Forecasting day ahead electricity price using ARMA methods

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1. Introduction

The liberalization of the Spanish electricity sector that took place in 1997 brought about the emergence of an electricity market where the trading of the energy delivered in the Iberian peninsular net is negotiated. This liberalization led to the creation of a market place on which participants can trade electricity forward contracts for different delivery periods[1].

Nowadays, Spanish market works through different trading channels and via different types of future contracts such as two-part-tariffs and contracts for differences, both of them called bilateral contracts[2]. This study focuses on day-ahead markets, where retailers and producers offer their bids, trying to optimize their profits. Due to this optimization, retailers and producers must conduct thorough studies to forecast electricity prices.

Electricity price forecasting (EPF) is complex due to the high volatility in prices because on one hand, energy is nonstorable, and on the other hand market demand must be continuously met. Day-ahead prices also follow a specific dynamic influenced by mean-reversion, seasonality and spikes [3,4].

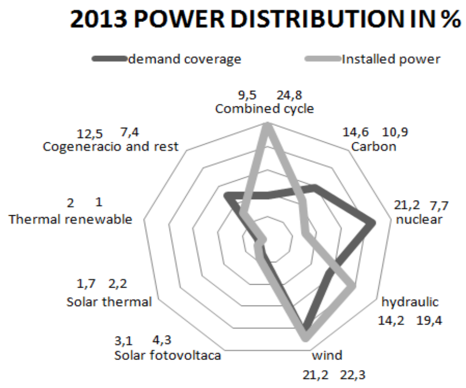
Unfortunately, the energy market differ from other financial markets, as it doesn't allow for continuous trading[3]. Most of the EPF studies are based on hourly prices, so time-series models are not available to use because they assume that the information set is updated by moving from one observation to the next in time. Hourly prices for next day are all determined at the same time, so the information used for pricing of delivery in hour 7 is the same as used in hour 18. Some forecasts are based on modeling daily average prices or they just study hours separately, in that case the market could be treated as continuous trading and a time series model would be right for the study[4]. In this study, in order to simplify the process using time series, the daily average price will be analysed.

Today various models of prediction based on different statistical methods exist, such as many heuristics optimization techniques like Auto-Regressive Integrated Moving Average (ARIMA), Artificial Neural Network(ANN), Adaptive Wavelet Neural Network (AWNN), hybrid PSO–ANFIS, gray model, Wavelet-ARIMA–RBF, hybrid intelligent, Fuzzy NNs (FNN) and Wavelet-FNN[5].

The main objective in this research is to predict day-ahead prices using ARMA. Box and Jenkins developed a regression model to identify, assess and diagnose dynamic time series models in which the time variable plays a key role[6]. In the area of EPF, several studies have already been conducted to estimate prices by ARMA: in some cases ARMA is combined with GARCH using a wavelet transform[7], in others a hybrid model with Radial Basis Function Neural Networks (RBFN) [8] or with Support Vector Regression (SVR) [9] is performed, and also a single ARIMA[10] or the addition of a Transfer Function and Dynamic Regression [11] have been used.

2. Spanish market

The electrical energy average generated in Spain has evolved in recent years from 56% of the energy generated in fossil fuel power stations [12] (mainly coal and fuel oil) in 2000 to an increase of renewable energy (which in 2013 covered 42.4% of demand).



Graphic 1. Spanish demand coverage and installed power in 2013. Font: Compilation based on data supplied by OMIE.

As indicated by [13] the peninsular electricity demand recorded its third consecutive annual decline, descending to 246 313 GWh, 2.3% less than 2012. Meanwhile, hydropower production varies with age (depending on rainfall), and coverage of demand from the rest of renewable progression is stimulated by successive governments. So [14] while wind energy in Spain in 2005 covering 7.7% of demand in 2013 reached 21%.

On the other hand, the six Spanish nuclear power plants have seen their participation in covering demand is gradually diminished due to this constant growth in recent decades and its stabilization due to the nuclear moratorium, from 35% in 1996 around 20% in late 2013.

According to [14] the peninsular installed capacity ended 2013 at 102.395 MW (699 MW greater than 2012). The greatest increase was recorded solar thermal (15% or 350 MW) and solar photovoltaic (3.3% or 103 MW).

This energy is sold in a complex market where the "market players" (electricity producers, distributors, marketers and qualified consumers) buy and sell electricity. The products traded are extremely varied, including sales of energy delivered during all hours of a quarter closed with half a year in advance (futures market) to transactions for energy delivered at a specific time, closed with a few hours notice. (daily and intraday).

2.1- Forwards

To avoid the risks introduced by the volatility of spot market prices [15], the natural tendency is to seek agreements of sale bilateral between generators and consumers. Therefore, in all mature electricity markets, futures markets exist (physical and / or financial) [16] whose volume or relevance is very important and obviously spot markets. As for the futures markets , there are two different ways to perform these contracts : from a regulatory platform by OMIP and OTC markets (unregulated markets) .

2.1.1- OMIP organized Market

This market is the result of collaboration agreements between the governments of Portugal and Spain , which crystallized in the creation of the Iberian Electricity Market (MIBEL) [17], consisting of two poles : the OMEL Spanish pole, responsible of spot market and OMIP Portuguese pole, responsible of futures market , both physical and financial .

The futures market management is performed by OMIP together with OMIClear (Clearing Company SA Energy Markets) [18]. The latter company, owned 100% by OMIP, serves as a clearing house and central counterparty for operations traded on OMIP and also can act as a central counterparty for operations closed in the OTC market. The products traded are future contracts or swaps with various time ranges (days, weeks, months, quarters, years) and time restrictions, since there is a difference between trading energy for the whole day than in peak hours (from 8:00 to 20:00 Monday to Friday).

All the negotiation process is anonymous, being unknown to the agents behind the operations. Moreover, all buying and selling orders are public to participants. In order to ensure market liquidity, it is promoted the existence of market makers. These agents guarantee the existence of a minimum volume of purchase and sale deals in return agreed with the managing body benefits.

As explained in more detail in [2] if the product is financial, a liquidation of the difference between the two prices is performed. For example, an agent bought 1 MWh leaving its open position at expiration, closed at a quote price of € 40 / MWh and the spot price was 45€/MWh, then OMIClear settled in their favour 5€. If the product is physical, the open positions are recorded in a physical trading account and they are sent to the OMEL daily market as accepting price offers. Assuming the same values of the previous example, the agent now buy 1 MWh in OMEL and pay for it € 45 / MWh, but OMIClear performs the aforementioned liquidation and pays 5 €. Therefore, the cost of energy purchased by the agent is 40 € / MWh, which corresponds to the closing price of the product in its last trading day.

2.1.2- The OTC market

In the OTC market trading is done through intermediaries or "brokers" to ensure confidentiality and anonymity of partners during the negotiations. Once an operation is closed, the "broker" brings together the participants of the contract to perfect them and to adopt preventing measures related to counterparty risk.

The most common closed transactions in the OTC market are financial products, which generally is a base product with durations of months, quarters, remaining years and complete years. However, because of the very nature of the OTC market, it may be possible to close specific, physical or financial operations, as the needs of the energy markets agents.

[19] As for specific data, the volume traded on the OTC market during the month of June 2013 stood around 26.7 TWh (+ 12.6% over the previous month). Also, the figure of 26.7 TWh traded in June 2013 is 8% higher than OTC volume traded during the same month last year (24.8 TWh in June 2012). The total volume traded in the first six months of 2013 (141.1 TWh) represents 56.8% of the negotiated throughout the previous year 2012, 9.9% higher than the volume traded in the first six months of 2012 (128.3 TWh).

To reference the OTC market liquidity, accumulated peninsular electricity demand in 2013 amounted, on 30 June, to 122.9 TWh, so the volume negotiated on OTC represents 114.8% of itself.

