MODELING OF THE BEHAVIOR IN HIGH-CYCLE FATIGUE BASED ON THE COUPLING PLASTICITY-DAMAGE IN A MESOSCOPIC SCALE

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Abstract. The methods of prediction of lifespan in high cycle fatigue are under development since decades and are used by engineers to dimension the structures. The purpose of the work presented in this paper is to establish a numerical tool of prediction for a polycrystalline metal subjected to complex multiaxial loadings in fatigue. In order to overcome a purely phenomenological description, a model based on the coupling plasticity-damage in a mesoscopic scale is formulated in the framework of thermodynamics of irreversible processes and by the introducing of the critical approach plan. Advanced numerical methods were exploited for the development of this tool, namely the Maximum Variance Method (MVM), the implicit and explicit diagrams of integration and the jump-in-cycles method. The confrontation of the results showed the relevance of the model most accurately to capture as closely as possible degradation mechanisms and to predict lifespan in concord with the experimental one.

1 INTRODUCTION

After decades of work in the field of fatigue of materials and structures, several approaches and models have been proposed and currently they reach a certain level of maturity and allow to effectively address many service situations, although no unified approach for all real situations that may experience a structure or all classes of materials. Current approaches present a wide range of applications and allows in most cases to meet the need sizing for metals. The vast majority of methods of lifespan calculation in High Cycle Fatigue (HCF) are based on the setting in equations of the mechanical quantities calculated on

a macroscopic scale. Among the approaches, this type of "multi-scale" (macro - meso / micro) seems to be unanimous where the important role of local plasticity on the appearance of a fatigue limit is widely accepted and fully justifies the using of this approach based on a passage from macroscopic to mesoscopic quantities.
The objective of this work is to contribute to the development of numerical tools for the lifespan prediction of healthy polycrystalline metals subject to complex multiaxial fatigue loadings inn high number of cycles. To go beyond a purely phenomenological description, we choose to respond to these objectives by basing on a multi-scale type of modeling approach.

2 MODELING OF PLASTICITY-DAMAGE COUPLING IN HIGH CYCLE FATIGUE

In HCF, the damage in metallic materials is indeed very localized and oriented in certain preferred directions. Local plasticity plays a fundamental role in the nucleation of cracks. The orientation of the damage is often related to the applied loading. During the initiation process, the plastic slide and propagation of microcracks occur in crystallographic slip systems positively oriented [1] and only some grains "misguided" undergo plastic slip. Activation of these slip systems depends on macroscopic loading orientation. In the light of these aspects, we propose a damage model at two scales dedicated to High cycle mutiaxial fatigue. It is mainly based on modeling proposed by Flacelière - Morel - Dragon [2] and the various changes made by Huy - Damien - Yves [3]. We propose to introduce the directional aspect of the damage, by attaching it to a specific system of slide characterized by maximum shear stress and defined by a normal plane $\underline{n}$ and a sliding direction $\underline{m}$ of the plan, the plan is called "critical level" hence the notion of critical plane approach (figure 1).

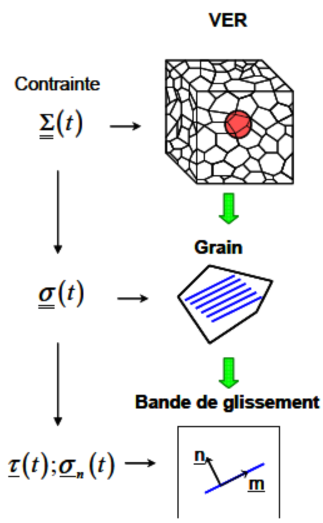


Figure 1: Critical plane illusatrion

2.1 Physical basis of modeling

The original idea proposed by Flacelière is a coupling between plasticity due to slippage in the bands and damage occurring by debonding at the mesoscopic scale. The initial model is formulated within the strict framework of thermodynamics of irreversible processes with internal variables for isothermal transformation, independent of time and in small deformations. Thermodynamic formulation presupposes the existence of a potential state, in this case the volume free energy.

