\{G\},\{G\})$ is a dominant strategy equilibrium. □

Considers a variation where $\mathfrak{d}$ acts as before but $\mathfrak{a}$ can act over a or M.

Theorem 4.2. *Let $\mathcal{U} = \langle \mathcal{E}, \mathcal{S}, \{a,M\}, \{T,G\}, 1, 1, Y, r\rangle$ where $\mathcal{S}$ is*

agent	a	b,c,d,e,f	T	M	G	P
$\mathfrak{a}$	$\delta_a \geq 0$	*0*	0	$\delta_M \geq 0$	*0*	0
$\mathfrak{d}$	0	0	$\delta_T \geq 0$	0	$\delta_G \leq 0$	0

it holds that $\Gamma(\mathcal{U})$ has always a dominant strategy equilibrium.

Proof Sketch. The following $\mathfrak{a}$-$\mathfrak{d}$ game $\Gamma(\langle \mathcal{E}, \mathcal{S}, \{a,M\}, \{T,G\}, 1, 1, Y, r\rangle)$ has dominant strategy equilibriums:

$\mathfrak{a}$ \ $\mathfrak{d}$	$\{T\}$	$\{G\}$
$\{a\}$	$u_{\mathfrak{a}} = Y + \frac{f}{g}(\delta_a - b\delta_T)$ $u_{\mathfrak{d}} = r + \frac{e}{g}(\delta_a - b\delta_T)$	$u_{\mathfrak{a}} = Y + \frac{f}{g}(\delta_a + \delta_G)$ $u_{\mathfrak{d}} = r + \frac{e}{g}(\delta_a + \delta_G)$
$\{M\}$	$u_{\mathfrak{a}} = Y + \frac{1}{g}(\frac{d}{P}\delta_M - fb\delta_T)$, $u_{\mathfrak{d}} = r - \frac{1}{g}((1-b)\frac{\delta_M}{P} + eb\delta_T)$	$u_{\mathfrak{a}} = Y + \frac{1}{g}(\frac{d}{P}\delta_M + f\delta_G)$, $u_{\mathfrak{d}} = r - \frac{1}{g}((1-b)\frac{\delta_M}{P} - e\delta_G)$

□

A case giving zero-sum games. Given $k > 0$ consider the *reduced utility* $u(a,d) = Y(a,d) - kr(a,d)$ and let $\mathcal{U} = \langle \mathcal{E}, \mathcal{S}, \mathcal{A}, \mathcal{D}, b_{\mathcal{A}}, b_{\mathcal{D}}, u \rangle$. Then $u_{\mathfrak{a}} = -u_{\mathfrak{d}} = u$ and $\Gamma(\mathcal{U})$ is a zero-sum. By similar techniques we have.

Theorem 4.3. *Given* $\mathcal{U} = \langle \mathcal{E}, \mathcal{S}, \{T,G\}, \{T,G\}, 1, 1, u \rangle$ *being* $\mathcal{S}$

agent	a,b,c,d,e,f	T	G	M,P
$\mathfrak{a}$	0	$\delta_T \leq 0$	$\delta_G \geq 0$	0
$\mathfrak{d}$	0	$\delta_T \geq 0$	$\delta_G \leq 0$	0

if $(f - ke)\delta = 0$ *the* $\mathfrak{a}$-$\mathfrak{d}$ *game has four* PNE *otherwise it has only one* PNE *either in* $(\{T\},\{T\})$ *or in* $(\{G\},\{G\})$.

5. Further developments

We are working towards understanding the structure of the Nash equilibria of $\mathfrak{a}$-$\mathfrak{d}$ games in more complex situations. Varian (1977) has studied the stability of the IS-LM model. As $(Y(a,d), r(a,d))$ can be seen as perturbations of the equilibrium point (Y, r) if will be of interest to study the relation of both models. Our preliminary result point out that it seems workable to apply the methods in this paper to others parametrized econometric models.

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