2.2- Day-ahead market:

The day-ahead market, (DA), aims to carry out electricity transactions for the next day by submitting bids for sale and purchase of electricity. All available production units that are not subject to a bilateral contract are required to submit offers for the day-ahead market, except units lower than 50 MW or that were not covered by the RD 1538/1987 [20] of Law 54/97.

2.2.1- Purchase and sale offers

The sellers in this market are required to be adhered to the rules of market power production [21] through the corresponding Support Agreement. The offers of the sellers will be presented to the market operator, and will be included in a matching procedure, considering the daily scheduling horizon to which they are intended. The sale bids electricity sellers submit to the market operator, can be simple or incorporate complex conditions. Simple bids are selling offers of energy where it's indicated a price and an amount of energy for each period and production unit owned by the sellers. Complex bids are those that meet the requirements governing simple bids, include all, some or one of the four following technical or economic conditions:

- The condition of indivisibility allows for the first few minutes of every hour a minimum value of operating power. This value can only be divided by applying load gradients declared by the same agent, or by applying distribution rules if the price is different of zero.
- The load gradient allows setting the maximum difference between start time and end time power production unit, limiting the maximum energy to match to avoid sudden changes in the production units.
- The minimum income condition allows the realization of offers at every hour, but it must be respected that the production unit doesn’t affect on the result of the match of the day if it doesn’t obtain, for all of its production in the day, a higher income than a fixed amount plus a variable remuneration for each matched kWh.
- The scheduled stop condition allows performing a scheduled stop on a maximum of three hours, with the only condition that the energy offered is decreasing in every hour,

If the production unit has been removed from the match for failure to meet the minimum income required conditions.

2.2.2- Matching process offers

The market operator will perform the matching of the purchase and sale offers of electricity by the method of simple or complex matching, depending on whether they attend simple or complex offers. In the simple matching method, the marginal price and the volume of electricity accepted for each production unit and for each hourly scheduling period are obtained. The complex match method obtains the matching result from simple matching method, in which the conditions of indivisibility and load gradient are added. Through an iterative process, different simple matchings are performed until every unit meets the minimum income and the scheduled stop condition.

With this iterative method, the first provisional final solution is obtained by considering an unlimited capacity in international interconnections. By another iterative process it is obtained the first definitive solution that respects the highest international interconnection capacity, considering both offers made to the daily market, as the executions of physical bilateral contracts with external interconnection involvement in the Iberian Market.

If there is a congestion on the interconnection between the Spanish and Portuguese Electrical systems [22] , the process described previously is repeated performing a separation of markets (market splitting) where a different price in each area of the Iberian Market is obtained. In case of market splitting, the price of the exporting country will be set as the price of the last offer matched localized in its area , and the price of the importing country will be set as the maximum of the prices obtained in the two corresponding matchings.

The Base Program Operation (PDBF) is the daily program, with an hourly breakdown, of the different production units corresponding to sales and acquisitions of energy in the Spanish peninsular system. This program, published at 14:00, is established by system operators from the schedule resulting of the daily market matching by OMIE and the communication and execution of bilateral contracts. After obtaining PDBF, system operators obtain before 16:00, the Viable Daily Schedule (PDV) by incorporating the amendments necessary for the resolution of technical constraints.

2.2.3- Patterns and significant factors in the price in the day-ahead market

The most easily identifiable fact in the behavior of electricity prices is that they are subjected to jumps, that is, abnormally high sudden variations [23]. The peaks are formed when after a sudden high jump in the price, shortly afterwards, it recovers suddenly his average. It have been recorded many episodes of extremely high prices in various parts of the world; and also some other extraordinarily low prices, even becoming negative in some occasion [24] .

Another important feature of the price series is that they show periodic fluctuations at various frequencies and scales along with "calendar effects" (holidays).

As for Spain, there is strong evidence of the existence of a daily and weekly pattern since there is a huge difference between the average prices in peak and base hours. In addition [25] discusses the evidence for the Spanish case of equal means for the days Monday to Friday and a sharp decrease, in the average price, during the weekend, especially on Sunday.

Additionally, in the day-ahead market there is evidence that jumps in prices tend to be correlated with the season, being more frequent and greater in the cold season. [26]

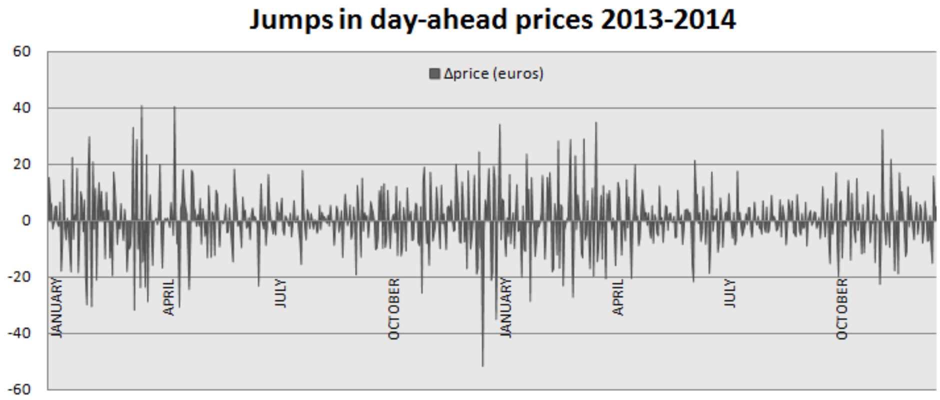
TABLA 1.A

PRECIOS MEDIOS DIARIOS DE LA ELECTRICIDAD EN ESPAÑA
Mercado diario de electricidad (1998-2004)

	Número de observaciones	Media	Mediana	Máximo	Mínimo	Desviación típica	Coeficiente de variación	Coeficiente de asimetría	Coeficiente de curtosis
P	2.541	3,01	2,80	10,69	0,56	0,96	32,07	1,32	7,97
ln P	2.541	1,05	1,03	2,37	–0,57	0,31	29,44	–0,06	3,65
Δ ln P	2.540	0,00	–0,01	1,20	–1,17	0,23	–	0,48	5,55
horas pico									
P	1.815	3,53	3,26	12,58	0,55	1,17	33,17	1,44	9,16
ln P	1.815	1,21	1,18	2,53	–0,59	0,32	26,58	–0,13	3,95
Δ ln P	1.814	0,00	0,00	1,39	–1,36	0,18	–	–0,17	12,02
horas valle									
P	2.541	2,36	2,29	7,20	0,39	0,65	27,62	1,05	6,30
ln P	2.541	0,82	0,83	1,97	–0,94	0,27	33,57	–0,41	5,42
Ain P	2.540	0,00	–0,01	1,78	–1,55	0,28	–	0,21	6,07
demanda alta									
P meses fríos	826	2,96	2,76	10,70	0,56	1,15	38,94	2,01	10,79
P meses cálidos	427	3,35	3,29	5,91	1,37	0,90	26,82	0,21	2,48
demanda baja									
P agosto	217	2,95	2,80	5,03	1,73	0,68	23,07	0,86	3,28

FUENTE: Elaboración propia con datos de OMEL.

Table 1. Daily average prices of electricity in Spain (1998-2004). Font: ANGEL PARDO, VICENTE MENEU, ENRIC VALOR, *Temperature and seasonality influences on Spanish electricity load*.



Graphic 2. Day-ahead price difference. Font: Compilation based on data supplied by OMIE.

To define the price of electricity, many variables may or not follow temporal patterns. Such as historical electricity prices, demand, energy mix, price of commodities, currencies, etc. Meanwhile, demand has a weekly pattern [27]. On weekdays is much higher than weekend and has an annual pattern, where December, January and February are considered high-demand months unlike June, July and August.