This model tends to describe the crack initiation phase and the first stage of crack propagation (stage 1 – mode II). The cumulated plastic strain is considered the main cause of cumulative damage. Damage is supposed to appear when the cumulated plastic strain reaches a threshold. The - above this threshold, the simultaneous changes in the plastic deformation and damage lead to degradation of the local properties of the material, especially in terms of plastic flow limit. After a number of cycles ($10^5 - 10^6$ cycles), the ruin of the crystal is achieved at a critical value of the variable effect of the damage d. Table 1 summarizes the set of state variables considered and associated forces. It should be noted that all these quantities are attached to a specific slip system (defined by a normal plane $\underline{n}$ and a plane of this sliding direction $\underline{m}$).

Table 1: thermodynamic variables of the model

State variable	Description	Associated force	Description
$\dot{\underline{\gamma}}^e$	Mesoscopic elastic deformation rate	$\underline{\tau}$	Mesoscopic resolved shear tensor
$\dot{\underline{\gamma}}^p$	Mesoscopic plastic deformation rate	$\underline{x}$	Kinematic hardening tensor
$\dot{p}$	Cumulated plastic deformation rate	r	Isotropic hardening
$\dot{d}$	Damage effect rate	F_d	Conjugate force
$\dot{\beta}$	Cumulated damage rate	k	Conjugate force

3.2 Passage to the mesoscopic scale

For a particular slip system, the Taylor-Lin Location law is given by:

$$\underline{\tau} = \underline{T} - \mu \underline{\gamma}^p \quad (1)$$

$$\underline{\sigma}_n = (\underline{n}.\underline{\underline{\sigma}}.\underline{n})\underline{n} = (\underline{n}.\underline{\underline{\Sigma}}.\underline{n})\underline{n} = \underline{\Sigma}_n \quad (2)$$

Where $\underline{\tau} = (\underline{m}.\underline{\underline{\sigma}}.\underline{n})\underline{m}$ and $\underline{T} = (\underline{m}.\underline{\underline{\Sigma}}.\underline{n})\underline{m}$ are mesocopic and macroscopic resolved shear in $\underline{m}$ direction, $\underline{\underline{\sigma}}$ and $\underline{\underline{\Sigma}}$ are mesoscopic and macroscopic stress tensor. μ shear module.

3.3 Free energy and volume dissipation

The free volume energy ω in a grain (or grains) is given by:

$$\omega(\underline{n},\underline{m}) = \frac{G}{2}\underline{\gamma}^e.\underline{\gamma}^e + \frac{1}{2}c\underline{\gamma}^p.\underline{\gamma}^p + \tilde{r}_\infty p.\exp(-sd) + \frac{\tilde{r}_\infty}{g}\exp(-gp)\exp(-sd) + \frac{1}{2}q\beta^2 \quad (3)$$

Where $c, \tilde{r}_\infty, s, g, G$ and q are material parameters.

Volume dissipation is given by:

$$\phi(\underline{n},\underline{m}) = \underline{\tau}.\dot{\underline{\gamma}}^p - \underline{x}.\dot{\underline{\gamma}}^p - r\dot{p} + F_d\dot{d} - k\dot{\beta} \geq 0 \quad (4)$$

3.4 State laws

Elastic behavior:
Isotropic linear elastic behavior is defined by the derivative of the free energy:

$$\underline{\tau} = \frac{\partial\omega}{\partial\underline{\gamma}^e} = G\underline{\gamma}^e \quad (5)$$

Meso-plasticity:
Thermodynamic forces associated with kinematic and isotropic hardening are given by:

$$\underline{x} = \frac{\partial\omega}{\partial\underline{\gamma}^p} = c\underline{\gamma}^p \quad (6)$$

$$r = \frac{\partial\omega}{\partial p} = \tilde{r}_\infty(1-\exp(-gp))\exp(-sd) \quad (7)$$

Meso-damage:

$$F_d = \frac{\partial\omega}{\partial d} = \tilde{r}_\infty s\exp(-sd)\left(p + \frac{\exp(-gp)}{g}\right) \quad (8)$$

$$k = \frac{\partial\omega}{\partial\beta} = q\beta \quad (9)$$

3.5 Evolution laws

Mesoscopic scale plasticity load surface f proposed by Huy [3] and adapted by [4] as follows:

$$f(\underline{\tau},\underline{x},r) = \sqrt{\frac{1}{2}\gamma_1(\underline{\tau}-\underline{x}).(\underline{\tau}-\underline{x}) + \gamma_2 J_{2,moy}^2 + \gamma_3 I_f(I_{1,a}, I_{1,m})} - (r+r_0) \leq 0 \quad (10)$$

Detailed expressions for the various parameters are given in the same reference.

