

A three-dimensional progressive damage model for fibre-composite materials

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Abstract

This note presents a damage model for fibre composite materials based in the approach by Matzenmiller et al. [Matzenmiller, A., Lubliner, J., Taylor, R.L., 1995. A constitutive model for anisotropic damage in fiber composites. *Mech. Mater.* 20, 125]. In this work, the model is developed in a three dimensional context with modified formulation for the constitutive law and damage evolution. An orthotropic composite subjected to mixed failure modes is assumed in this development. Its formulation and implementation details are provided.

Keywords: Fibre composites; Damage; Failure modes

1. Introduction

The number of engineering applications using composite materials in light-weight structures and, in particular, in the aerospace industry has exploded in the last decade, mainly due to the large strength-weight ratio that provide. The modelling of these materials and their damage evolution is also of an increasing interest. Nevertheless, the mixed mode of failure in composite materials often result in sophisticated damage models. So far, the schemes are mainly centred on two tendencies. The first one is concerned with the development of quadratic stress-based criteria that characterised the failure after an initial elastic behavior (see [Hinton and Soden, 1998](#); [Puck and Schurmann, 1998](#)). The second trend proposes a progressive evolution of the damage, although most of these last approaches are limited to two-dimensional models, assuming generally a plane stress state in the laminae (see [Edlund and Volgers, 2004](#); [Hufenbach et al., 2004](#)). The present study follows this last direction but in a three-dimensional framework. The relevant formulation is presented below.

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2. 3D Fibre-composite damage model

The proposed 3D damage model contains the formulation for the following modes of failure: fibre rupture, fibre buckling or kinking, matrix cracking and matrix crushing. These types are modelled by means of a combination of growth functions Ψ_i and damage directors \mathbf{v}_i . The model defines a set of state variables ω_{kj} that represents the state of damage in the composite. The main steps concerning the implementation of the damage model are outlined as follows. An effective stress tensor $\hat{\boldsymbol{\sigma}}$ is assumed following the strain equivalence principle by [Lemaitre and Chaboche \(1990\)](#):

$$\begin{aligned}\hat{\boldsymbol{\sigma}} &= \mathbf{M} \cdot \boldsymbol{\sigma}; \quad \hat{\boldsymbol{\varepsilon}} = \mathbf{C}_0 \cdot \boldsymbol{\varepsilon}; \quad \boldsymbol{\sigma}^T = [\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{23}, \sigma_{31}] \\ \boldsymbol{\sigma} &= [\mathbf{M}]^{-1} \cdot \mathbf{C}_0 \cdot \boldsymbol{\varepsilon} = \mathbf{C}(\omega) \cdot \boldsymbol{\varepsilon}\end{aligned}\quad (1)$$

where \mathbf{C}_0 is the stiffness matrix, $\mathbf{C}(\omega)$ denotes the so-called ‘*damaged*’ non-symmetric stiffness tensor and \mathbf{M} is the second-order diagonal tensor containing the damage variables

$$\mathbf{M} = \text{diag} \left[\frac{1}{1 - \omega_{11}}, \frac{1}{1 - \omega_{22}}, \frac{1}{1 - \omega_{33}}, \frac{1}{1 - \omega_{12}}, \frac{1}{1 - \omega_{23}}, \frac{1}{1 - \omega_{31}} \right]$$

The damage rule $\dot{\omega}$ is given by

$$\dot{\omega} = \sum_{i=1}^{n_{\text{modes}}} \Psi_i \mathbf{v}_i \quad \text{with} \quad \Psi_i = \langle \nabla_{\boldsymbol{\varepsilon}} g_i / \|\nabla_{\boldsymbol{\varepsilon}} g_i\|, \dot{\boldsymbol{\varepsilon}} \rangle_+ \quad (2)$$

$$g_i = \boldsymbol{\varepsilon}^T \cdot \mathbf{G}_i \cdot \boldsymbol{\varepsilon} - c_i \Rightarrow \nabla_{\boldsymbol{\varepsilon}} g_i = \boldsymbol{\varepsilon}^T \cdot (\mathbf{G}_i^T + \mathbf{G}_i) \quad (3)$$