2.3- Intraday market

As already mentioned above the intraday market (ID) is a mechanism to adjust the deviations between the demand calculated the day before and the one calculated in various periods of the working day. Intraday is organized in six auctions also called (here-in-after sessions), with different closing times and different periods of action. The energy sold at these auctions is used between 3.25 and 6.25 hours after the closure of the auction.

Table 1
Timing of the Spanish intraday sessions.

	1° Intraday	2° Intraday	3° Intraday	4° Intraday	5° Intraday	6° Intraday	7° Intraday
Gate-closure	18:45 (D-1)	21:45 (D-1)	1:45 (D)	4:45 (D)	8:45 (D)	12:45 (D)	18:45 (D)
Trading hours	1–24 (D)	1–24 (D)	5–24 (D)	8–24 (D)	12–24 (D)	16–24 (D)	21–24 (D)

Table 2. Auction schedules of intraday market. Font: J.P. CHAVES, C. FERNANDES, The Spanish intraday market design: A successful solution to balancerenewable generation?

Table 1 presents the closure and the action period of each intraday session, where 'D-1' refers to the day before of the operation and 'D' is the day of operation.

Although the ID Spanish market is divided into six auctions, the action time of the first session is divided in two. The first period action takes place between 21:00 and 24:00 of 'D-1' and corresponds to the last session for the day of the operation (D-1). The second period corresponds to the first identification session for the day of the operation 'D' where electricity is auctioned for all hours of 'D'.

OMIE manages the DA and ID, regardless of the technical restrictions, which are managed later through additional markets managed independently by each SO (In the case of Spain, by Red Electrica Española). These technical restrictions are intended to adapt the production programs resulting from physical bilateral contracts, the DA and ID, to ensure compliance with quality and safety conditions required for delivery.

According to established in Article 2 of Royal Decree 2019/1997 [28], as amended by Royal Decree 134/2010 of February 12 and Article 13 of Royal Decree 2019/1997, as amended by Royal Decree 1544 / 2011 of October 31, the adjustment services of the system are:

- Resolution of restrictions by guarantee of supply.
- Resolution of technical constraints. (daily, intraday, and in real time)
- Additional services:
  - Raise additional power reserve.
  - Frequency-power regulation (primary, secondary and tertiary regulations).
- Deviation management process.

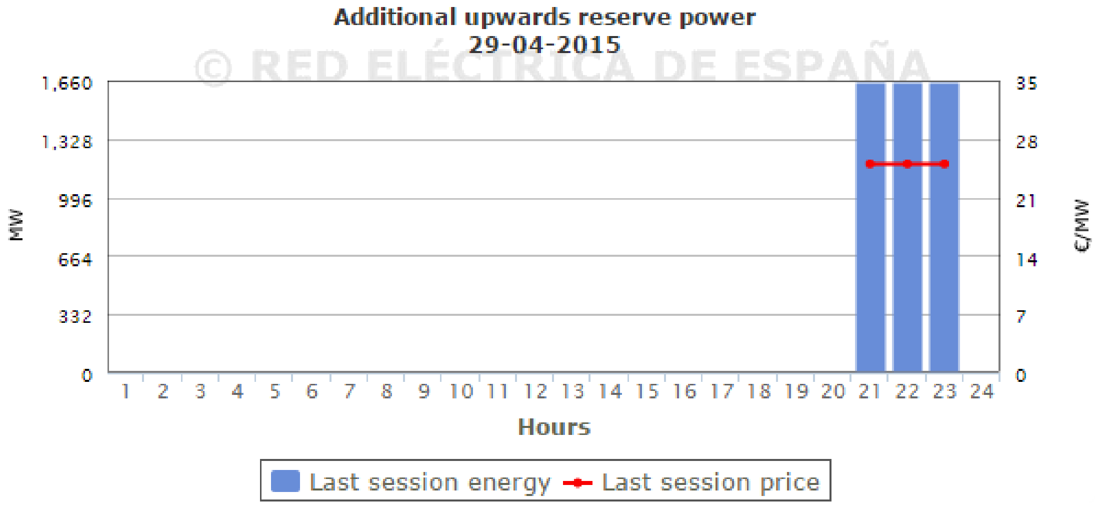
The 25th of February of 2011 started the implementation of Royal Decree 134/2010 of February 12 [29], as amended by Royal Decree 1221/2010, of October 1, whereby is established the constraint solving process guarantee of supply, mechanism that encourages the consumption of national coal by market mechanisms set out in detail in the PO3.10 "Solving guarantee supply constraints".

Article 11.4 of Directive 2003/54 / EC allows the dispatch priority energy combustion with local materials if it does not more than 15% coverage of total demand and without exceeding the individual limits of each generator set in Article 25.1 of Law 54/1997 of the Electricity Sector.

This adjustment is made with the technical constraints of the DA (PDBF), where necessary changes for the safety of power grid determined in the P.O.1.1 "Performance criteria and security" [30] are made.
Each of these procedures has two parts. Phase I or restrictions solution, consists in increase or decrease the energy scheduled determined in the DA in order to comply with the restrictions. Phase II or program settings, is performed from program changes (generation units, energy imports and/or pumping consumption units) to obtain a new generation and demand balanced program that respects the limitations for safety established in phase 1 of the process.

Furthermore, an adjustment of the technical restrictions is also produced after each ID without any added cost and an adjustment for technical restrictions in real time, where only phase 1 is performed and where the complaint process lies with the final demand.

When the Provisional Viable Program (PVP) is published, resulting of the resolution of guarantee of supply and technical constraints of PDBF, the System Operator (SO) calculates for each hour of the next day how many manageable thermal units, which fulfill the premises specified in P.O.3.9 [30], could increase their power. At the same time it calculates the upwards power needed in the Spanish peninsular electricity system as set forth needed in P.O.1.5. "Establishment of the reserve for the regulation frequency-power" [30]. After these two calculations it is realized an auction where the price to be paid in case of using this electricity is set.



Graphic 3. Allocated and price additional upwards reserve. Font: Red Electrica Española (REE).

In order to maintain security and control of system frequency stability, the operator system does three auctions, known as primary regulation, secondary regulation and tertiary regulation.

The primary regulation aims to maintain the stability of system frequency in front of instant imbalances produced between generation and consumption (sudden loss of generation, demand or disruption of international trade). This service is mandatory and unpaid for each generator coupled to the electrical network which are arranged to vary up to 1.5% its rated power P.O.7.1. “Complementary service of primary regulation” [30].
Secondary regulation is a frequency-power control system that aims to maintain the generation-demand balance correcting interconnection deviations between Spain and France

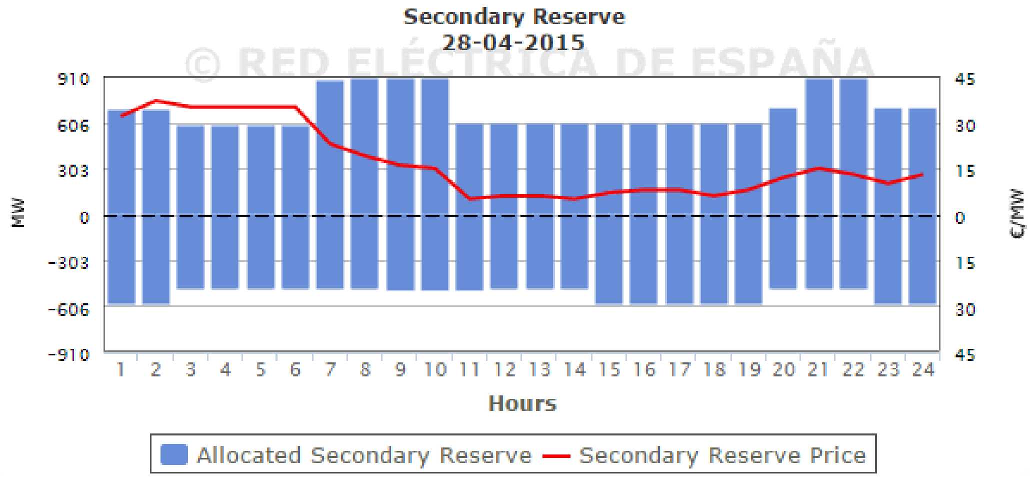
The current recommendation of the ENTSO-E (R5.8 Chapter5_ EWEA) [31] provides that the minimum value of secondary regulation is:

$$R = \sqrt{aL_{max} + b^2 - b}$$

Where:

L_{max} = Level of demand expected in the Spanish Mainland Electricity System.

a = 10 MW // b = 150 MW



Graphic 4. Allocated and price secondary reserve. Font: Red Electrica Española (REE).