The evolution of the plastic slip $\underline{\gamma}^p$ and accumulated plastic strain p, respecting the non-associated law, are defined by:

$$F(\underline{\underline{\sigma}},\underline{\underline{x}},r) = J_2(\underline{\tau}-\underline{x}) - r \quad (11)$$

$$\underline{\dot{\gamma}}^{p} = -\dot{\lambda}^{p}\left(\frac{\partial f}{\partial \underline{x}}\right) = \frac{1}{2}\dot{\lambda}^{p}\,\frac{\underline{\tau}-\underline{x}}{r+r_0} \tag{12}$$

$$\dot{p} = -\dot{\lambda}^{p}\left(\frac{\partial F}{\partial r}\right) = \dot{\lambda}^{p} \tag{13}$$

The function damage threshold h and the damage dissipation potential H are given by:

- Function damage threshold :

$$h(F_d, k) = F_d - (k + k_0) \leq 0 \tag{14}$$

- Damage dissipation potential :
  - Initiation phase $(0 \leq d < d_p)$:

$$H_1(F_d, k) = aF_d - k \tag{21}$$

  - Propagation phase $(d_p \leq d < d_c)$:

$$H_2(F_d, k; \sigma_n) = F_d(1 + b\langle\sigma_n\rangle) - k \tag{22}$$

- Evolution law :
  - Initiation phase $(0 \leq d < d_p)$:

$$\dot{d} = \dot{\lambda}^{d}\left(\frac{\partial H_1}{\partial F_d}\right) = a\dot{\lambda}^{d} \tag{23}$$

$$\dot{\beta} = -\dot{\lambda}^{d}\left(\frac{\partial H_1}{\partial F_d}\right) = \dot{\lambda}^{d} \tag{24}$$

  - Propagation phase $(d_p \leq d < d_c)$:

$$\dot{d} = \dot{\lambda}^{d}\left(\frac{\partial H_2}{\partial F_d}\right) = \dot{\lambda}^{d}(1 + b\langle\sigma_n\rangle) \tag{25}$$

$$\dot{\beta} = -\dot{\lambda}^{d}\left(\frac{\partial H_2}{\partial F_d}\right) = \dot{\lambda}^{d} \tag{26}$$

The accumulations become:

$$\dot{p} = \dot{\lambda}^{p} = \frac{\left(\frac{\partial f}{\partial \underline{\tau}}\right) : \dot{\underline{\tau}}}{A} \tag{27}$$

- Initiation phase :

$$A = \frac{1}{4} c \frac{\gamma_1^2 (\underline{\tau} - \underline{x}).(\underline{\tau} - \underline{x})}{(r + r_0)^2} + \frac{\partial^2 \omega}{\partial p^2} - \left(\frac{\partial^2 \omega}{\partial d \partial p} \right)^2 \frac{a}{a \dfrac{\partial^2 \omega}{\partial d^2} + q} \tag{28}$$

$$\dot{d} = a \dot{\lambda}^d = a \frac{-\dfrac{\partial^2 \omega}{\partial d \partial p}}{a \dfrac{\partial^2 \omega}{\partial d^2} + q} \dot{p} \tag{29}$$

- Propagation phase :

$$A = \frac{1}{4} c \frac{\gamma_1^2 (\underline{\tau} - \underline{x}).(\underline{\tau} - \underline{x})}{(r + r_0)^2} + \frac{\partial^2 \omega}{\partial p^2} - \left(\frac{\partial^2 \omega}{\partial d \partial p} \right)^2 \frac{(1 + b \langle \sigma_n \rangle)}{(1 + b \langle \sigma_n \rangle) \dfrac{\partial^2 \omega}{\partial d^2} + q} \tag{30}$$

$$\dot{d} = (1 + b \langle \sigma_n \rangle) \dot{\lambda}^d = (1 + b \langle \sigma_n \rangle) \frac{-\dfrac{\partial^2 \omega}{\partial d \partial p}}{(1 + b \langle \sigma_n \rangle) \dfrac{\partial^2 \omega}{\partial d^2} + q} \dot{p} \tag{40}$$

4 NUMERICAL IMPLEMNETATION

The numerical implementation of the model developed is by determining the different values of internal variables at each moment of the loading cycle from the constitutive equations models. For this, we need to solve a system of nonlinear differential equations of the first order. This is a local integration at a point Gauss law behavior. The deformation story is supposed to be known. The aim of local integration is to determine the mechanical condition at the time $t_{n+1} = t_n + \Delta t$ assuming known him at the moment t_n; Δt represents the known time interval.