Above, i denotes a mode of damage, n_{modes} denotes the total number of failure modes modelled and c_i is an empirical parameter defining the damage surface. $\langle \cdot \rangle_+$ is the non-negative inner product vanishes for negative values accounting for the trespassing on the damage surface. This ensures that there is not growth of damage if the damage surface is not reached. $\nabla_{\boldsymbol{\varepsilon}}$ denotes the strain gradient $\frac{\partial}{\partial \boldsymbol{\varepsilon}}$ and $\dot{\boldsymbol{\varepsilon}}$ the strain rate. g_i are the evolving damage surfaces in the strain space. \mathbf{G}_i are obtained from the constitutive law (1) and from the following equivalence of the quadratic forms in stress and strain spaces

$$\boldsymbol{\sigma}^T \cdot \mathbf{F}_i \cdot \boldsymbol{\sigma} = \boldsymbol{\varepsilon}^T \cdot \mathbf{G}_i \cdot \boldsymbol{\varepsilon} \quad (4)$$

The \mathbf{F}_i second-order tensors are derived from stress-based criteria of failure with the introduction of the damage variables ω_{ij} . For example, \mathbf{F}_{\parallel} corresponds to the failure in the longitudinal direction of the fibres and is derived from

$$\boldsymbol{\sigma}^T \cdot \mathbf{F}_{\parallel} \cdot \boldsymbol{\sigma} = \frac{\sigma_{11}^2}{(1 - \omega_{11}^2)X_{11}^2} + \frac{\sigma_{12}^2}{(1 - \omega_{12}^2)S_{12}^2} + \frac{\sigma_{31}^2}{(1 - \omega_{31}^2)S_{31}^2}$$

where X_{11} denotes the strength in the direction of the fibres, and S_{12} and S_{31} denote the shear strength values in the corresponding directions. The modelling of the unitary damage directors \mathbf{v}_i is based upon the stiffness components that are degraded when a particular mode of damage occurs. For instance, fibre rupture \mathbf{v}_{\parallel} affects to the stiffness degradation in 11, 12 and 31 directions,

$$\mathbf{v}_{\parallel} = \left[\lambda_{\parallel}^{(11)} \ 0 \ 0 \ \lambda_{\parallel}^{(12)} \ 0 \ \lambda_{\parallel}^{(31)} \right]^T$$

The weights $\lambda_{\parallel}^{(pq)}$ are chosen taking in account the experimental observations for that particular mode of failure.

3. Concluding remarks

A progressive damage model for composites was presented in this note. The 3D model is based on a previous successful work by [Matzenmiller et al. \(1995\)](#) with modified formulation for simulation of additional modes of damage associated not only to in-plane directions but rather in a three-dimensional context. It is expected that the future simulations using this model will result in a more physically-based assessment of

the damage evolution. The formulation is susceptible of implementation into an explicit finite element code which is currently ongoing. As a work in progress, expressions of opinion are highly appreciated.

References

- Edlund, U., Volgers, P., 2004. A composite ply failure model based on continuum damage mechanics. *Compos. Struct.* 65, 347.
- Hinton, M.J., Soden, P.D., 1998. Predicting failure in composite laminates: the background to the exercise. *Compos. Sci. Technol.* 58, 1001–1010.
- Hufenbach, W., Bohm, R., Kroll, L., Langkamp, A., 2004. Theoretical and experimental investigation of anisotropic damage in textile reinforced composite structures. *Mech. Compos. Mater.* 40 (6), 519–532.
- Lemaitre, J., Chaboche, J. L., 1990. *Mechanics of Solids Materials*. Cambridge University Press.
- Matzenmiller, A., Lubliner, J., Taylor, R.L., 1995. A constitutive model for anisotropic damage in fiber composites. *Mech. Mater.* 20, 125–152.
- Puck, A., Schurmann, H., 1998. Failure analysis of FRP laminates by means of physically based phenomenological models. *Compos. Sci. Technol.* 58, 1045–1067.