The secondary reserve to reduce is established according to the increasing or decreasing trend of the demand curve, between the 40% and 100% of the reservation to rise.

The service of secondary regulation is liquidated on two concepts that are detailed in the P.O.14.4. "Collection rights and obligations of payment services system setting" [30]. Each production unit with assigned band is entitled to charge the hourly marginal price for offering only band availability. In addition, if this energy is used, it is paid the hourly marginal price of secondary use, calculated from the marginal cost of using tertiary regulation.

In P.O.7.3. “Tertiary regulation” [30], the tertiary regulation reserve is defined as the maximum power variation that can perform a production or consumption unit in less than 15 minutes, and can be maintained for at least two hours. The tertiary regulation is a complementary service used to address unforeseen imbalances between generation and demand, reflected in a net use, either reduce or rise, of the secondary regulation reserve.

The tertiary reserve allocations to rise or reduce will be economically valued only by the energy required in the time interval that has remained the assignment. Depending on the direction of the allocation, to rise or reduce, every programming unit is entitled to collect or has a payment obligation respectively, as contained in the P.O.14.4. "Collection rights and payment obligations for services system setting" [30]. The cost is assigned to units which are deviate of the program. If the expected deviation between two sessions of ID is greater than 300 MWh and is maintained for several hours, the SO does not use the allocations from tertiary reserve and convene the imbalance management market .

In the Imbalances Management market rising or reducing bids may be performed for the pumping and manageable production units, whether ordinary or special regime. These market players have less than 30 minutes to present bids for each of their programming units. At the same price, there is an order of allocation set out in the P.O.3.3. “Management of generation-consumption” [30].

The generation and the pumping cost will be valued at the marginal price of the bids allocated in each period. Collection rights or payment obligations of each unit are defined in the P.O.14.4. "Collection rights and payment obligations for services system setting" [30].

The hourly overrun that occurs as a result of the appearance of detours that had to be managed by the SO then has repercussions on market players who have acted against the system needs.

3. Time Series and ARMA:

The objective of this section is to introduce the concepts of time series and ARMA models, without delving into the equations of the theory and mathematics, as our study is based more on determining which factors most influence the spot market price and elaborate the model using EViews software package. Most of the information contained in this section was compiled from the following books, which provide detailed theory and the development of the equations used: Applied Econometric Time Series[32], Introduction to Time Series Analysis and Forecasting[33],Analise de Series Temporais[34] and Forecasting[35]. As it will be seen, the following Section is to show the tests of each of the concepts explained below.

Before we get into the world of time series, stochastic processes should be explained as a time series is a partial realization of a stochastic process.

A stochastic process is simply a collection of random variables indexed by time. The possible values that can take the random variable are properly named states, so you can have a space of discrete states and continuous state space. On the other hand, the time variable may be discrete type or continuous type. In the case of discrete time it could be taken as an example that state changes occur every day, every month, every year, etc. In the case of continuous time, state changes could be made at any time. Below a summary table where the four possible combinations as to the nature of our time variable and differ is shown.

	t **Discrete**	*t* **Continuous**
X **Discrete**	Discrete state and discrete time process (**Chain**) (Monthly produced units of a product)	Discrete state and continuous time process (**Pure Jump Process**) (Produces units until instant t)
X **Continuous**	Continuous state and discrete time process (Daily produced tones of a product)	Continuous state and continuous time process (**Continuous Process**) (Velocity of a vehicle at the instant t)

The data analysed in the studio is the average price of the Spanish day-ahead market of energy, so the time parameter is diary, that is to say, discrete. On the other hand, the state of the variable is continuous since price is of a continuing nature, so we have a continuous state and discrete time process. As mentioned above, a time series is a partial realization of a stochastic process, and in particular with discrete time parameter, therefore a time series is a collection of observations of well-defined data items obtained through repeated measurements over time. For example, measuring the level of unemployment each month of the year would comprise a time series. This is because employment and unemployment are well defined, and consistently measured at equally spaced intervals. Data collected irregularly or only once are not time series.

It is fair to say that time-series econometrics is concerned with the estimation of difference equations containing stochastic components. In its most general form, a difference equation expresses the value of a variable as a function of its own lagged values, time, and other variables.

In order to solve the set of difference equations, an iterative process is performed. Sometimes alternative processes due to the slowness associated with the iteration are used, but in our case, as discussed above, Eviews software package is used, and therefore this slowness does not become a problem for us.

The traditional use of time-series analysis was to forecast the time path of a variable. Uncovering the dynamic path of a series improves forecasts since the predictable components of the series can be extrapolated into the future. The growing interest in economic dynamics has given a new emphasis to time-series econometrics. Stochastic difference equations arise quite naturally from dynamic economic models. Appropriately estimated equations can be used for the interpretation of economic data and for hypothesis testing.

Time series forecasting consists of estimating the unknown parameters in the appropriate model and using these estimates, projecting the model into the future to obtain a forecast. For example, imagine an equation Eq.(3.1) that represents a model with a linear trend,

$$x_t = a_1 + a_2 t + \epsilon_t \tag{3.1}$$

let $\hat{a}_1$ and $\hat{a}_2$ in Eq.(3.2) be estimates of the unknown parameters a_1 and a_2 in Eq.(3.1). If we currently are at the end of period T, the forecast of the expected value of the observation in some future period $T+\alpha$ would be

$$\hat{x}_{T+\alpha}(T) = \hat{a}_1 + \hat{a}_2(\mathrm{T} + \alpha) \tag{3.2}$$

Thus, the forecast simply projects the estimate of the trend component, $\hat{a}_2$, α periods into the future.

In most of the forecasting techniques, errors and observations are assumed to be independent, but this is frequently unwarranted. That is, there are many time series in which successive observations are highly dependent. If this is the case, there are available forecasting techniques which are designed to exploit this dependency and which will generally produce superior results. Many of these forecasting techniques are based on recent developments in time series analysis recently consolidated and presented by Box and Jenkins, and are called *Box-Jenkins models*.

One of this models is the path that is going to be followed to analyse our time series, AutoRegressive Moving Average (ARMA) models. Another possible way is an ARIMA model, where an integrator term is added. The difference between the two models lies in the stationary behaviour of our time series. An expanded Dickey - Fuller test[36] is performed to detect unit roots. In case of rejecting the null hypothesis that affirms the existence of unit roots in our series, it can be said that it is stationary and therefore can be modelled by an ARMA model. In case the hypothesis cannot be rejected, an integrator term must be added turning the ARIMA model.