4.1 Searching for critical planes

In the case of the numerical implementing the proposed model, research of orientations of a critical plan is a crucial step. The approach assumes that critical plane, since a crack priming site, the material plane where fatigue damage reaches its maximum value, is when the amplitude of the shear stress is maximum; this method is called the method of Maximum Variance (MMV). [5] Based on this definition, we have implemented an algorithm to search for the critical plane orientations and integrate into the overall process of calculating fatigue lifespan. Determining the direction of the plane in question is made by solving a classic problem of maximizing the variance function of shear stress $\left(\max\left(Var[\tau_q(t)]\right)\right)$. For this, we have used the deterministic method called "Gradient Ascent Method".

4.2 Discretization and integration of behavior laws

To discretize and integrate the laws of behavior, we have exploited the implicit integration schemes namely generalized midpoint method and asymptotic method. The purpose of these methods is to convert, in a first step, the laws of variation data in differential form in algebraic equations. To these algebraic equations will be added that of the yield criterion and damage threshold in order to build a complete system of algebraic equations which, after resolution to update all the variables behavior model.

The system of algebraic equations reduces to solve, contains the following equations:

$$\begin{cases} f_{n+1} = 0 \\ h_{n+1} = 0 \end{cases} \Rightarrow \begin{cases} f_{n+1} + \left.\dfrac{\partial f}{\partial \Delta p}\right|_{n+1} \partial \Delta p + \left.\dfrac{\partial f}{\partial \Delta d}\right|_{n+1} \partial \Delta d = 0 \\ h_{n+1} + \left.\dfrac{\partial h}{\partial \Delta p}\right|_{n+1} \partial \Delta p + \left.\dfrac{\partial h}{\partial \Delta d}\right|_{n+1} \partial \Delta d = 0 \end{cases}$$

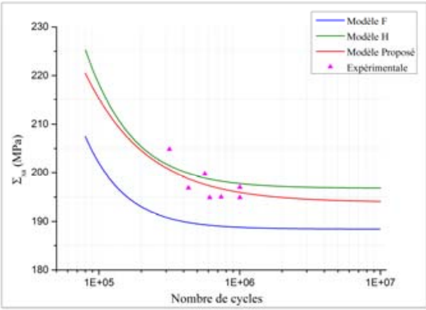
In order to speed up the calculation codes as they require considerable processing time, we used a numerical integration method based on the jumping in cycles.

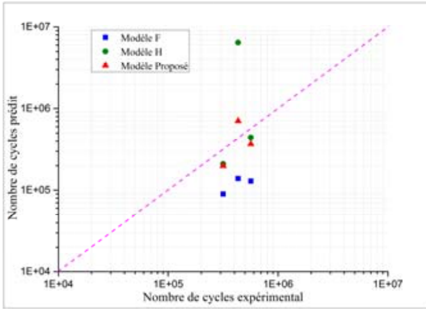
5 RESULTS AND DISCUSSION

To validate the proposed model and methods, we simulated tests available in the literature to determine the lifespan by the application of the proposed model and compared them with experimental lifetimes and those found by the application of models originals. Model validation is carried on C35 steel. The test loads are different modes and paths. Note that in the following figures, the F denotes the model Flacelière- Morel-Dragon, H for the model of Huy-Damien-Yves and finally the proposed model. The signals applied are of constant amplitude sinusoidal type.

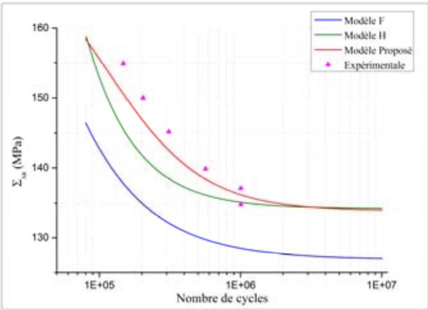
5.1 Alternate Tension - Torsion in phase (Proportional loading)