It is possible to combine a moving average process with a linear difference equation to obtain an ARMA model. An ARMA (p,q) would be a model with *p* Autoregressive(AR) and *q* Moving Average(MA) terms Eq. (3.3):

$$x_t = \delta + \emptyset_1 x_{t-1} + \emptyset_2 x_{t-2} + \cdots + \emptyset_p x_{t-p} - \theta_1 \epsilon_{t-1} - \theta_2 \epsilon_{t-2} - \cdots - \theta_q \epsilon_{t-q} + \epsilon_t \tag{3.3}$$

Or

$$\emptyset_p(B) x_t = \delta + \theta_q(B) \epsilon_t$$

Where

$$\epsilon_t \sim N(0, \sigma^2) \rightarrow \text{(White Noise)}$$

and B is the backward shift operator, defined such that

$$B^j \epsilon_t = \epsilon_{t-j}$$

ARMA processes must meet two conditions indisputably: stationarity and invertibility.

The stationarity of a time series is related to its statistical properties in time. That is, in the more strict sense, a stationary time series exhibits similar "statistical behaviour" in time and this is often characterized as a constant probability distribution in time. However, it is usually satisfactory to consider the first two moments of the time series and define stationarity (or weak stationarity) as follows: (1) the expected value of the time series does not depend on time and (2) the autocovariance function defined as Cov(y_t ,y_{t+k}) for any lag k is only a function of k and not time: that is γ_y (k) = Cov (y_t,y_{t+k}).

In a crude way, the stationarity of a time series can be determined by taking arbitrary "snapshots" of the process at different points in time and observing the general behaviour of the time series. If it exhibits "similar" behaviour, one can then proceed with the modelling efforts under the assumption of stationarity.

The stationarity of an ARMA process is related to the AR component in the model and can be checked through the roots of the associated polynomial, which is developed from Eq.(3.4):

$$m^p - \emptyset_1 m^{p-1} - \emptyset_2 m^{p-2} - \cdots - \emptyset_p = 0 \tag{3.4}$$

If all the roots of Eq. (5.64) are less than one in absolute value, then ARMA (p, q) is stationary.

It is usual to call a moving average model invertible if it has an equivalent autoregressive representation of infinite order. The importance of the invertibility requirement is that a non-invertible moving average model cannot be used to forecast. The invertibility condition is also used to select one of many alternative moving average models that have the same autocovariances [37].

Similar to the stationarity condition, the invertibility of an ARMA process is related to the MA component and can be checked through the roots of the associated polynomial,

$$m^q - \theta_1 m^{q-1} - \theta_2 m^{q-2} - \cdots - \theta_q = 0 \tag{3.5}$$

If all the roots of Eq. (3.5) are less than one in absolute value, then ARMA (p, q) is said to be invertible.

The software package shows immediately all the roots, so it is very simple to identify both conditions and reject different models in case they are not fulfilled.

So far, ARMA (p, q) processes have been explained, but obviously, the values of the parameters p and q must be specified. To do this, different models must be performed obtaining various statistical coefficients that provide information related to their quality on the statistical model. Once you have checked the conditions of stationarity and invertibility have been checked, the most appropriate model should be selected. The two factors that help us choose the most appropriate model are Akaike information criterion (AIC) and Schwarz Information Criterion, also known as Bayesian Information Criterion (BIC) [38].

In order to obtain the values of p and q , the correlation between continuous observations can also be analyzed. For that, the functions of simple and partial autocorrelation must be studied. Anyway, this functions are just a estimating aid as ARMA models can present complex correlation functions.

The autocorrelation function (ACF) is a set of coefficients that have a measure between observations separated by k periods in time series. Instead, the partial autocorrelation function (PFAC) measures the correlation between periods of separation removing the effects caused by intermediate observations between these periods. The tool used in the analysis of these functions is the correlogram. By PFAC correlogram, you can get the value of p and q, seeking the last significant coefficient that exceeds the limit. Then you should check if the AR (p) and MA (q) coefficients are significant in the model.

The residuals of our ARMA(p,q) must be also analyzed. They should be white noise, that is to say, zero mean and random. Thus, residues do not contain any information in our model. To check this, you must analyze the Ljung-Box q-statistic (LBQ) to test whether a series of residuals of a model are random and independent. If residuals are not independent, one residual can be correlated with a different observation k time units later. Autocorrelation can decrease the accuracy of a time-based predictive model, such as time series plot, and lead to misinterpretation of the data.

In order to improve the model fit, a set of exogenous variables wil be added corresponding to the significant factors in the market price discussed in the previous section. To do this, the statistical test should be checked to verify that the variable actually affects the model. The model will be also tried to be improved by adding terms of seasonality [39], making our model Seasonal Autoregressive Moving Average (SARMA), ARMA (p, q) x (P, Q $)_s$, whose equation is defined as:

$$\emptyset_p(B)\emptyset_p(B^s)x_t = \theta_q(B)\theta_q(B^s)\epsilon_t$$

In the next section, all tests in the process of modelling and detailed results as well as the final model chosen are shown.

4- Analytical process and results

Coming up next, the study to determine our prediction model is described in detail. This study consists of two parts: the first is the development of a pure ARMA model and the second is an ARMA model in which exogenous variables are used just to identify the most significant factors in the energy price performance.

The behavior of our time series should be the first analysis to perform, since as explained in the previous section, it should be checked that our data do not contain a unit root. For that, an Augmented Dickey-Fuller test has been done:

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(PMEDIO)

Method: Least Squares
Date: 06/11/15 Time: 15:57

Sample (adjusted): 1/08/2013 12/31/2014
Included observations: 723 after adjustments

Variable	Coefficient	Std. Error	t-Statistic
PMEDIO(-1)	-0.089759	0.024411	-3.676930
D(PMEDIO(-1))	-0.176548	0.041123	-4.293209
D(PMEDIO(-2))	-0.300794	0.040058	-7.508937
D(PMEDIO(-3))	-0.254833	0.040962	-6.221259
D(PMEDIO(-4))	-0.115481	0.040063	-2.882516
D(PMEDIO(-5))	-0.218923	0.037586	-5.824588
D(PMEDIO(-6))	-0.125463	0.037090	-3.382694
C	3.819037	1.108929	3.443896

R-squared	0.189697	Mean dependent var
Adjusted R-squared	0.181763	S.D. dependent var
S.E. of regression	9.227243	Akaike info criterion
Sum squared resid	60876.54	Schwarz criterion
Log likelihood	-2628.492	Hannan-Quinn criter.
F-statistic	23.91221	Durbin-Watson stat
Prob(F-statistic)	0.000000	

Null Hypothesis: PMEDIO has a unit root
Exogenous: Constant
Lag Length: 6 (Automatic - based on SIC, maxlag=19)

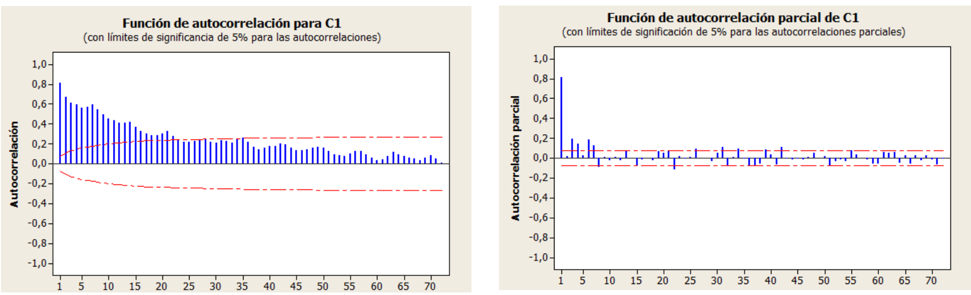
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.676930	0.0046
Test critical values:	1% level	-3.439167	
	5% level	-2.865321	
	10% level	-2.568840	

*MacKinnon (1996) one-sided p-values.

Table 3. Augmented Dickey-Fuller Test. Font: Compilation based on data supplied by MIBEL and analysed with Eviews.

As it can be observed, our statistic test allows us to reject the null hypothesis that asserts the existence of a unit root, and therefore it can be said that our data series is stationary, for any level of confidence.

The next study to make respecting to the average price time series is the correlation over time. It will be used one of the tools most commonly used in econometric studies: the correlogram, already mentioned in the previous section. Although the simple correlogram does not attenuate very quickly, it can be seen in the PFAC that the time series is clearly dependent on the seven days preceding the date (corresponding to the previous week). Also in this same graph it is shown that the previous day greatly influences the current date, and therefore the first step in predicting the price will be from an AR (1).



Graphic 5. EACF and PACF of the model. Font: Compilation based on data supplied by OMIE.

From this first step, it starts an iterative game where AR (p) and MA (q) terms are added contemplating the criteria explained in the previous section.

Once it has been added an AR or MA and have been found to meet the above criteria, it is time to add other AR or MA until LBQ correlograms become unsignificant, and therefore, the residuals do not contain any information.

The image on the right shows the FAC and the PFAC of the residuals of the final model that was found in this study, and as we can see, none of the waste comes significant.

Date: 06/12/15 Time: 15:51
Sample: 1/08/2013 12/31/2014
Included observations: 723
Q-statistic probabilities adjusted for 6 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.010	0.010	0.0800	
		2	-0.012	-0.012	0.1881	
		3	0.034	0.034	1.0070	
		4	-0.002	-0.003	1.0109	
		5	0.020	0.021	1.2915	
		6	-0.005	-0.007	1.3118	
		7	0.029	0.030	1.9332	0.164
		8	-0.007	-0.009	1.9665	0.374
		9	0.005	0.007	1.9859	0.575
		10	-0.045	-0.048	3.4966	0.478
		11	0.061	0.064	6.2481	0.283
		12	-0.035	-0.040	7.1276	0.309
		13	0.007	0.014	7.1636	0.412
		14	0.061	0.054	9.9364	0.270
		15	-0.008	-0.004	9.9810	0.352
		16	-0.036	-0.039	10.930	0.363
		17	-0.007	-0.005	10.971	0.446
		18	-0.040	-0.046	12.131	0.435
		19	-0.042	-0.037	13.453	0.413
		20	-0.002	-0.005	13.456	0.491
		21	-0.003	0.004	13.461	0.567
		22	-0.041	-0.046	14.697	0.547
		23	-0.040	-0.030	15.886	0.532
		24	-0.048	-0.045	17.581	0.484
		25	-0.032	-0.034	18.327	0.501
		26	0.034	0.039	19.217	0.508
		27	-0.017	-0.012	19.432	0.557
		28	0.052	0.048	21.461	0.492
		29	-0.044	-0.043	22.927	0.465
		30	-0.050	-0.039	24.810	0.416
		31	0.112	0.111	34.381	0.100
		32	0.025	0.027	34.840	0.115

Table 4. Residual correlogram. Font: Compilation based on data supplied by OMIE.

The definite model which can approximate the time series of electricity prices in the Day-ahead market is:

Dependent Variable: PMEDIO
Method: Least Squares
Date: 06/12/15 Time: 15:49
Sample (adjusted): 1/08/2013 12/31/2014
Included observations: 723 after adjustments
Convergence achieved after 5 iterations
MA Backcast: 12/18/2012 1/07/2013

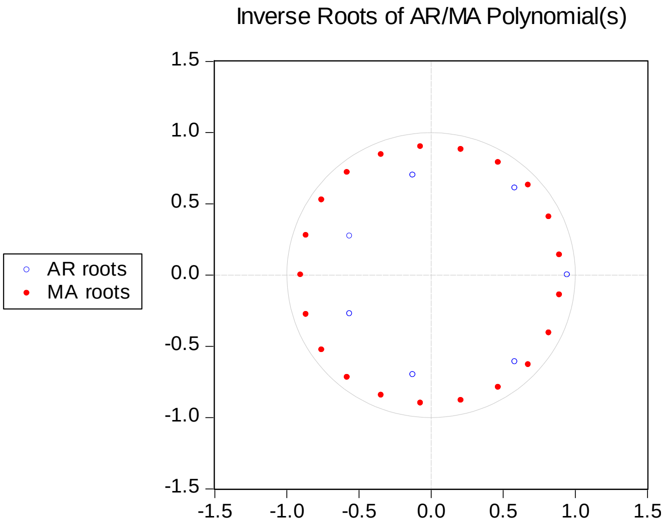
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	42.69995	3.482469	12.26140	0.0000
AR(1)	0.738009	0.036880	20.01108	0.0000
AR(7)	0.132951	0.037080	3.585536	0.0004
AR(2)	-0.090034	0.037325	-2.412203	0.0161
AR(6)	0.090152	0.036387	2.477572	0.0135
MA(4)	0.181436	0.038412	4.723450	0.0000
MA(21)	0.147086	0.036897	3.986402	0.0001

R-squared	0.702293	Mean dependent var	43.09712
Adjusted R-squared	0.699798	S.D. dependent var	16.63258
S.E. of regression	9.113106	Akaike info criterion	7.266939
Sum squared resid	59462.87	Schwarz criterion	7.311315
Log likelihood	-2619.998	Hannan-Quinn criter.	7.284067
F-statistic	281.5080	Durbin-Watson stat	1.976428

Table 5. Statistical parameters of the model. Font: Compilation based on data supplied by OMIE.

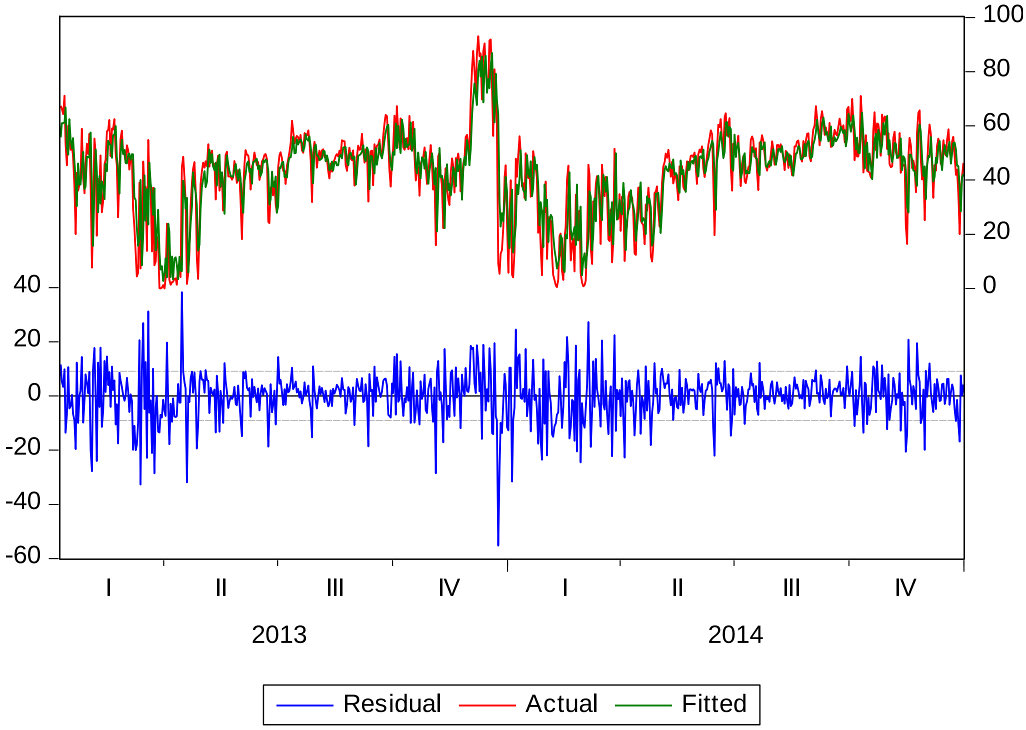
As shown in the following table interacting variables are: constant, AR (1), AR (2), AR (6), AR (7), MA (4) and MA (21).