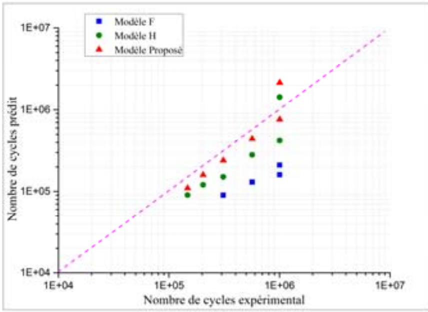
Stress ratio $k = 0.5$, *phase angle* $\delta = 0°$





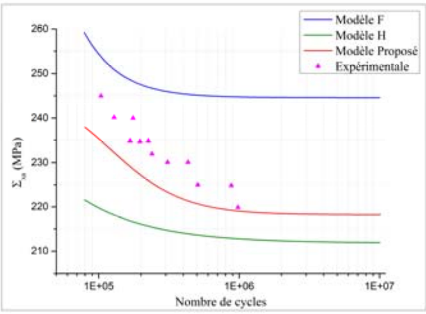
Stress ratio $k = 1$, *phase angle* $\delta = 0°$

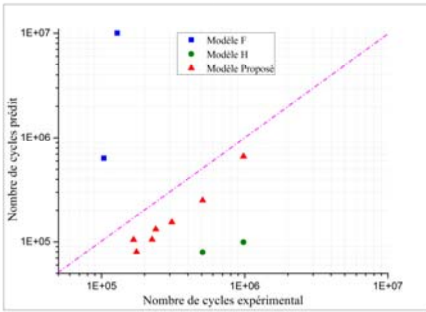




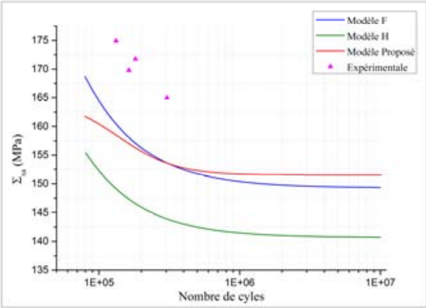
5.2 Alternate Tension - Torsion in out of phase (Non proportional loading)

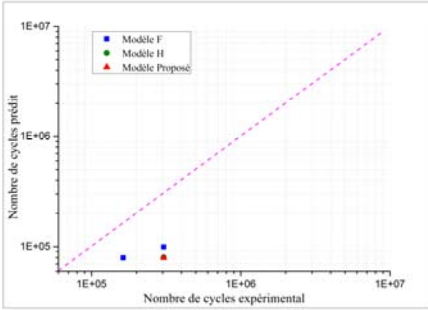
Stress ratio $k = 0.5$, *phase angle* $\delta = 90°$





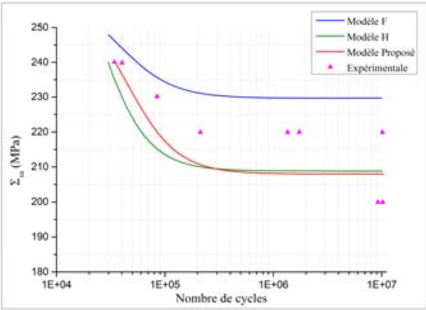
Stress ratio $k = 1$, *phase angle* $\delta = 90°$