As predicted earlier, the AR (1) has much more influence than the others AR or MA. The R-squared indicates that the model has a 70% adjustment for the values of real average price. It is also observed that the final model has no unit root.



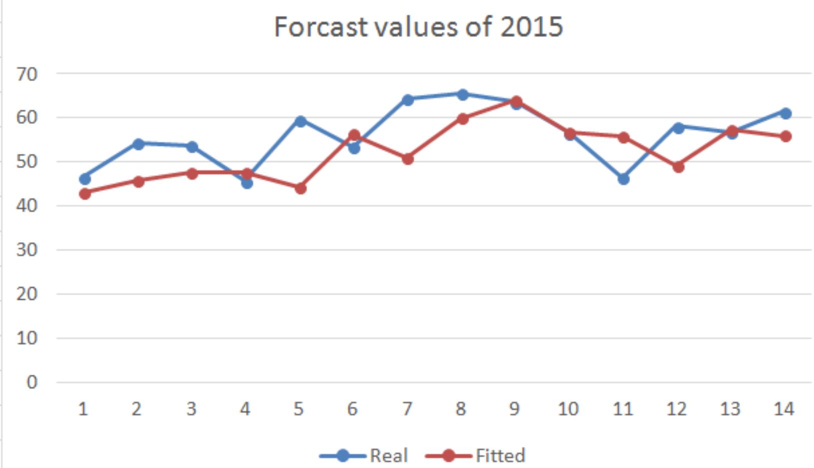
Graphic 6. Unity roots of the model. Font: Compilation based on data supplied by OMIE.

Finally the fitted model overlapped with the time series is shown, so as the residuals. As shown in the graph, errors depend on the jumps that electricity price has, that is, the greater is the difference between two prices the biggest the residual is. As discussed in section two, the biggest price variability is in the cold months, and as a result, is also where worst predictions are obtained.



Graphic 7. Actual, fitted, residual plot. Font: Compilation based on data supplied by OMIE.

Real values belonging to the first two weeks of 2015 and their respective are shown in the chart below. As it can be seen, it is possible to obtain a tracking. Furthermore it can be seen that the predicted values are delayed one day compared with the actual values



Graphic 8. Forecasted and real values of the 5 first months of 2015. Font: Compilation based on data supplied by OMIE

As mentioned above, the second part of the study is to improve the accuracy of our model by adding exogenous variables in our ARMA model. To do this, we have obtained the daily values of a set of factors that are considered that may have a major influence on the behavior of energy prices. Initially, these factors are: the amount of diary energy sold produced by coal, combined cycle consumption in pumping, eolic, hydro, nuclear, solar photovoltaic, solar thermal, renewable thermal and fuel gas; energy balances (difference between exported and imported energy with neighboring countries) respective to France, Portugal, Morocco and Andorra; the total daily demand; finally, the price of Brent oil and the euro-dollar conversion.

Then several models have been designed, some of which have been discarded directly due to the presence of unit roots or very high values of AIC and BIC, discussed in the previous section. Furthermore, various external factors were removed as they were not significant. Include the difference between the previous model since in the new model the auto regressive terms AR (7), AR (6), AR (2) do not appear and the moving average term becomes an MA (1). Also they vary the terms of seasonality, where a SAR term (7) appears and the corresponding moving average term becomes SMA (7).

The following table shows the final model, which provides a really precise estimate of our dependent variable.

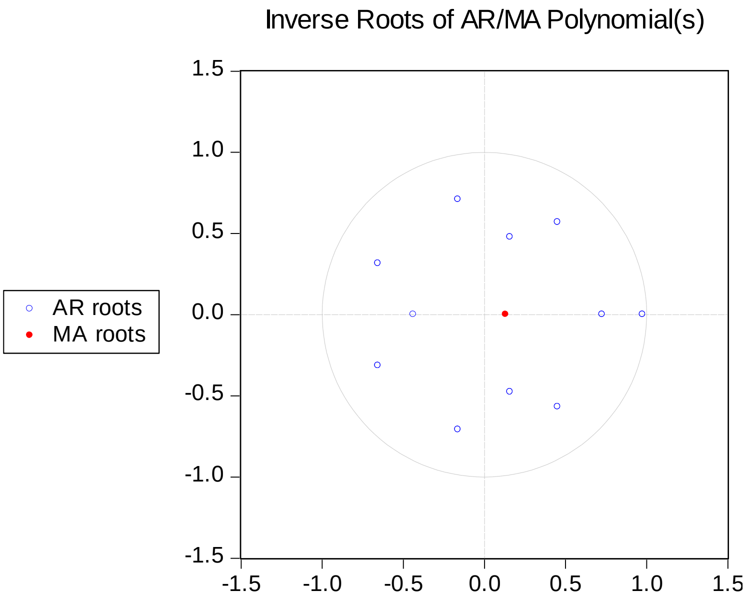
Dependent Variable: PMEDIO
Method: Least Squares
Date: 06/11/15 Time: 16:47
Sample (adjusted): 1/12/2013 12/31/2014
Included observations: 719 after adjustments
Convergence achieved after 13 iterations
MA Backcast: 1/11/2013

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	20.52795	4.871744	4.213677	0.0000
CARBON	6.47E-05	5.28E-06	12.27186	0.0000
CICLO_COMBINADO	9.64E-05	1.37E-05	7.020888	0.0000
CONSUMOS_EN_BOMBEO	0.000488	1.96E-05	24.92501	0.0000
FUEL___GAS	0.000683	0.000200	3.411204	0.0007
HIDRAULICA	0.000135	8.21E-06	16.38771	0.0000
SALDO_FRANCIA	0.000104	1.64E-05	6.338281	0.0000
SALDO_PORTUGAL	6.99E-05	1.16E-05	6.036309	0.0000
SOLAR_TERMICO	0.000134	4.29E-05	3.123504	0.0019
ENERGIA	-1.38E-05	3.40E-06	-4.062687	0.0001
AR(4)	0.107928	0.033281	3.242939	0.0012
AR(1)	0.860546	0.035217	24.43554	0.0000
SAR(7)	0.107464	0.038428	2.796482	0.0053
MA(1)	-0.132652	0.053548	-2.477277	0.0135

R-squared	0.957399	Mean dependent var	42.96247
Adjusted R-squared	0.956613	S.D. dependent var	16.57917
S.E. of regression	3.453352	Akaike info criterion	5.335847
Sum squared resid	8407.577	Schwarz criterion	5.424985
Log likelihood	-1904.237	Hannan-Quinn criter.	5.370262
F-statistic	1218.758	Durbin-Watson stat	1.990861
Prob(F-statistic)	0.000000		

Table 6. Statistical parameters of the model. Font: Compilation based on data supplied by OMIE