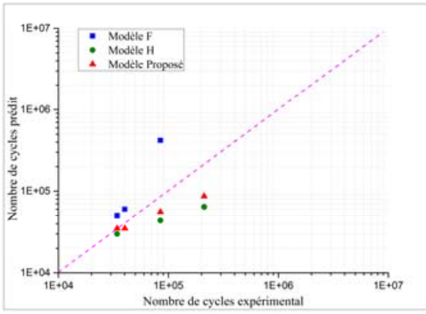




5.3 Loading with mean stress

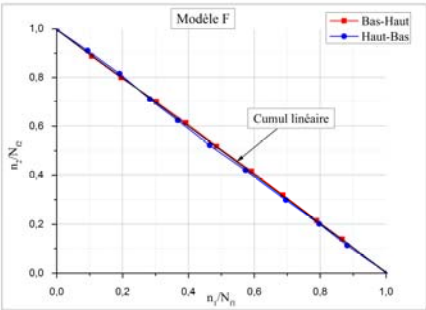
Repeated tension

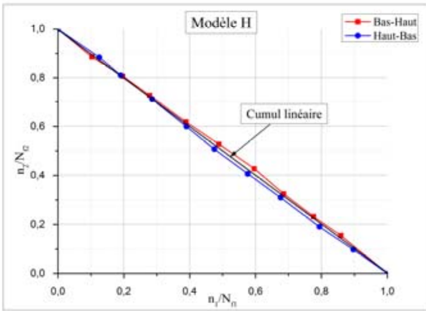


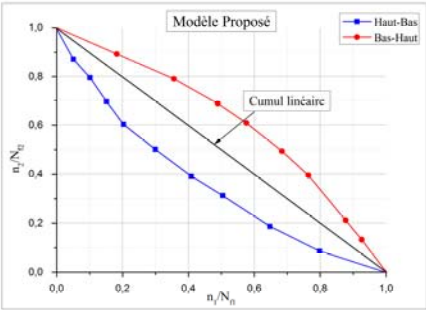


5.4 Simple loads at two levels high-low, low-high

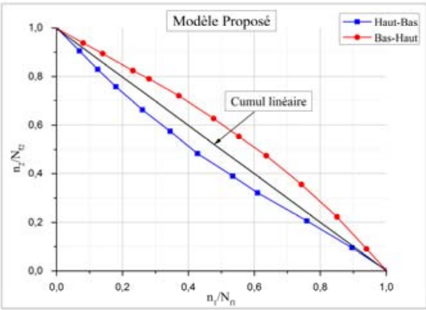
Alternate torsion

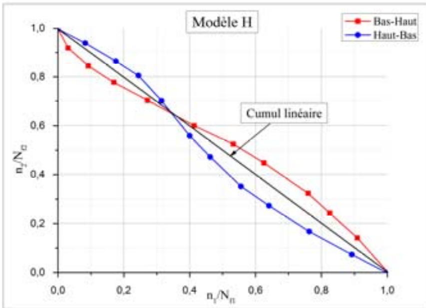


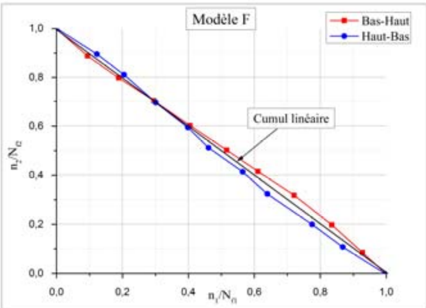




Alternate tension







5.5 Discussion

Through the trials for different modes and paths loads and two different types of polycrystalline materials, several conclusions can be drawn:

- The comparison of the results allowed us to validate the models and methods proposed earlier this memory;

- We have shown the capabilities and limitations of models depending on the mode and loading path;

- Several points have been improved in the proposed model, namely the quality of predictions, especially for out of phase loads, loads with mean stress, solicitations block, which fully justifies our contribution by introducing the approach critical level.

6 CONCLUSIONS

Based on existing models in the literature, we proposed a damage model for fatigue in many cycles. This model highlights the crucial role of the coupling mechanisms of plasticity and damage at the local (micro or meso) in the initiation and propagation of cracks. The localized nature of the damage to the micro / meso-scale justifies multiscale formalism of existing models in this area.

Our main contribution from the original model was the introduction of the concepts of critical plane approach. The attachment of physical quantities to a specific slip system defined by a normal plane and a sliding direction, enabled us to consider the dimensionality of the damage, this feature greatly improved model prediction capabilities base. The identification of the parameters of these models for a given material, require some experimental data: two curves S-N (alternate tension, alternate torsion for example) and a curve of the evolution of crack length in alternate tension.

REFERENCES

1. Suresh, S. (1998). Fatigue of Materials. Cambridge, University Press. Weibull.
2. Flacelière, L. et Morel, F. (2004). A constitutive high cycle fatigue damage model based on the interaction between microplasticity and local damage. Proceedings 7th international conference on Biaxial/Multiaxial fatigue and fracture, Berlin, Germany.
3. Huy, V., Damien, H., et Yves, N. (2009). Une approche par invariants de la fatigue multiaxiale sous chargement complexe : prédiction de la durée de vie. 19ème Congrès Français de Mécanique.
4. Messellek, A. (2014). Modélisation du comportement en fatigue à grand nombre de cycles basée sur le couplage plasticité / endommagement à l'échelle mésocopique. Thèse de magistère. Ecole Militaire Polytechnique.
5. Susmel, L. (2010). A simple and efficient numerical algorithm to determine the orientation of the critical plane in multiaxial fatigue problems. International Journal of Fatigue 32 1875–1883.