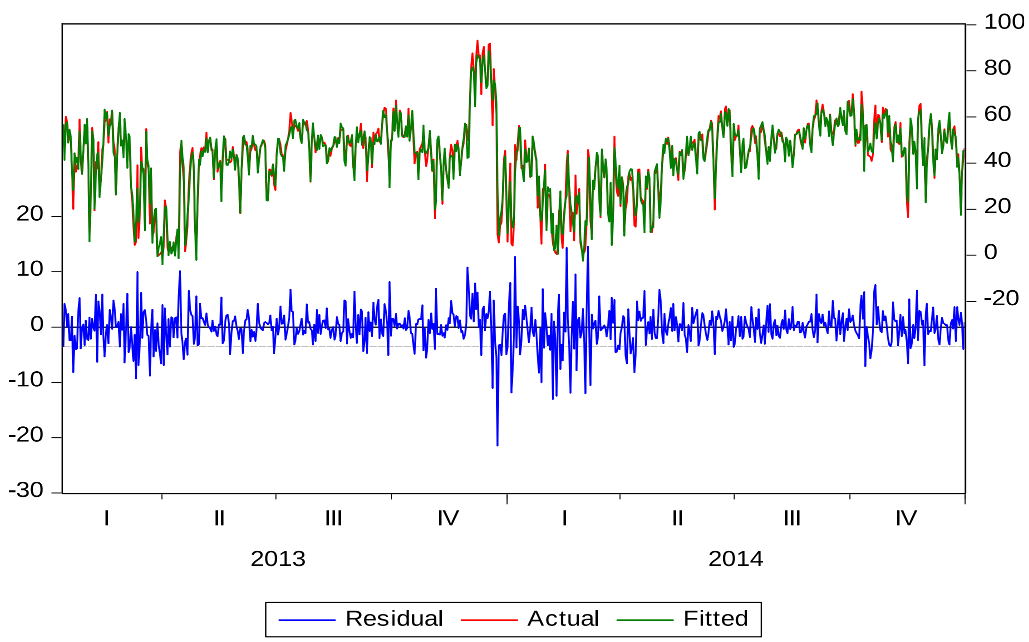
It can be seen that our model has no unit root, both in the auto regressive and the moving average part and therefore it can be confirmed that the model is stationary and invertible. Below it is shown a graphic where the roots of the model are represented:



Graphic 9. Unity roots of the model. Font: Compilation based on data supplied by OMIE.

It can be seen that all roots lie within the complex unit circle.

Should also be analyzed the model residuals. These should have a random behavior and present an average with a value of 0, ie, should be sought that do not contain any information. Below the residuals, which are the result of the subtraction between the estimated value and the actual value, are shown.



Graphic 10. Actual, fitted, residual plot. Font: Compilation based on data supplied by OMIE.

It can be seen that the average of residuals resides in 0. On the other hand, to check that they have no effect on the model, a correlogram has been performed where LBQ statistic tests are shown:

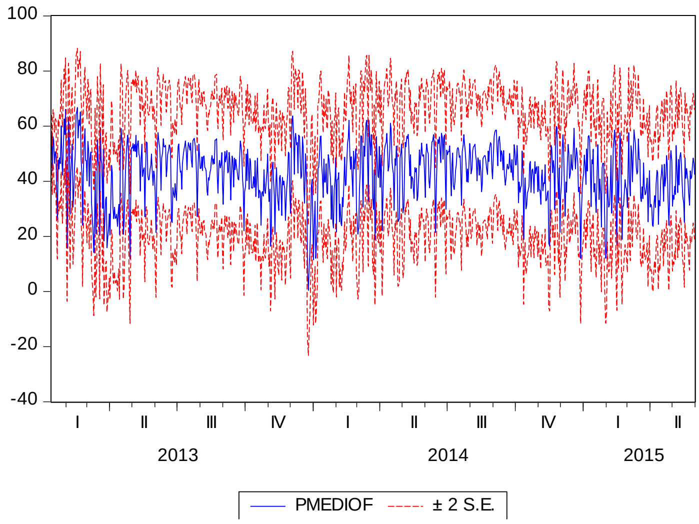
Date: 06/11/15 Time: 16:49
Sample: 1/12/2013 12/31/2014
Included observations: 719
Q-statistic probabilities adjusted for 4 ARMA term(s)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.004	0.004	0.0102	
		2	-0.029	-0.029	0.6279	
		3	0.023	0.024	1.0274	
		4	-0.022	-0.023	1.3673	
		5	0.004	0.005	1.3781	0.240
		6	0.019	0.017	1.6276	0.443
		7	0.008	0.009	1.6699	0.644
		8	0.030	0.030	2.3319	0.675
		9	0.023	0.023	2.7134	0.744
		10	-0.053	-0.051	4.7327	0.579
		11	0.085	0.086	10.021	0.187
		12	0.044	0.040	11.446	0.178

Table 7. Q-statistic correlogram. Font: Compilation based on data supplied by OMIE.

As we see, for a lag k of 12 delays, errors are not significant and therefore contain no information in our model.

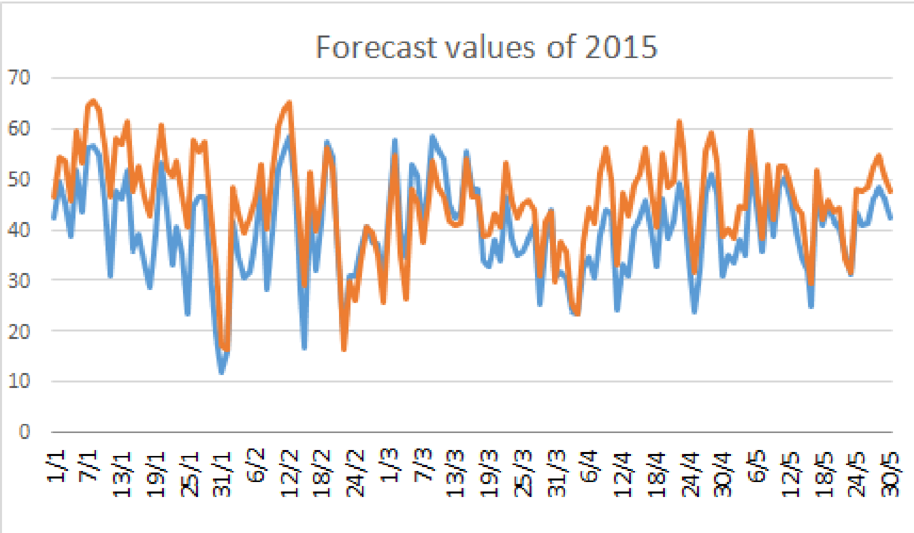
Finally, a prediction of data has been made to prove the accuracy of the model and to demonstare that the significant variables really are determining factors in the price. Unfortunately, this model is not useful to predict the prices as it needs the value of each of the variables for the following day. In this study, it has been possible to perform a prediction since the persecuted values belong to the first 5 months of 2015, and therefore, as of today we have these values. Real values(blue) belonging to the first 5 months and their respective prediction (orange) of 2015 are shown in the chart below. As it can be seen, it is possible to obtain a precise tracking.



Forecast: PMEDIOF	
Actual: PMEDIO	
Forecast sample: 1/01/2013 5/31/2015	
Adjusted sample: 1/12/2013 5/31/2015	
Included observations: 719	
Root Mean Squared Error	11.96904
Mean Absolute Error	8.753362
Mean Abs. Percent Error	97.11935
Theil Inequality Coefficient	0.132084
Bias Proportion	0.000908
Variance Proportion	0.259516
Covariance Proportion	0.739576

Graphic 11. Forecasting of Spanish day-ahead electricity price. Font: Compilation based on data supplied by MIBEL.

Real values(blue) belonging to the first 5 months and their respective prediction (orange) of 2015 are shown in the chart below. As it can be seen, it is possible to obtain a precise tracking.



Graphic 12. Forecasted and real values of the 5 first months of 2015. Font: Compilation based on data supplied by OMIE.

5. Conclusions

Time series prediction techniques have been successfully used for electricity demand and price forecast in the Spanish market. Nowadays as a result of the increasing introduction of renewable energies into the energy market, a study must be conducted to see how they influence the final price. The technique used in this file is an ARMA process, a technique that, as it can be seen, works well but not excellent due to the high volatility of our time series.

Moreover it has been found that exogenous variables that influence the price are the amounts of energy sold right on the same auction and exchanges produced between Spain, Portugal and France, for this reason it is advisable to make a study of the temporal evolution of each of the energies involved in the day-ahead market in order to improve the final price of the auction prediction.

All these features make this approach appealing and with plenty of potential for improving.

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